Two models for the simulation of multiphase flows in oil and gas pipelines

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Multiphase production network

From the reservoir to the process installations
Offshore oilfields

Multiphase production

- **Marginal fields** near large decreasing reservoirs:
  - small accumulations of hydrocarbons
  - financially worthwhile if low cost developments
  - long subsea tiebacks to existing infrastructures
  - ex: North sea

- **Deep water fields**
  - great water depths ($\approx 1000m$): no other choice
  - Gulf of Mexico, West Africa

- Water, oil and gas
  only two-phase flow in the following
Two-phase flow regimes

ex: Horizontal pipe

Stratified Flow →

Stratified wavy flow →

Annular Flow with droplets →

Bubbly Dispersed Flow →

Slug Flow →

Small Slug Flow →

Large Bubble Flow →

Liquid  Gas
with droplets

Flow
Annular
Stratified
wavy
flow
Stratified
Flow
Dispersed
Bubbly
Flow
Flow
Slug
Large
Bubble 
Flow
Flow
Small Slug

 Depends on fluid velocities, gas fraction ...
Gravity

- Large differences in flow behavior between horizontal, inclined, and vertical pipe flow

- Gas density and gas/liquid distribution change as a function of pressure (depth)

- Non uniform elevation of the pipeline
  - pipe lying on the seabed
  - riser reaching the platform
  - can induce large scale instabilities: terrain slugging, severe slugging
Severe-slugging phenomenon

It occurs for low velocity of gas and liquid phases.

Cyclic phenomenon that can be broken down into 4 parts:

1. liquid accumulates at the low point
2. blockage until the pressure becomes sufficient to lift the liquid column
3. the liquid slug starts to go upward along the riser, the gas begins to flow

- **Consequences**:
  - Large surges in the liquid and gas production rates
  - Equipment trips and unplanned shutdowns if processing facilities are not adequately sized

- **Terrain slugging**: low points in the topography of the pipe
Industrial objectives

Simulation tool for the design of multiphase production networks

- Maximise the production, minimize the risks and the costs
  - it is difficult to avoid severe-slugging (shut-downs ...)
  - over dimension the processing facilities

- Simulate transient phenomena induced either by
  - operating conditions: flowrates variations at the inlet, pressure variations at the outlet
  - the non uniform topography of the pipeline

- Characteristics of the flow
  - Long distance (10 to 100 kms), “low” velocities ≃ m/s
  - Importance of gravity and compressibility: large variations in the gas density, gas fraction

- Accurate estimation of outlet flowrates
Two models for oil and gas pipelines

- I - Pipe and fluid representations
- II - Drift Flux model
- III - No pressure wave model
Pipe and Fluid

• **Pipe**
  - large length, small diameters (10 to 30 cm)
  - 1D model with variable inclination (here: constant diameter)

• **Fluid description**:
  - Two-phase Immiscible Flow: compressible gas and liquid, no mass transfer between phases
  - Multiphase compositionnal Flow:
    - mass transfer between phases assuming thermodynamical equilibrium
    - lumping preprocessing procedure to reduce the number of components: 2 to 10 components
Drift flux model

- **Immiscible two-phase flow**: 
  - Mass conservation of each phase
    \[
    \frac{\partial}{\partial t} (\rho_\alpha R_\alpha) + \frac{\partial}{\partial x} (\rho_\alpha R_\alpha V_\alpha) = 0 \quad \alpha = G, L
    \]
  - **Thermo**: Phase properties \(\rho_\alpha\) as a function of \((P, T)\)

- **Compositional two-phase flow**: 
  - Mass conservation of each component \(k\)
    \[
    \frac{\partial}{\partial t} \left( \sum_{\alpha=G,L} C^\alpha_k \rho_\alpha R_\alpha \right) + \frac{\partial}{\partial x} \left( \sum_{\alpha=G,L} C^\alpha_k \rho_\alpha R_\alpha V_\alpha \right) = 0 \quad k = 1 \ldots N
    \]

  \(R_\alpha\) volumetric fraction, \(V_\alpha\) velocity, \(C^\alpha_k\) mass fraction of component \(k\) in phase \(\alpha\)
  - **Thermo**: 
    Phase properties \(\rho_\alpha, R_\alpha, C^\alpha_k\) as a function of \((P, T, C_k, k = 1 \ldots N)\)
Drift flux model

- **Momentum conservation equation for the mixture**

\[
\frac{\partial}{\partial t} \left( \sum_{\alpha} \rho_\alpha R_\alpha V_\alpha \right) + \frac{\partial}{\partial x} \left( \rho_\alpha R_\alpha V_\alpha^2 + P \right) = T_w - \rho g \sin \theta
\]

- **Algebraic slip equation**: \( dV = V_G - V_L \)

\[
\Phi(V_M, x_G, \Gamma(P), dV, x) = 0
\]

- **Temperature**

\( T = \text{cste} \) or one mixture energy balance
Characteristics of the DFM Model

• **Non linear hyperbolic system of conservation laws**

\[
\frac{\partial}{\partial t} W(x, t) + \frac{\partial}{\partial x} F(x, W(x, t)) = Q(x, W(x, t))
\]

- no algebraic expression of \( F(x, W) \) and \( Q(x, W) \)
- Jacobian \( DF(x, W) \) computed numerically

• **Isothermal gas-liquid flow**: \( \lambda_1 < \lambda_2 < \lambda_3 \)

  - under simplifying assumptions (S. Benzoni):
    \[
    \lambda_1 = v_L - w, \lambda_3 = v_L + w, \lambda_2 = v_G
    \]
    \( w \approx \text{sound velocity from about 50 m/s to several 100m/s} \)
    - \( \lambda_1 < 0, \lambda_3 > 0 \): pressure pulses (sonic waves)
    - \( \lambda_2 \): gas volume fraction waves (fluid transport)

\( \lambda_2 \) positive or negative, \( \approx 1 \text{ m/s} \)

- \(|\lambda_1| > > |\lambda_2|, |\lambda_3| > > |\lambda_2| \)
Characteristics of the DFM Model

- **Isothermal compositional flow:**
  Hyperbolic system \((N + 1) \times (N + 1)\):
  Eigenvalues \(\lambda_1 < ... \lambda_k ... < \lambda_{N+1}\)
  - \(\lambda_1 < 0, \lambda_{N+1} > 0\) : “pressure waves”
  - \(\lambda_k\) : composition waves: m/s
  - \(|\lambda_1| >> |\lambda_k|, |\lambda_{N+1}| >> |\lambda_k|\)

- **Low Mach Number Flows**
  Main interest: fluid transport “waves”
  responsible for the main dynamics in the pipeline
• **Boundary Conditions** (Two-phase immiscible flow)
  
  - Inlet Boundary \( x = 0 \) : flow rates for each phase or for each component
    
    \[
    \rho_G R_G V_G(0, t) = \frac{Q_G(t)}{\text{Sect}} \\
    \rho_L R_L V_L(0, t) = \frac{Q_L(t)}{\text{Sect}}
    \]
  
  - Outlet boundary \( x = L \) : pressure, liquid can not go back into the pipe
    
    \[
    P(L, t) = P_{\text{outlet}}(t) \\
    R_L(L, t) = 0 \text{ si } V_L < 0
    \]

• **Initial condition**
  
  Steady flow with BC at \( t = 0 \) \( Q_G(0), Q_L(0), P_{\text{outlet}}(0) \)

• **Transient Flow** induced by BC variations, instabilities (severe-slugging)
Numerical scheme

- **Cell centered Finite Volume scheme**
  
  cell $]x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}[$, unknown $W_i \simeq W(x_i)$
  
  $\Delta x_i \frac{d}{dt} W_i + H_{i+\frac{1}{2}} - H_{i-\frac{1}{2}} = \Delta x_i Q_i$

- **Numerical flux**
  
  - difficult to use algebraically constructed approximate Riemann solver like Roe scheme
  
  - Rusanov : uniform dissipation on all the waves, too dissipative on void fraction waves
  
  - Idea : introduce enough numerical dissipation but preserves the different orders of magnitude

  $H(U, V) = \frac{1}{2}(F(U) + F(V)) + \frac{1}{2}D(U, V)(U - V)$

  $D(U, V) = |DF(U)| + |DF(V)|$
Time discretisation

- **Explicit scheme**
  - CFL based on sonic waves
  - time steps too small / time of fluid transport

- **Linearly implicit scheme**
  Linear implicitation of the source term and of the flux

\[
H(U^{n+1}, V^{n+1}) \approx H(U^n, V^n) + \text{approx} \left( \frac{\partial}{\partial U} H(U^n, V^n) \right) (U^{n+1} - U^n) \\
+ \text{approx} \left( \frac{\partial}{\partial V} H(U^n, V^n) \right) (V^{n+1} - V^n)
\]

\[
\frac{\partial}{\partial U} H(U, V) \approx \frac{1}{2} (DF(U) + D(U, V))
\]

Does not account for the different wave velocities
Semi-implicit scheme

- explicit on the slow waves and linearly implicit on the fast waves
- CFL based on fluid transport waves, time steps in agreement with the main phenomena
- Splitting of the flux into a “slow waves” part and a “fast waves” one
  - cf. G. Fernandez PHD thesis for the Euler equations
  - The small eigenvalues are cancelled in the flux derivatives, similar to the “diagonal” approach of Fernandez
  ex: $DF(U)$ replaced by $\overline{DF}(U) = T\Lambda T^{-1}$,
  $\overline{\Lambda}_k = \lambda_k(U)$ for $k = 1, N + 1$, $\overline{\Lambda}_k = 0$ for $k = 2, ..., N$
- CFL:
  $\Delta t < min(CFL_v \frac{\Delta x}{\Delta x_{max, \lambda_k, k=2, ..., N+1}}, CFL_p \frac{\Delta x}{\Delta x_{max, \lambda_k, k=1, N+1}})$,
  $CFL_v = 0.8, CFL_p = 20$
Second order scheme

Wall friction and gravitational terms induce large $\frac{\partial}{\partial x}(P)$: second order scheme necessary

- **MUSCL approach**:
  - linear reconstruction on “physical” variables: $P, c_i, V_M$
  - minmod limiter

- **RK2 type scheme** to enable CFL 0.8 on slow waves
  - second order time scheme on the slow waves
  - first order on the fast waves
Boundary conditions (immiscible flow)

- **Inlet BC** imposed on the numerical fluxes
  - Fictitious cell: $W_0$
  
  \[
  \begin{align*}
  H_G(W_0, W_1) &= Q_G(t) \\
  H_L(W_0, W_1) &= Q_L(t) \\
  L_1^t W_0 &= L_1^t W_1
  \end{align*}
  \]
  
  - Non linear system, sometimes very difficult to solve

- **Outlet BC**
  
  - Pressure BC: fictitious $I + 1$ cell, $W_{I+1}$
  
  \[
  \begin{align*}
  P(W_{I+1}) &= P_{outlet}(t) \\
  L_2^t W_{I+1} &= L_2^t W_I \\
  L_3^t W_{I+1} &= L_3^t W_I
  \end{align*}
  \]
• Change in the flow direction at the outlet: $v_L < 0$
  - failure of any numerical treatment based on the sign of the eigenvalues or on a adequate fictitious state ($R_{I+1} = 0$)
  - Empirical and simple approach:
    if $H_L(W_I, W_{I+1}) < 0$ then $(H_L)_{I+\frac{1}{2}} = 0$
    - approach justified theoretically on a scalar model (T. Gallouët): leads to a monotone continuous numerical flux and satisfies the BC.

• Time discretization
  - BC solved during the “explicit” step of the scheme
  - $W_0, W_{I+1}$ kept constant during the semi-implicit step
  Loss of accuracy as $H^{n+1}$ does not satisfy the BC
  Reasonable results
Application: Inlet Gas Flowrate Increase

Horizontal pipeline of 5000\(m\) length, immiscible two-phase flow

Oil Mass Flowrate at different times

Explicit, Implicit and Semi-implicit

\[ \text{Nb Step Expli} = 50 \times \text{nbStep Semi-Impli} \]

\[ \text{CPU Expli} = 45 \times \text{CPU Semi-Impli} \]
Application : 7 components

Ascending pipeline of $5000m$ length and $0.146m$ diameter.
Boundary conditions : $P_{outlet}(t) = 10$ bar
At the inlet: $Q_1..Q_5$ constant, $Q_6,Q_7$ increased from 0 to 2 in 50 s

$Q_1$ and $Q_7$ at different times

Eigenvalues at time $t = 400s$
Vertical pipe: liquid fraction and flowrate
“No Pressure Wave” Model

Two phase immiscible flow

- Modify the DFM model to account for the low Mach Number flow

- Analytical study for a simplified slip law (H. Viviand)
  \[ \epsilon = \frac{U}{a_G}, \quad \frac{U}{a_L} = \frac{\epsilon}{K} \]
  where \( a_{\alpha} \) sound velocity in phase \( \alpha \), \( U \) characteristic velocity

- asymptotic expansion of the solution with respect to \( \epsilon \)

- Simplified momentum conservation without momentun time derivative and flux terms
NPW

- **Mass and simplified momentum conservation**

\[
\frac{\partial}{\partial t} (\rho_\alpha R_\alpha) + \frac{\partial}{\partial x} (\rho_\alpha R_\alpha V_\alpha) = 0 \quad \alpha = G, L
\]

\[
\frac{\partial}{\partial x} (P) = T_w - \rho g \sin \theta
\]

- thermo, slip laws

- **Properties**

\[
B \frac{\partial}{\partial t} W + A \frac{\partial}{\partial x} W = Q
\]

B is singular. Find \((a, b)\) s.t. \(\det(aB - bA) = 0\):

- \(\lambda = \frac{a}{b} \simeq u\) : one “fluid wave” velocity
- \(b = 0\) : one “double” infinite wave velocity

**Sonic waves approximated by infinite velocity waves**

Mixed parabolic/hyperbolic system of PDE
Compositional NPW Model

- **Mass conservation of each component** $i$
  \[
  \frac{\partial}{\partial t} \left( \sum_{\alpha=G,L} C_i^{\alpha} \rho_{\alpha} R_{\alpha} \right) + \frac{\partial}{\partial x} \left( \sum_{\alpha=G,L} C_i^{\alpha} \rho_{\alpha} R_{\alpha} V_{\alpha} \right) = 0 \quad i = 1 \ldots N
  \]

- **Thermo**: Phase properties $\rho_{\alpha}$, $R_{\alpha}$, $C_i^{\alpha}$ as a function of $(P, T, C_i)$

- **Simplified momentum conservation**
  \[
  \frac{\partial}{\partial x} (P) = T_w - \rho g \sin \theta
  \]

- **Algebraic slip equation**: $dV = V_G - V_L$
  \[
  \Phi(V_M, x_G, \Gamma(P), dV, x) = 0
  \]

- **Same initial and Boundary conditions as DFM**
Numerical scheme

- **VF scheme on staggered mesh**
  - implicit centered scheme for the parabolic part (sonic waves)
  - explicit upwind scheme for the hyperbolic part (slow waves)
  - CFL based on phase velocities (0.4)

- **Mass conservation** + thermo: cell \([x_i-\frac{1}{2}, x_i+\frac{1}{2}]\)

\[
\frac{\Delta x_i}{\Delta t} \left( (C_k^\alpha \rho_\alpha R_\alpha)_{i+1}^n - (C_k^\alpha \rho_\alpha R_\alpha)_i^n \right) + H_{\alpha,i+\frac{1}{2}} - H_{\alpha,i-\frac{1}{2}} = 0
\]

\[
H_{\alpha,i+\frac{1}{2}} = \begin{cases} 
(C_k^\alpha \rho_\alpha R_\alpha)_i^n V_{\alpha,i+\frac{1}{2}}^{n+1} & \text{if} \quad (V_\alpha)_i+\frac{1}{2} > 0 \\
(C_k^\alpha \rho_\alpha R_\alpha)_{i+1}^n V_{\alpha,i+\frac{1}{2}} & \text{otherwise}
\end{cases}
\]

- **Momentum conservation** + slip law: cell \([x_i, x_{i+1}]\)

\[
P_{i+1}^{n+1} - P_i^{n+1} = \Delta x_i Q_{i+\frac{1}{2}}^{n+1} \quad i=1..I-1
\]
Boundary conditions

Two-phase immiscible flow

- **Inlet Boundary**: given mass flowrates
  \[ H_{\alpha}^{\frac{1}{2}} = Q_{\alpha} \]

- **Outlet Boundary**
  - Mass Fluxes:
  \[ H_{LI+\frac{1}{2}} = \begin{cases} 
  \rho_{LI}(R_L)_I V_{LI+\frac{1}{2}} & \text{if } (V_L)_{I+\frac{1}{2}} > 0 \\
  0 & \text{otherwise} 
\end{cases} \]
  - Pressure
  \[ P_{outlet} - P_I = \frac{\Delta x_I}{2} Q_{I+\frac{1}{2}} \]
Application : Inlet Gas Flowrate Increase

Horizontal pipeline of $5000m$ length and $0.146m$ diameter.

Oil Mass Flowrate in the pipe : NPW/DFM comparison
Severe Slugging

- 60m long horizontal pipe, 14m long riser
- D=5cm
- Constant inlet mass flowrates and outlet pressure
  - $Q_G = 1,9610^{-4} \text{kg/s}$, $Q_L = 2,8510^{-4} \text{kg/s}$
  - $P_{\text{outlet}} = 1\text{bar}$
Severe Slugging : $R_L$

NPW
\[ R_L(t) \]
\[ x = 0, 60, 74m \]

DFM
\[ R_L(t) \text{ pour} \]
\[ x = 0, 60, 74m \]

DFM CPU $\simeq$ 9 NPW CPU (single-phase flow)
Severe Slugging: $R_L$ in the riser

NPW

DFM
Conclusion

- Two models and schemes adapted to low Mach Number compositional two-phase flow
  - DFM: scheme explicit for the “slow” wave and “implicit” for the fast waves
  - NPW: acoustic waves approximated by infinite velocity waves, semi-implicit scheme
    Larger time steps for NPW, less CPU time

- Extension to
  - more complex fluid flow:
    - Three-phase flow: water phase
    - Four-phase flow: “solid” phase (hydrate, wax) for cold deep sea production
    - Complex networks
The Girassol (West Africa) production network