Ghost Fluid Method for Interfaces flow computations


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Interfaces flow

For interfaces flow, it is assumed that a characteristic volume is always occupied by a pure phase fluid.

Numerical strategies

- “Explicit” interface tracking (Front tracking)
- “Implicit” interface tracking (Level Set)
- Interface reconstruction (VOF)
- Diffusive Interface
Outline

1. Interfaces flow Model
   - Level Set for Interface tracking
   - Governing equations

2. Ghost Fluid Methods
   - Finite volume
   - Applications: 1D Cases

3. Extension to unstructured meshes
   - Finite Volume
   - 2D Applications

4. Conclusions, Coming and Future work
   - DG approach for the level set equation
   - Finite Element
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1. **Interfaces flow Model**
   - Level Set for Interface tracking
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Interfaces flow model

Interface Model: bi-fluid case

The interface $S$ is the zero of a single level set function $\phi$. 
Interfaces flow model

Interface Model: bi-fluid case

The interface $S$ is the zero of a single level set function $\phi$.

Example

$S(t) = \left\{ x \text{ such that } \phi(t, x) = 0 \right\}$

$\Omega_1(t) = \left\{ x \text{ such that } \phi(t, x) < 0 \right\}$

$\Omega_2(t) = \left\{ x \text{ such that } \phi(t, x) > 0 \right\}$
Level Set for Interface tracking

Dynamic of the Level set function

Transport Equation formulation

\[ \partial_t \phi + u(\phi) \cdot \nabla \phi = 0 \]

where \( \phi \) is any regular function such that \( \phi(t, x) = 0 \) for \( x \in S(t) \)

\( u(\phi) \), the velocity field, function of the interface motion.
Level Set for Interface tracking

Dynamic of the Level set function

Transport Equation formulation

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\( u(\phi) \), the velocity field, function of the interface motion.

Hamilton-Jacobi formulation

\[ \partial_t \phi + F(\nabla \phi)|\nabla \phi| = 0, \quad \text{where} \quad F(\nabla \phi) = \frac{u(\phi) \cdot \nabla \phi}{|\nabla \phi|} \]

In this context, \( \phi \) is usually a signed distance function.
Governing equations

**General Case**: \( \Omega = \Omega_1(t) \cup \Omega_2(t) \cup S(t) \).

**Mathematical Model**

\[
\begin{align*}
\mathcal{L}_1 \omega_1 &= 0 \quad \text{for } \phi(t, x) < 0 \quad (1) \\
\mathcal{L}_2 \omega_2 &= 0 \quad \text{for } \phi(t, x) > 0 \quad (2) \\
\mathcal{G}_1 \omega_1 - \mathcal{G}_2 \omega_2 &= \Sigma(\phi, \omega_1, \omega_2) \quad \text{for } \phi(t, x) = 0 \quad (3) \\
\partial_t \phi + \mathbf{u}(\phi, \omega_1, \omega_2) \cdot \nabla \phi &= 0 \quad \text{for } (t, x) \in [0, T] \times \Omega \quad (4)
\end{align*}
\]

**Definitions**: (\( \mathcal{L}_k, \omega_k, \mathcal{G}_k, \mathbf{u}(\phi, \omega_1, \omega_2), \Sigma(\phi, \omega_1, \omega_2) \))

- \( \mathcal{L}_k \) and \( \omega_k \) are differential operators and the set of the unknown relevant for the flow description in the region \( \Omega_k \).
- \( \mathcal{G}_k, \mathbf{u}(\phi, \omega_1, \omega_2) \) and \( \Sigma(\phi, \omega_1, \omega_2) \) are associated to jump conditions and waves transmission at interfaces.
Governing equations

**General Case** : \( \Omega = \Omega_1(t) \cup \Omega_2(t) \cup \mathcal{S}(t) \).

### Mathematical Model

\[
\mathcal{L}_1 \omega_1 = 0 \quad \text{for } \phi(t, x) < 0 \\
\mathcal{L}_2 \omega_2 = 0 \quad \text{for } \phi(t, x) > 0 \\
\mathcal{G}_1 \omega_1 - \mathcal{G}_2 \omega_2 = \Sigma(\phi, \omega_1, \omega_2) \quad \text{for } \phi(t, x) = 0 \\
\partial_t \phi + \mathbf{u}(\phi, \omega_1, \omega_2) \cdot \nabla \phi = 0 \quad \text{for } (t, x) \in [0, T] \times \Omega
\]

### Definitions : \((\mathcal{L}_k \text{ and } \omega_k, \mathcal{G}_k, \mathbf{u}|_\mathcal{S} \text{ and } \Sigma)\)

1. \(\mathcal{L}_k \text{ and } \omega_k\) are differential operator and the set of the unknown, relevant for the flow description in the region \(\Omega_k\).

2. \(\mathcal{G}_k, \mathbf{u}(\phi, \omega_1, \omega_2)\) and \(\Sigma(\phi, \omega_1, \omega_2)\) are associated to jump conditions and waves transmission at interfaces.
Governing equations

**General Case:** \( \Omega = \Omega_1(t) \cup \Omega_2(t) \cup S(t) \).

**Mathematical Model**

\[
L_1 \omega_1 = 0 \quad \text{for } \phi(t, x) < 0 \quad (1)
\]
\[
L_2 \omega_2 = 0 \quad \text{for } \phi(t, x) > 0 \quad (2)
\]
\[
G_1 \omega_1 - G_2 \omega_2 = \Sigma(\phi, \omega_1, \omega_2) \quad \text{for } \phi(t, x) = 0 \quad (3)
\]
\[
\partial_t \phi + u(\phi, \omega_1, \omega_2) \cdot \nabla \phi = 0 \quad \text{for } (t, x) \in [0, T] \times \Omega \quad (4)
\]

**Definitions:** (\( L_k \) and \( \omega_k \), \( G_k \), \( u|_S \) and \( \Sigma \))

1. \( L_k \) and \( \omega_k \) are differential operator and the set of the unknown, relevant for the flow description in the region \( \Omega_k \).
2. \( G_k \), \( u(\phi, \omega_1, \omega_2) \) and \( \Sigma(\phi, \omega_1, \omega_2) \) are associated to jump conditions and waves transmission at interfaces.
Compressible/Compressible Interfaces

Assumptions
We assume that pure fluids are inviscid, compressible and flow described anywhere by the conservative Euler Equations.

Definitions: (\( \omega \), Fluid Model, Interfaces Model, \( \Sigma \))

1. \( \omega_k = \omega = (\rho, \rho u, \rho e)^T \)
2. \( \mathcal{L}_k \omega_k = \partial_t \omega + \nabla \cdot f(\omega, p_k) \);
   Pressures are given by equations of state \( p_k = p_k(\omega) \)
3. \( \Sigma(\phi, \omega_1, \omega_2) \equiv 0 \) in absence of tension forces, chemical reaction and phase transition
4. How are defined \( u | S \) and \( p | S \) for Shock or CDS
Compressible/Compressible Interfaces

Assumptions

We assume that pure fluids are inviscid, compressible and flow described anywhere by the conservative Euler Equations.

Definitions: (States, Fluid Model, Interfaces Model and Σ)

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3. \( \mathcal{G}_k(\phi)\omega_k = (u|_S \cdot \nabla \phi)\omega + \nabla \phi \cdot f(\omega, p|_S) \)

4. \( \Sigma(\phi, \omega_1, \omega_2) \equiv 0 \) in absence of tension forces, chemical reaction and phase transition.

5. How are defined \( u|_S \) and \( p|_S \)? Shock or CD?
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Definitions : (States, Fluid Model, Interfaces Model and $\Sigma$)

1. $\omega_k \equiv \omega = (\rho, \rho u, \rho e)^T$
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   *Pressures are given by equations of state $p_k = p_k(\omega)$*
3. $\mathcal{G}_k(\phi) \omega_k = (u \mid_S \cdot \nabla \phi) \omega + \nabla \phi \cdot f(\omega, p \mid_S)$
4. $\Sigma(\phi, \omega_1, \omega_2) \equiv 0$ *in absence of tension forces, chemical reaction and phase transition.*
5. *How are defined $u \mid_S$ and $p \mid_S$? Shock or CD?*
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4. $\Sigma(\phi, \omega_1, \omega_2) \equiv 0$ in absence of tension forces, chemical reaction and phase transition.
5. How are defined $\mathbf{u}|_S$ and $p|_S$? Shock or CD?
Governing equations

**Equation of State (EOS)**

**Mie-Gruneisen family of equation of state**

\[ \Gamma_k(\rho)p_k + \pi_k(\rho) = \rho \varepsilon \quad \text{with} \quad \varepsilon = e - \frac{u \cdot u}{2}, \]

where \( \Gamma(\rho) \) and \( \pi_k(\rho, \varepsilon) \) are given functions.

**Example (Perfect Gas EOS)**

\[ \Gamma(\rho) = \frac{1}{\gamma - 1}, \quad \pi(\rho) = 0, \quad c^2 = \frac{\gamma \rho}{\rho} \]

**Example (Modified Tait’s EOS)**

\[ \Gamma(\rho) = \frac{1}{m - 1}, \quad \pi(\rho) = \frac{m(\pi_* - \pi_0)}{m - 1}, \]

Water: \( m = 7.15, \quad \pi_* = 3.3110^8 Pa, \quad \pi_0 = 10^5 Pa \)
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Finite volume formulation

Explicit scheme

- \[ a_i \omega_i^{n+1} = a_i \omega_i^n - \sum_{j \in \nu(i)} \Phi \left( n_{ij}, \omega_i^n, \omega_j^n \right) - \sum_{j \in \kappa(i)} \Phi_S \left( n_{ij}, \omega_i^n, \omega_j^n \right) \]
- \[ a_i \phi_i^{n+1} = a_i \phi_i^n - R_i \left( \phi^n, \omega^{n+1} \right) \]

1. \( j \in \nu(i) \) is a neighbor cell of \( i \) such that \( \phi_i^n \phi_j^n > 0 \).
   The flux \( \Phi \) is a classical one (Roe, HLL, HLLC).

2. \( j \in \kappa(i) \) is a neighbor cell of \( i \) such that \( \phi_i^n \phi_j^n < 0 \).
   The flux \( \Phi_S \) have to be consistent with jump conditions and wave transmission at the interface.
**Finite volume**

**Ghost Fluid Method Principle**

Use a classical Flux with ghost states

\[ \Phi_S \left( \mathbf{n}_{ij}, \omega_i^n, \omega_j^n \right) \simeq \Phi \left( n_{ij}, \tilde{\omega}_i^n, \tilde{\omega}_j^n \right) \text{ where } \tilde{\omega} = \tilde{\omega} (\tilde{\rho}, \tilde{\mathbf{u}}, \tilde{p}) \]

**Properties of ghost states**

The ghost states should be such as the flux \( \Phi \left( n_{ij}, \tilde{\omega}_i^n, \tilde{\omega}_j^n \right) \) be consistent with

- the jump conditions (static constraint),
- the wave transmission (dynamic constraint).
Finite volume

Original Ghost Fluid Method (Fedkiw et al. 99)

Strategy based on static constraint: jump conditions

\[ \tilde{\omega}_i = \omega_i, \quad \text{and} \quad \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\rho_{ij}, u_j, p_j) \]

where \( \rho_{ij} \) is given by

\[ s(\rho_i, p_i) = s'(\rho_{ij}, p_j) \]

\( \rho_{ij} \) is an evaluation of the density close to the interface.

Implicit assumptions

- Equation with entropy functions \( s \) and \( s' \), is invertible. OK for perfect gas and some Mie-Grunieson EOS.
- Entropy can be extrapolated “near” the interface.
Isobaric fix (Fedkiw et al. JCP, 1999)

In order to prevent overheating errors in the GFM method,

\[
\tilde{\omega}_i = \tilde{\omega}_i (\rho_i, u_i, p_i), \quad \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\rho_{ij}, u_j, p_j), \quad s(\rho_i, p_i) = s'(\rho_{ij}, p_j)
\]

Neighbor cells in 1D Case: \(i- = i - 1, j = i + 1, j+ = j + 1\).
Isobaric fix Ghost Fluid Method (Fedkiw et al. 99)

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Neighbor cells in 1D Case: \( i- = i - 1, j = i + 1, j+ = j + 1 \).
Finite volume

Isobaric fix Ghost Fluid Method (Fedkiw et al. 99)

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In order to prevent overheating errors in the GFM method

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\tilde{\omega}_i = \tilde{\omega}_i (\rho_{i-}, u_i, p_i), \quad \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\rho_{ij}, u_j, p_j), \quad s(\rho_{i-}, p_i) = s'(\rho_{ij}, p_j)
\]

Neighbor cells in 1D Case: \( i- = i - 1, j = i + 1, j+ = j + 1 \).
Modified Ghost Fluid Method (Liu et al. 03)

Strategy mixing static and dynamic constraints.

Wave transmission: two-shock bi-fluid solver

\[ dp + \tilde{\rho}_+ \tilde{c}_+ du = 0 \quad \text{along} \quad \frac{dx}{dt} = \tilde{u} + \tilde{c}_+ \quad (5) \]

\[ dp - \tilde{\rho}_- \tilde{c}_- du = 0 \quad \text{along} \quad \frac{dx}{dt} = \tilde{u} - \tilde{c}_- \quad (6) \]

\[ u = u \cdot \nabla \phi / |\nabla \phi|, \text{is the normal velocity.} \]

Jump conditions on material interfaces

\[ p_{ij} = p_{ji} = \tilde{p}, \quad u_{ij} = u_{ji} = \tilde{u} \]
Finite volume

Modified Ghost Fluid Method (Liu et al. 03)

Jump conditions in a pure fluid gives \( \tilde{\rho} \) as a function of \( \tilde{\rho} \)

\[
\left[ \frac{1 + \Gamma_k(\rho)}{\rho} p_k + \pi_k(\rho) \right.\left. + p_k \right] = 0 \rightarrow \left\{ \begin{array}{l}
\tilde{\rho}_i (\tilde{\rho}, \omega_{i-}), \\
\tilde{\rho}_j (\tilde{\rho}, \omega_{j+})
\end{array} \right.
\]

Nonlinear system defining \( \tilde{\rho} \) and \( \tilde{u} \)

\[
\tilde{u} - u_{i-} = \int_{p_{i-}}^{\tilde{p}} \frac{dp}{\tilde{a}_-(p)} \quad \text{and} \quad \tilde{u} - u_{j+} = -\int_{p_{j+}}^{\tilde{p}} \frac{dp}{\tilde{a}_+(p)}.
\]

\( \tilde{a}_- \) and \( \tilde{a}_+ \) are approximated acoustic impedances.
Modified Ghost Fluid Method (Liu et al. 03)

\[
\tilde{\omega}_i \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\tilde{u}}_i, \tilde{\tilde{p}}) \\
\tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{\tilde{u}}_j, \tilde{\tilde{p}})
\]

\[
\tilde{\tilde{u}}_i = (\tilde{\tilde{u}} - u_i \cdot n)n + u_i
\]

\[
\tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\tilde{u}}_i, \tilde{\tilde{p}}) \\
\tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{\tilde{u}}_j, \tilde{\tilde{p}}) \\
s(\tilde{\rho}_i, \tilde{\tilde{p}}) = s(\rho_i, p_i)
\]
**Finite volume**

**Modified Ghost Fluid Method (Liu et al. 03)**

- **M-GFM**
  \[ \tilde{\omega}_i \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{u}_i, \tilde{p}) \]
  \[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{u}_j, \tilde{p}) \]

- **H-GFM**
  \[ \tilde{\omega}_i = \tilde{\omega}_i (\check{\rho}_i, \check{u}_i, \check{p}) \]
  \[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\check{\rho}_j, \check{u}_j, \check{p}) \]
  \[ s(\check{\rho}_i, \check{p}) = s(\check{\rho}_i, \check{p}_i) \]

\[ \tilde{u}_i = (\check{u} - u_i \cdot n)n + u_i \]
Modified Ghost Fluid Method (Liu et al. 03)

**M-GFM**

\[ \tilde{\omega}_i \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{u}_i, \tilde{p}) \]

\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{u}_j, \tilde{p}) \]

\[ \tilde{\omega}_i = (\tilde{u} - u_i \cdot n)n + u_i \]

**H-GFM**

\[ \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{u}_i, \tilde{p}) \]

\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{u}_j, \tilde{p}) \]

\[ s(\tilde{\rho}_i, \tilde{p}) = s(\rho_i, p_i) \]
Finite volume

Modified Ghost Fluid Method (Liu et al. 03)

\[
\begin{align*}
\omega_i & \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\mathbf{u}}_i, \tilde{p}) \\
\tilde{\omega}_{ij} & = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{\mathbf{u}}_j, \tilde{p}) \\
\tilde{\mathbf{u}}_i &= (\tilde{\mathbf{u}} - \mathbf{u}_i \cdot \mathbf{n}) \mathbf{n} + \mathbf{u}_i
\end{align*}
\]

H-GFM

\[
\begin{align*}
\tilde{\omega}_i &= \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\mathbf{u}}_i, \tilde{p}) \\
\tilde{\omega}_{ij} &= \tilde{\omega}_{ij} (\tilde{\rho}_i, \tilde{\mathbf{u}}_j, \tilde{p}) \\
s(\tilde{\rho}_i, \tilde{p}) &= s(\rho_i, p_i)
\end{align*}
\]
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\[ \tilde{u}_i = (\tilde{u} - u_i \cdot n)n + u_i \]

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**H-GFM**

\[ \tilde{\omega}_i = \tilde{\omega}_i (\rho_i, \tilde{u}_i, \tilde{p}) \]
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\[ \tilde{\mathbf{u}}_i = (\tilde{\mathbf{u}} - \mathbf{u}_i \cdot \mathbf{n})\mathbf{n} + \mathbf{u}_i \]
Finite volume

Modified Ghost Fluid Method (Liu et al. 03)

\[ \tilde{\omega}_i \equiv \tilde{\omega}_j = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\vec{u}}_i, \tilde{p}) \]
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\[ \tilde{\vec{u}}_i = (\tilde{\vec{u}} - \vec{u}_i \cdot \vec{n})\vec{n} + \vec{u}_i \]

M-GFM

H-GFM

\[ \tilde{\omega}_i = \tilde{\omega}_i (\rho_i, \tilde{\vec{u}}_i, \tilde{p}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\rho_j, \tilde{\vec{u}}_j, \tilde{p}) \]
\[ s(\tilde{\rho}, \tilde{p}) = s(\rho, p) \]

Update Fluid 1
Finite volume

Modified Ghost Fluid Method (Liu et al. 03)

\[ \tilde{\omega}_i \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\tilde{u}}_i, \tilde{\tilde{p}}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{\tilde{u}}_j, \tilde{\tilde{p}}) \]
\[ \tilde{\tilde{u}}_i = (\tilde{\tilde{u}} - u_i \cdot n)n + u_i \]

\[ \tilde{\omega}_i = \tilde{\omega}_i (\rho_i, \tilde{u}_i, \tilde{p}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\rho_j, \tilde{u}_j, \tilde{p}) \]
\[ s(\tilde{\rho}, \tilde{p}) = s(\rho, p) \]
Finite volume

Modified Ghost Fluid Method (Liu et al. 03)

\[ \tilde{\omega}_i \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\mathbf{u}}_i, \tilde{p}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{\mathbf{u}}_j, \tilde{p}) \]
\[ \tilde{\mathbf{u}}_i = (\tilde{\mathbf{u}} - \mathbf{u}_i \cdot \mathbf{n}) \mathbf{n} + \mathbf{u}_i \]

H-GFM

\[ \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{\mathbf{u}}_i, \tilde{p}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{\mathbf{u}}_j, \tilde{p}) \]
\[ s(\bar{\rho}_i, \bar{p}) = s(\rho_i, p_i) \]
\[ \tilde{\mathbf{u}}_i = (\tilde{\mathbf{u}} - \mathbf{u}_i \cdot \mathbf{n}) \mathbf{n} + \mathbf{u}_i \]
Finite volume

Modified Ghost Fluid Method (Liu et al. 03)

M-GFM
\[ \omega_i \equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{u}_i, \tilde{p}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{u}_j, \tilde{p}) \]
\[ \tilde{u}_i = (\tilde{u} - u_i \cdot n)n + u_i \]

H-GFM
\[ \tilde{\omega}_i = \tilde{\omega}_i (\rho_i, \tilde{u}_i, \tilde{p}) \]
\[ \tilde{\omega}_{ij} = \tilde{\omega}_{ij} (\rho_i, \tilde{u}_j, \tilde{p}) \]
\[ s(\tilde{\rho}_i, \tilde{p}) = s(\rho_i, p_i) \]
Modified Ghost Fluid Method (Liu et al. 03)

\( \tilde{u}_i = (\tilde{u} - u_i \cdot n)n + u_i \)

M-GFM
\[
\begin{align*}
\omega_i &\equiv \tilde{\omega}_i = \tilde{\omega}_i (\tilde{\rho}_i, \tilde{u}_i, \tilde{p}) \\
\tilde{\omega}_{ij} &\equiv \tilde{\omega}_{ij} (\tilde{\rho}_j, \tilde{u}_j, \tilde{p})
\end{align*}
\]

H-GFM
\[
\begin{align*}
\tilde{\omega}_i &\equiv \tilde{\omega}_i (\tilde{\rho}_i, \tilde{u}_i, \tilde{p}) \\
\tilde{\omega}_{ij} &\equiv \tilde{\omega}_{ij} (\tilde{\rho}_i, \tilde{u}_j, \tilde{p}) \\
\end{align*}
\]

\[ s(\tilde{\rho}, \tilde{p}) = s(\rho, p) \]
Applications: 1D Cases

Sod shock tube

![Graph showing the comparison of different methods for the Sod shock tube problem. The graph plots density against a dimensionless variable, with three curves representing GFM, M-GFM, and Exact solutions. The graph demonstrates the accuracy of the Ghost Fluid Methods (GFM) and Modified Ghost Fluid Methods (M-GFM) in approximating the exact solution.]
Applications: 1D Cases

Helium-Air shock tube

![Graph showing density over time for different methods: GFM, M-GFM, H-GFM, and Exact. The graph compares the performance of these methods against the exact solution in a Helium-Air shock tube scenario.]
Applications: 1D Cases

Shock interaction with Water-Air interface

![Shock Interaction Graph]

- M-GFM
- H-GFM
- Exact
Applications: 1D Cases

Stronger shock interaction with Water-Air interface
Outline

1. Interfaces flow Model
   - Level Set for Interface tracking
   - Governing equations

2. Ghost Fluid Methods
   - Finite volume
   - Applications: 1D Cases

3. **Extension to unstructured meshes**
   - Finite Volume
   - 2D Applications

4. Conclusions, Coming and Future work
   - DG approach for the level set equation
   - Finite Element
Multi-D Finite volume schemes

Directions Based Schemes

\[ a_i \omega_i^{n+1} = a_i \omega_i^n - \sum_{j \in \nu(i)} \Phi \left( n_{ij}, \omega_i^n, \omega_j^n \right) - \sum_{j \in \kappa(i)} \Phi_S \left( n_{ij}, \omega_i^n, \omega_j^n \right) \]

\[ \Phi_S \left( n_{ij}, \omega_i^n, \omega_j^n \right) = \Phi \left( n_{ij}, \tilde{\omega}_i^n, \tilde{\omega}_j^n \right) \]

\[ \tilde{\omega}_i^n = \tilde{\omega} \left( \omega_i, \omega_j, \nabla \phi_{ij}, \omega_{i-}, \omega_{j+} \right) \]

\[ \tilde{\omega}_j^n = \tilde{\omega} \left( \omega_i, \omega_j, \nabla \phi_{ij}, \omega_{i-}, \omega_{j+} \right) \]

Need to be defined

- The interface normal \( \nabla \phi_{ij} \)
- The states \( \omega_{i-} \) and \( \omega_{j+} \)
Interfaces parameters for GFM

Stencil for interface normal computation: $N_{ij} = \nabla\tilde{\varphi}_{ij}$
Interfaces parameters for GFM

Stencil for “isobaric 1” $\omega_{i-}$ and $\omega_{j+}$
Interfaces parameters for GFM

Stencil for “isobaric 2” $\omega_{i-}$ and $\omega_{j+}$
Interfaces parameters for GFM

Stencil for “isobaric 3” $\omega_i^-$ and $\omega_j^+$
Interfaces parameters for GFM

Stencil for “isobaric 4” $\omega_{i-}$ and $\omega_{j+}$
2D Applications

Shock/bubble interaction: Air-Helium
2D Applications

Shock/bubble interaction: Air-water
2D Applications

Shock/bubble interaction: Air-Helium
2D Applications

Shock/bubble interaction: Air-water
2D Applications

Shock/bubble interaction: Air-water
# Outline

1. **Interfaces flow Model**
   - Level Set for Interface tracking
   - Governing equations

2. **Ghost Fluid Methods**
   - Finite volume
   - Applications: 1D Cases

3. **Extension to unstructured meshes**
   - Finite Volume
   - 2D Applications

4. **Conclusions, Coming and Future work**
   - DG approach for the level set equation
   - Finite Element
DG approach for the level set equation

DG for the transport of the level set
DG approach for the level set equation

DG for the transport of the level set
DG approach for the level set equation

Shock/bubble interaction: Barth vs. DG (P1)
Compressible/Incompressible Interfaces

Compressible Model: $\omega_1 = (\rho, \rho u, \rho e)^T$ Hyperbolic system.
Compressible component is defined as previously.

Incompressible Model: $\omega_2 = (\pi, u)^T$

1. $\partial_t u + (u \cdot \nabla) u + \frac{1}{\rho} \nabla \pi = 0$
2. $\nabla \cdot u = 0$
Finite element formulation

Stabilized Galerkin method (with mass lumping)

\[ a_i \omega_i^{n+1} = a_i \omega_i^n - \sum_{\tau \in T(i)} \Phi(\omega^{n}_{\tau}) - \sum_{\tau \in K(i)} \Phi_S(\omega^{n}_{\tau}) \]

\[ a_i \phi_i^{n+1} = a_i \phi_i^n - R_i(\phi^n, \omega^{n+1}) \]

Stabilizations techniques

- SUPG for convection
- PSPG to handle LBB condition
- Grad-Div to enforce the incompressible constraint.

Ghost Fluid

\[ \Phi_S(\omega^{n}_{\tau}) = \Phi(\tilde{\omega}^{n}_{\tau}) \]
Finite Element

Other Issues

Example

- Deflagation Detonation (Fedkiw 1999)
- Eulerian Fluid/Lagrangian Solid (Fedkiw 2002, Cirak 2004)
- Thin flame and LES premixed combustion (Moureau et al. 2005)
- Phase transition (Gibou et al. 2006)
- Surface tension

Main points to deal with

1. Define the equation for the level set function.
2. Set out appropriate jump conditions at the interfaces.
Les bonnes choses ont une fin !!!!!!!!! merci aux organisateurs !!!!!!!!!

Thanks

and farewell!