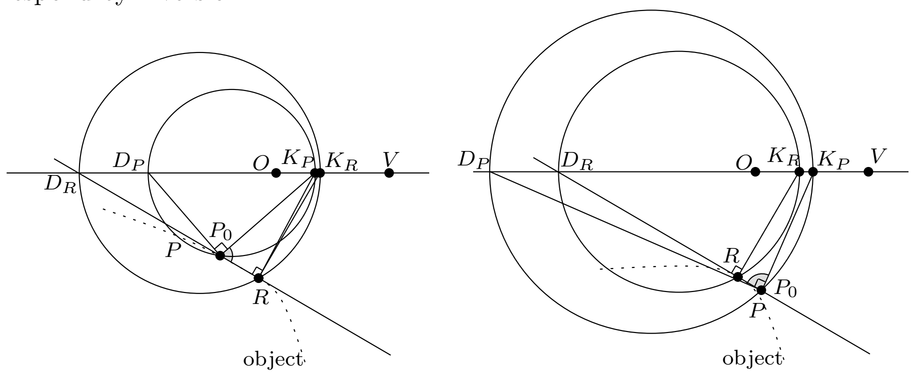
Claim: *The vector B'_P goes in the direction of R, that is $B'_P \cdot PR \geq 0$.*
Proof:

$$\text{sign}(B'_P \cdot PR) = \text{sign}((B_P - N_P) \cdot PR) = \text{sign}(B_P \cdot PR - N_P \cdot PR)$$

— $N_P \cdot PR$ is negative, It can be decomposed in $N_P = N'_P + N'^{\perp}_P$ such that N'_P belongs to the plane PRN_R and $N'^{\perp}_P$ is orthogonal to that plane. Then, $N'^{\perp}_P \cdot PR$ is zero by definition and $N'_P \cdot PR$ is negative by convexity of the cross-section of the object by plane PRN_R.

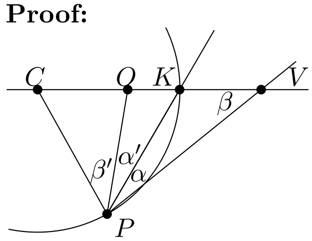
— It suffices to prove that $B_P \cdot PR$ is positive to ensure the result. This is equivalent to prove that $PK_P \cdot PR$ is positive, being K_P the intersection of line $P + B_P$ and line OV. We remark (lemma below) that the set of points P such that $K_P = K$, for a given K, is the sphere through K such that O and V correspond by inversion.



Choose P_0 on a line tangent to the object in R in plane OVR and such that $K_{P_0} = K_P$. Then, if the angle RP_0K_P is smaller than $\frac{\pi}{2}$, moving from P_0 to P on the sphere of points having the same point K and on the relevant side of the tangent plane to the object in R (since the object is convex) will only make this angle decrease.

The fact that RP_0K_P is smaller than $\frac{\pi}{2}$ is granted since it is smaller than $D_PP_0K_P = \frac{\pi}{2}$, if P_0 is on the right of P (see Fig. right) and smaller than $\pi - D_PP_0K_P = \frac{\pi}{2}$, if P_0 is on the left of P (see Fig. left). □

Lemma: The set of points P such that $K_P = K$ for a given K is the sphere through K such that O and V correspond by inversion.
Proof:



We consider colinear points C, O, V, K such that $CO \cdot CV = CK^2$ and a point P on the sphere of center C through K, The bissector of OPV actually goes through K. Let denote α the angle OPK, α' the angle KPV β the angle OVP and β' the angle CPO. Considering triangle PKV, we get $OKP = \alpha + \beta$. Considering the isocele triangle CKP, we get $OKP = \alpha' + \beta' = \alpha + \beta$. The triangles CPO and CPV have a common angle in C and the same length ratio $\frac{CP}{CO} = \frac{CK}{CO} = \frac{CV}{CK} = \frac{CV}{CP}$ thus, they are isothetic and $\beta = \beta'$. Therefore, we conclude that $\alpha = \alpha'$, that is PK is the claimed bissector. □