Mathematical morphology

- with emphasis on analysis of hyperspectral images and remote sensing applications

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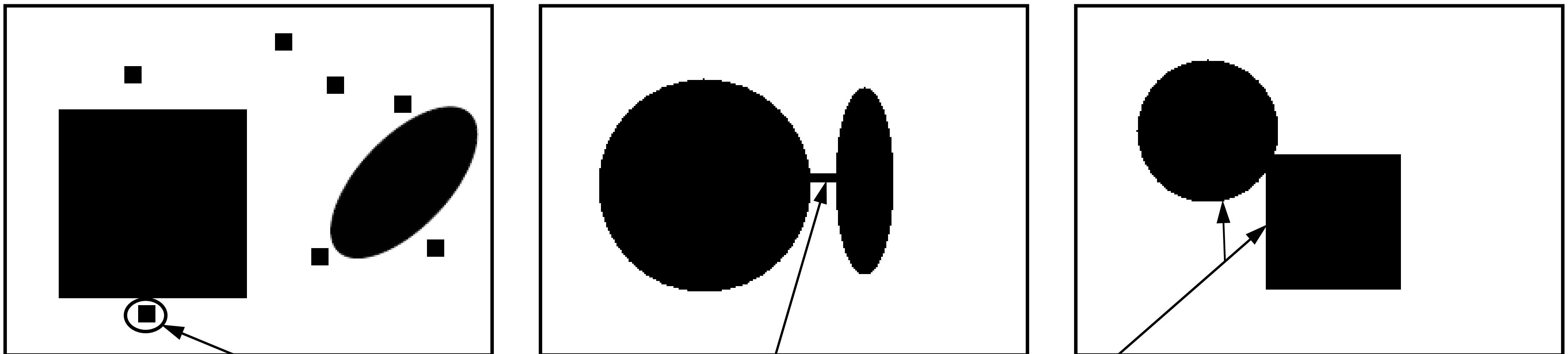
Outline

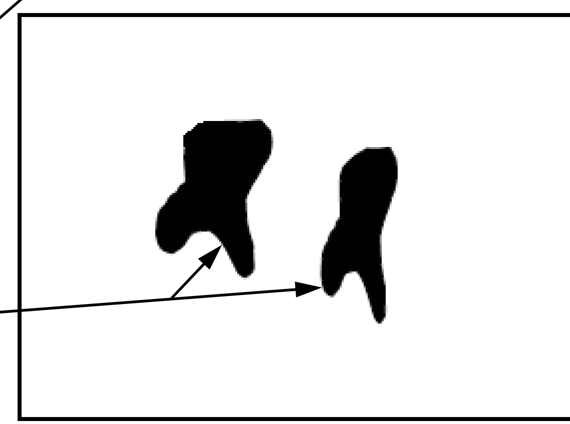
- Introduction
- Basic concepts of mathematical morphology
- Mathematical morphology for grey-scale and hyperspectral images
- Remote sensing application 1: Classification of hyperspectral images of an urban area using morphological profiles
- Remote sensing application 2: Segmentation and classification of hyperspectral images using watershed
- Practical session on mathematical morphology

Basic concepts of mathematical morphology

Mathematical morphology: why to use?



- How to remove this noise?
- How to separate these two components?
- How to label differently these two connected shapes?
- How to compare these two shapes?



Mathematical morphology (MM)

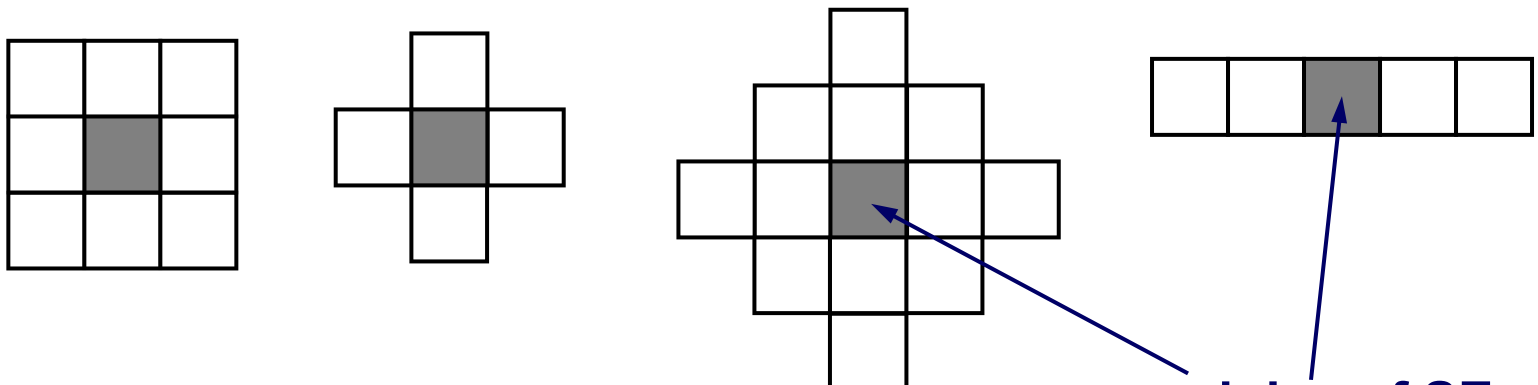
- A theory for the ***analysis of spatial structures***
- ***Morphology:*** aims at analysing the shape and form of objects.
- ***Mathematical:*** analysis is based on:
  - set theory
  - integral geometry
  - lattice algebra

- ***Non-linear*** processing operators (do not blur the edges as convolutions do)
- We'll concentrate on MM for digital images: binary, grey-scale and hyperspectral

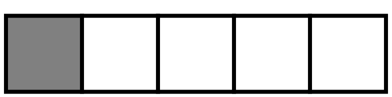
- ***Basic idea:*** locally compare structures within the image with a reference shape called the ***Structuring Element*** (SE)

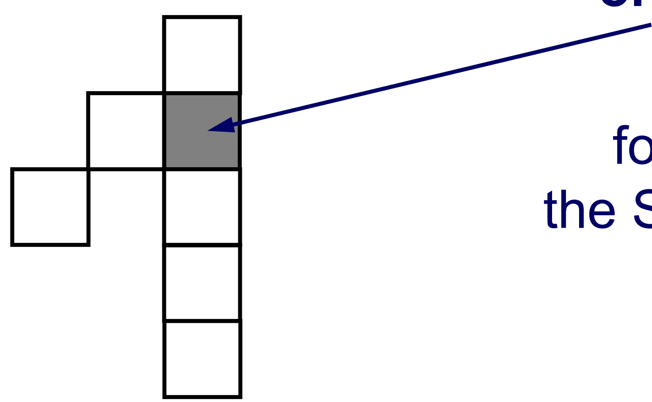
Structuring element (SE)

- A small set used to analyse locally the image
- ***Shape*** and ***size*** of SE ← a priori knowledge about the geometry of relevant/irrelevant image structures
- Usually symmetrical, connected, and convex



- But not always





Dilation and erosion

Fundamental morphological operators = 2 letters of the morphological alphabet

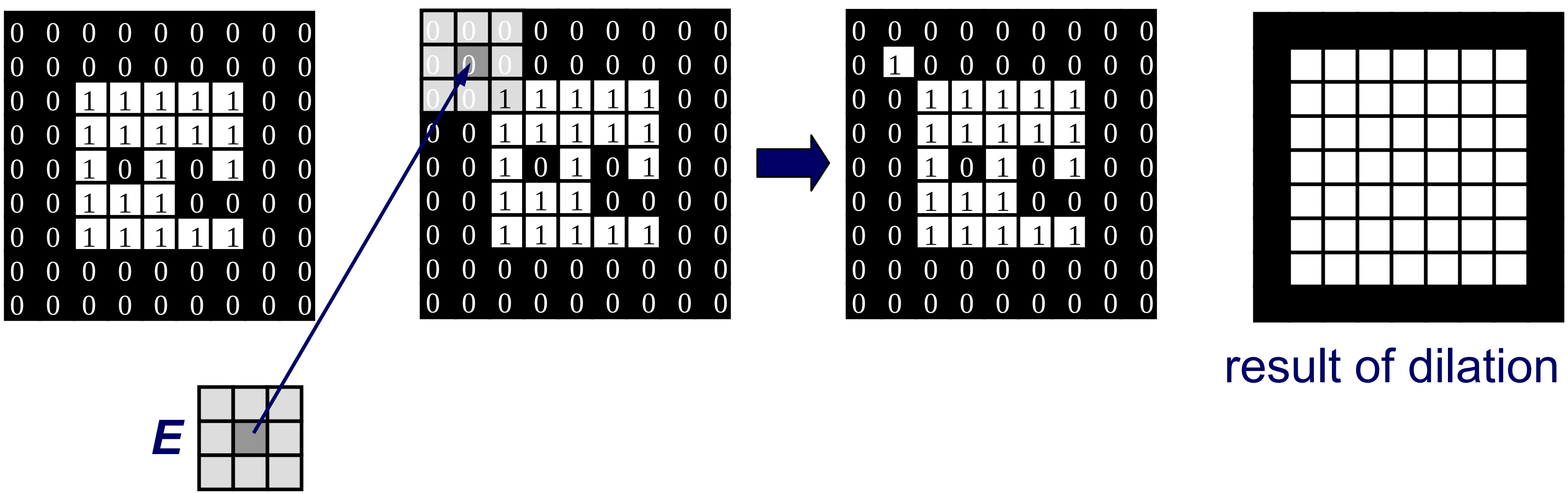
⬇

All other operators are expressed in terms of
dilations and erosions

Dilation for binary images

- "Does the SE ***hit*** the set?"
- ***Dilation*** of a set X by a structuring element E is defined as the locus of points **x** such that E ***hits*** X when its origin is placed at **x**:

$$\delta_E(X) = \{\mathbf{x} \in R^d \mid E_{\mathbf{x}} \cap X \neq \varnothing\}$$



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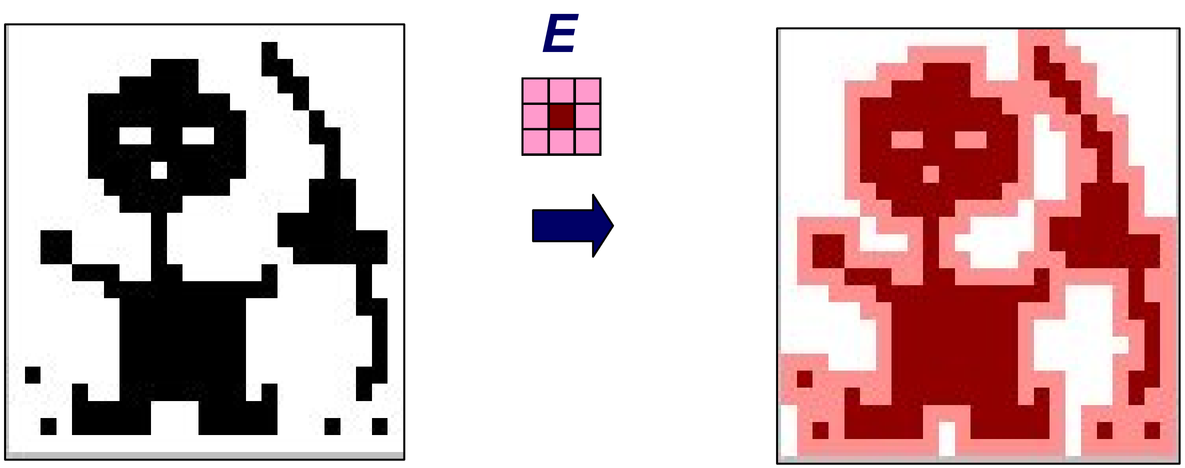
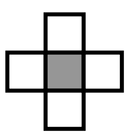


result of dilation

Dilation: properties and use

- Basic property: $X \subseteq \delta_E(X)$
- Consequences:
  - Fill in the holes smaller than *E*
  - Enlarge capes
  - Connect two close shapes
- Example of application: bridging gaps

E

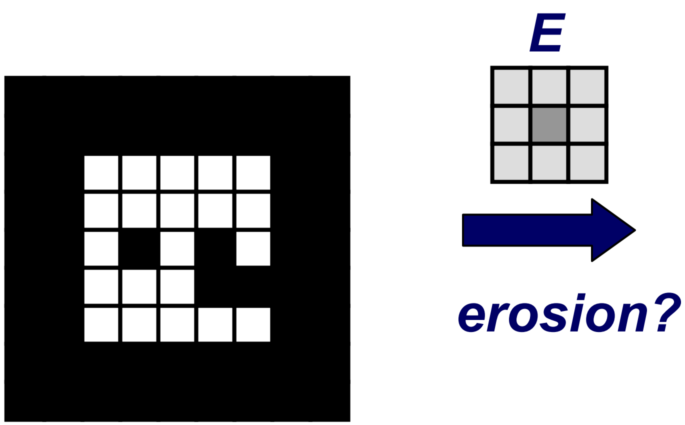


R. C. Gonzalez, R. E. Woods, Digital Image Processing. Prentice Hall, 2002.

Erosion for binary images

- “Does the SE ***fit*** the set?”
- ***Erosion*** of a set X by a structuring element E is defined as the locus of points $\mathbf{x}$ such that E is ***included*** in X when its origin coincides with $\mathbf{x}$:

$$\varepsilon_E(X) = \{\mathbf{x} \in R^d \mid E_{\mathbf{x}} \subseteq X\}$$



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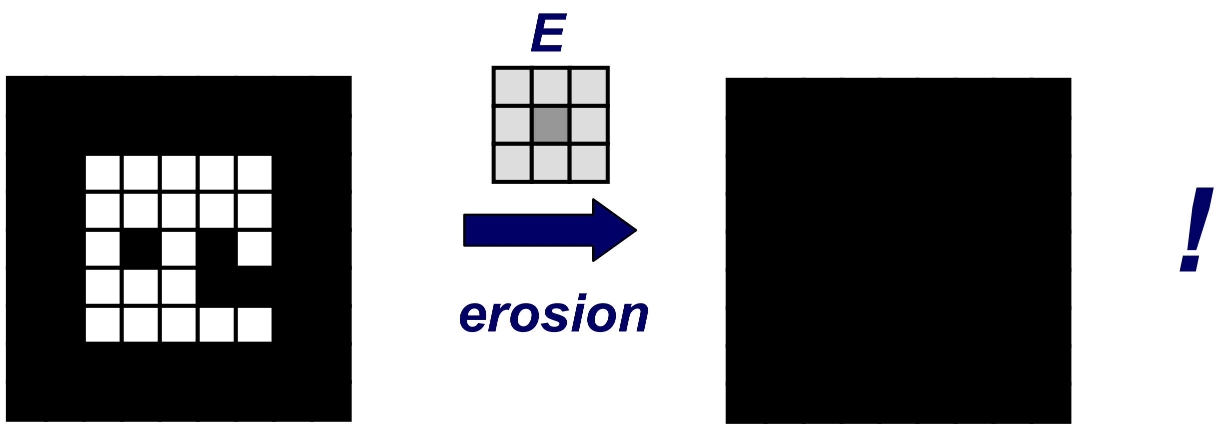

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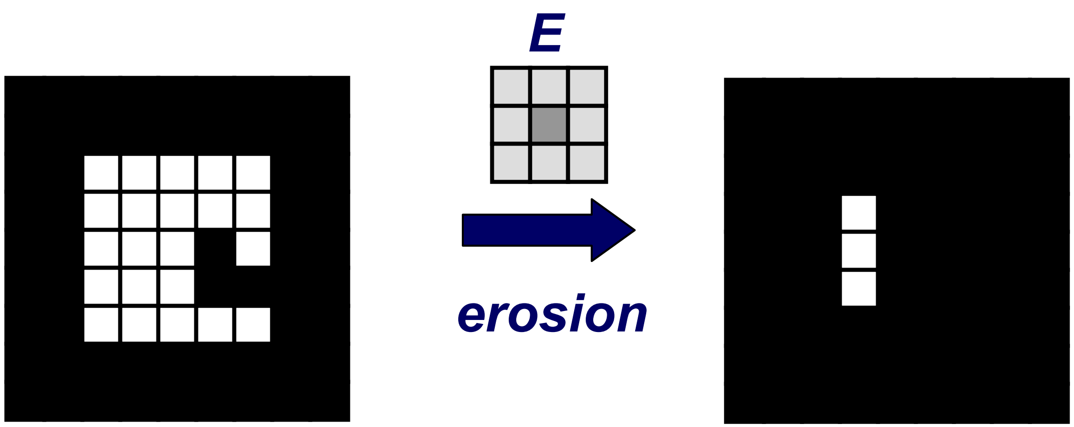

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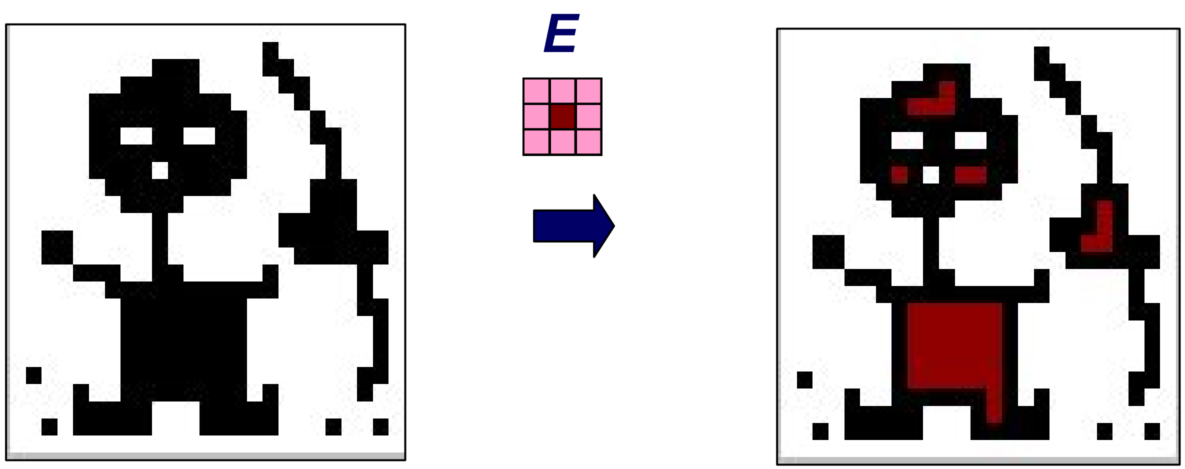
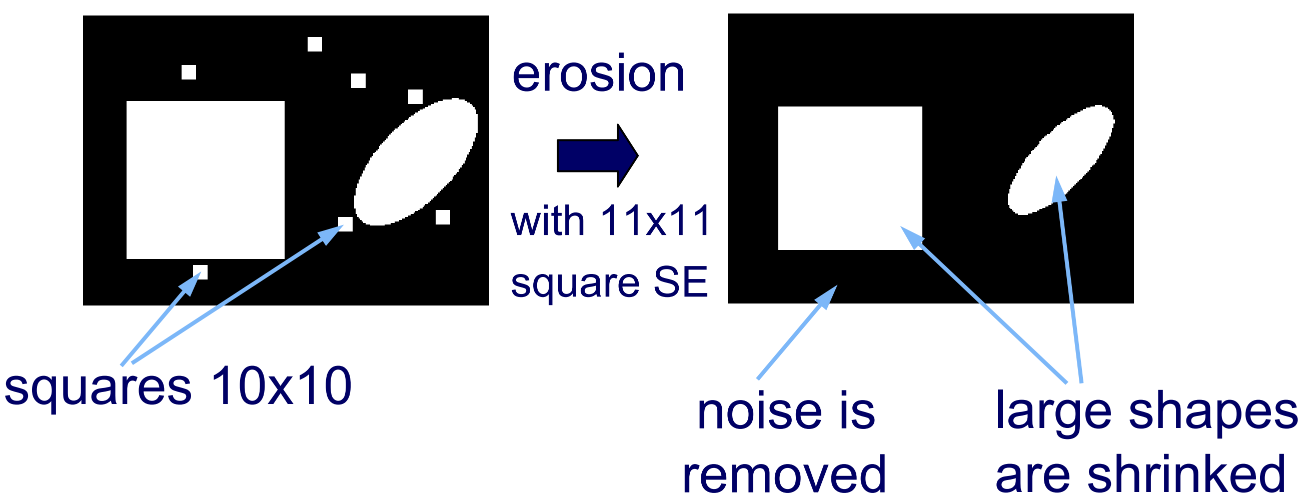


result of erosion

Erosion: properties and use

- Basic property: $\varepsilon_E(X) \subseteq X \subseteq \delta_E(X)$
- Consequences:
  - Eliminate connected components smaller than *E*
  - Eliminate narrow capes
  - Enlarge holes
- Example of application: eliminating irrelevant details (noise)



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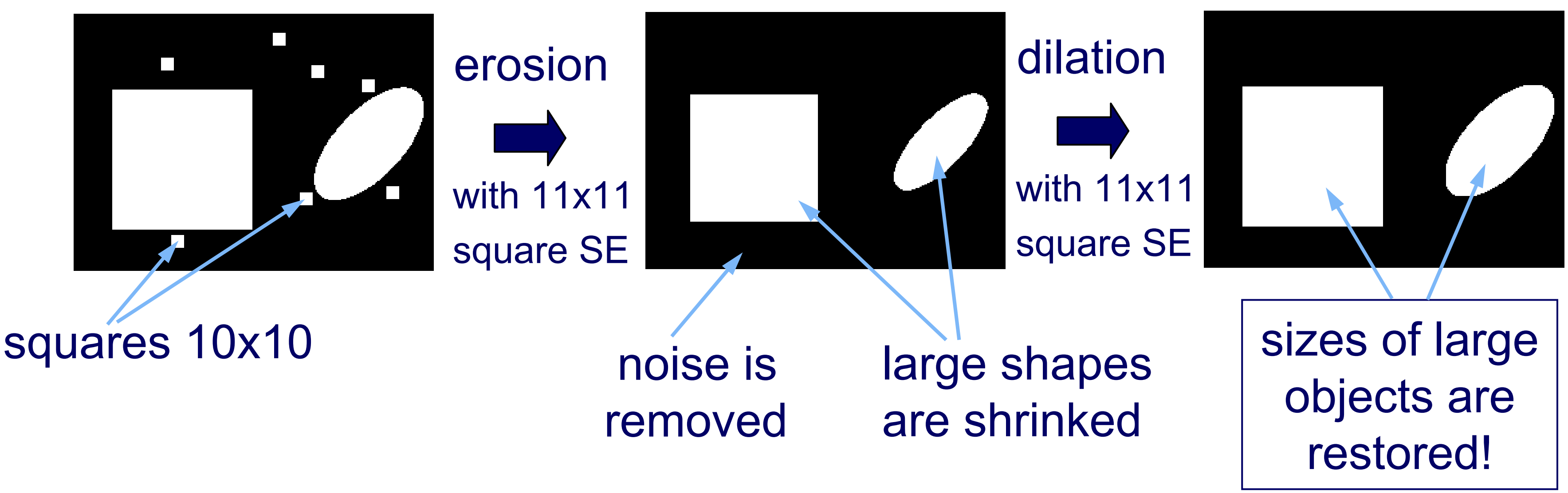
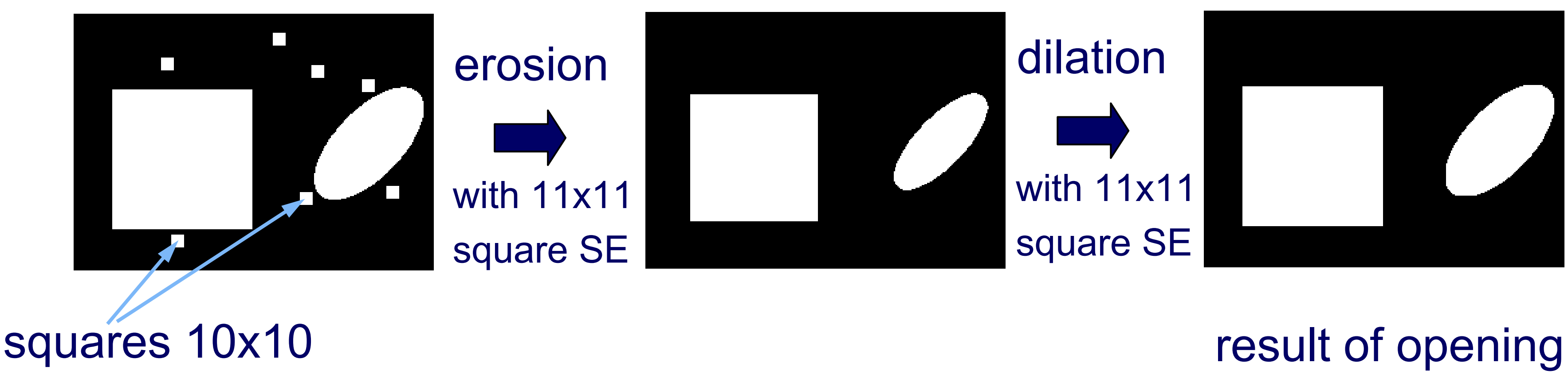


Opening for binary images

- ***Opening*** of a set X by a structuring element E is defined as the erosion of X by E followed by the dilation with the reflected (symmetric with respect to the origin) SE $\tilde{E}$:

$$\gamma_E(X) = \delta_{\tilde{E}}[\varepsilon_E(X)]$$

- Consequences:
  - Objects smaller than E disappear
  - Other objects remain "unchanged"



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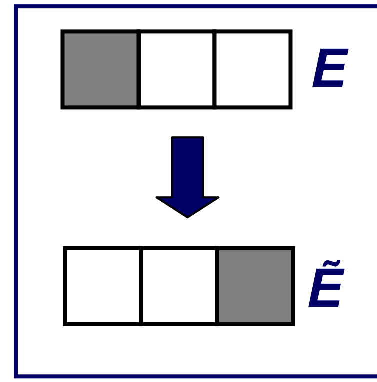

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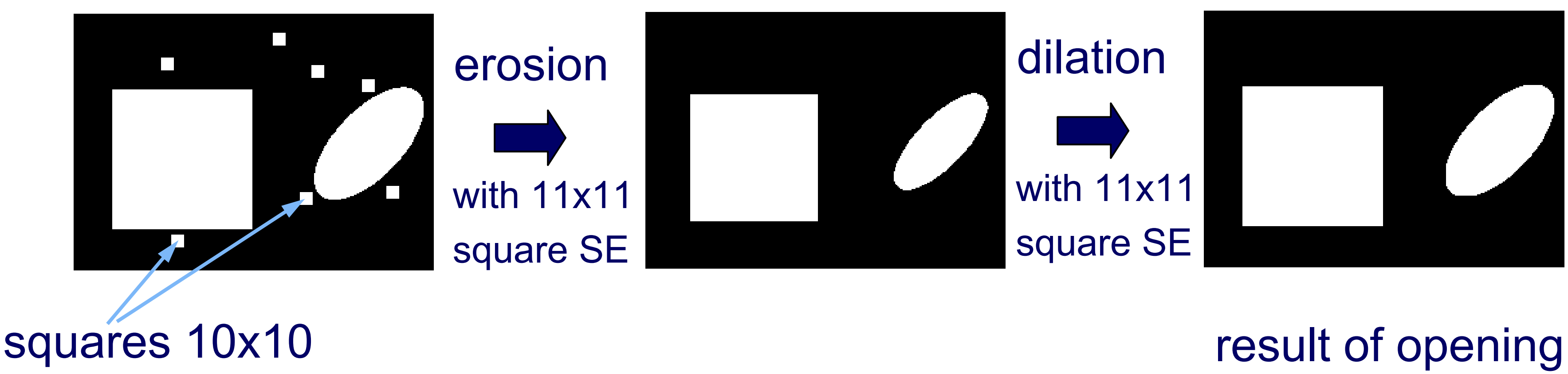

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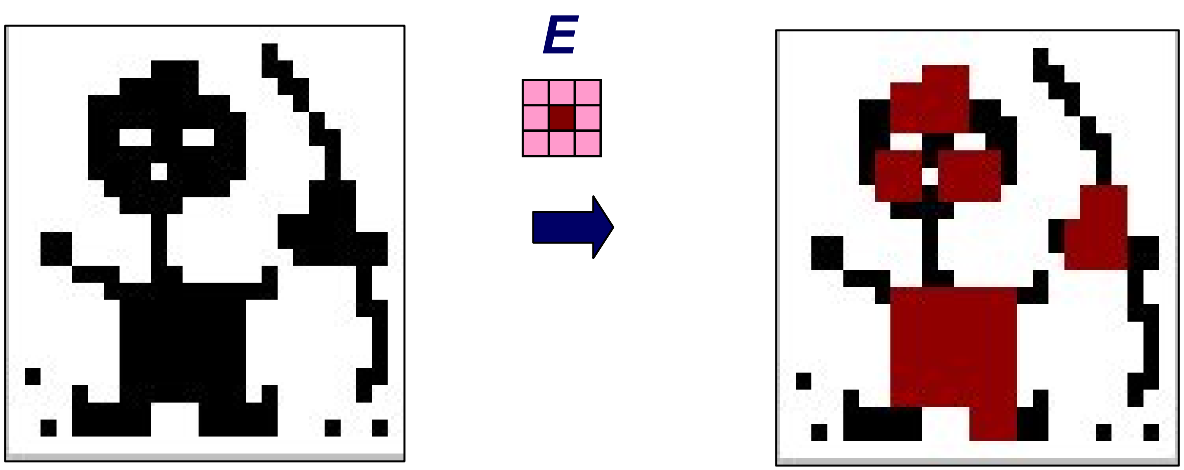


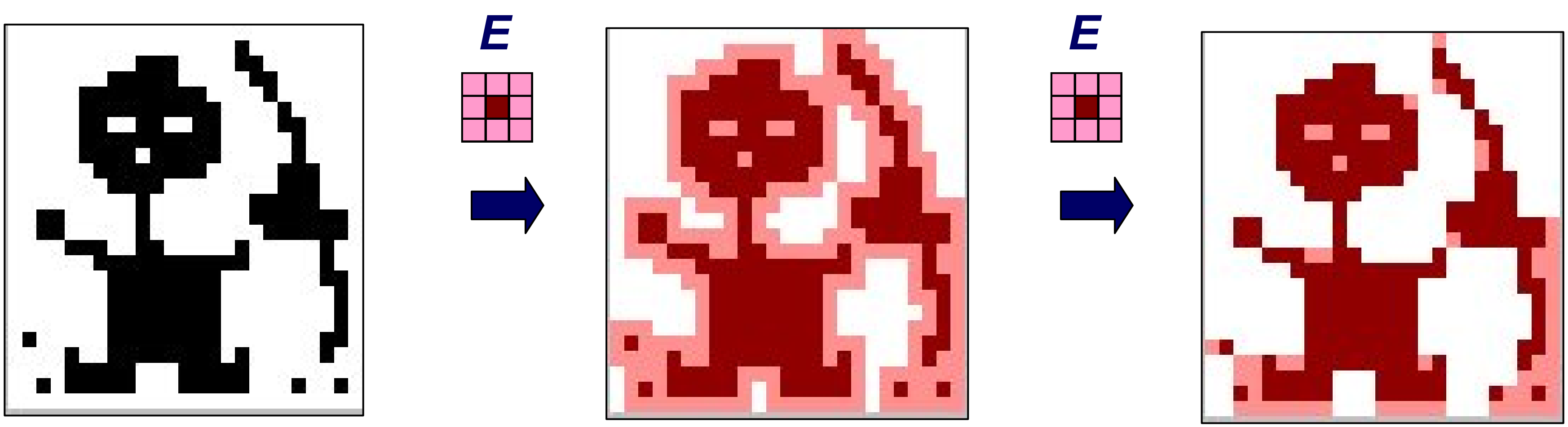
result of opening

Closing for binary images

- ***Closing*** of a set X by a structuring element E is defined as the dilation of X by E followed by the erosion with the reflected SE $\tilde{E}$:

$$\varphi_E(X) = \varepsilon_{\tilde{E}}[\delta_E(X)]$$

- Order: $\gamma_E(X) \subseteq X \subseteq \varphi_E(X)$
- Consequences:
  - Holes smaller than E are eliminated
  - Other objects remain “unchanged”



result of dilation

result of closing

Other MM operators: Top-hat

- ***Top-hat*** transformation of an image *X* is defined as the difference between the original image *X* and its opening γ:

$$TH(X) = X - \varphi(X)$$

- Consequences:
  - Objects smaller than *E* are extracted
  - Other objects disappear

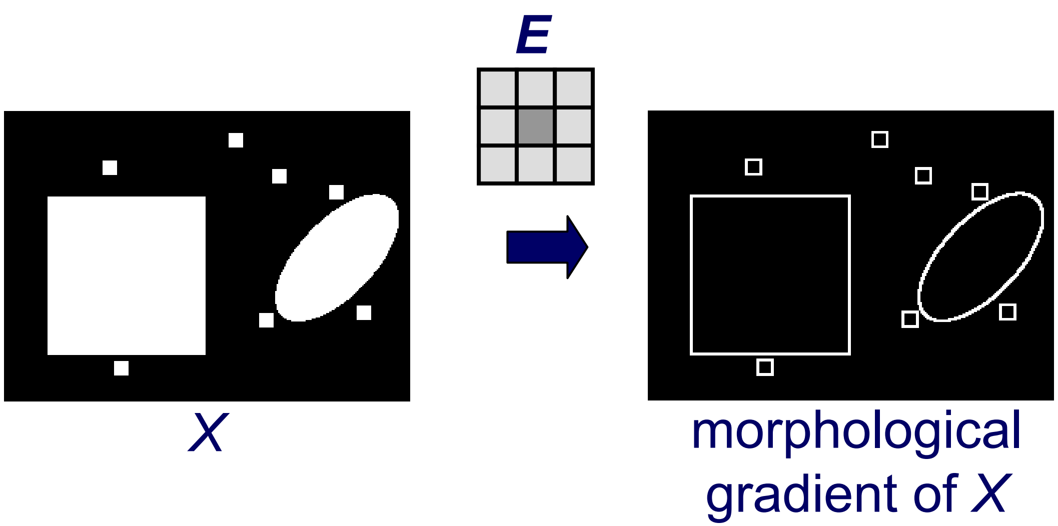
X $\varphi_{11x11}(X)$ $TH(X)$

Morphological gradient

- The basic ***morphological gradient*** of an image *X* is defined as the arithmetic difference between the dilation and the erosion of *X* by the elementary SE *E* :

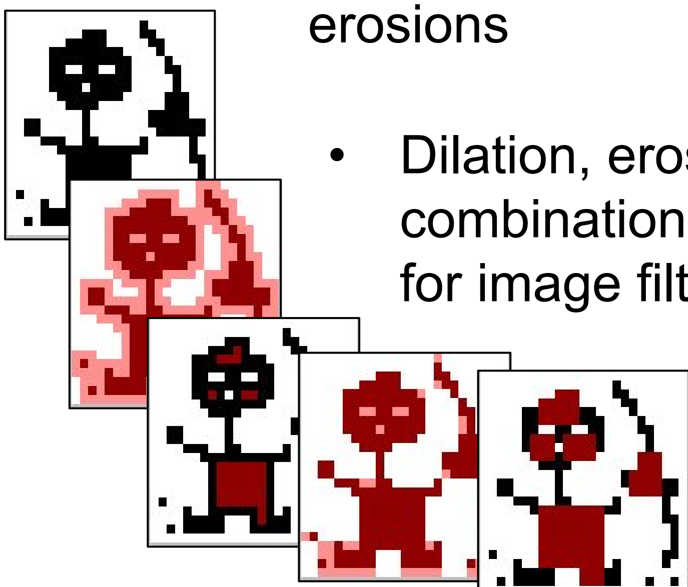
$$\rho_E(X) = \delta_E(X) - \varepsilon_E(X)$$

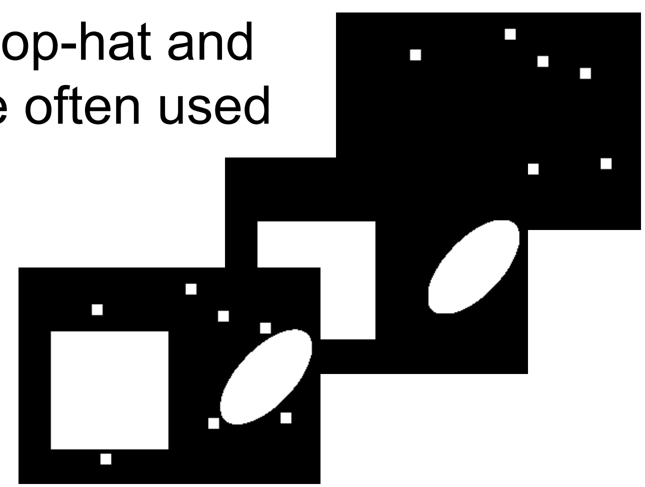
- Only symmetric SEs are considered
- Tends to depend less on edge directionality (when compared to first derivatives)



Summary: Basic concepts of MM

- Mathematical morphology can be defined as a theory for the analysis of spatial structures
- Basic idea of MM: locally compare structures within the image with a reference shape called the Structuring Element
- Dilation and erosion are two fundamental MM operators
  ➔ All other operators are expressed in terms of dilations and erosions
  - Dilation, erosion, opening, closing, top-hat and combinations of these operators are often used for image filtering

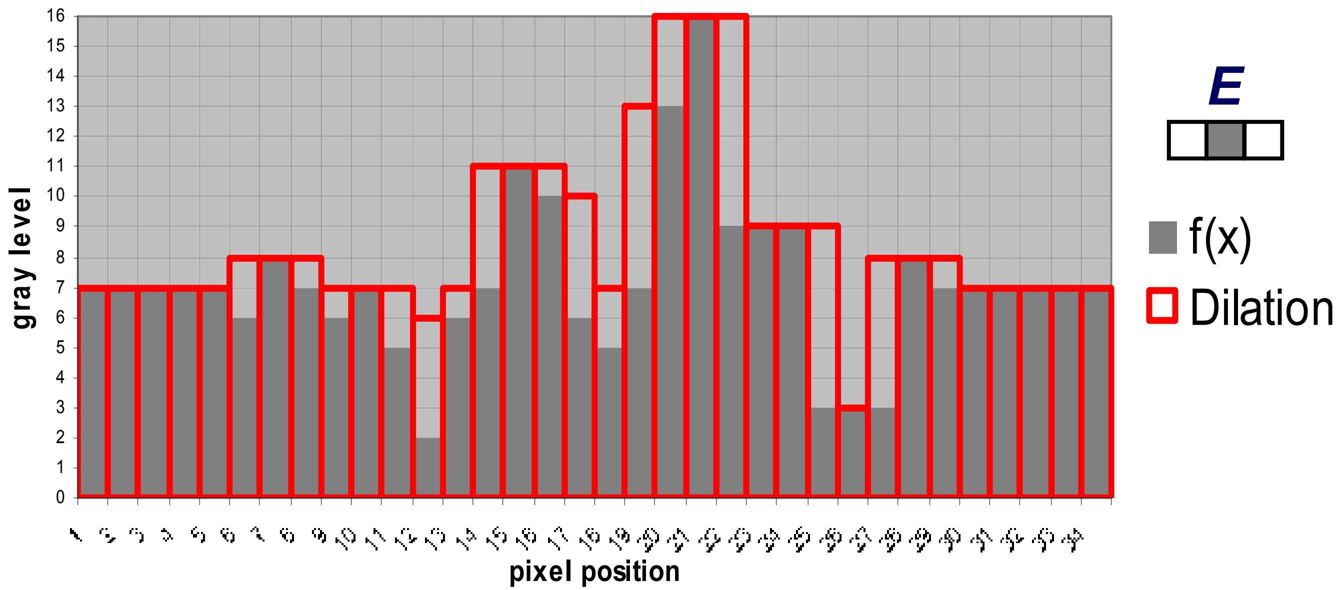




Mathematical morphology for grey-scale and hyperspectral images

Dilation for grey-scale images

- ***Dilation:*** replace every pixel by the *maximum* value computed over the neighborhood defined by the structuring element



Dilation for grey-scale images

- ***Dilation:*** replace every pixel by the *maximum* value computed over the neighborhood defined by the structuring element

- Consequence:
  - Features that are *brighter* than their immediate surroundings are *enlarged*
  - Features that are *darker* than their immediate surroundings are *shrinked*
  - Effect driven by the size and shape of the SE

$\delta_5(X)$

SE: disk of radius = 5

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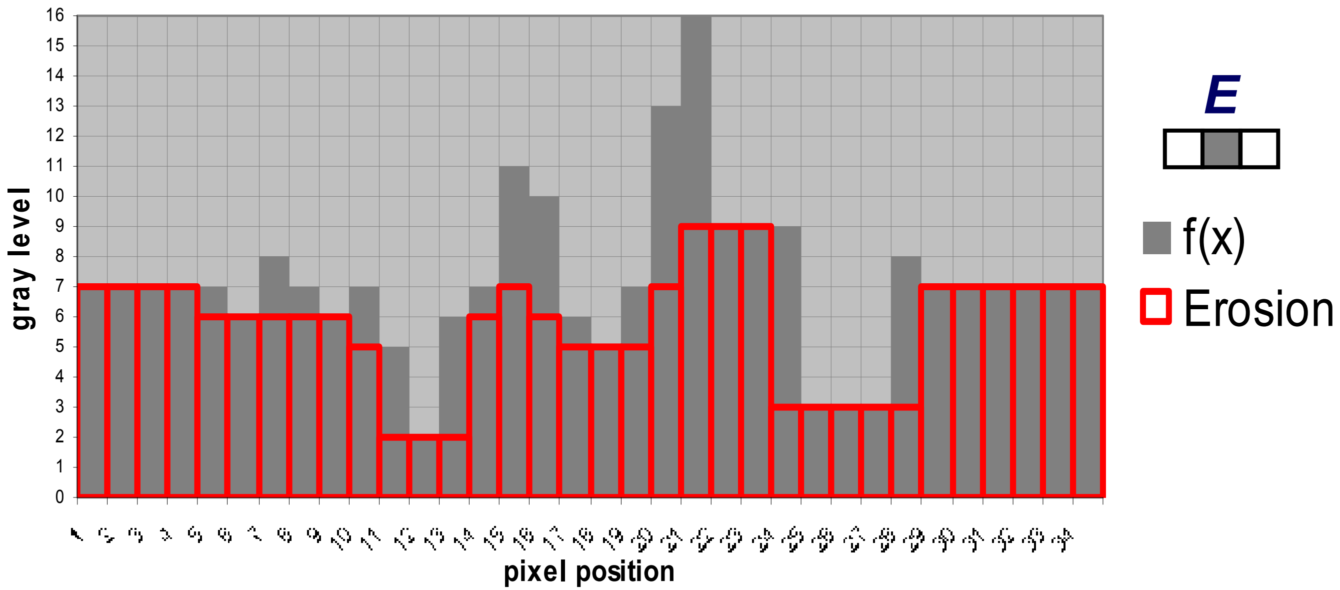
$\delta_{15}(X)$ ➡

SE: disk of radius = 15

Erosion for grey-scale images

- ***Erosion:*** replace every pixel by the *minimum* value computed over the neighborhood defined by the structuring element



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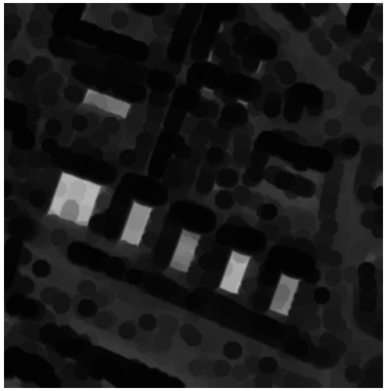
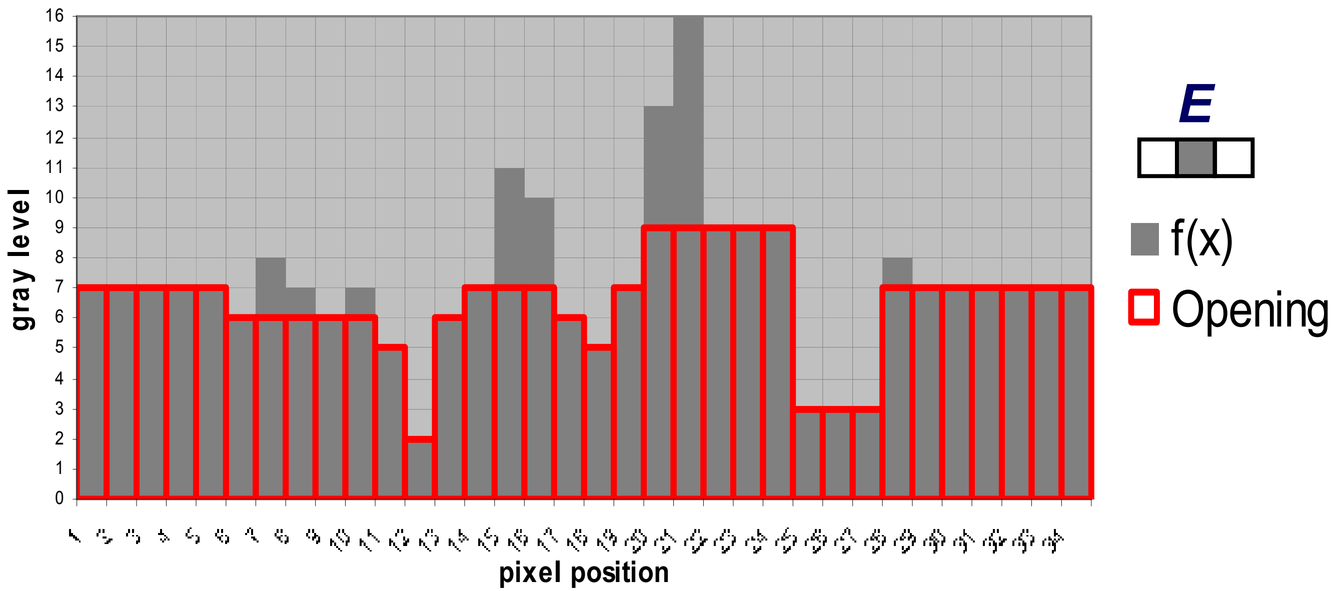


Opening for grey-scale images

- ***Opening:*** erosion followed by a dilation with the symmetrical structuring element



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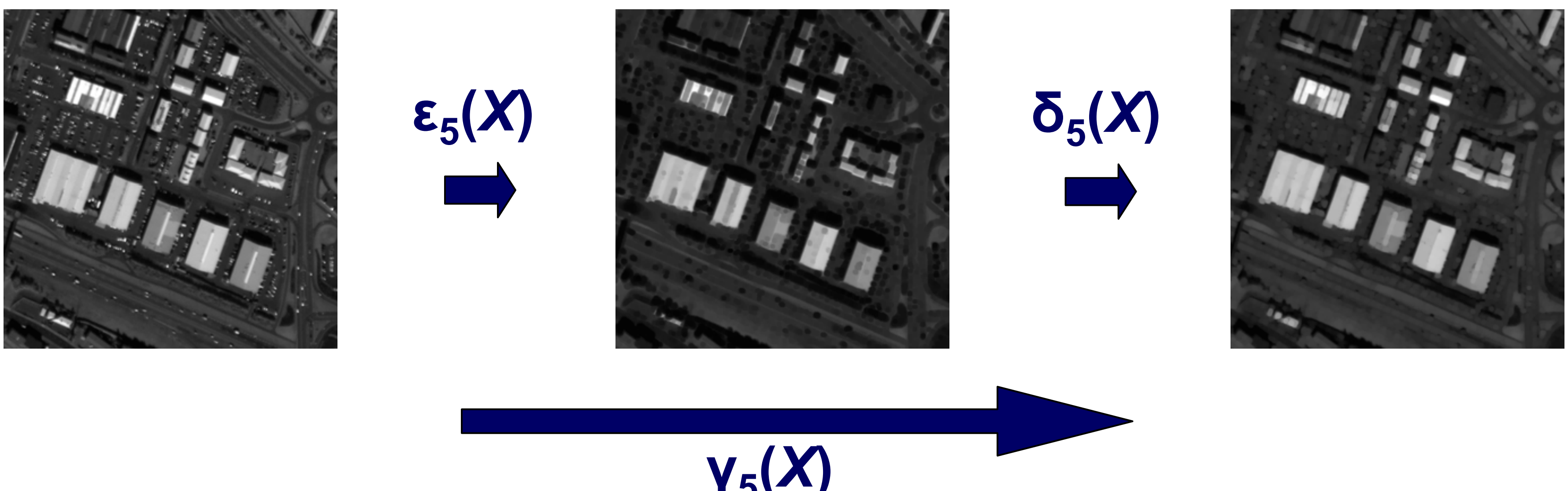
  - Other features (dark, or bright and large) remain "unchanged"



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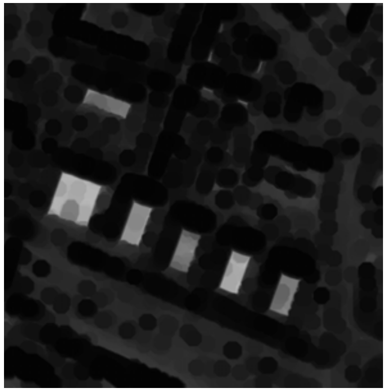
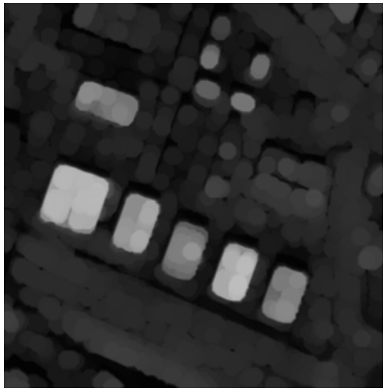
$\varepsilon_{15}(X)$



$\delta_{15}(X)$



$\gamma_{15}(X)$

Closing for grey-scale images

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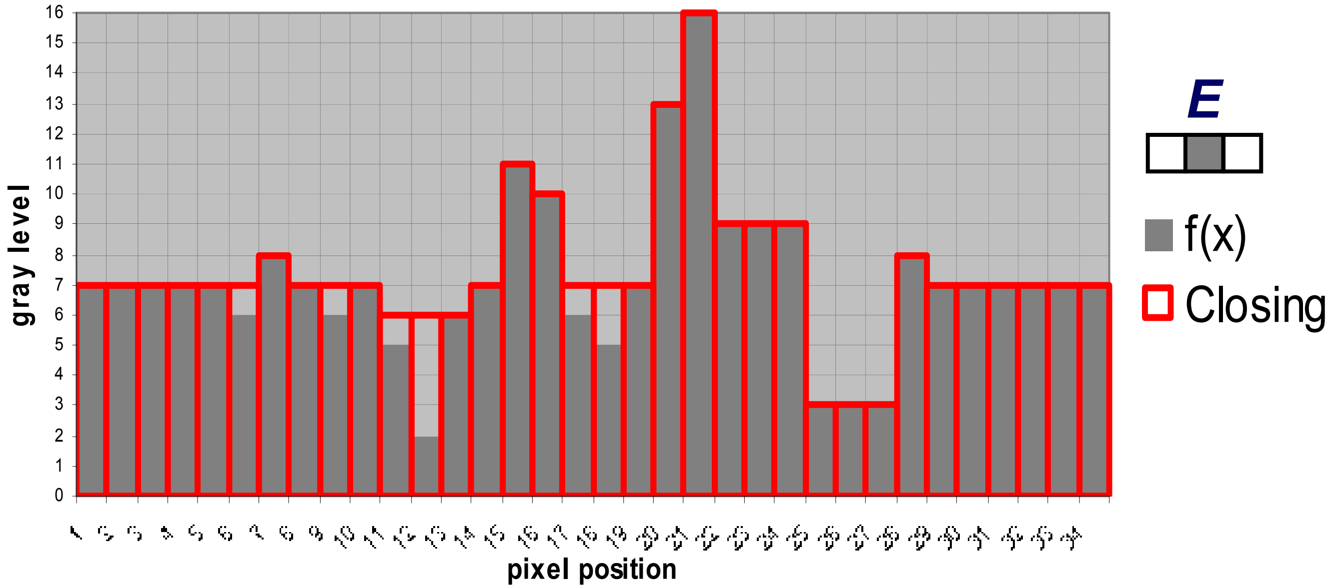

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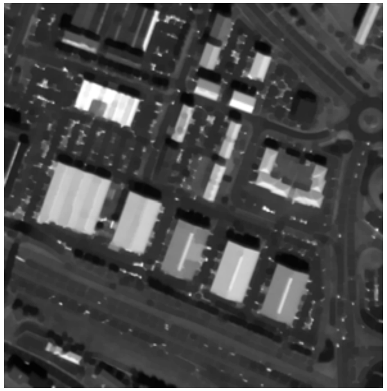
$\delta_5(X)$

$\varepsilon_5(X)$



$\varphi_5(X)$

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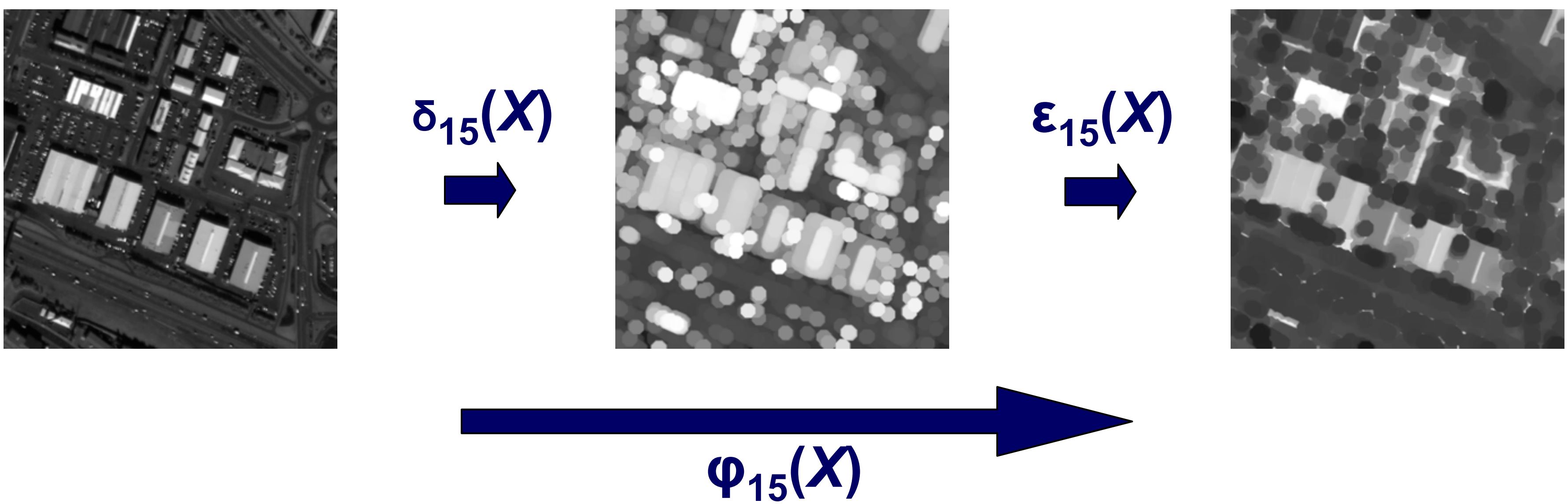


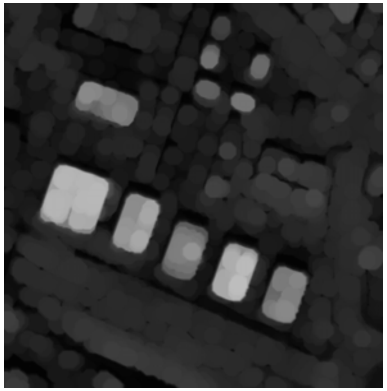
Geodesic reconstruction

- *Connected* operators
- Same properties, with no shape noise

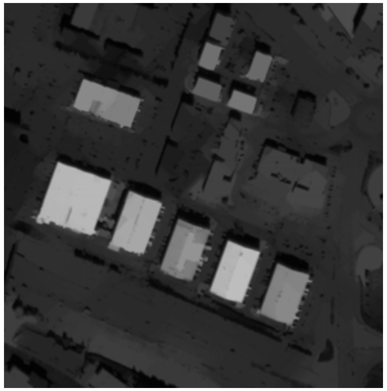
- Opening by reconstruction:
  - Preserves the shape of the objects that are not removed by erosion

Original image: X
Im1 := ε_{15}(X)
Im2 := δ_{15}(Im1)
Im3 := min(Im2, X)
CV Im1 := Im3

$\gamma_{15}(X)$ ➡



opening ➡ **by reconstruction**



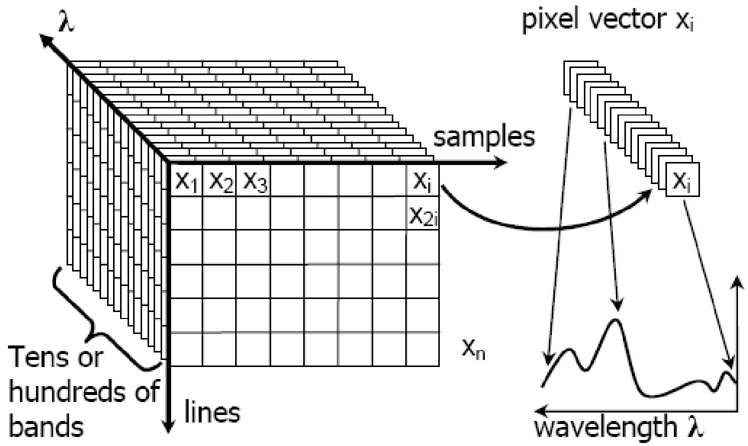
To read: P. Soille, Morphological Image Analysis, 2nd ed. Springer-Verlag, 2003.

Mathematical morphology for hyperspectral images

Multivariate mathematical morphology

- ***Hyperspectral image:***

  every pixel = spectrum
  = vector of ***very high dimension***



- ***Problem:***
  Mathematical morphology requires a ***complete lattice*** structure.
  Every set of pixels has one ***infimum*** and one ***supremum***.

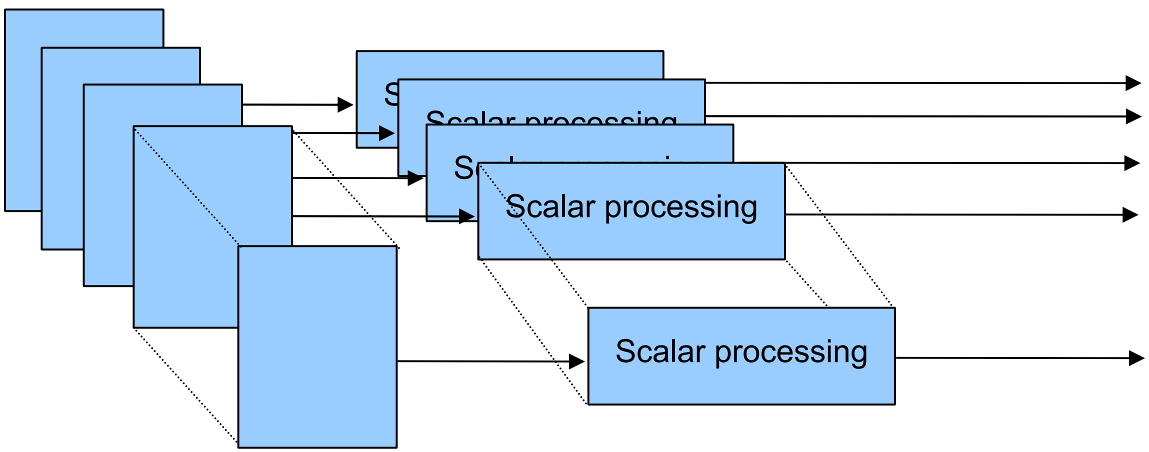
Shall one process hyperspectral data in a vector way ?

If yes : ***How can one order vectors ?***

If no : ***How shall one proceed ?***

Multivariate mathematical morphology

- ***Marginal*** approach:
  Hyperspectral image = ***set of grey level images*** that are processed ***separately***



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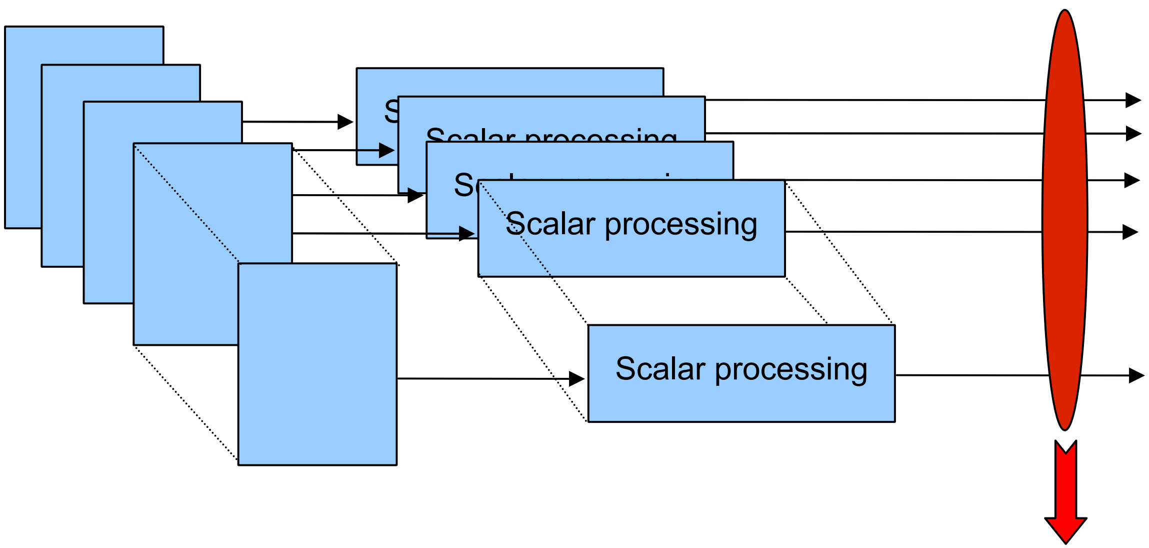

New vectors appear
(loss of inter-band correlation)

Multivariate mathematical morphology

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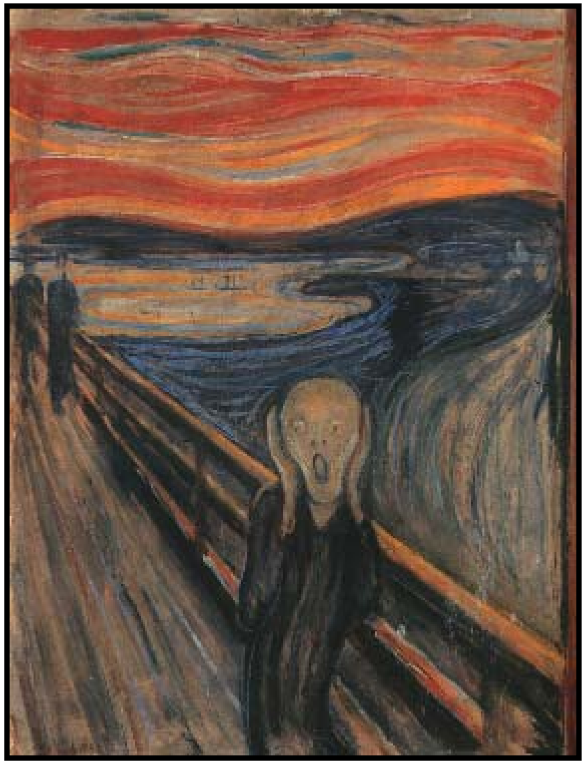

Original image

Marginal opening

Multivariate mathematical morphology

- ***Marginal*** approach:
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Original image

Marginal opening

Multivariate mathematical morphology

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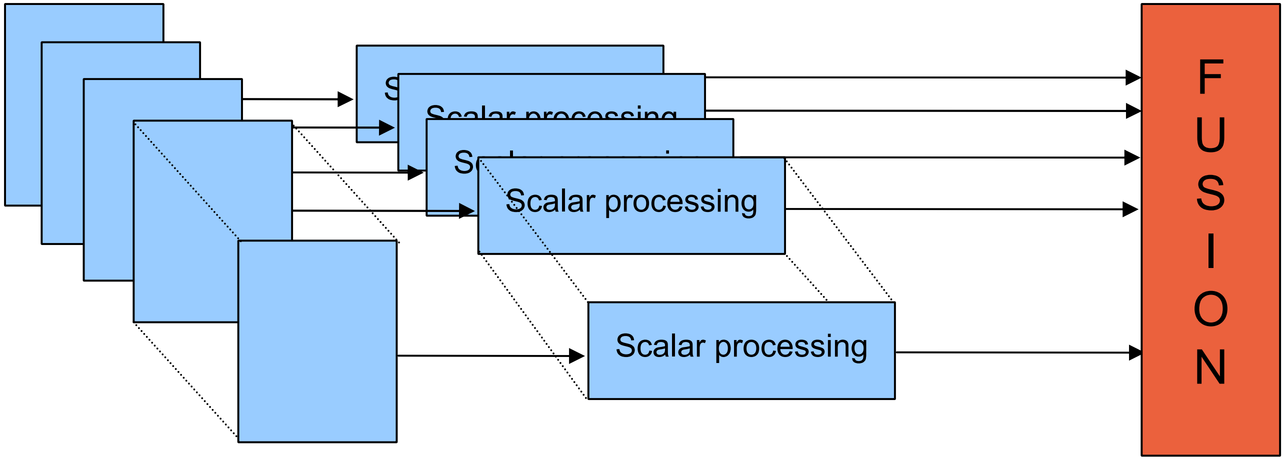
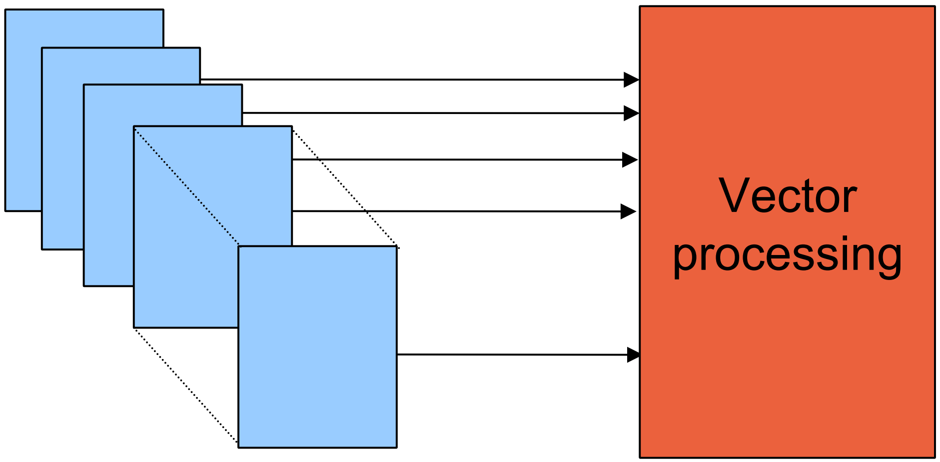


Multivariate mathematical morphology

- ***Vector*** approach:
  hyperspectral image = one ***single image***, every pixel is one (very long) vector

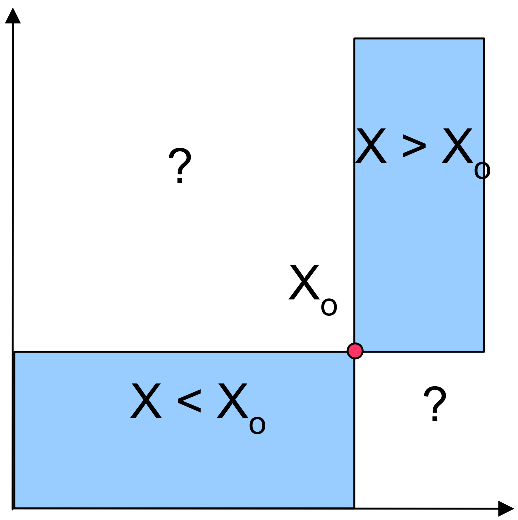


Multivariate mathematical morphology

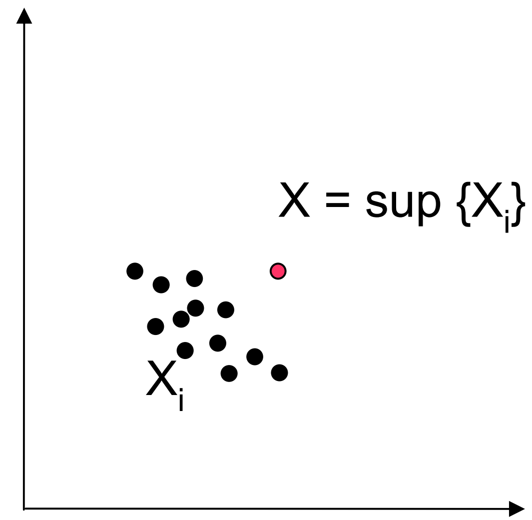
- ***Vector*** approach

  define an order on vectors?

  ➢ Canonical order $(X \leq Y) \Leftrightarrow (X(i) \leq Y(i)\ , \forall i)$



Partial order



False “colors”

Multivariate mathematical morphology

- ***Vector*** approach

define an order on vectors?

- Canonical order $(X \le Y) \Leftrightarrow (X(i) \le Y(i),\ \forall i)$
- General formalism :

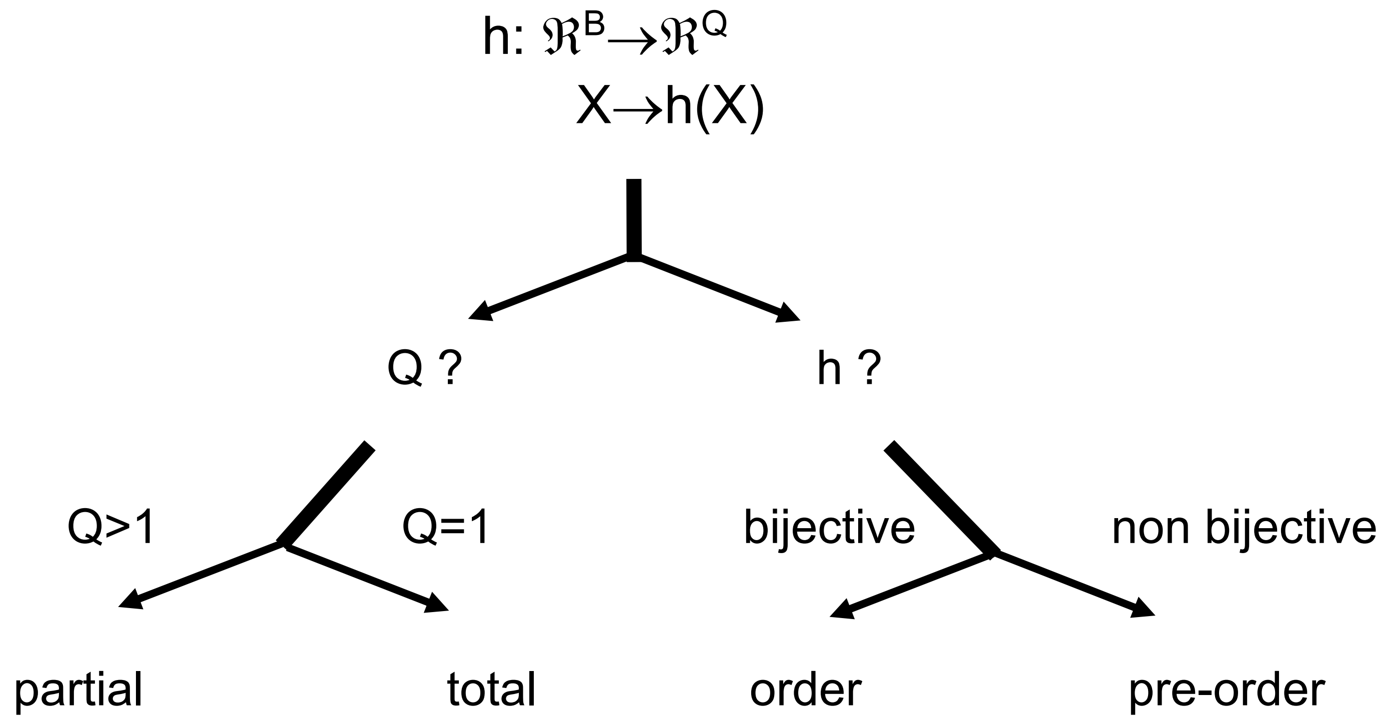
Change space for ordering

transform *h* such that:

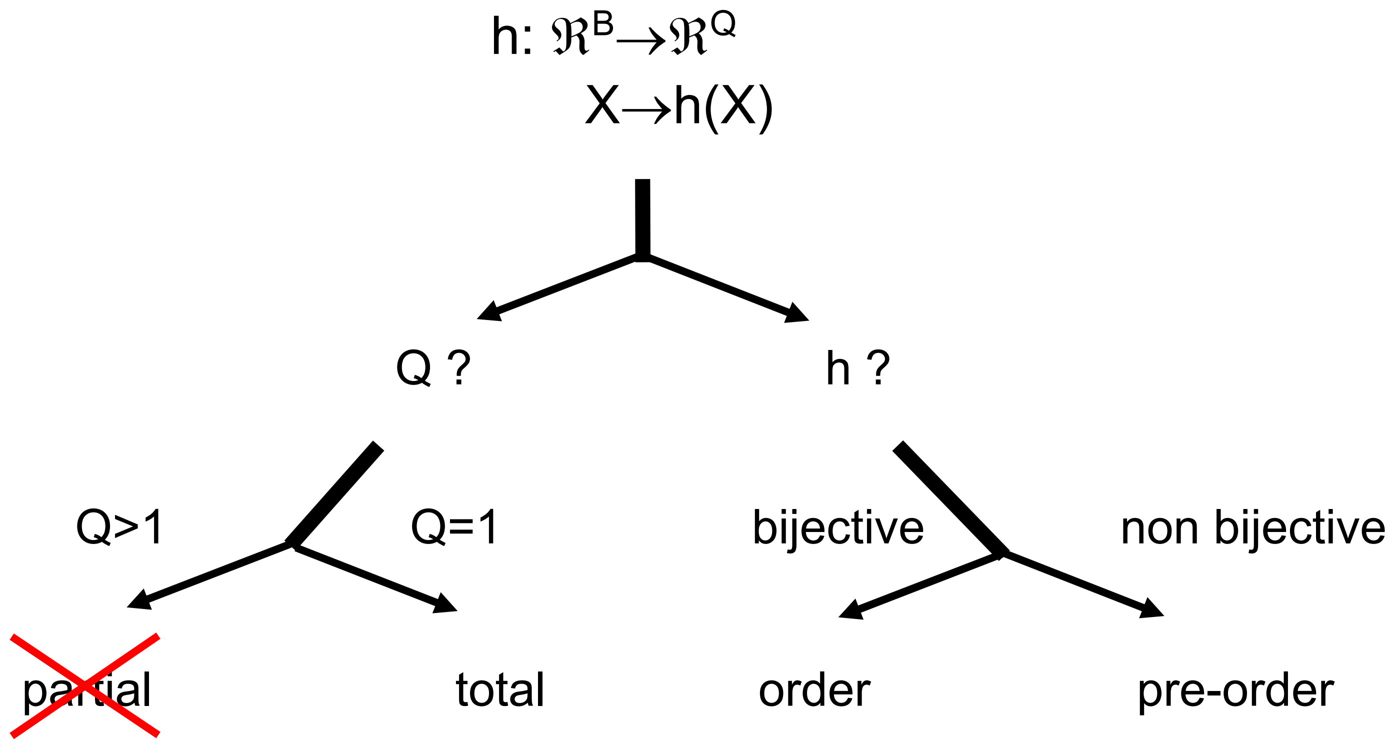
$$h: \Re^B \to \Re^Q$$
$$X \to h(X)$$

$$(X \le Y) \Leftrightarrow (h(X) \le h(Y))$$

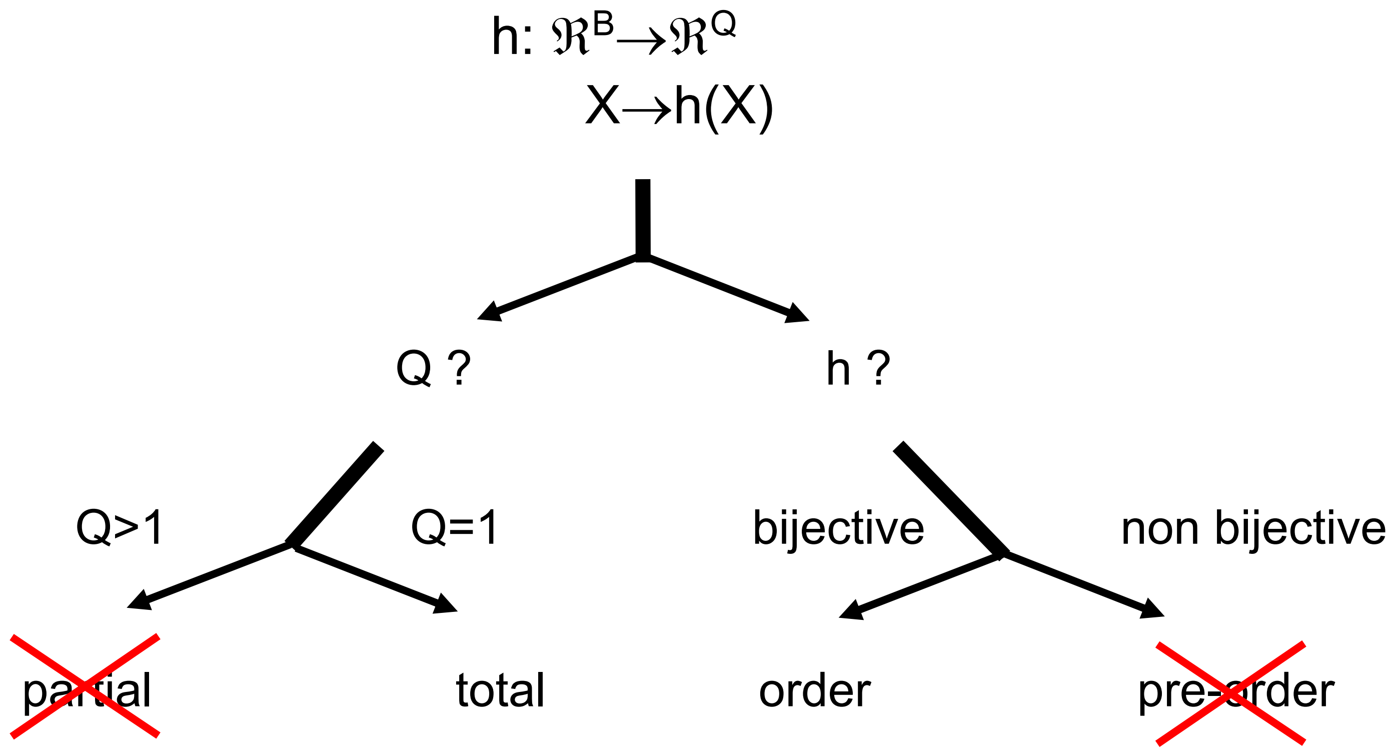
Multivariate mathematical morphology



Multivariate mathematical morphology



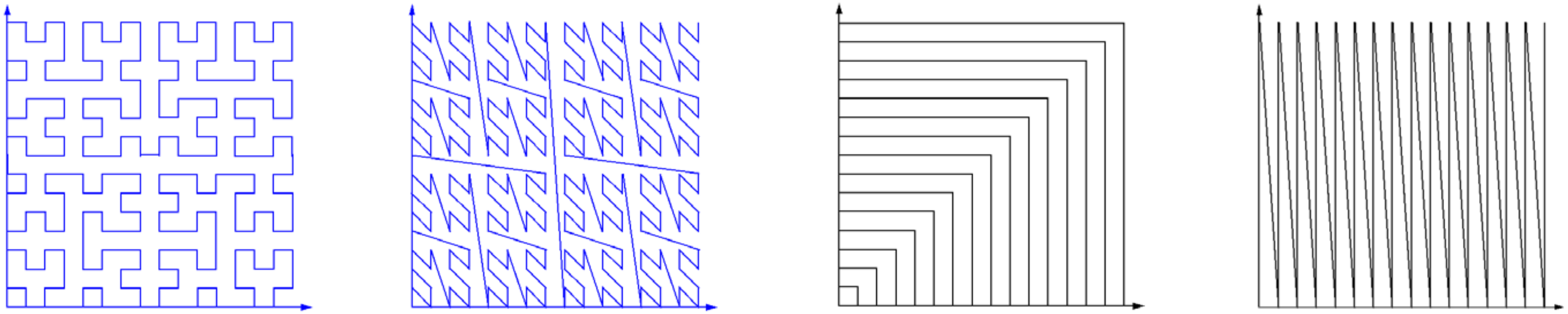
Multivariate mathematical morphology



Multivariate mathematical morphology

- Total ordering relation ⇔ bijective function *h*, *Q=1*

  ⇔ space filling curve

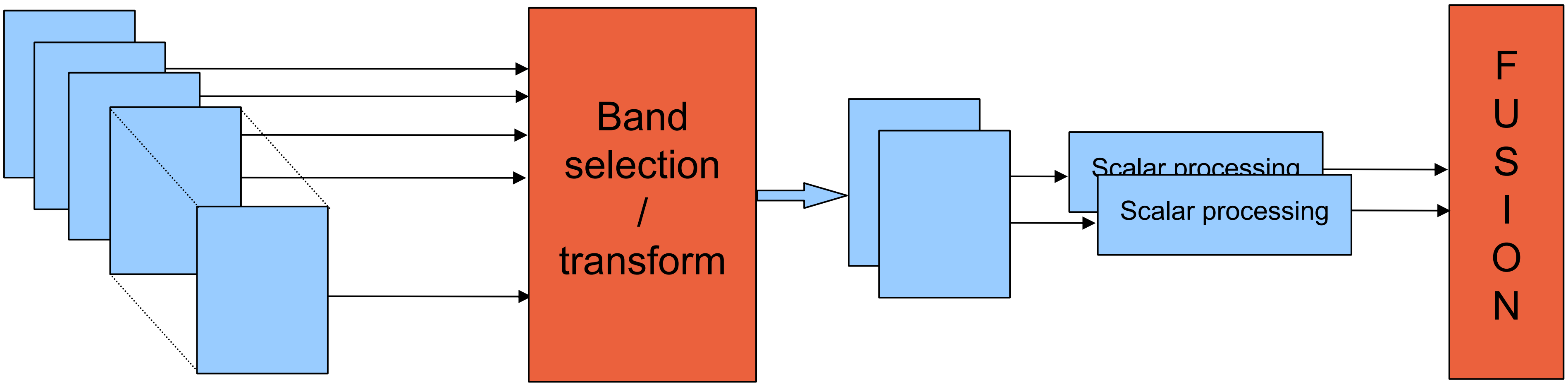


- Problem: such a mapping *h* cannot be linear

  distortion of the space topology

Multivariate mathematical morphology

- ***Mixed*** approach



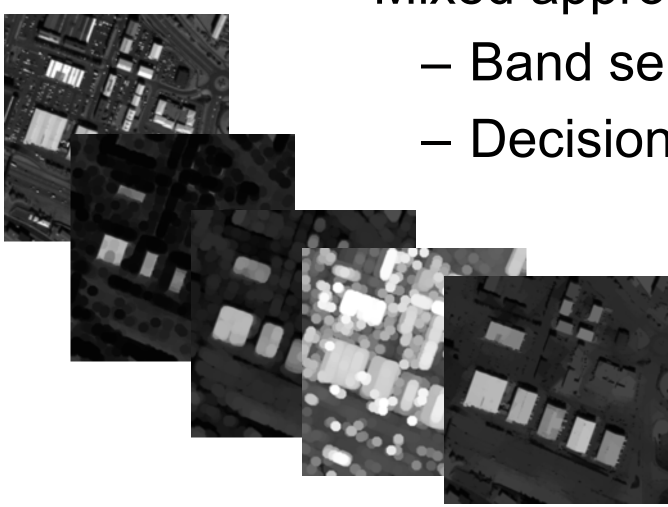
- Band selection/transform: Principal Component Analysis (PCA)
  Independent Component Analysis (ICA)
  Decision Boundary Feature Extraction (DBFE)

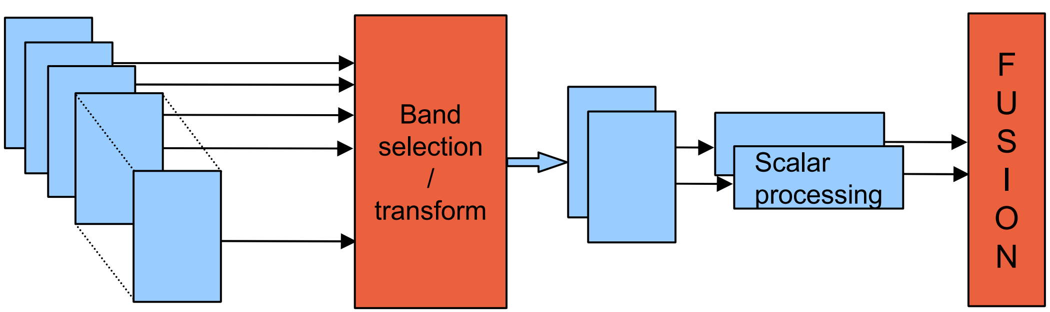
- Fusion: Concatenate marginal results
  Decision fusion

Summary: MM for grey-scale and hyperspectral images

- MM operators can be directly extended to grey-scale images using max (sup) and min (inf) operators

- MM requires a complete lattice structure: total ordering is required
  - ➢ How shall one extend MM to hyperspectral images?
    - Marginal approach: each band is processed separately
    - Vector approach: each pixel is one vector
    - Mixed approach
      - – Band selection/transform
      - – Decision fusion



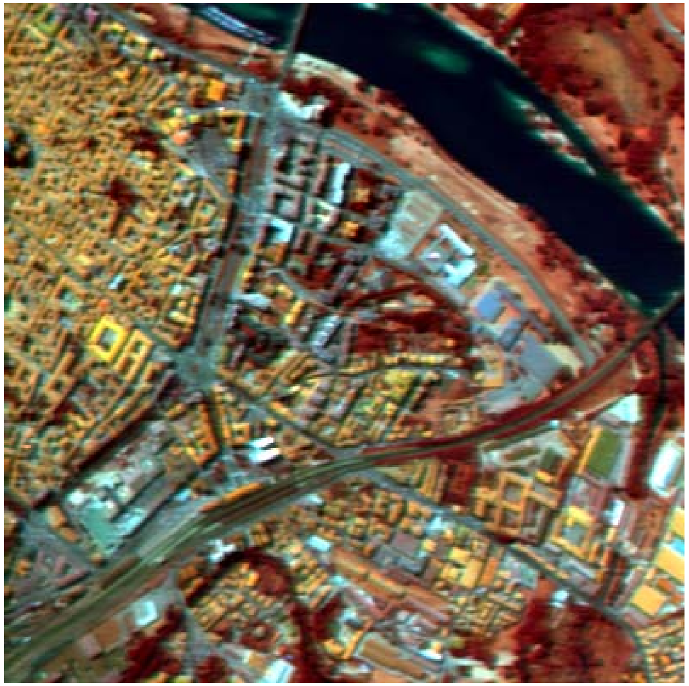


Remote sensing application 1:

Classification of hyperspectral images of an urban area using morphological profiles

Pavia data

- DAIS7915 airborne imaging spectrometer from DLR
- Pavia, ITALY
  - Date: 08/07/2002
- Spatial resolution 2.6 m
- 80 channels
  - 73 channels, 0.496 – 4.315 μm
    - Only use channels 1-40 (0.496 – 1.756 μm) due to noise in higher bands
  - 7 thermal infrared bands

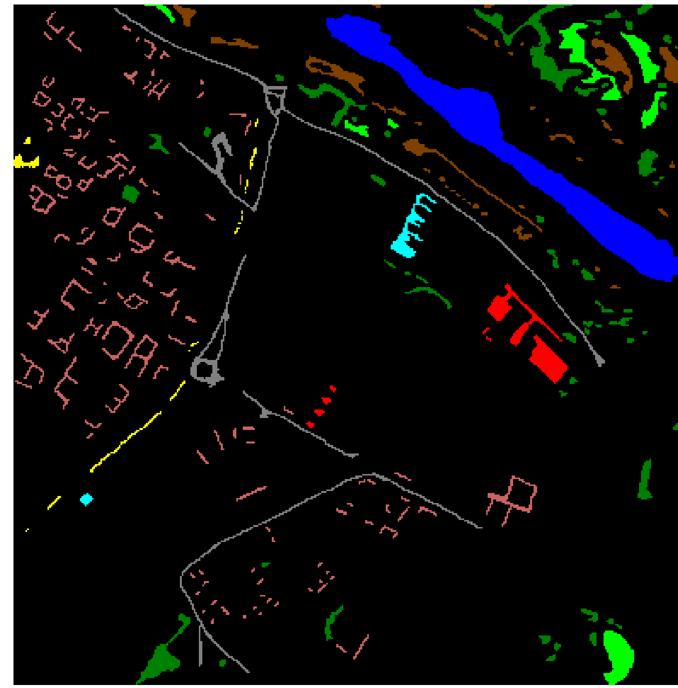


Channels 35(r), 8(g), and 1(b)
(400 x 400 pixels)

Classification problem

- ***Task:*** Assign ***every*** pixel to ***one*** of the ***nine*** information classes:

Shadows
Roofs
Parking lots
Asphalt
Trees
Meadow
Soil
Bitumen
Water

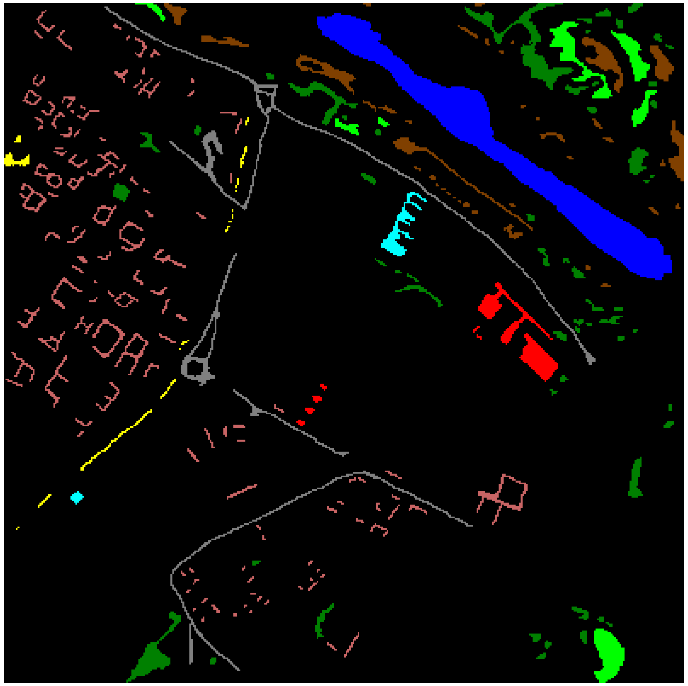


Ground truth reference

Classification problem

- ***Task:*** Assign ***every*** pixel to ***one*** of the ***nine*** information classes:

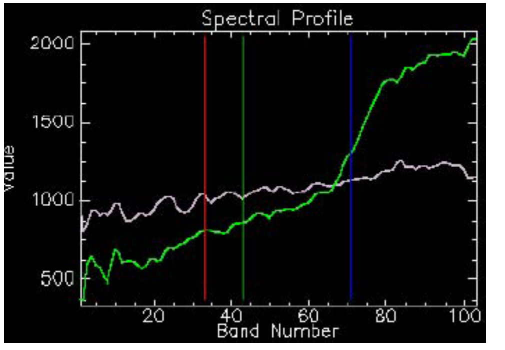
	Training samples	Test samples
Shadows	61	180
Roofs	500	1682
Parking lots	74	213
Asphalt	429	1275
Trees	500	1817
Meadow	311	940
Soil	366	1109
Bitumen	165	520
Water	500	3214



Ground truth reference

Classification approaches

- ***Only spectral*** information
  - ➢ Spectrum of each pixel is analysed
  - ➢ Directly accessible
  - ➢ Classification methods*:
    - Gaussian maximum likelihood
    - Neural networks
    - Kernel-based methods (e.g. SVM) → good performances



- ***Spectral + spatial*** information
  - ➢ Info about spatial structures included
  - ➢ How to define spatial structures?

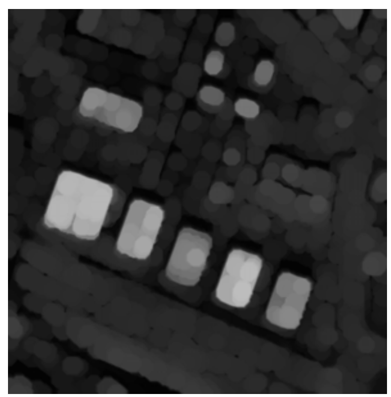
Mathematical morphology deals with the analysis of spatial structures

****To read: R. O. Duda et al., Pattern Classification, 2nd ed. Wiley, 2001.***

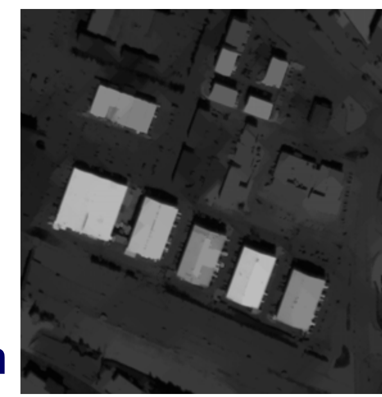
Morphological opening by reconstruction

- Consequence:
  - Features that are ***brighter*** than their immediate surroundings and ***smaller*** than the SE ***disappear***
  - Other features (dark, or bright and large) remain "unchanged"

$\gamma_{15}(X)$ ➡



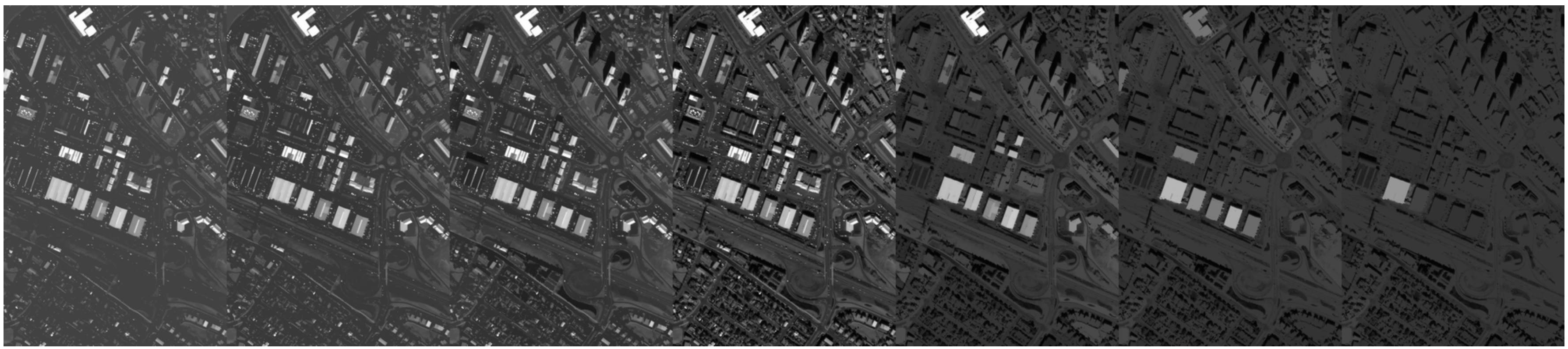
opening ➡ **by reconstruction**



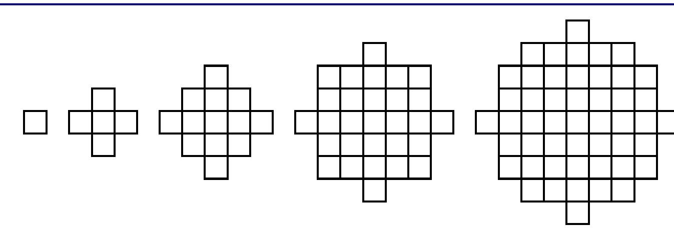
Morphological profile

Closings (increasing SE)

Openings (increasing SE)



- Structuring Element: disc
- Segmentation Variables
  - Number of openings/closings
  - Radius increment (step size)



Feature extraction

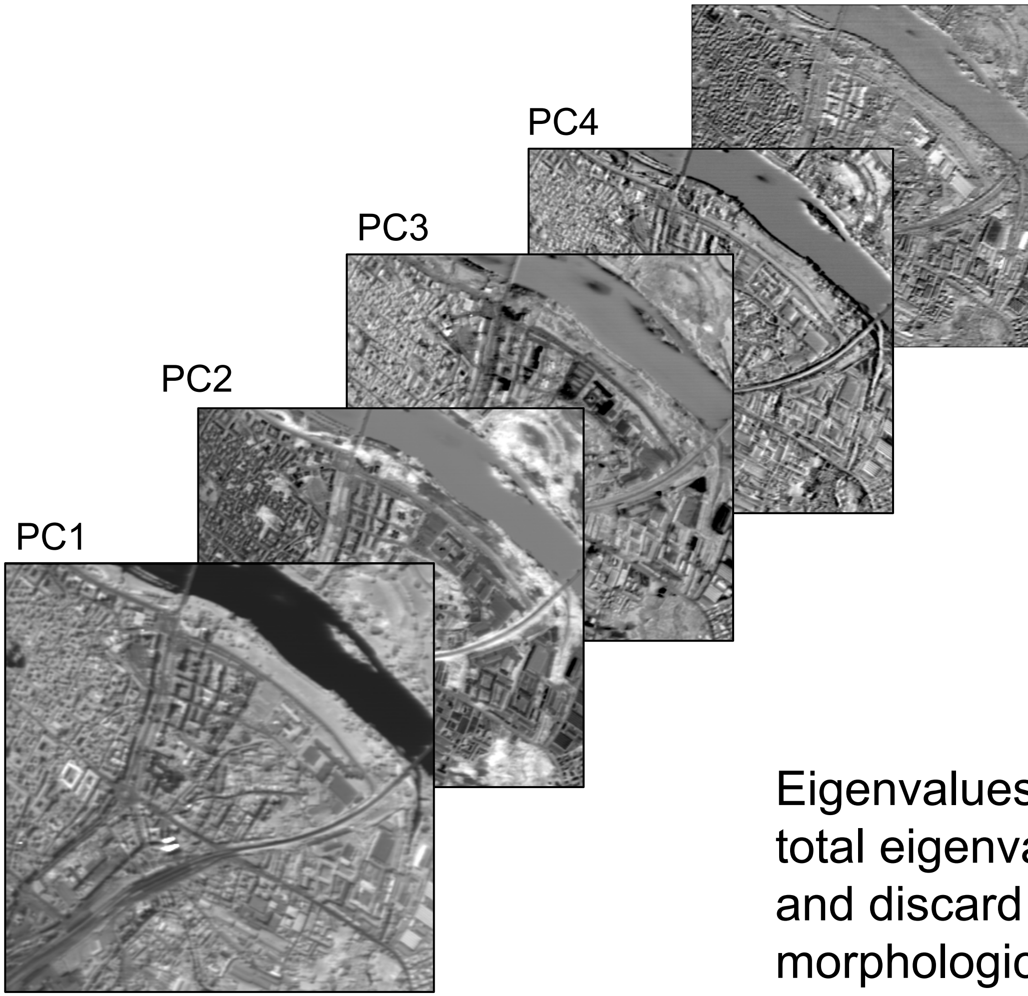
- Method previously applied to high resolution panchromatic data (grey-scale images)

- Here we consider the use of the method for high resolution hyperspectral data

- ***Need to create a morphological profile from the hyperspectral data***

- Use feature extraction: Principal Component Analysis (optimum for signal representation)

Pavia data: principal components

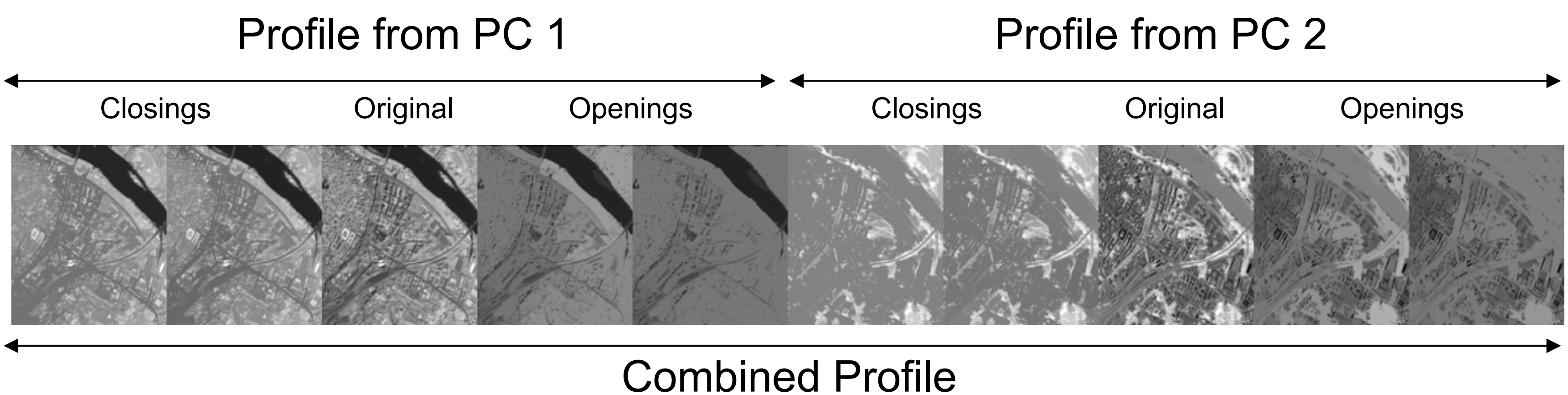


	Value	$\lambda_i / \sum\lambda$
λ_1	$2.27 \cdot 10^5$	78,2%
λ_2	$5.22 \cdot 10^4$	18.0%
λ_3	$8.39 \cdot 10^3$	2.9%
λ_4	$1.16 \cdot 10^3$	0.4%
$\lambda_5 + ... + \lambda_{40}$		<0.5%

Eigenvalues λ_1 and λ_2 make up >96% of the total eigenvalue sum. We keep PC1 and PC2 and discard other components for morphological processing.

Combined morphological profile

- 2nd principal component contains too much information to be discarded
- Morphological profiles for 1st and 2nd principal components combined in one profile



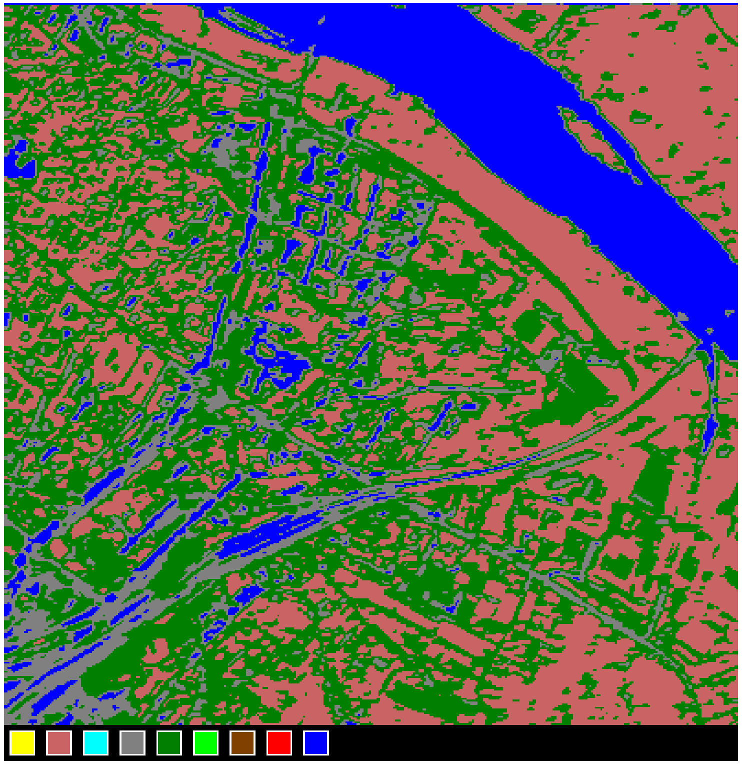
Classification of Pavia data

- Morphology settings
  - The use of 1st and 2nd Principal Components with four openings/closings and step size of 2 resulted in good classification accuracies

- Neural network (NN) classifier
  - Classification performed using a neural network with one hidden layer
  - Decision Boundary Feature Extraction (DBFE) for the NN was tested on the morphological profile

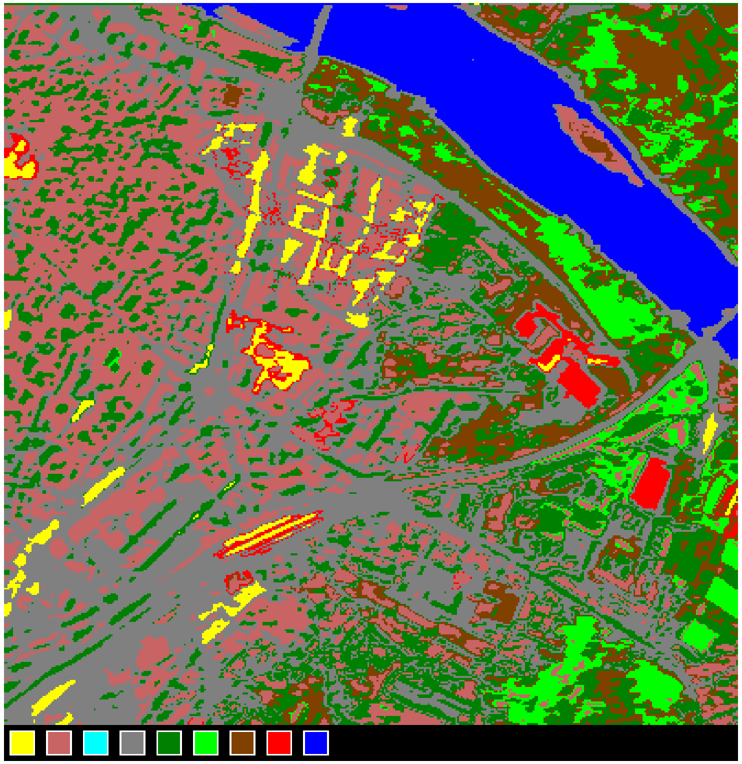
Pavia data: classification results

- Only the 1st principal component (1 input feature) is used for NN
- 3 hidden neurons
- Overall test accuracy: ***56.2%***



Pavia data: classification results

- Morphological profile of the 1st PC (9 input features)
  - 4 openings/closings, radius increment: 2
- 9 hidden neurons
- Overall test accuracy: ***77.0%***

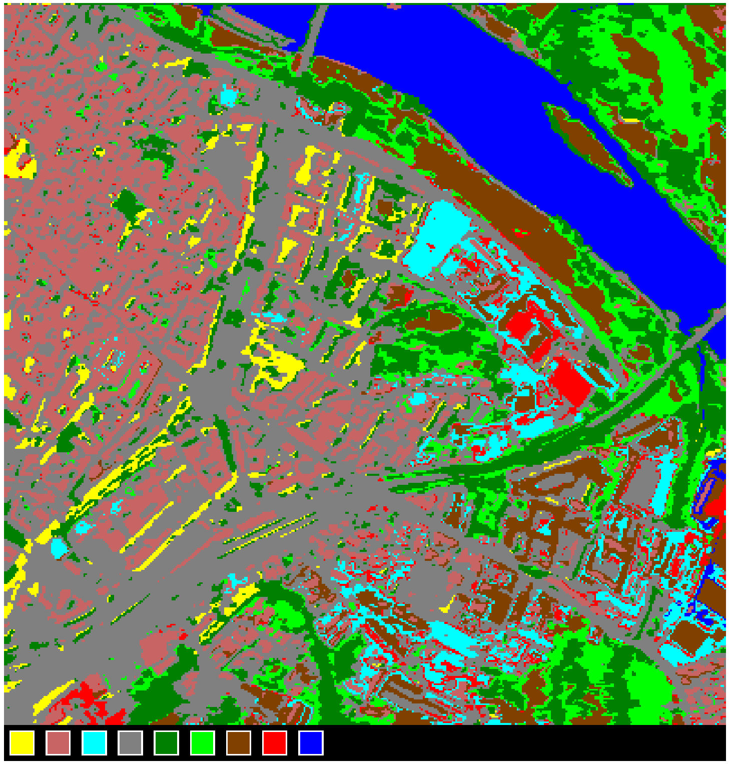


Pavia data: classification results

- Morphological profile of the 1st and 2nd PCs (18 input features)
  - PC1: 4 openings/closings, radius increment: 2
  - PC2: 4 openings/closings, radius increment: 2
- 13 hidden neurons
- Overall test accuracy: ***91.5%***

Pavia data: classification results

- Morphological profile of the 1st and 2nd PCs (18 input features)
  - PC1: 4 openings/closings, radius increment: 2
  - PC2: 4 openings/closings, radius increment: 2
- DBFE
  - Reduced to 8 features (99%)
- Overall test accuracy: ***95.0%***



Pavia data: overall test accuracies for NN (%)

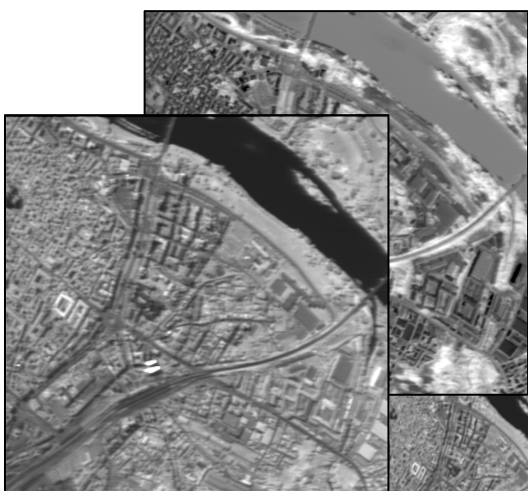
Index		PC1	PC1 Morph. profile	PC1+PC2 Morph. profiles	PC1+PC2 Morph. profiles with DBFE
1	Shadows	0.0	73.3	40.0	**83.9**
2	Roofs	89.1	91.6	84.8	**89.8**
3	Parking lots	0.0	0.0	87.3	**97.2**
4	Asphalt	39.6	88.5	95.5	**96.4**
5	Trees	51.3	46.2	94.1	**94.2**
6	Meadow	0.0	53.5	**94.3**	94.1
7	Soil	0.0	57.4	79.0	**95.3**
8	Bitumen	0.0	83.7	82.5	**84.4**
9	Water	**100.0**	**100.0**	**100.0**	**100.0**
	Overall Acc.	56.2	77.0	91.5	**95.0**

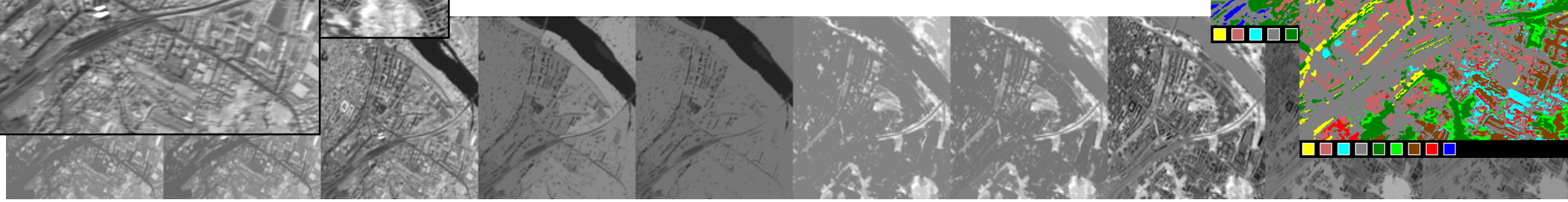
Summary: classification using MM profiles

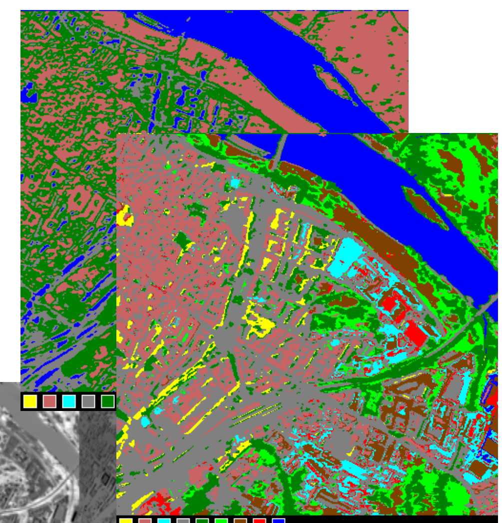
- Spatial information captured by morphological profiles
  ➔ Great potential for classification of images of urban areas

- The use of morphological profiles for hyperspectral data
  ➔ Use feature extraction: Principal Component Analysis

- Spectral-spatial approaches for image analysis to be developed







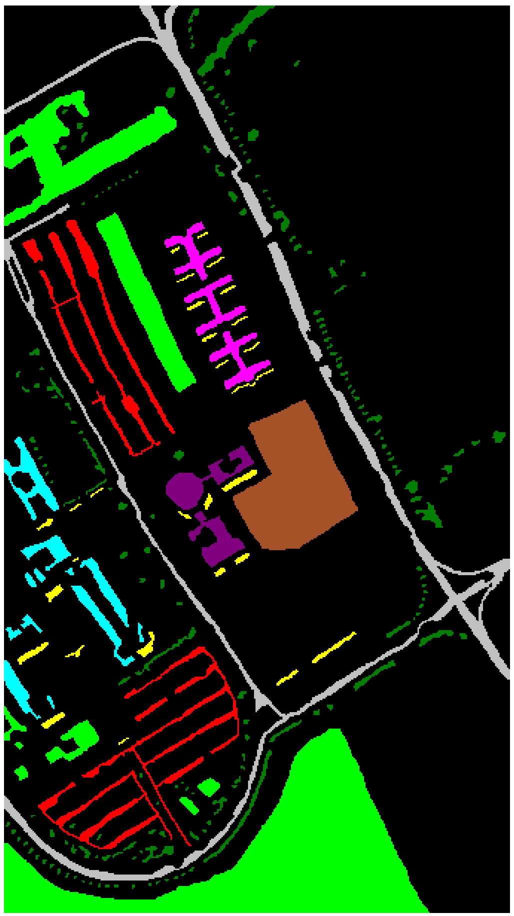
Remote sensing application 2:

Segmentation and classification of hyperspectral images using watershed

Classification problem

Input ROSIS image
610 x 340 pixels,
103 bands

Ground truth
reference



Task

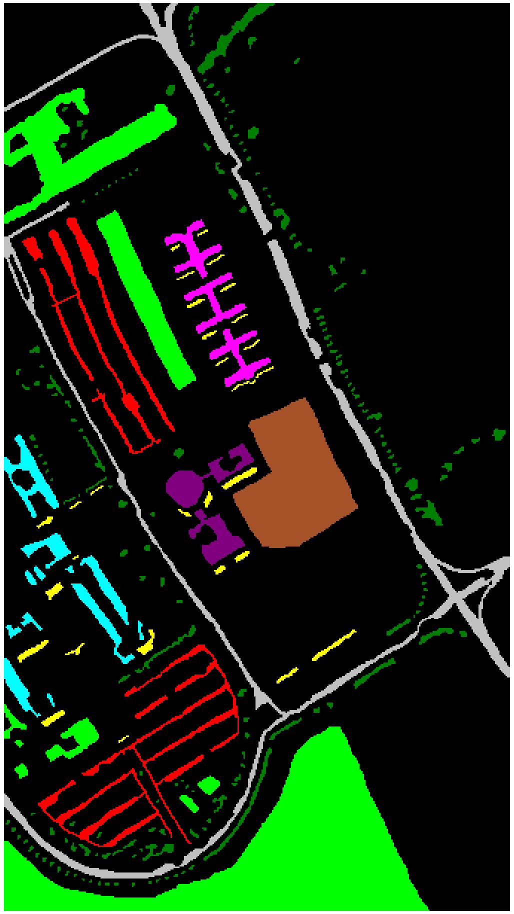
Assign ***every*** pixel to ***one*** of the ***nine*** information classes:

alphalt
meadows
gravel
trees
metal sheets
bare soil
bitumen
bricks
shadows

Classification problem (103-band ROSIS data)

Class	Training samples	Test samples
Asphalt	548	6641
Meadows	540	18649
Gravel	392	2099
Trees	524	3064
Metal sheets	265	1345
Bare soil	532	5029
Bitumen	375	1330
Bricks	514	3682
Shadows	231	947

Ground truth reference



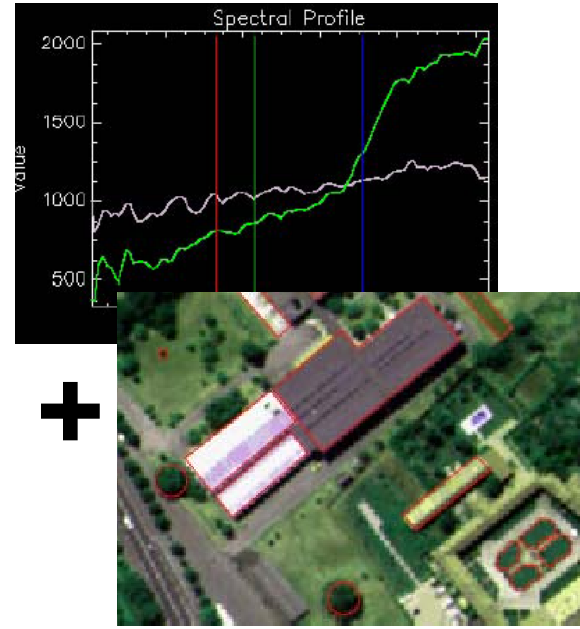
Task

Assign ***every*** pixel to ***one*** of the ***nine*** information classes:

alphalt
meadows
gravel
trees
metal sheets
bare soil
bitumen
bricks
shadows

Spectral-spatial classification

- Spectral + spatial information for more accurate classification
- How to define spatial structures?
  - ➢ Closest neighborhood (e.g. morphological profiles) → done before
  - ➢ Adaptive neighborhood (segmentation map) → ***currently investigated***



Spectral-spatial classification

- Spectral + spatial information for more accurate classification

- How to define spatial structures?
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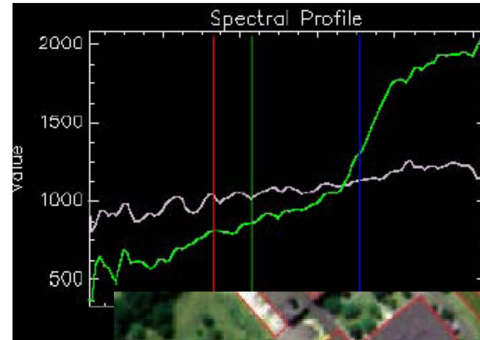


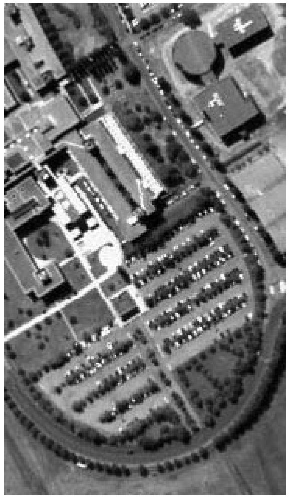
+

Objective:

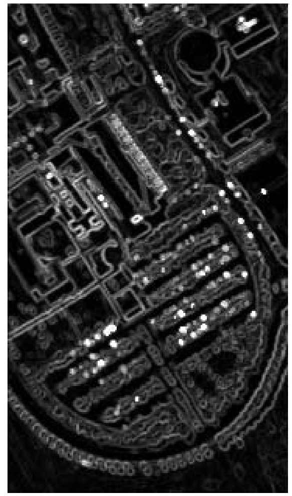
- ***Segment*** a hyperspectral image = find an exhaustive partitioning of the image into homogeneous regions
  - ➢ Use MM approach to segmentation: ***watershed transformation***

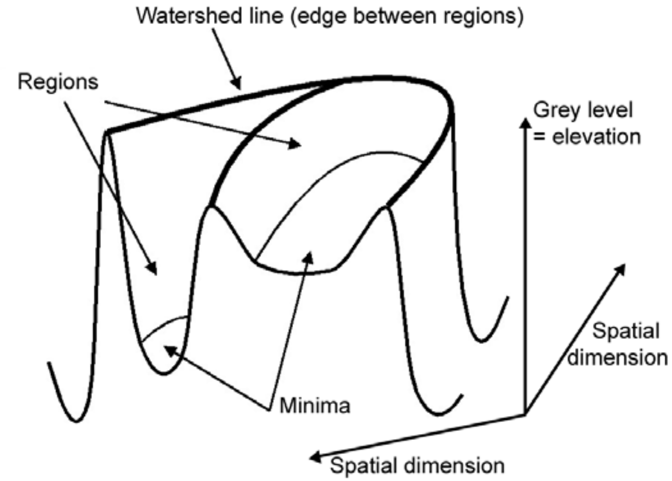
- ***Spectral*** info + ***spatial*** info → classify image

Watershed segmentation



gradient



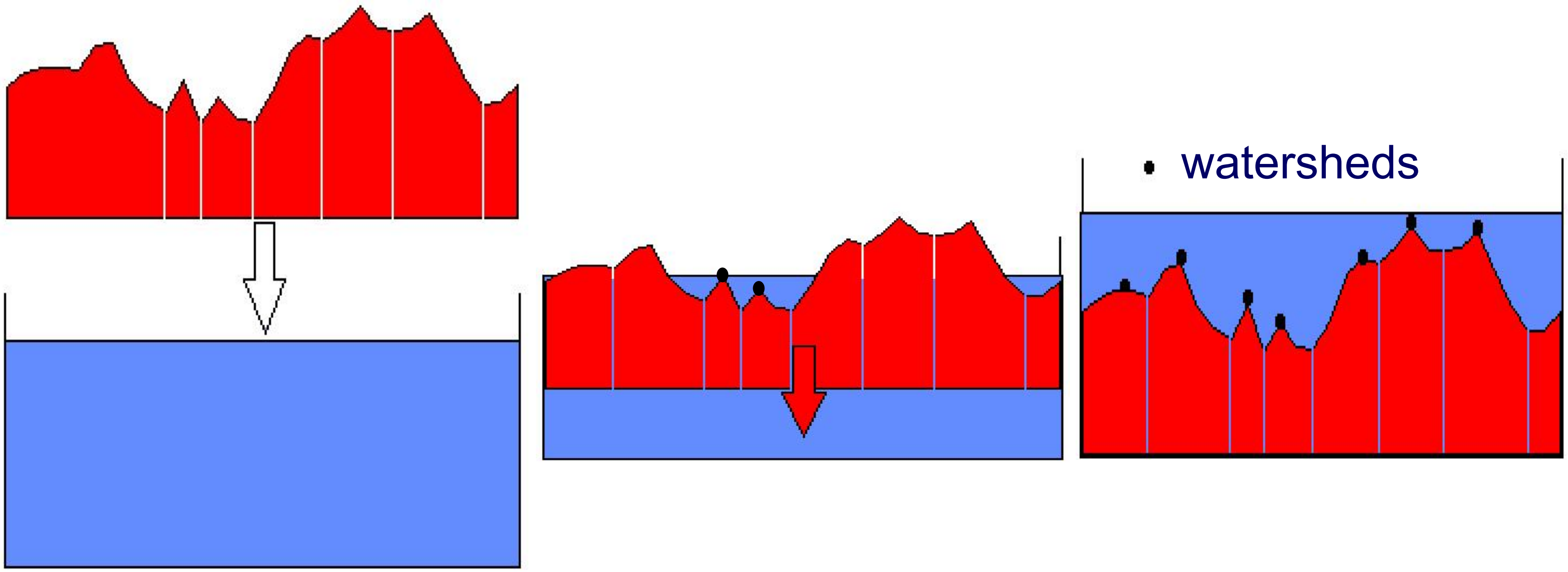


Region growing + edge detection method:

- ***Minimum*** of a gradient = core of a homogeneous region
- ***1 region*** = set of pixels connected to 1 local minimum of the gradient
- ***Watershed lines*** = edges between adjacent regions

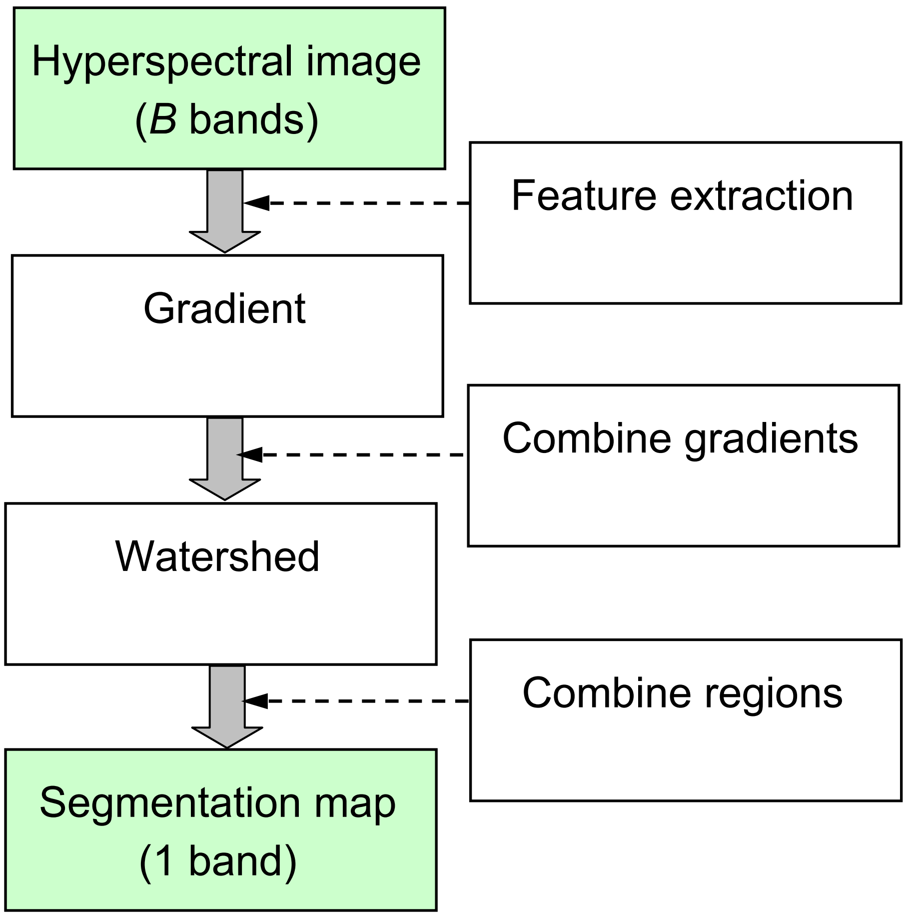
Watershed algorithm

- L. Vincent and P. Soille, “Watersheds in digital spaces: an ***efficient*** algorithm ***based on immersion simulations***,” *IEEE Trans. Pattern Analysis and Machine Intel*., vol. 13, no. 6, pp. 583–598, June 1991.



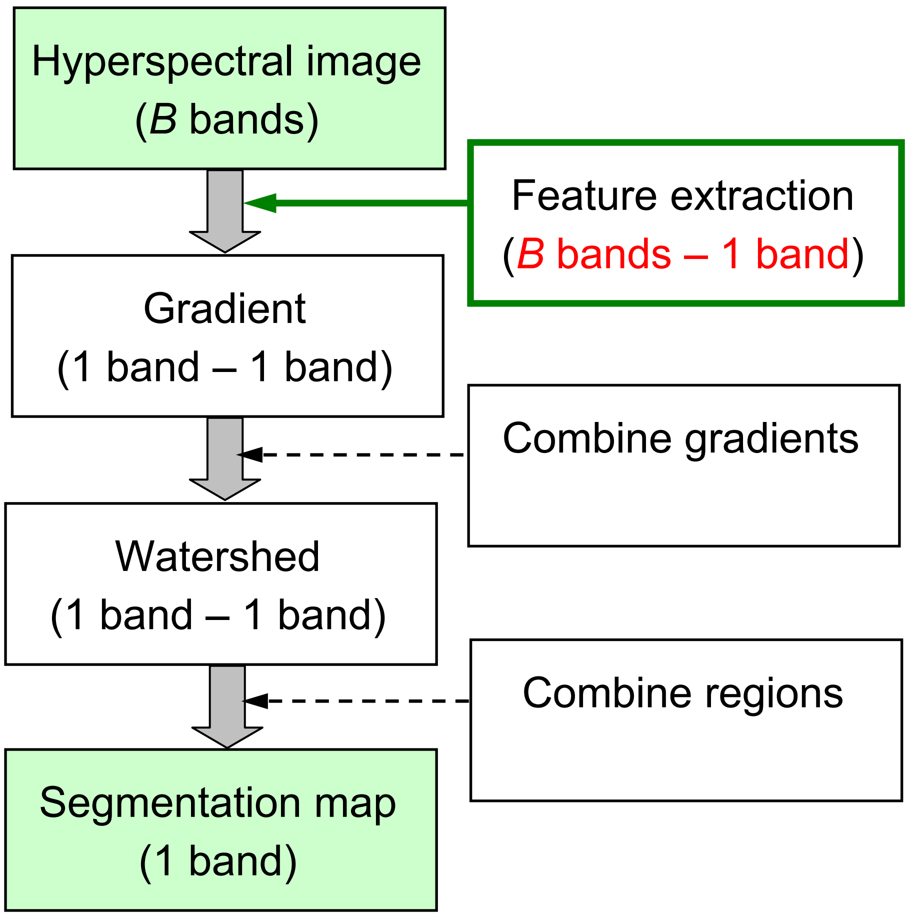
Watershed for a hyperspectral image

- From *B*-band image → 1-band segmentation map:



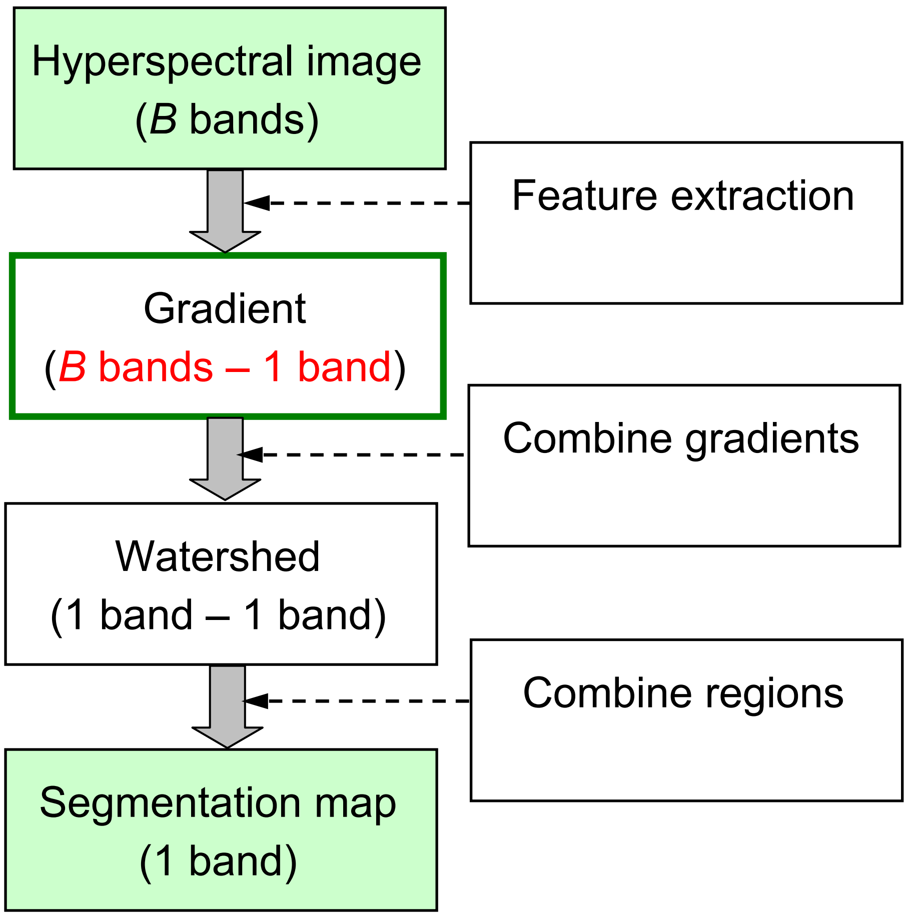
Watershed for a hyperspectral image

- From *B*-band image → 1-band segmentation map:
  - ➢ Feature extraction (PCA, ICA, …)?



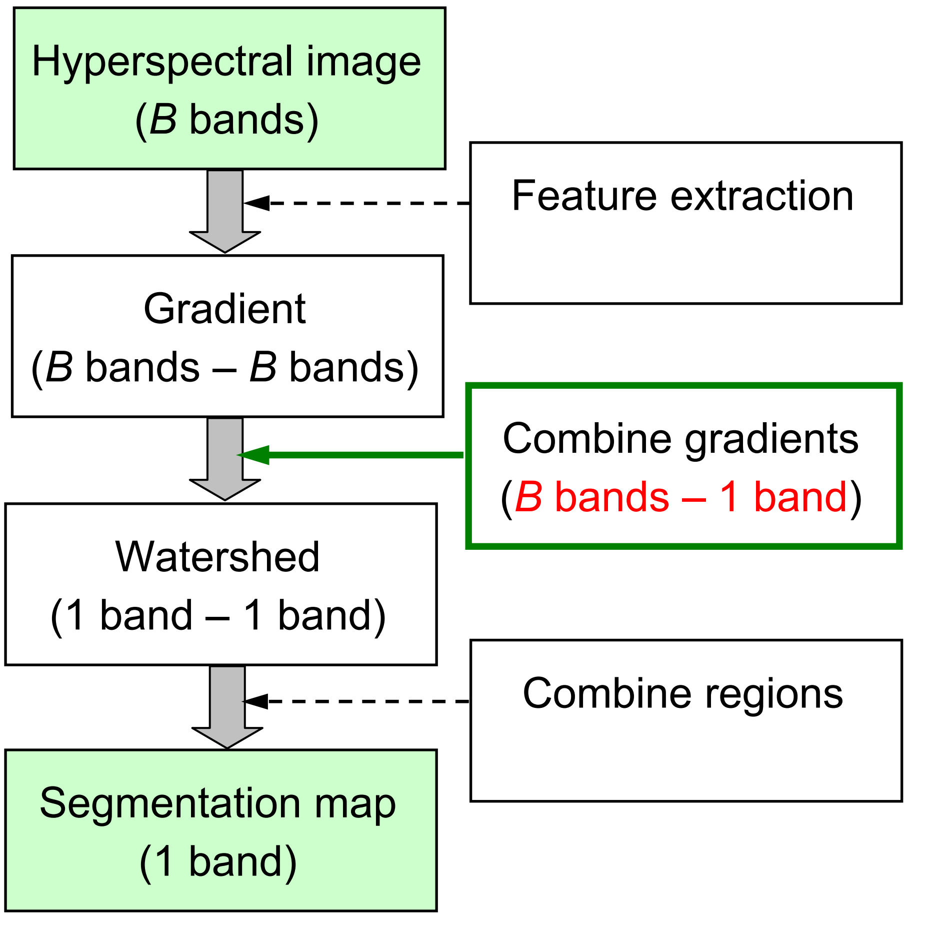
Watershed for a hyperspectral image

- From *B*-band image → 1-band segmentation map:
  - Feature extraction (PCA, ICA, …)?
  - Vectorial gradient?



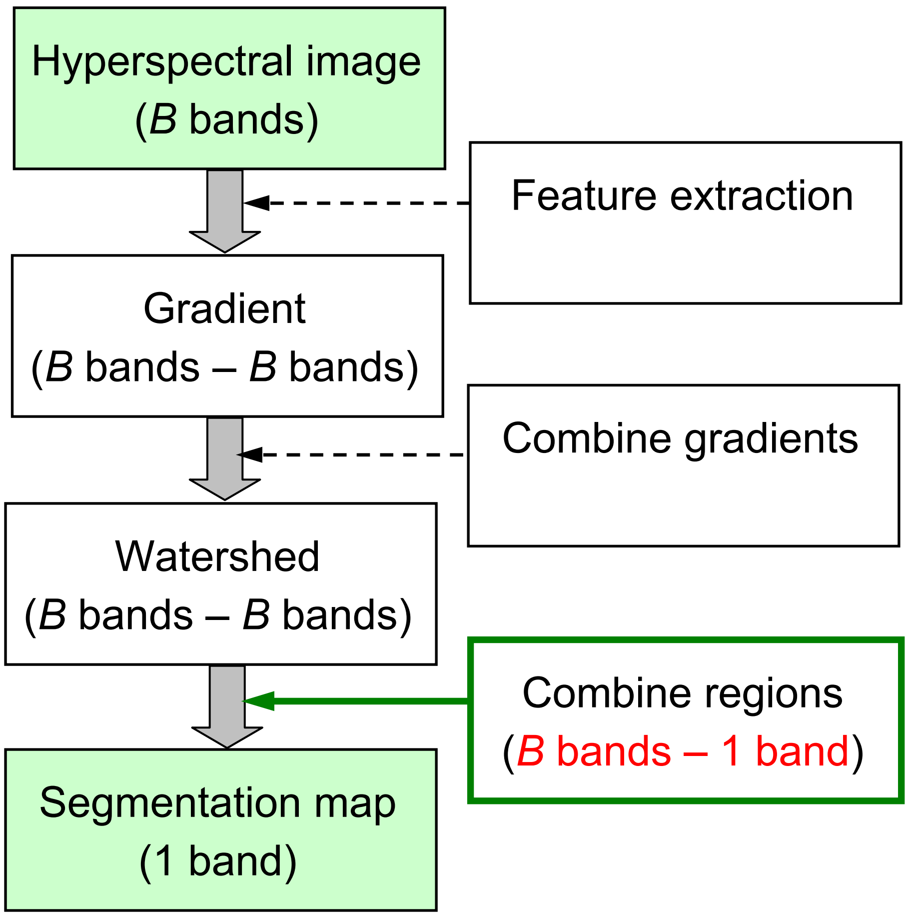
Watershed for a hyperspectral image

- From *B*-band image → 1-band segmentation map:
  - Feature extraction (PCA, ICA, …)?
  - Vectorial gradient?
  - Combine *B* gradients?



Watershed for a hyperspectral image

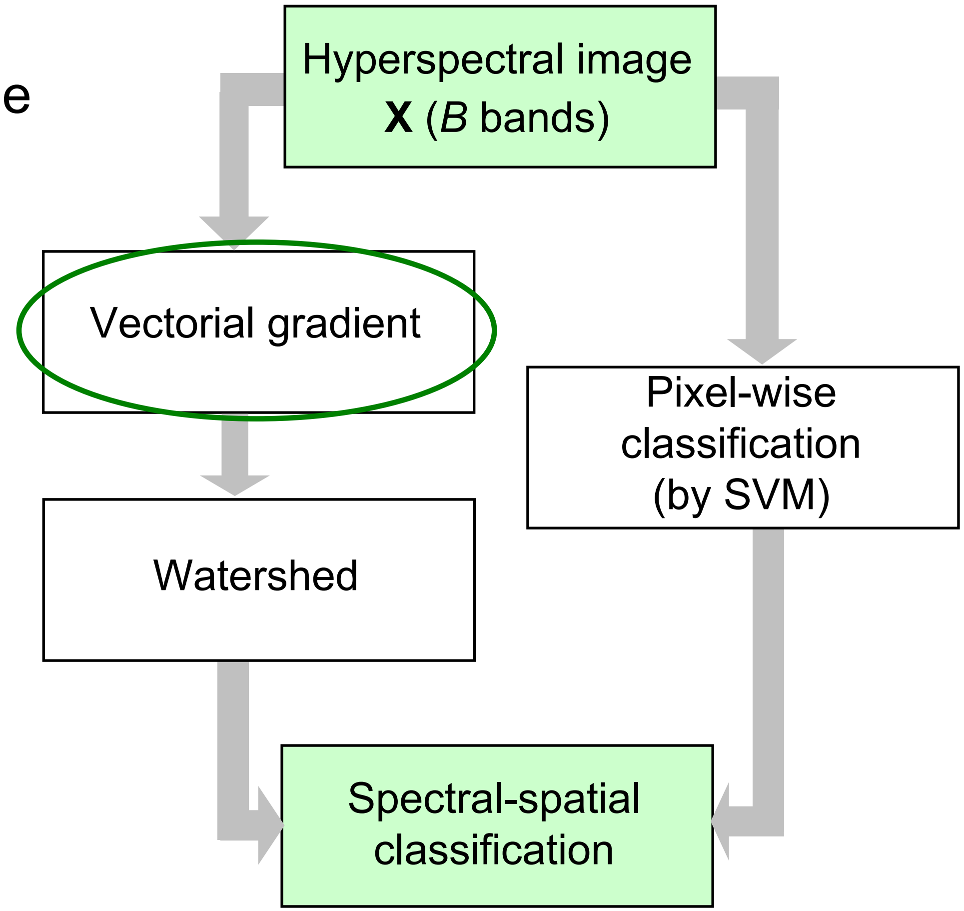
- From *B*-band image → 1-band segmentation map:
  - ➢ Feature extraction (PCA, ICA, …)?
  - ➢ Vectorial gradient?
  - ➢ Combine *B* gradients?
  - ➢ Combine *B* watershed regions?



Segmentation and classification of data

- Input
  - B-band hyperspectral image $\mathbf{X} = \{x_j \in R^B, j = 1, 2, \ldots, n\}$
  - B ~ 100

- Vectorial gradient



Robust Color Morphological Gradient

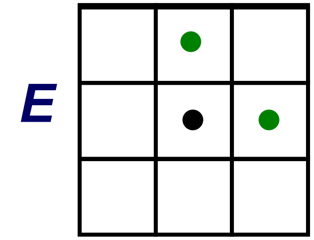
- For each pixel $\mathbf{x}_p$, $\chi = [\mathbf{x}_{p1}, \mathbf{x}_{p2}, \ldots, \mathbf{x}_{pe}]$ is a set of *e* vectors within *E*

- Color Morphological Gradient (CMG):

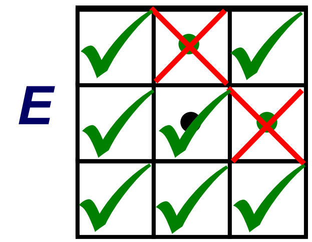
$$CMG_E(\mathbf{x}_p) = \max_{i,\, j \in \chi}\{\| \mathbf{x}_{pi} - \mathbf{x}_{pj} \|_2\}$$

- Robust Color Morphological Gradient (RCMG):
  - $\mathbf{x}_{pi\_max}$, $\mathbf{x}_{pj\_max}$ – pixels that define the CMG of $\mathbf{x}_p$
  - $$RCMG_E(\mathbf{x}_p) = \max_{i,\, j \in \{\chi - [\mathbf{x}_{pi\_max},\, \mathbf{x}p_{j\_max}]\}}\{\| \mathbf{x}_{pi} - \mathbf{x}_{pj} \|_2\}$$

If two pixels marked by green define the CMG:



The RCMG is computed using all the pixels except these two:

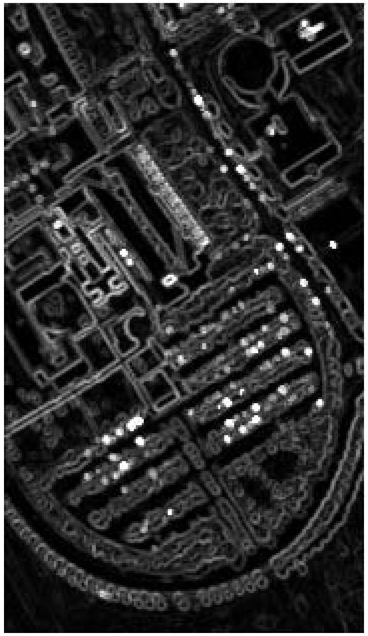


RCMG of the University of Pavia image

- *B*-band image → one-band gradient
- Principal borders are defined
- Presence of “noisy” edges
  - Filter image → “noisy” borders reduced, but info about details lost

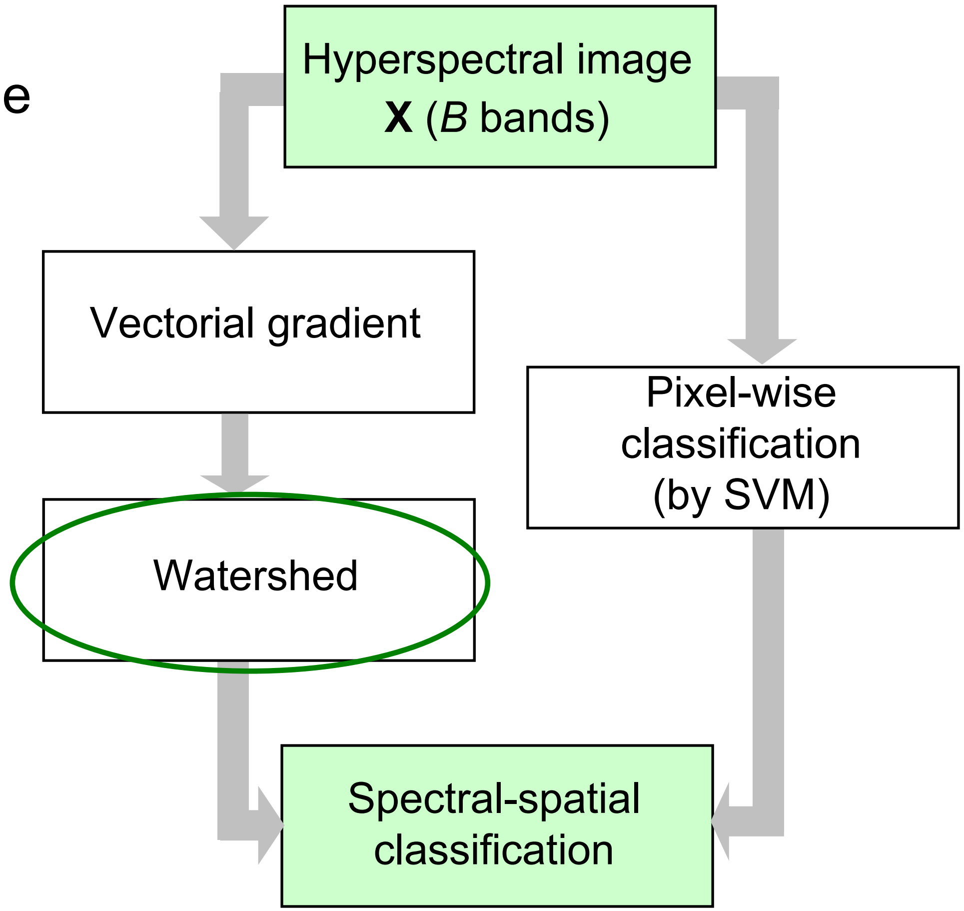
Input image

RCMG



Segmentation and classification of data

- Input
  - *B*-band hyperspectral image $\mathbf{X} = \{x_j \in R^B, j = 1, 2, \ldots, n\}$
  - B ~ 100
- Vectorial gradient
- ***Watershed***

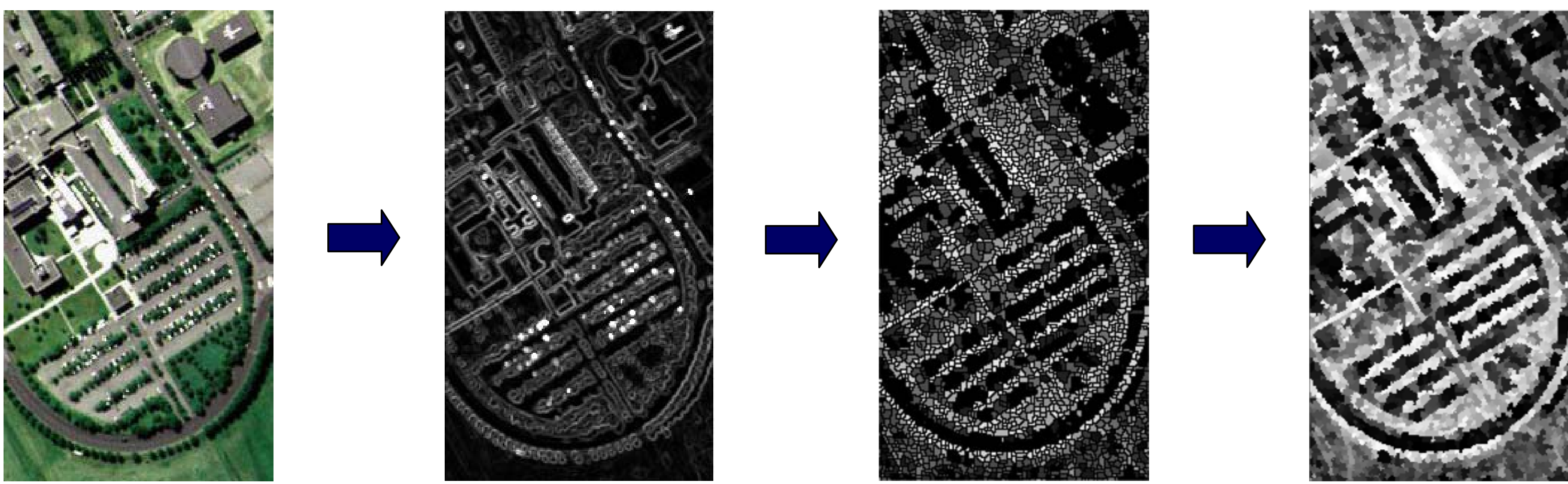


Watershed

- Algorithm of Vincent and Soille (1991)
- For every region **S** Standard Vector Median:

$$\mathbf{s}_{VM} = argmin_{\mathbf{s}\in\mathbf{S}} \left\{ \sum_{j=1}^{m} \|\mathbf{s} - \mathbf{s}_j\|_1 \right\}$$

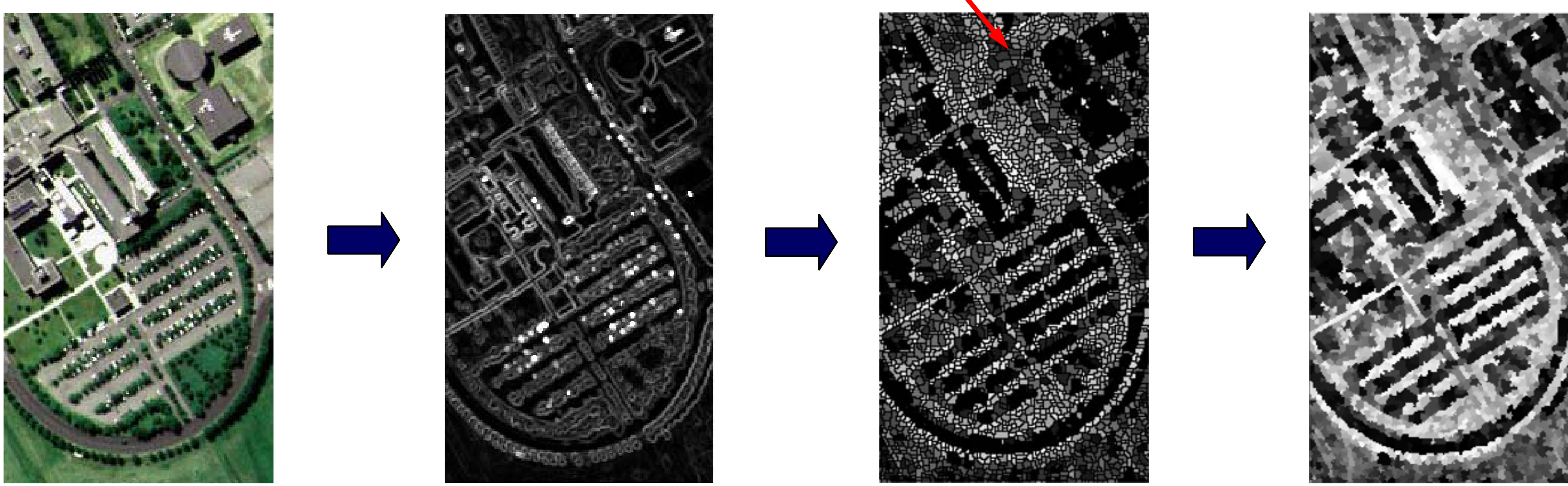
- Every watershed pixel → to the neighboring region with the "closest" median



Watershed

- Oversegmentation
  - Merging of regions

- Obtained regions → to improve classification

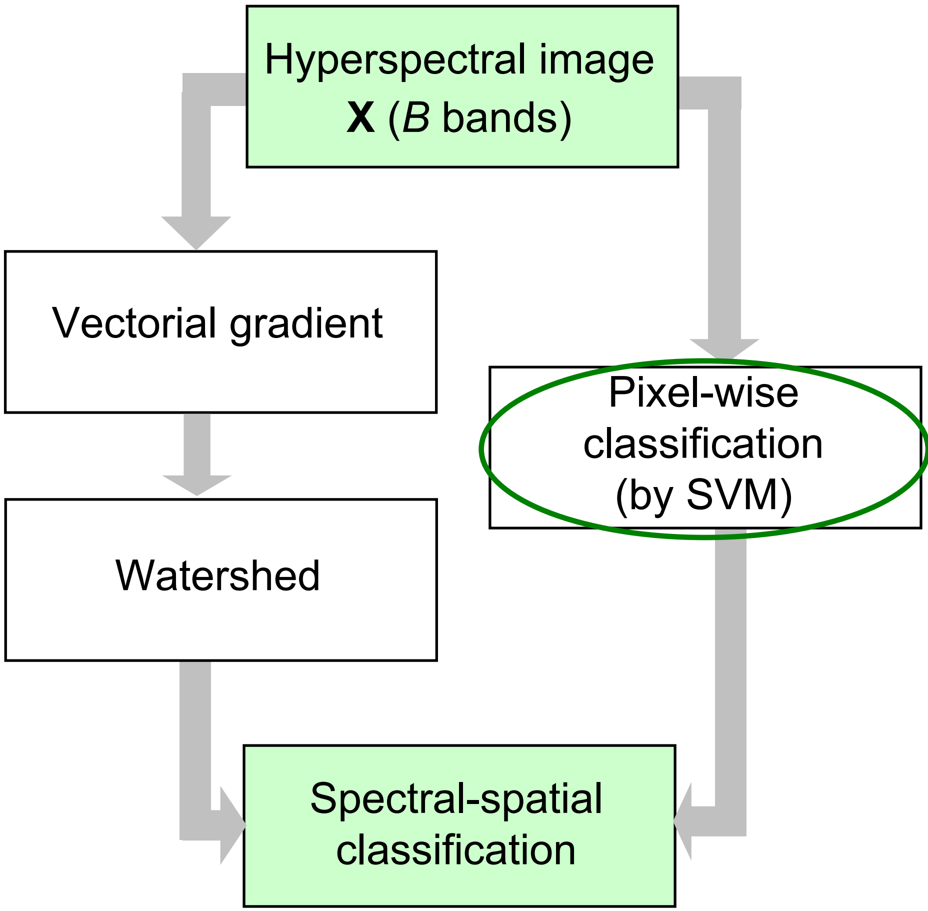


Y. Tarabalka, J. A. Benediktsson, J. Chanussot

Mathematical morphology

Segmentation and classification of data

- Input
  - *B*-band hyperspectral image $\mathbf{X} = \{x_j \in R^B, j = 1, 2, \ldots, n\}$
  - B ~ 100
- Vectorial gradient
- Watershed
- ***Pixel-wise classification (by SVM)***



Pixel-wise SVM classification

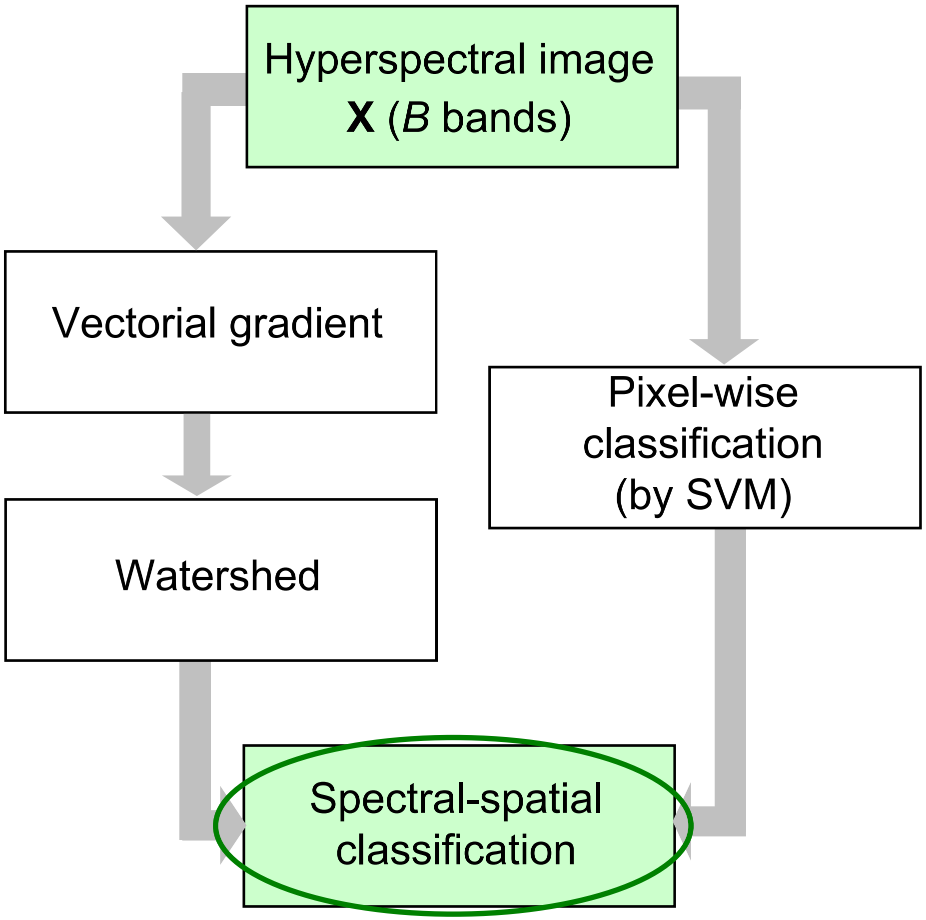
- Milti-class pairwise (one *versus* one) classification, with Gaussian Radial Basis Function was performed

- Optimal parameters were determined by 5-fold cross-validation: C = 128, γ = 0.125

- Overall test accuracy: ***82.1%***

Segmentation and classification of data

- Input
  - *B*-band hyperspectral image
    $\mathbf{X} = \{x_j \in R^B, j = 1, 2, \ldots, n\}$
  - B ~ 100
- Vectorial gradient
- Watershed
- Pixel-wise classification (by SVM)
- ***Spectral-spatial classification***



Spectral-spatial classification

Pixel-wise
classification map

Pixel-wise
classification map
+
segmentation map

Majority vote within the watershed regions

No WHEDs
(watershed pixels not processed)

With WHEDs
(watershed pixels assigned to neighboring regions)

Spectral-spatial classification

Classification accuracies (%):

Accuracy	Pixel-wise SVM	SVM + Majority vote	
		No WHEDs	*With WHEDs*
Overall accuracy	82.08	84.41	**86.64**
Average accuracy	89.11	90.70	**92.13**
Kappa coefficient κ	77.49	80.32	**83.05**
Asphalt	85.48	89.82	**94.28**
Meadows	71.56	74.03	**76.41**
Gravel	**70.70**	69.99	69.89
Trees	97.88	98.04	**98.30**
Metal sheets	99.55	**99.78**	**99.78**
Bare soil	93.46	95.37	**97.51**
Bitumen	91.95	94.74	**97.14**
Bricks	92.97	96.31	**98.29**
Shadows	**98.42**	98.20	97.57

Spectral-spatial classification

Pixel-wise classification map

Majority vote within the watershed regions

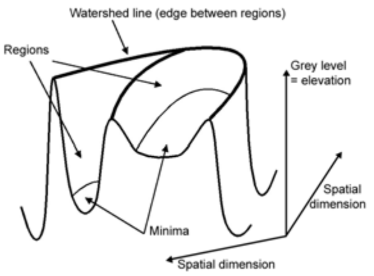
No WHEDs
(watershed pixels not processed)

With WHEDs
(watershed pixels assigned to neighboring regions)

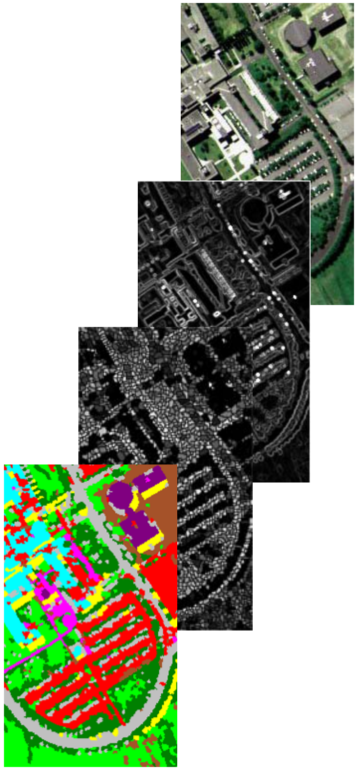
- Noise reduced
- Rough borders

Summary: segmentation and classification using watershed

- Segmentation by morphological watershed combines region growing and edge detection techniques
  - ➢ Extension of a watershed to hyperspectral data is feasible



- A further step forward towards the integration of spatial and spectral information for the classification of hyperspectral data:
  use of ***adaptive neighborhoods*** (***segmentation map***)
  - ➢ Results are promising

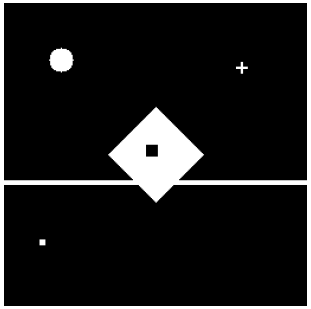


- ***Perspectives:***
  Reduce oversegmentation 🡐 incorporate *a priori* knowledge into segmentation
  - ➢ Marker-controlled watershed segmentation

Practical session on mathematical morphology

using Image Processing Toolbox of Matlab

Binary mathematical morphology



- Download the image test.tif: http://www3.hi.is/~yut2/files/MMImage/test.tif
- For the considered image:
  - Test elementary morphological operators (erosion and dilation) with different structuring elements: horizontal segments (line) or vertical segments (column), square, cross

Help:
- *IM2 = imdilate(IM, SE)* performs dilation of image IM by a structuring element SE.
- Function *se = strel(shape, parameters)* constructs structuring elements with a variety of shapes and sizes. Examples:
  SE = strel('line', LEN, DEG)
  SE = strel('square', W)
- To perform erosion, use function *imerode*.

Binary mathematical morphology

- For the considered image:
  - Combine elementary operators to construct the following operations:
    - Opening (erosion followed by dilation with the same structuring element, for example line or square) and closing (dilation followed by erosion). What is the effect of these filters?
    - "Top-hat" (initial image minus opening). How this operator can be useful?

Help:

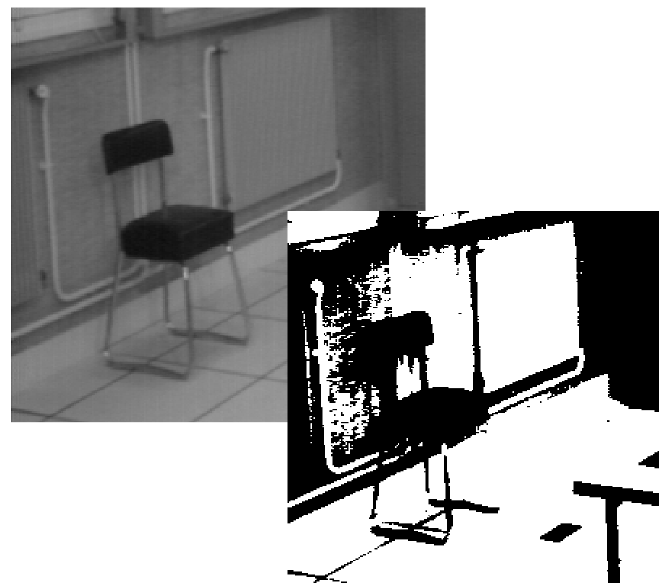
- Opening and closing are implemented in Image Processing Toolbox of Matlab with functions *imopen* and *imclose*.
- Top-hat filtering can be performed using function *IM2 = imtophat(IM,SE)* or function *imsubtract* together with opening function.

Binary mathematical morphology

- For the considered image:
  - With the help of different morphological operators:
    - Try to isolate pixels belonging to one or another structure of the image (diamond formation, circle, line, cross, black square or white square)
    - Delete the white line, preserving all the other structures intact
  - Implement and test different morphological gradients (dilation-image, image-erosion, dilation-erosion)
    - Which differences do you observe between these gradients (finesse, localisation)?
    - What is the influence of the structuring element (size and shape)?

Binary mathematical morphology

- You can investigate the influence of these different operators for the “natural” binary images obtained by thresholding of grey-scale images



MM for grey-scale images

- Download grey-scale images femme.tif and chaise.tif:
  http://www3.hi.is/~yut2/files/MMImage/femme.tif
  http://www3.hi.is/~yut2/files/MMImage/chaise.tif
  You can also use any other grey-scale images

- Repeat, for the grey-scale images, the study fulfilled for the binary images:
  - Erosion (minimum value of the image in the window defined by the structuring element)
  - Dilation (maximum value of the image in the window defined by the structuring element)
  - And combinations of these operators
    - Opening and closing
    - "Top-hat" operator
    - Morphological gradients