

Image Processing

Traitement d'images

Yuliya Tarabalka

yuliya.tarabalka@hyperinet.eu

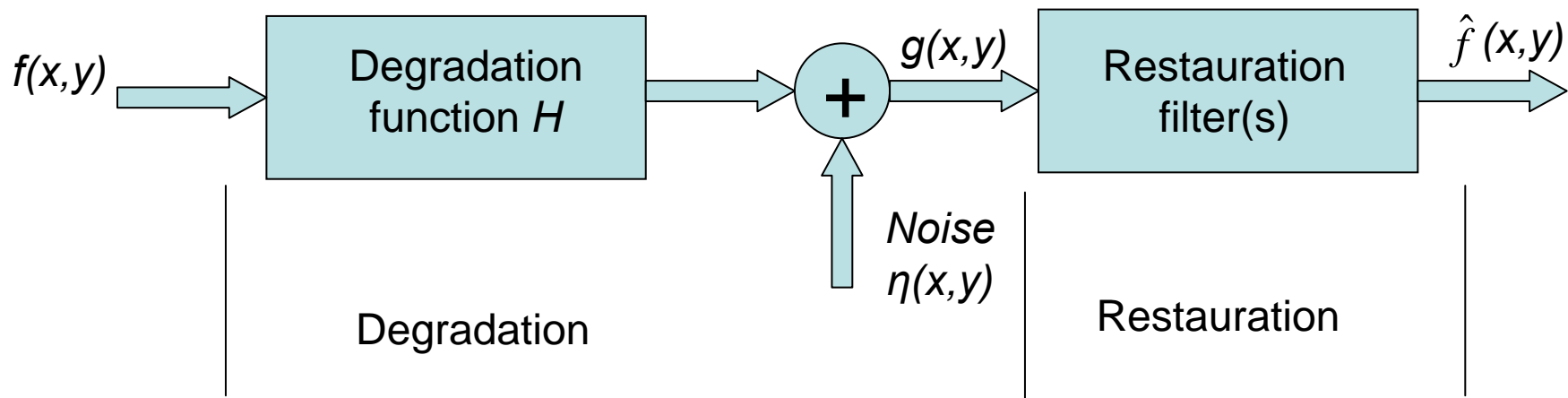
yuliya.tarabalka@gipsa-lab.grenoble-inp.fr

Tel. 04 76 82 62 68

Noise reduction

Image restoration

- **Restoration** attempts to *reconstruct* an image that has been degraded by using a priori knowledge of the degradation phenomenon
 - Modeling the degradation
 - Applying the inverse process in order to recover the original image



Noise models (Models of noisy images) Grenoble INP

Gaussian noise

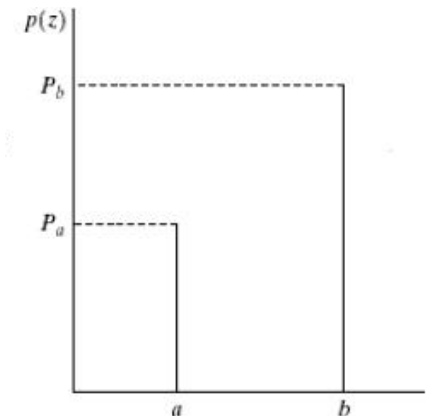
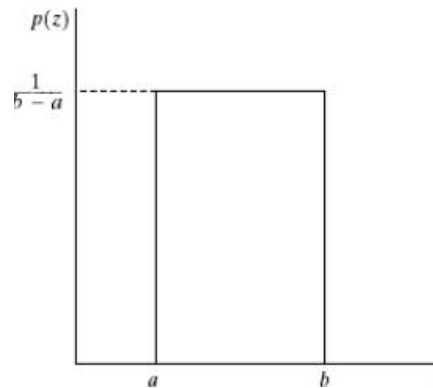
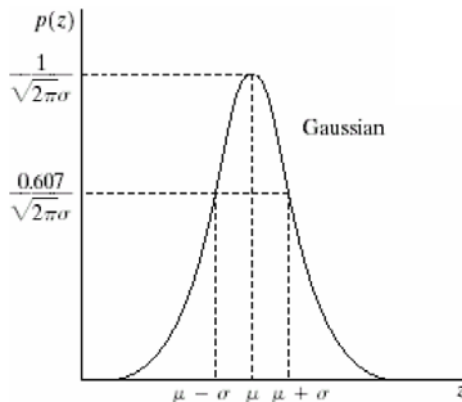
Uniform noise

Impulse (salt-and-pepper) noise

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(z - \mu)^2}{2\sigma^2}\right\}$$

$$p(z) = \begin{cases} 1/(b - a) & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



- used frequently in practice
- macroscopic effect of many microscopic effects

`imnoise(I, 'gaussian', ...)`

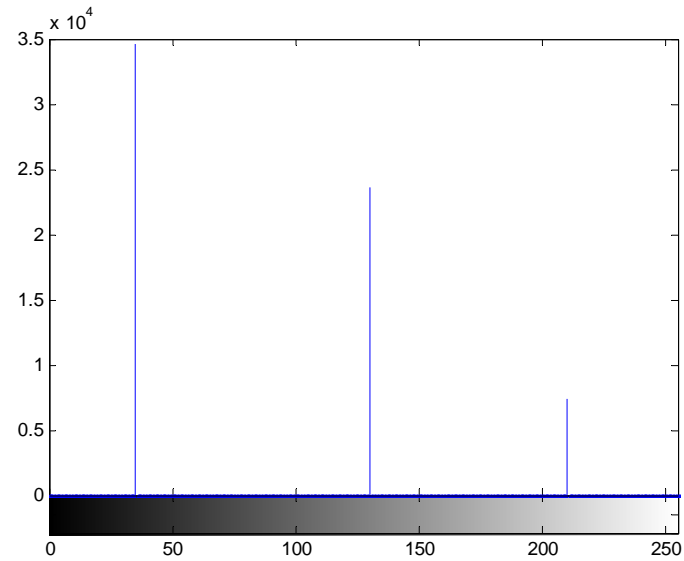
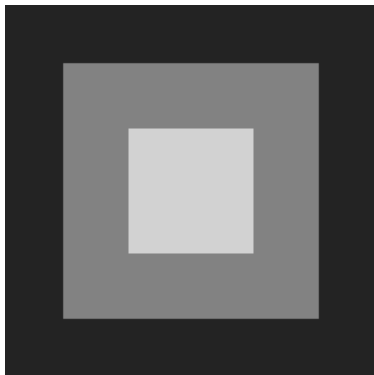
- in acquiring images with a CDD camera

`imnoise(I, 'salt & pepper', ...)`

Image Processing

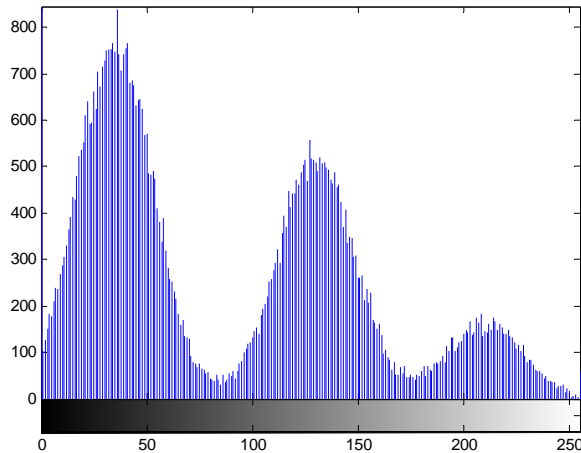
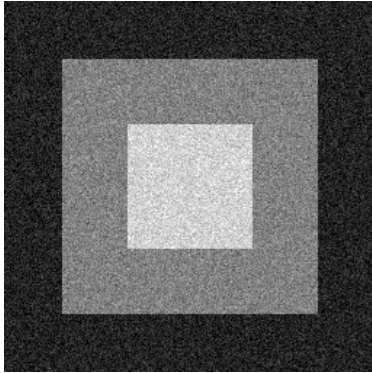
Noise models (Models of noisy images)

- Test pattern

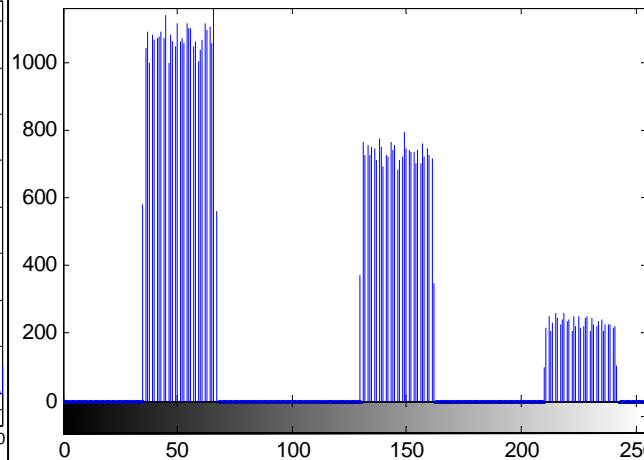
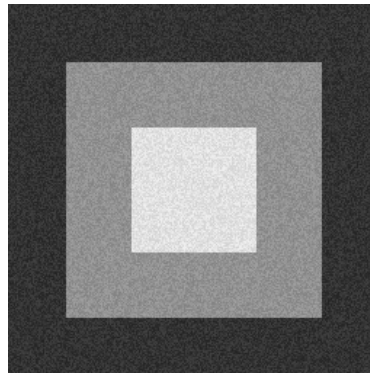


Noise models (Models of noisy images) Grenoble INP

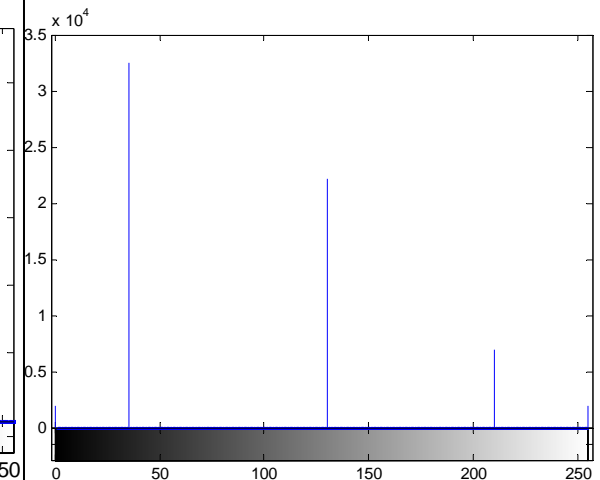
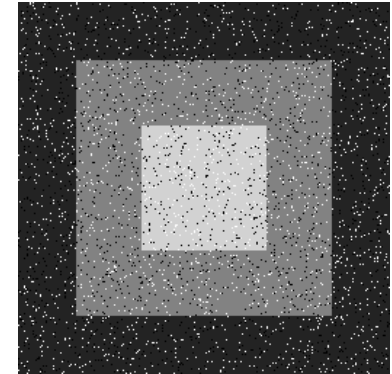
Gaussian noise



Uniform noise

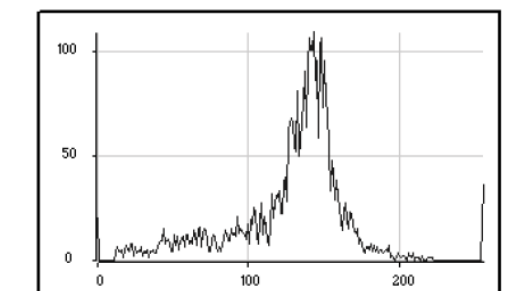
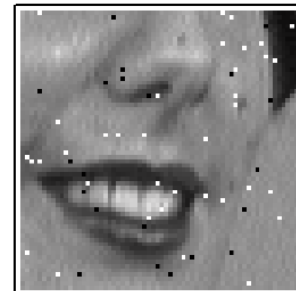
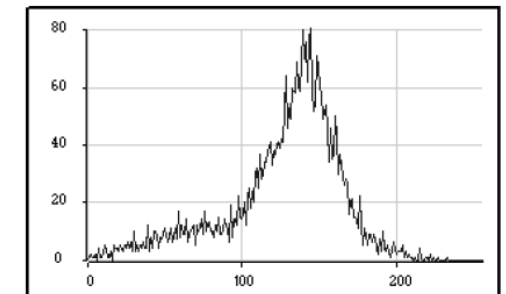
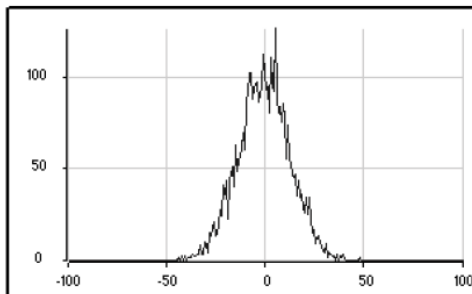
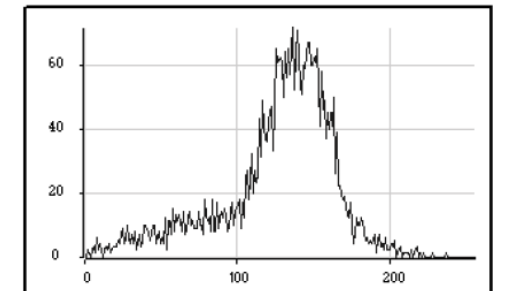
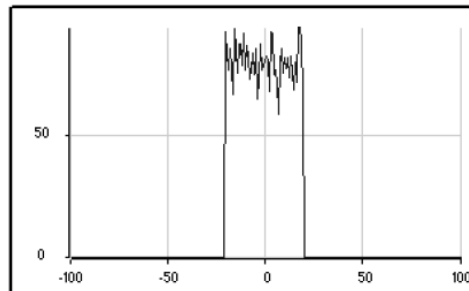
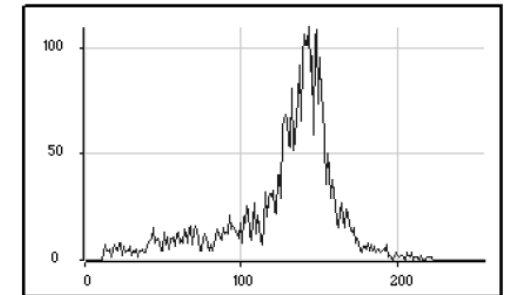


Impulse (salt-and-pepper) noise



Noise models (Models of noisy images)

Original image →



Convolution

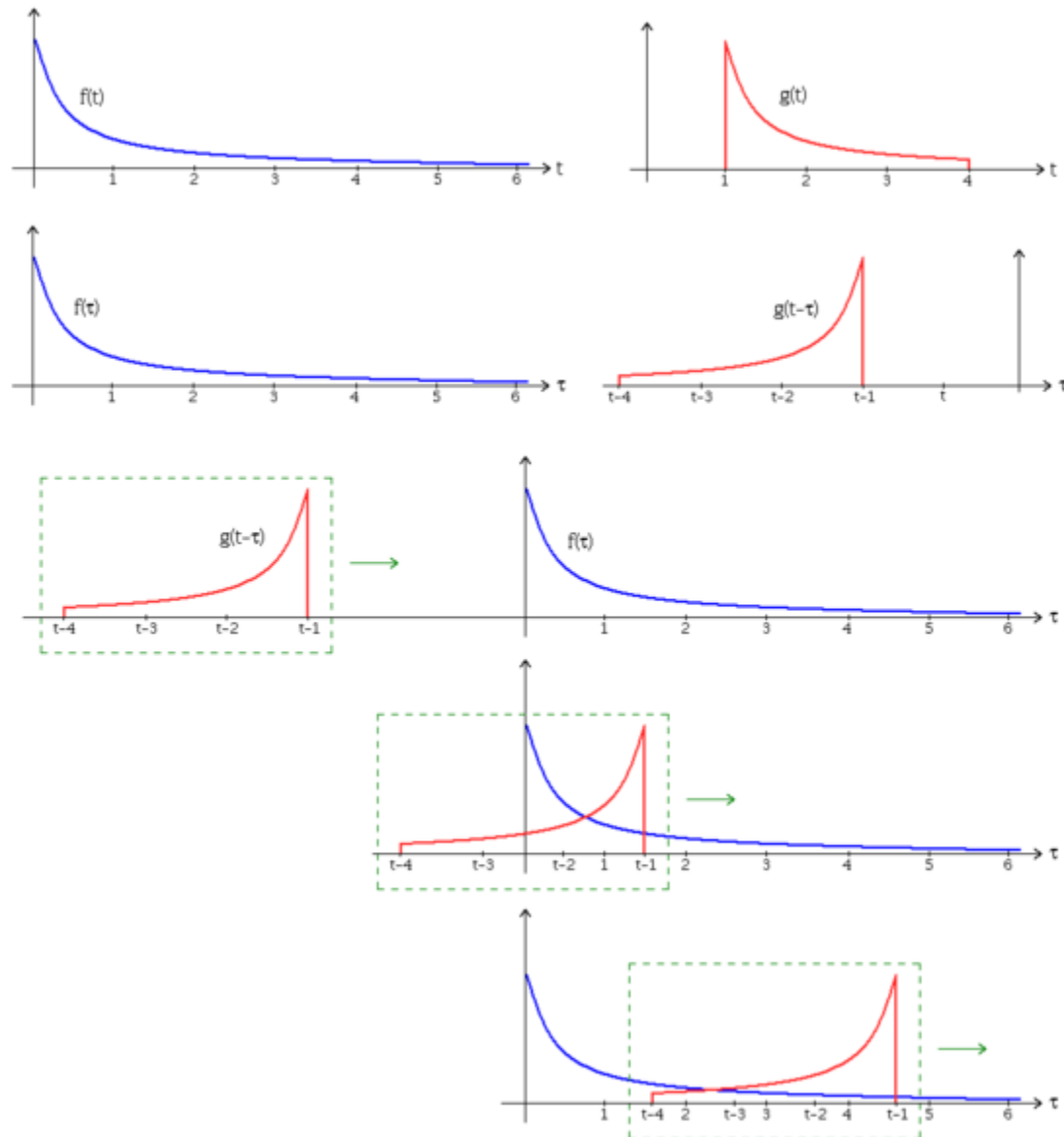
1 D, continue (continuous space) : $f * h(t) = \int_{-\infty}^{\infty} (f(x)) \cdot (h(t - x)) dx$

2 D, continue : $f * h(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (f(x, y)) \cdot (h(u - x, v - y)) dx dy$

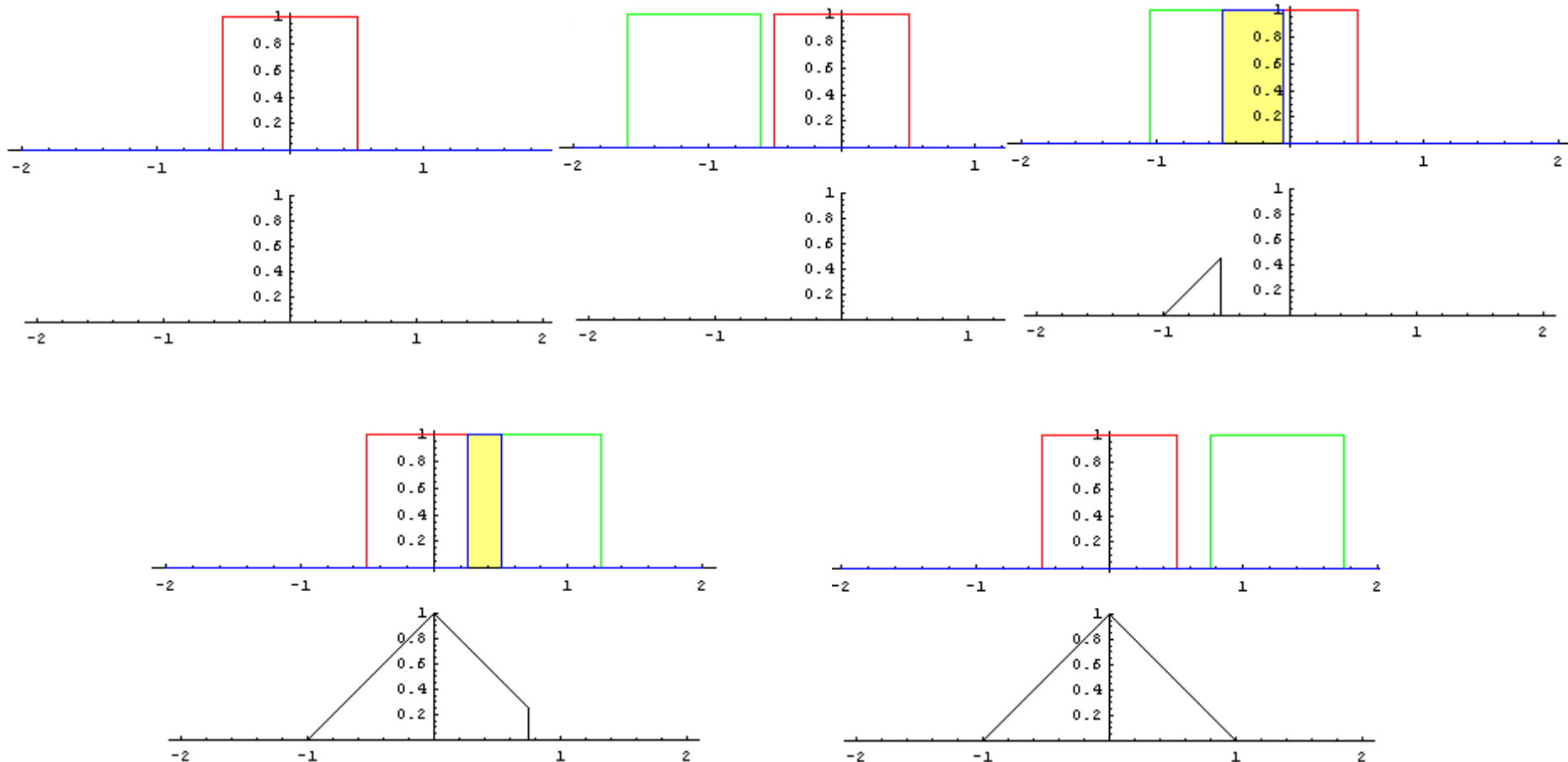
2D, discrete space:

$$f(x, y) * h(x, y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) h(x-m, y-n)$$

Convolution

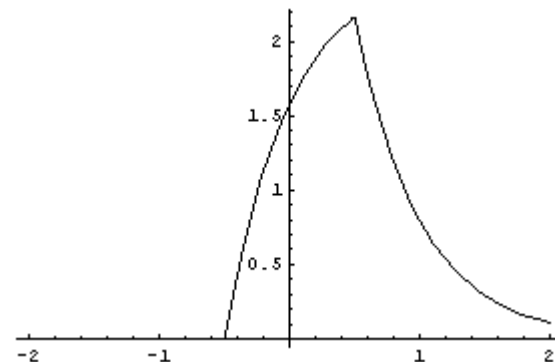
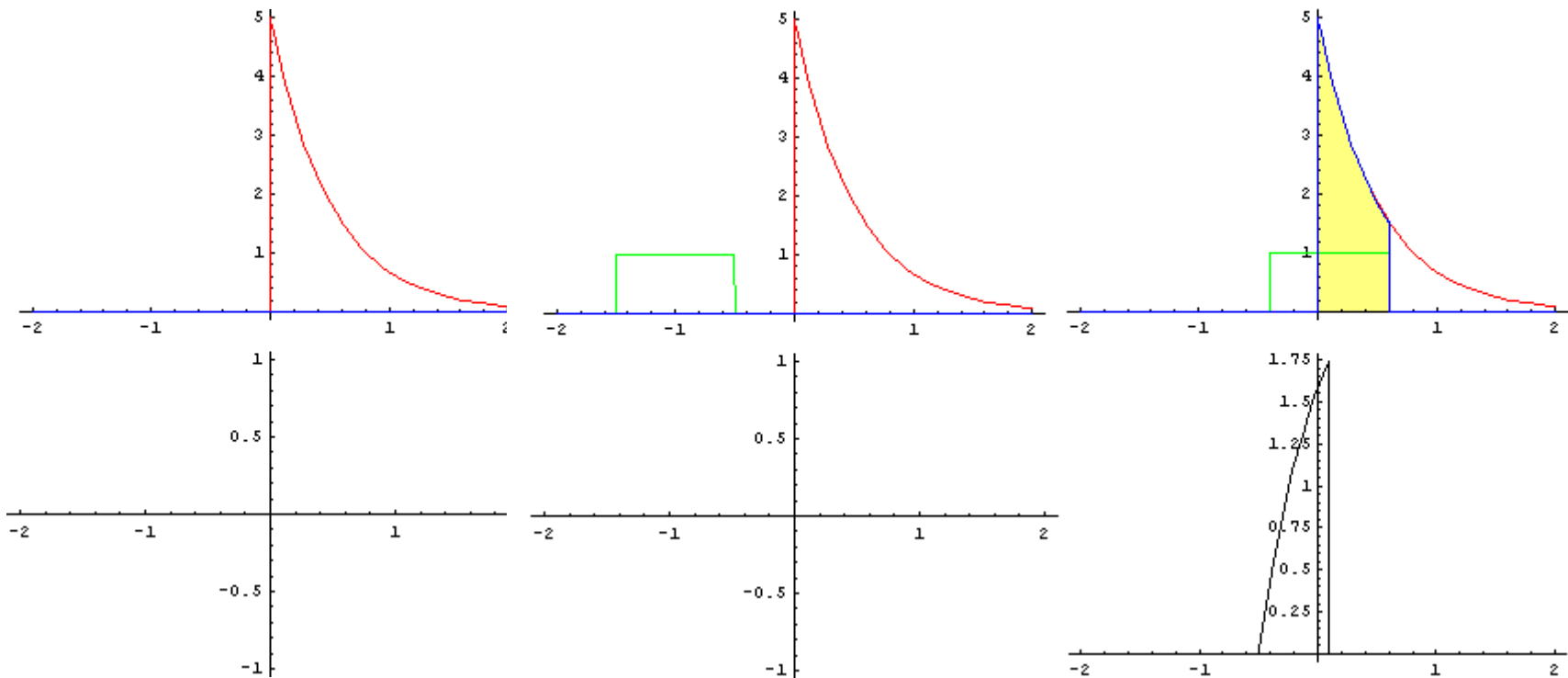


Convolution



Convolution of two square pulses: the resulting waveform is a triangular pulse. The integral of their product is the area of the yellow region

Convolution

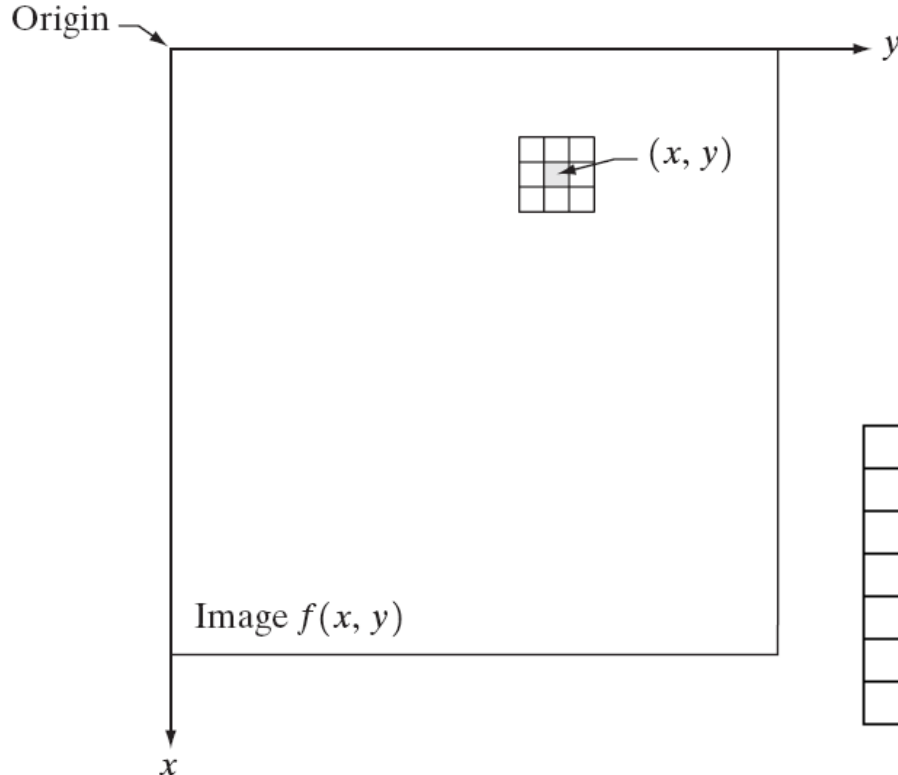


Spatial filtering (neighborhood operations)

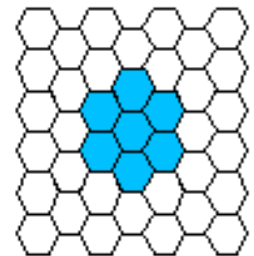
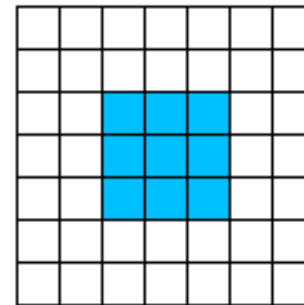
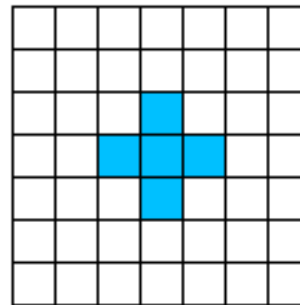


- The value of each pixel (x,y) in the output image is computed from the values of the pixels in the **neighborhood** of (x,y) :

$$g(x, y) = T[f(x, y)]$$



Sliding filtering windows:



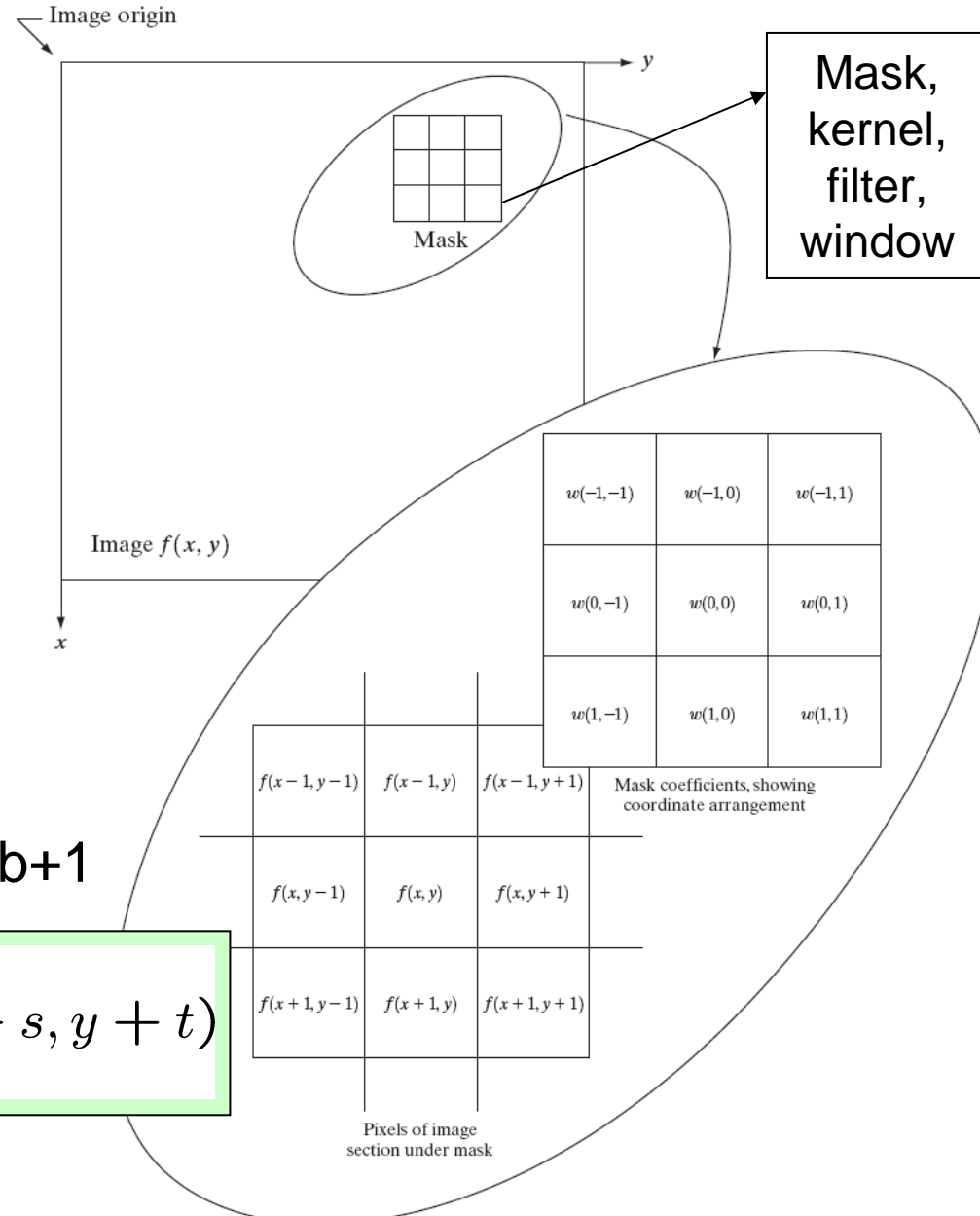
Spatial filtering (neighborhood operations)



- **Linear filtering:** the response of the filter is given by a sum of products of the filter coefficients and the corresponding image pixels within the filter mask

- The mask of size $m \times n$ is centered at (x, y)
- We assume $m=2a+1$, $n=2b+1$

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$



Spatial filtering (neighborhood operations)

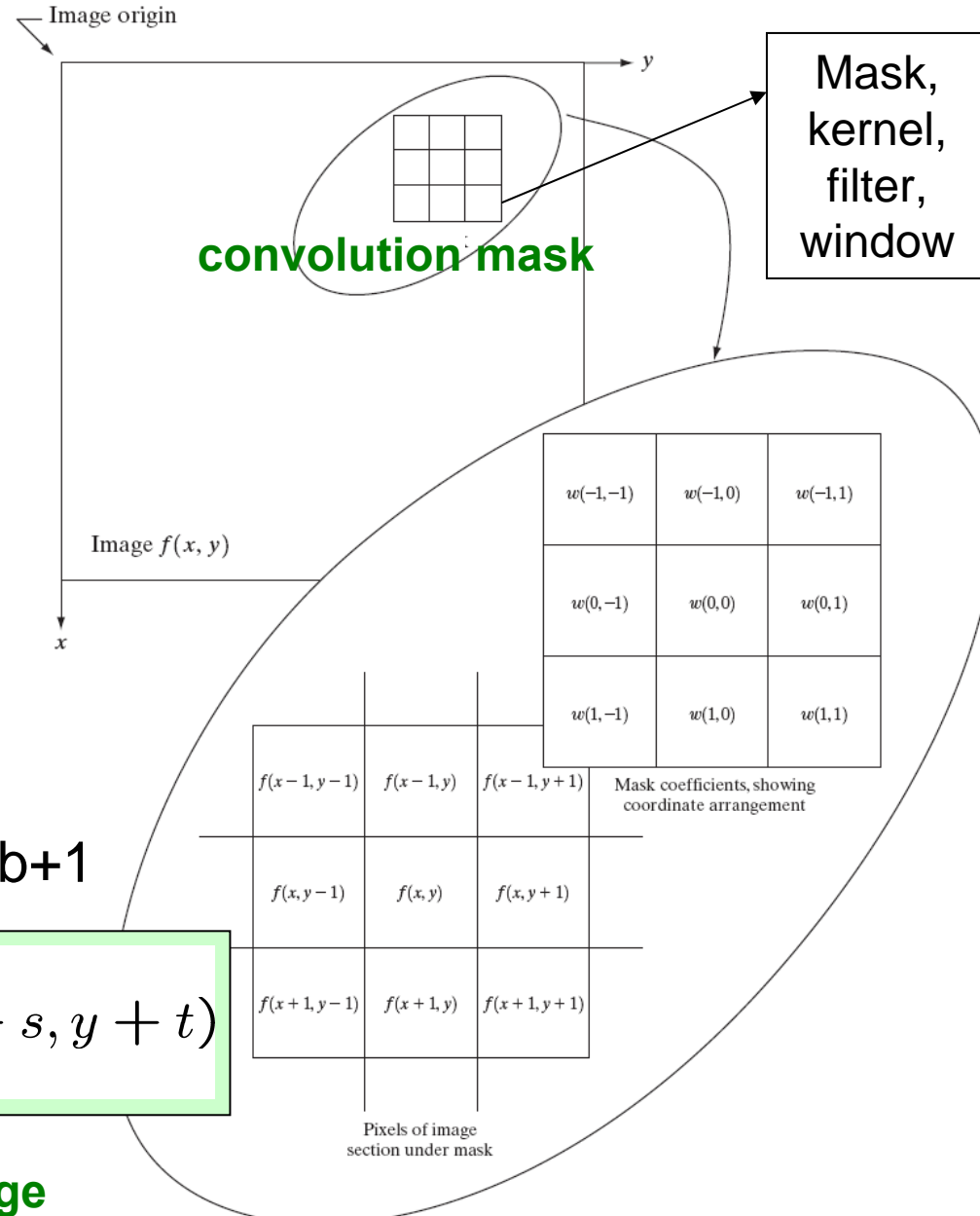


- **Linear filtering:** the response of the filter is given by a sum of products of the filter coefficients and the corresponding image pixels within the filter mask

- The mask of size $m \times n$ is centered at (x, y)
- We assume $m=2a+1$, $n=2b+1$

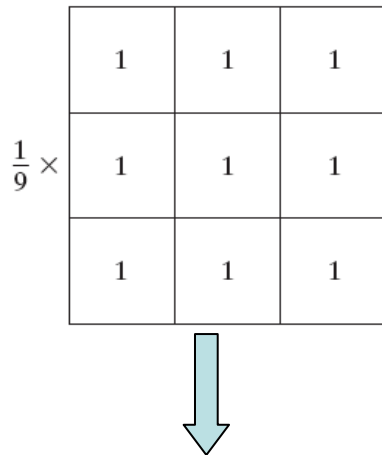
$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

convolving a mask with an image



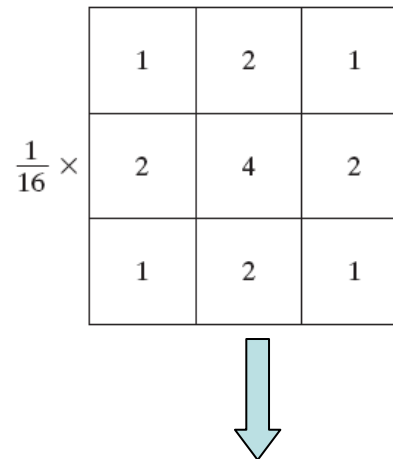
Smoothing linear filters

- The output: (weighted) average of the pixels contained in the neighborhood of the filter mask
- Averaging, lowpass filters



Mean filter: replace each pixel by the arithmetic mean of its neighbors and itself

- **reduce noise** (e.g., gaussian)
- but **blur edges**

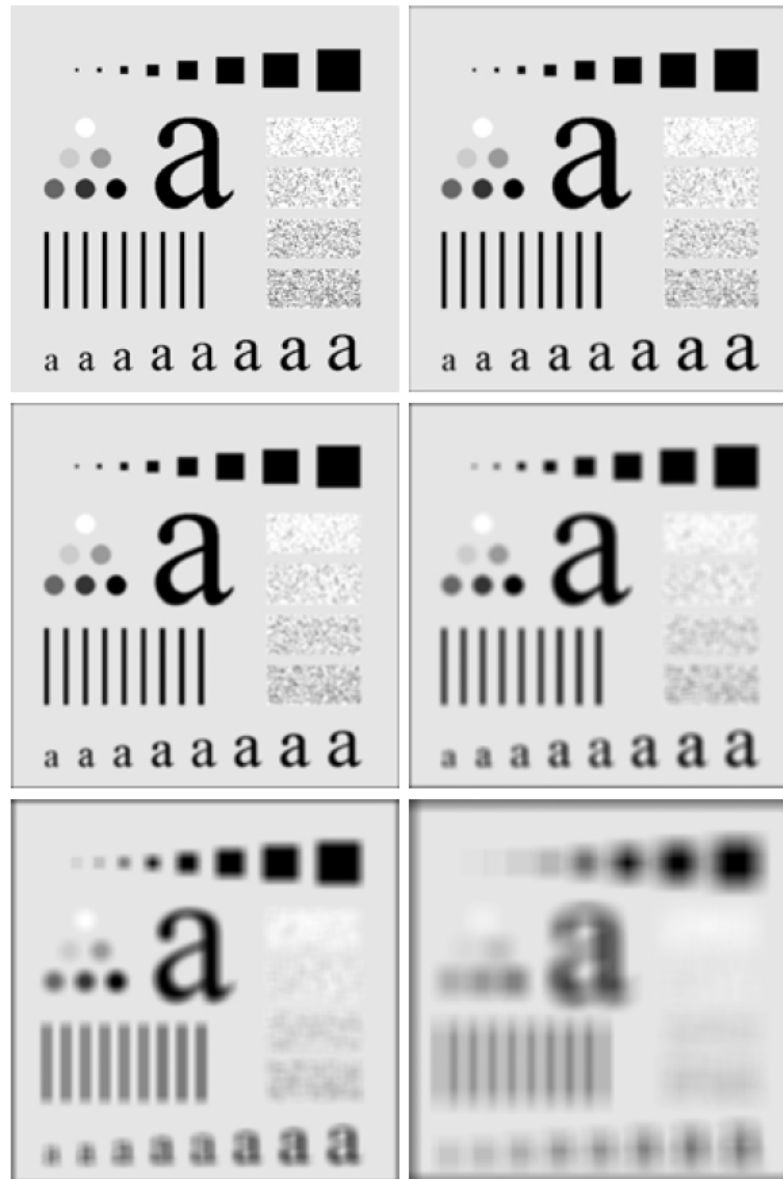


Weighted average: reduce blurring in the smoothing process

Smoothing linear filters

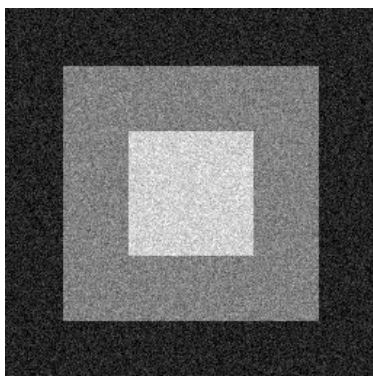
- Matlab:
conv2(), *imfilter()*

Results of smoothing
with square averaging
Filter masks of sizes n
 $= 3, 5, 9, 15,$ and 35

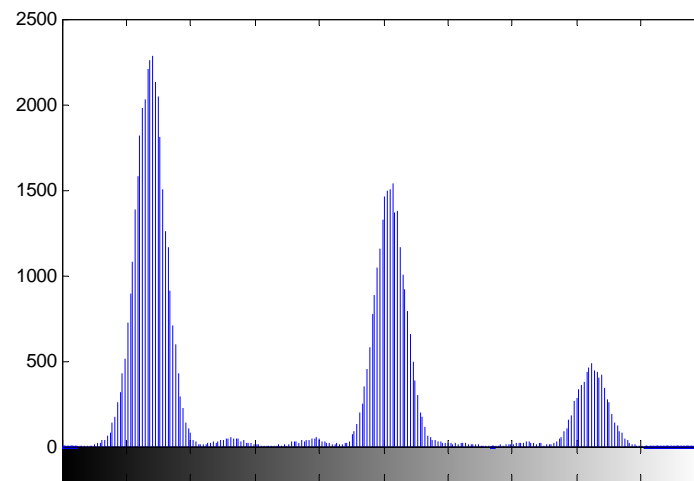
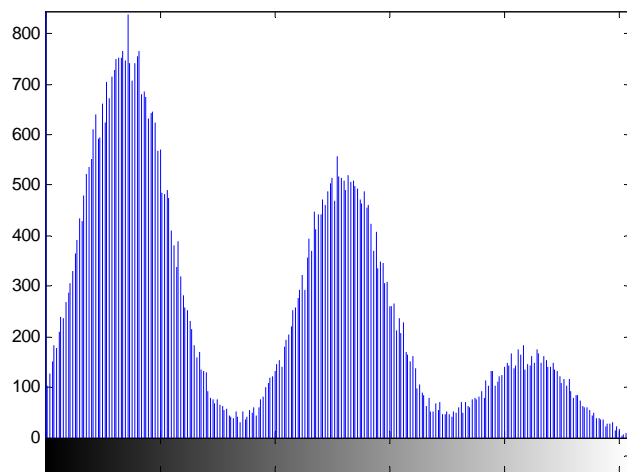
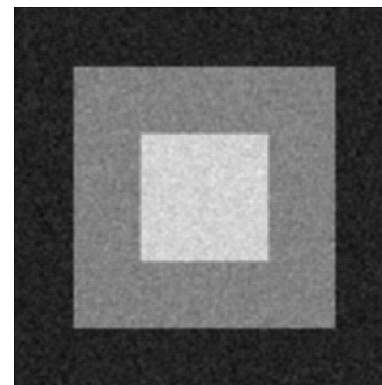


Smoothing linear filters

Image corrupted by
Gaussian noise



Mean filtering
(3x3 mask)



Smoothing non-linear filtering

- **Order filters:** response is based on ordering (ranking) the pixels contained within the considered window

$$\begin{bmatrix} 10 & 134 & 124 \\ 152 & 45 & 51 \\ 61 & 51 & 88 \end{bmatrix}$$

Smoothing non-linear filtering

Order filters: response is based on ordering (ranking) the pixels contained within the considered window

$$\begin{bmatrix} 10 & 134 & 124 \\ 152 & 45 & 51 \\ 61 & 51 & 88 \end{bmatrix}$$

$$10 < 45 < 51 < 51 < 61 < 88 < 124 < 134 < 152$$

- **Median filter** replaces the value of a pixel by the median of the gray levels in its neighborhood
 - For our example it is 61:

$$10 < 45 < 51 < 51 < \mathbf{61} < 88 < 124 < 134 < 152$$
 - Especially effective for impulse (salt-and-pepper) noise

Smoothing non-linear filtering

Order filters: response is based on ordering (ranking) the pixels contained within the considered window

$$\begin{bmatrix} 10 & 134 & 124 \\ 152 & 45 & 51 \\ 61 & 51 & 88 \end{bmatrix}$$

$$10 < 45 < 51 < 51 < 61 < 88 < 124 < 134 < 152$$

- **Middle filter:** the value of a pixel is computed as the average of the maximum and minimum values in its neighborhood
 - For our example it is 81:

$$10 < 45 < 51 < 51 < 61 < 88 < 124 < 134 < 152$$
 - Particularly effective for uniform noise

Smoothing non-linear filtering

Image corrupted by salt-and-pepper noise

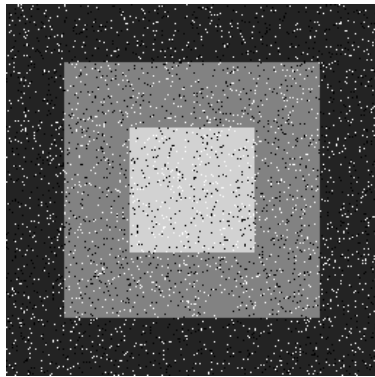


Image after median filtering (Matlab: *medfilt2()*)

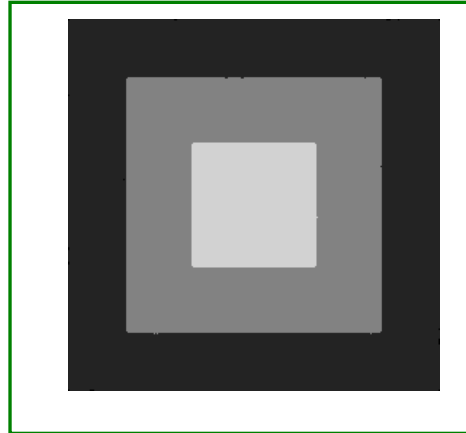
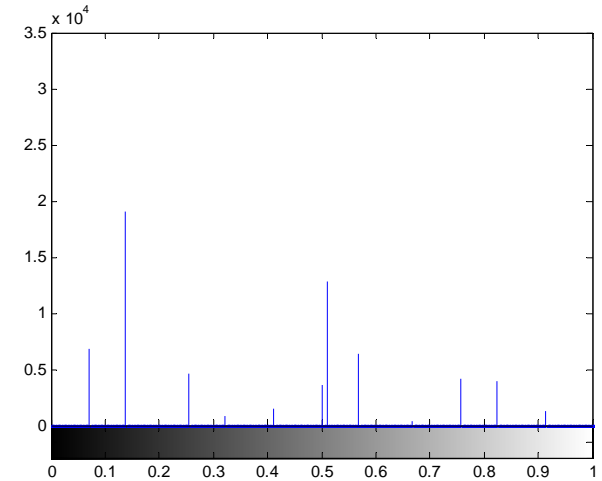
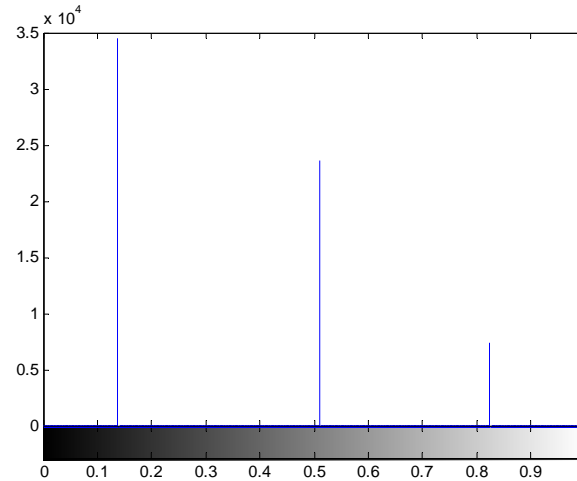
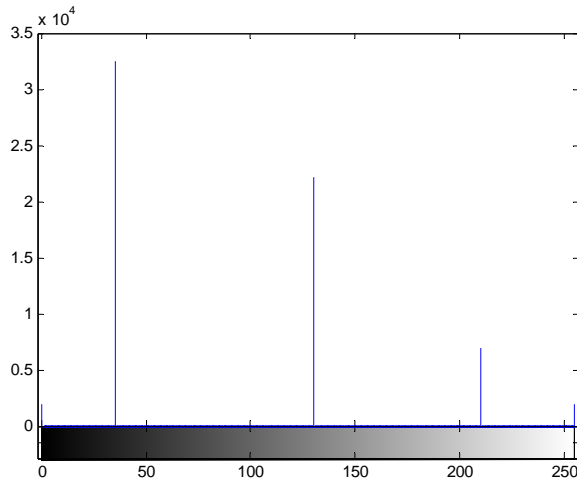
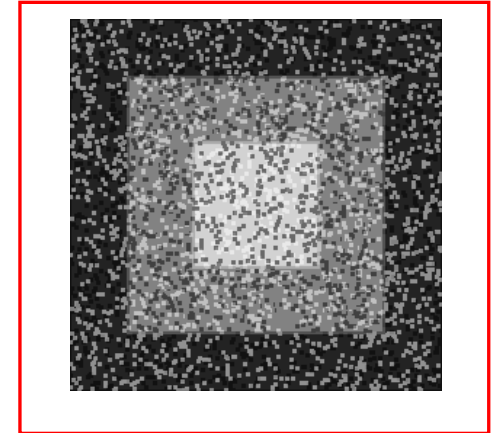


Image after middle filtering



Contrast enhancement

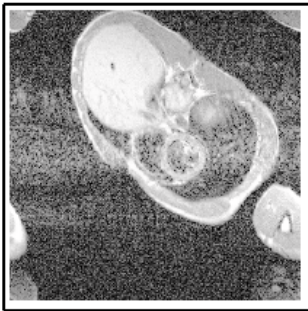
If $\text{mean} > \text{middle}$,
 $\text{max}(\text{mean}, \text{middle})$;
otherwise $\text{min}(\text{mean}, \text{middle})$



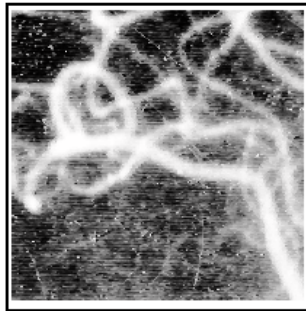
- Contrast enhancement accentuates edges, but also noise

Real images

- **Reference, non-noisy image is not available**
- **One must assume (guess) a type of noise**
 - Choose the part of an image that is assumed to be uniform (homogeneous) without the noise
 - Test the hypothesis and choose the best filter



MRI



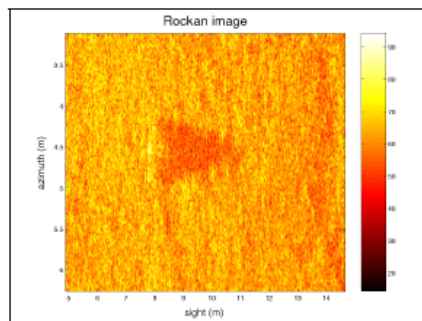
angiography (X-Ray)



echography (Ultra-Sound)



CCD



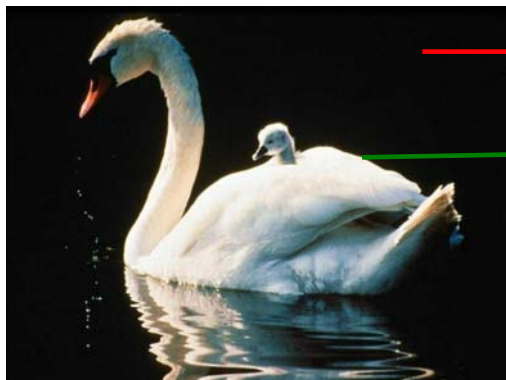
sonar

Edge detection

Segmentation & edge detection

- **Segmentation** partitions an image into its constituent regions or objects
- Image segmentation algorithms are based on one of two (or two) basic properties of intensity values: discontinuity and similarity
- **Edge detection** is used to isolate objects based on abrupt changes in intensity

Edge = fast change in intensity



→ No edge

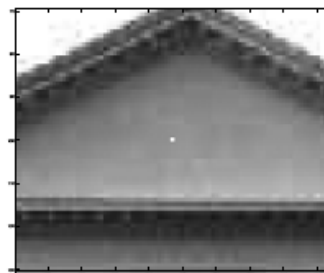
→ Edge

Detection of discontinuities

- The most common way to look for discontinuities is to convolve a mask with an image
- **Point detection:**
 - filtering an image with a mask

-1	-1	-1
-1	8	-1
-1	-1	-1

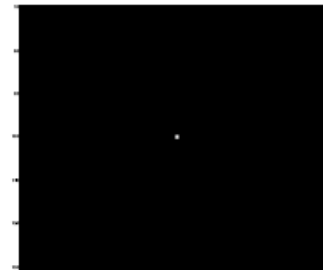
- thresholding



Original image



After filtering



After thresholding

Edge detection

- **Edge** is a set of connected pixels that lie on the boundary between two regions
- **Derivative** is a measure of how a function changes as its input changes
- First-order derivative of a one-dimensional function $f(x)$:

$$\frac{\partial f}{\partial x} = f(x) - f(x - 1)$$

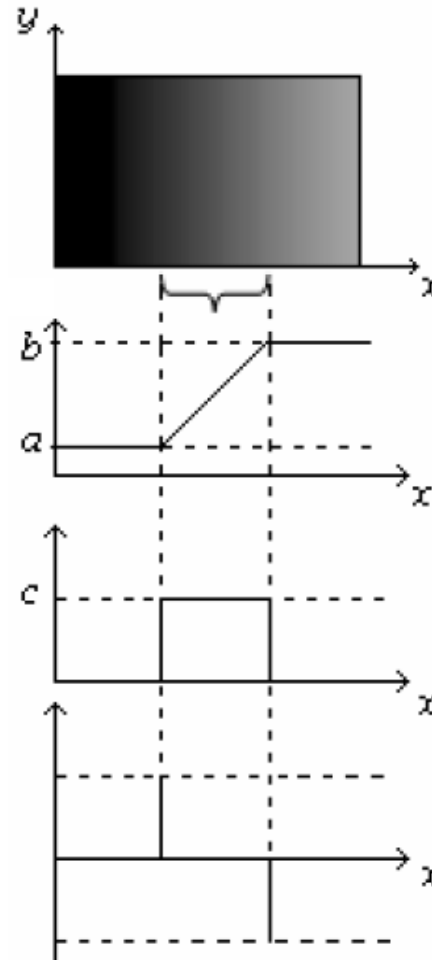
- Second-order derivative:

$$\frac{\partial^2 f}{\partial x^2} = f(x + 1) + f(x - 1) - 2f(x)$$

Edge detection



Model of an ideal edge

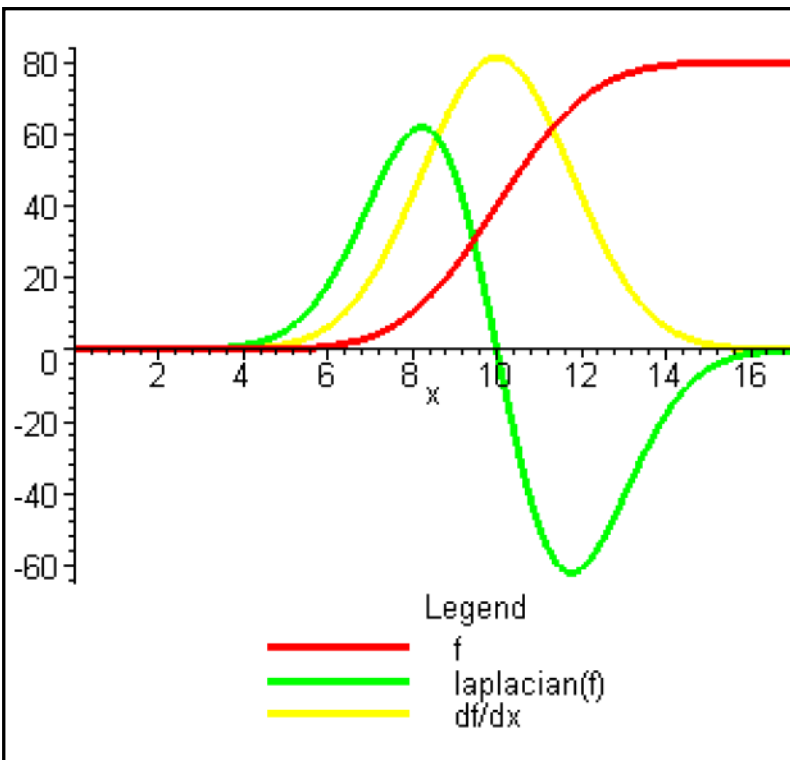


Model of a ramp edge

Grey-level profile

First derivative

Second derivative



Edge detection

- The gradient of an image $f(x,y)$ at location (x,y) is defined as the vector

$$\nabla \vec{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- Magnitude of the gradient $\|\nabla \vec{f}\| = \sqrt{G_x^2 + G_y^2}$

- Convolution masks:

$$G_x$$

0	0	0
-1	1	0
0	0	0

$$G_y$$

0	-1	0
0	1	0
0	0	0

Edge detection

- Convolution masks:

0	0	0
0	-1	0
0	0	1

Roberts

0	0	0
0	0	-1
0	1	0

*“Cross”
operator*

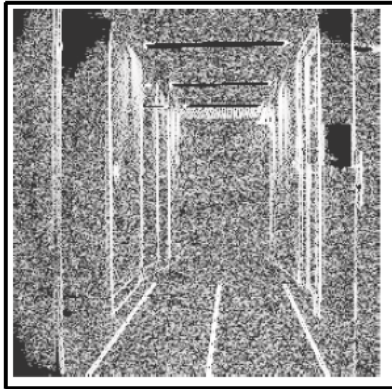
$$G_x$$

0	0	0
-1	0	1
0	0	0

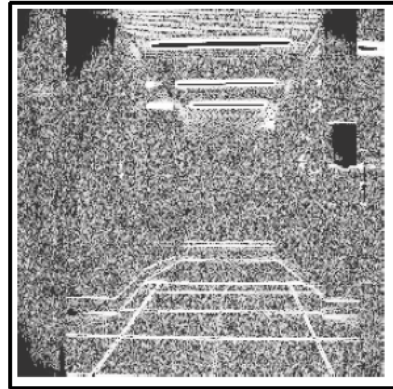
$$G_y$$

0	-1	0
0	0	0
0	1	0

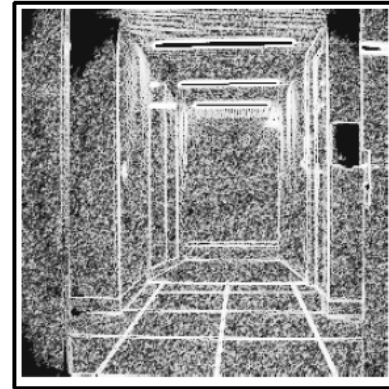
Edge detection



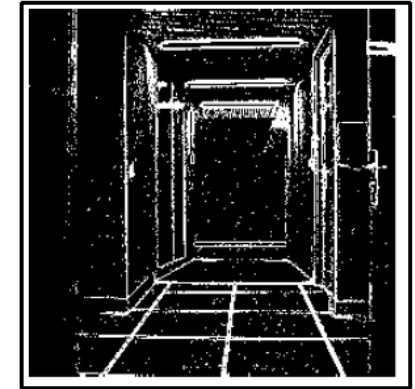
$|\text{grad1}|$
Horizontal gradient



$|\text{grad2}|$
Vertical gradient



$\max(|\text{grad1}|, |\text{grad2}|)$



Threshold

- We define a point in an image as being an **edge point** if its two-dimensional first-order derivative is greater than a specific threshold
- **Problem 1**: gradient is sensitive to noise!

Edge detection

- Filtering in one dimension + detection in another dimension

Prewitt:

$$G_x$$

-1	0	1
-1	0	1
-1	0	1

$$G_y$$

-1	-1	-1
0	0	0
1	1	1

Sobel:

$$G_x$$

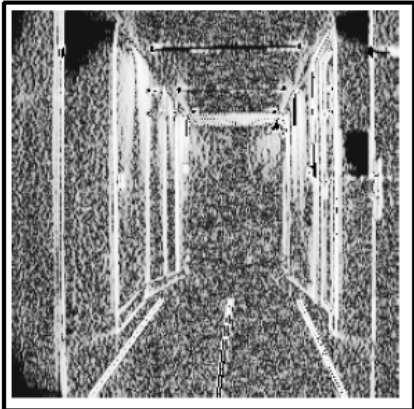
-1	0	1
-2	0	2
-1	0	1

$$G_y$$

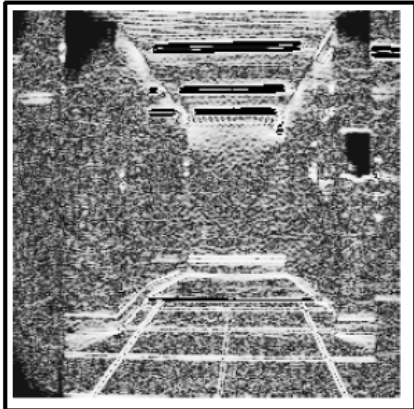
-1	-2	-1
0	0	0
1	2	1

Edge detection

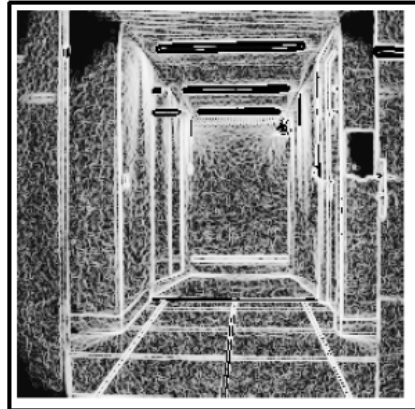
- Filtering in one dimension + detection in another dimension



|sobel1|



|sobel2|



max



threshold

- **Problem 2:** Some edges are not closed → post-processing to link (close) edges

Edge detection

- The Laplacian of a 2-D function $f(x, y)$ is a second-order derivative defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

The Laplacian mask is isotropic

0	-1	0
-1	4	-1
0	-1	0

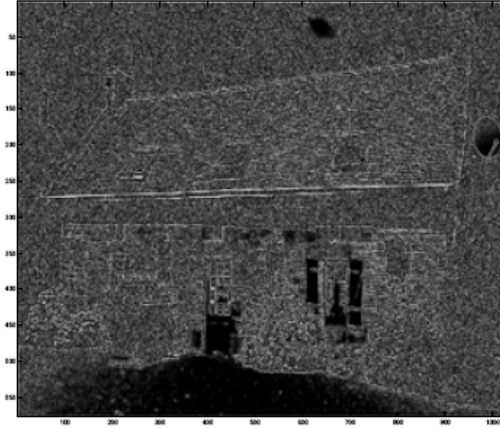
-1	-1	-1
-1	8	-1
-1	-1	-1

- Edge points are defined as the zero-crossings of the Laplacian
- **Problem:** the Laplacian is very sensitive to noise

Edge detection



Original noisy image



After Laplacian filtering



After Prewitt filtering

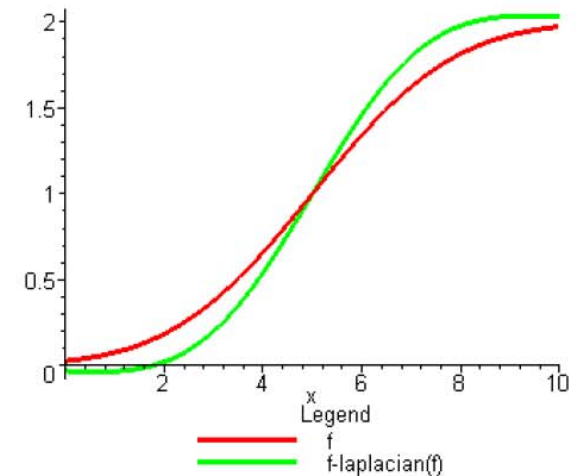
- **There are other advanced edge detection algorithms:**
 - Canny edge detector

Enhancement

- Edges can be accentuated by applying $f(x, y) - \Delta f(x, y)$

0	-1	0
-1	5	-1
0	-1	0

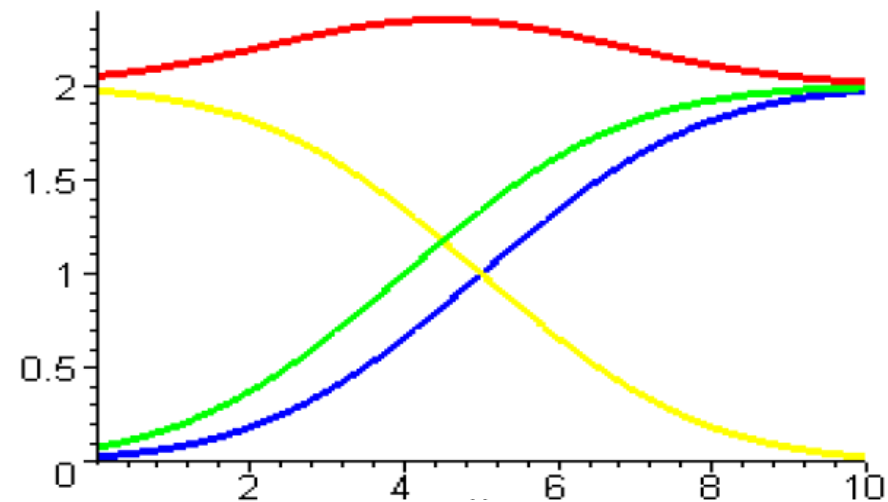
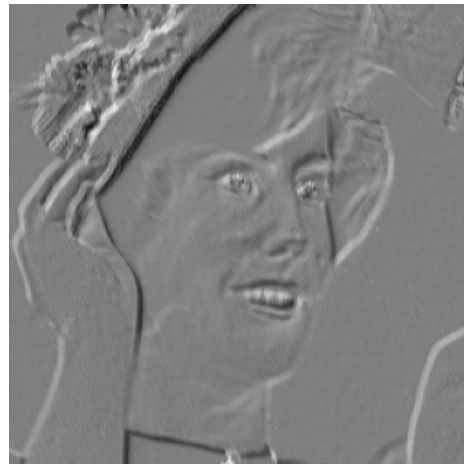
-1	-1	-1
-1	9	-1
-1	-1	-1



- Sensitive to noise**

Relief effect (“emboss”)

- The **relief effect** can be obtained by summing an image negative with the translated original image



Orientation analysis

