Image Processing

Traitement d'images

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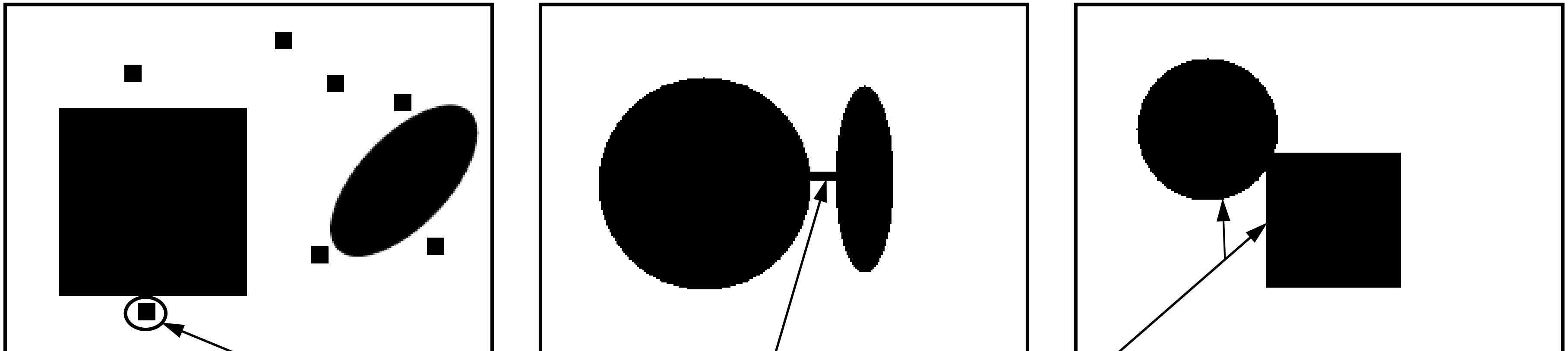
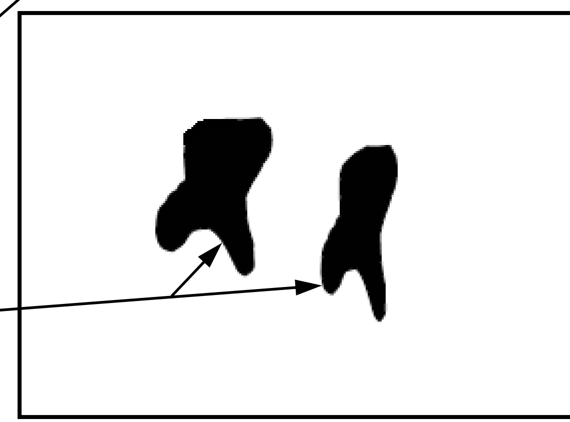
Bases of mathematical morphology

Mathematical morphology: why to use?



- How to remove this noise?
- How to separate these two components?
- How to label differently these two connected shapes?
- How to compare these two shapes?



Mathematical morphology (MM)

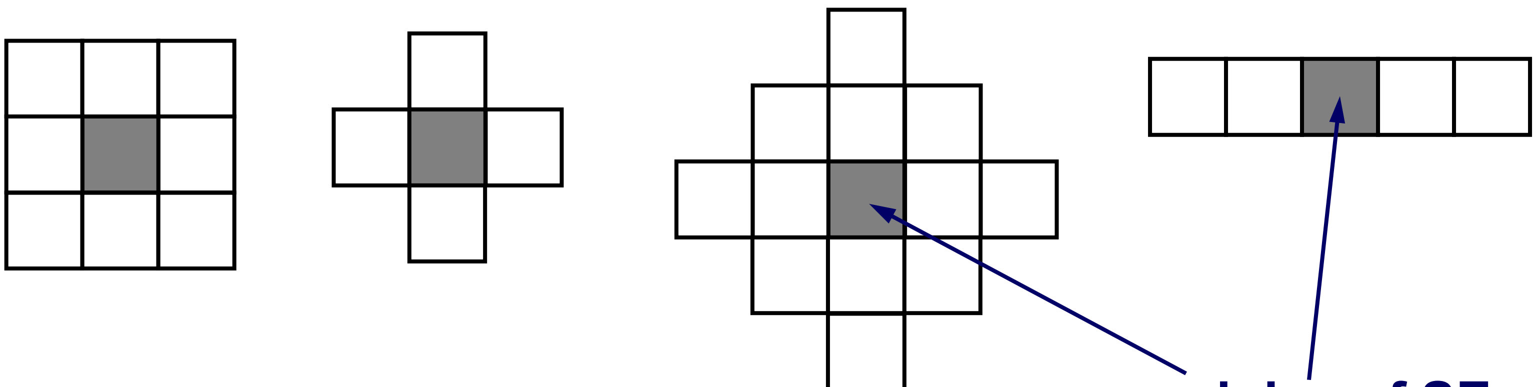
- A theory for the ***analysis of spatial structures***
- ***Morphology:*** aims at analysing the shape and form of objects.
- ***Mathematical:*** analysis is based on:
  - set theory
  - integral geometry
  - lattice algebra

- ***Non-linear*** processing operators (do not blur the edges as convolutions do)
- We'll concentrate on MM for digital images: binary and grey-scale

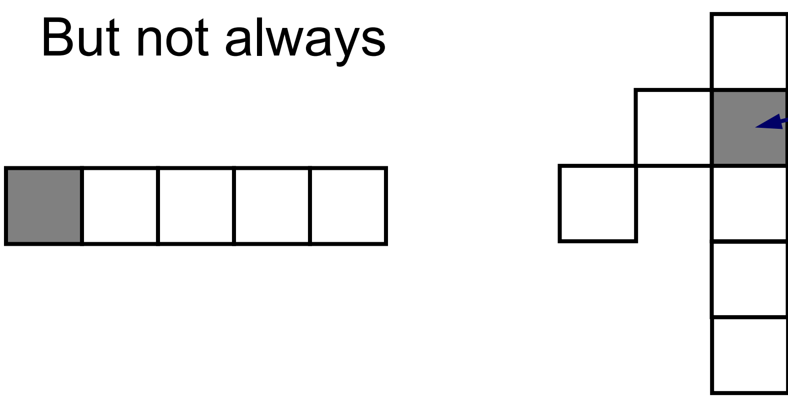
- ***Basic idea:*** locally compare structures within the image with a reference shape called the ***Structuring Element*** (SE)

Structuring element (SE)

- A small set used to analyse locally the image
- ***Shape*** and ***size*** of SE ← a priori knowledge about the geometry of relevant/irrelevant image structures
- Usually symmetrical, connected, and convex



- But not always



Dilation and erosion

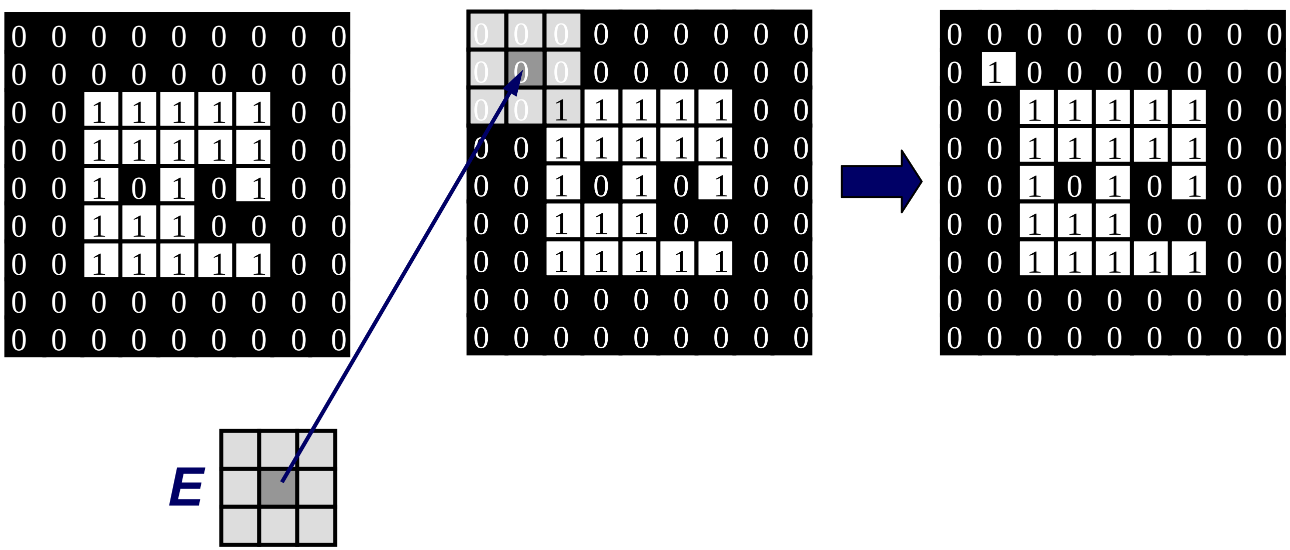
Fundamental morphological operators =
2 letters of the morphological alphabet

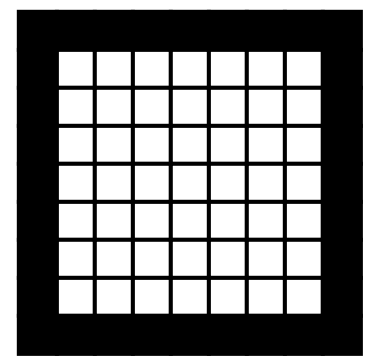
All other operators are expressed in terms of
dilations and erosions

Dilation for binary images

- “Does the SE ***hit*** the set?”
- ***Dilation*** of a set X by a structuring element E is defined as the locus of points $\mathbf{x}$ such that E ***hits*** X when its origin is placed at $\mathbf{x}$:

$$\delta_E(X) = \{\mathbf{x} \in R^d \mid E_{\mathbf{x}} \cap X \neq \varnothing\}$$





result of dilation

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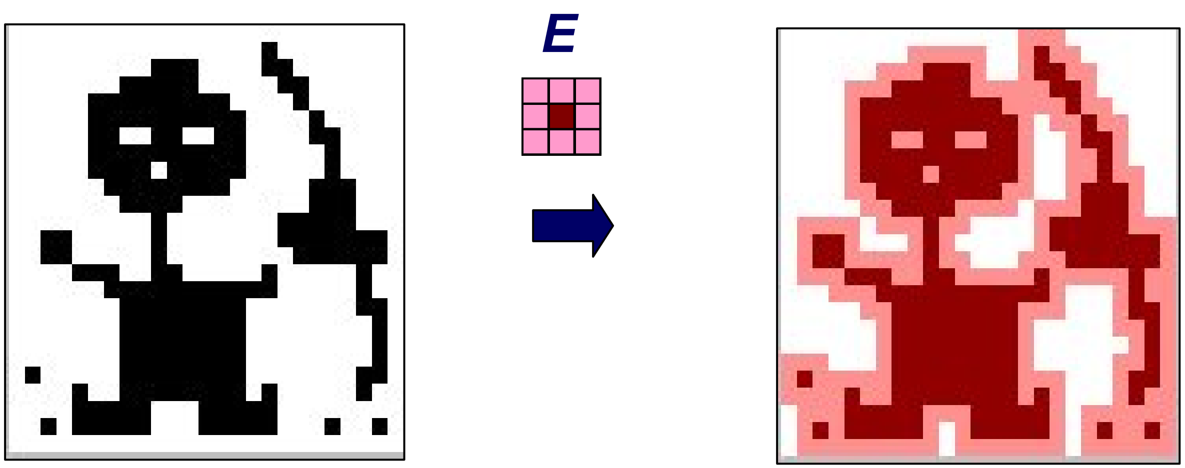
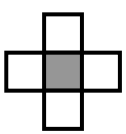


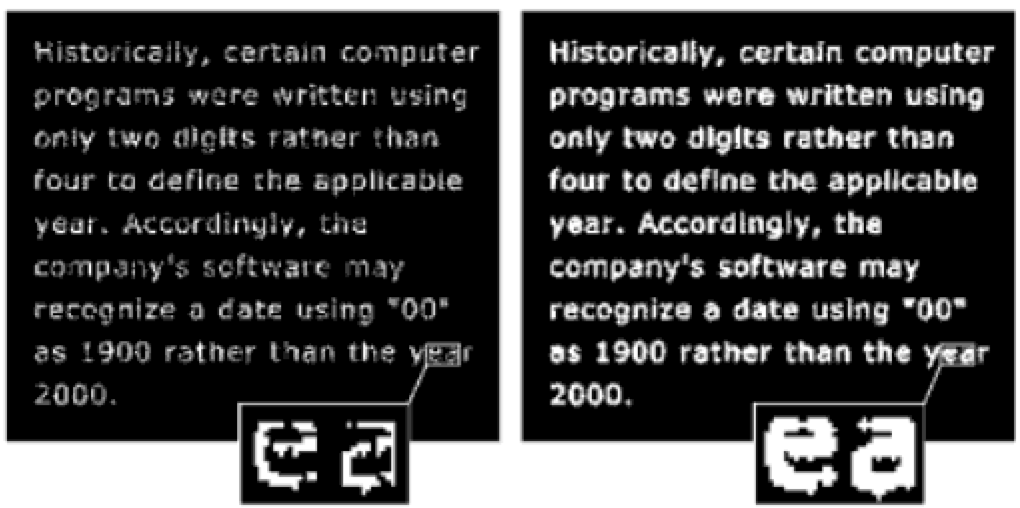
result of dilation

Dilation: properties and use

- Basic property: $X \subseteq \delta_E(X)$
- Consequences:
  - ➢ Fill in the holes smaller than *E*
  - ➢ Enlarge capes
  - ➢ Connect two close shapes
- Example of application: bridging gaps

E



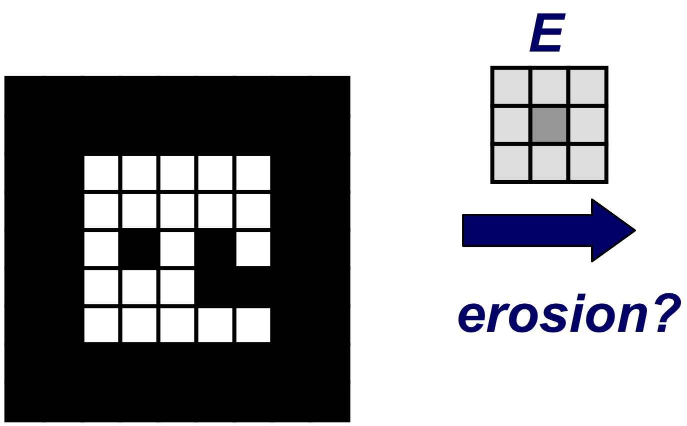


R. C. Gonzalez, R. E. Woods, Digital Image Processing. Prentice Hall, 2002.

Erosion for binary images

- “Does the SE ***fit*** the set?”
- ***Erosion*** of a set X by a structuring element E is defined as the locus of points **x** such that E is ***included*** in X when its origin coincides with **x**:

$$\varepsilon_E(X) = \{\mathbf{x} \in \mathrm{R}^d \mid E_\mathbf{x} \subseteq X\}$$



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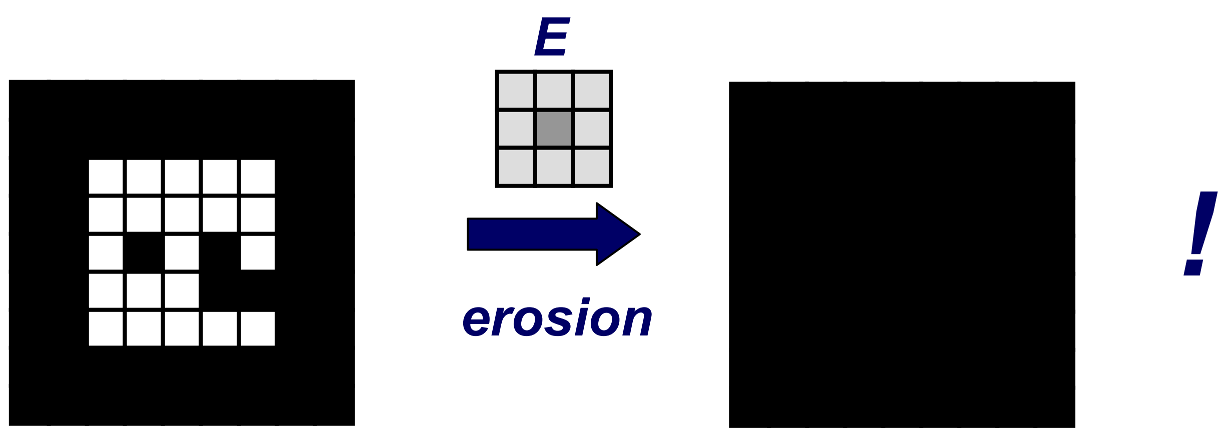

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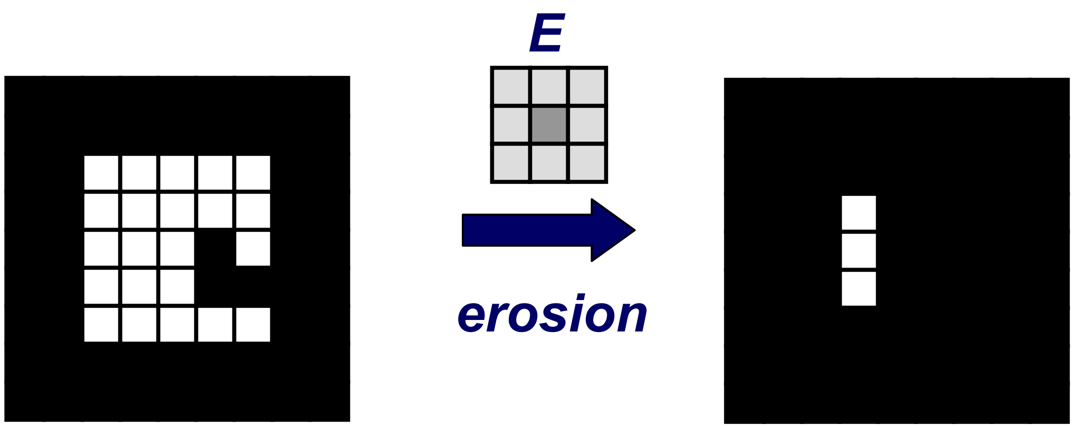

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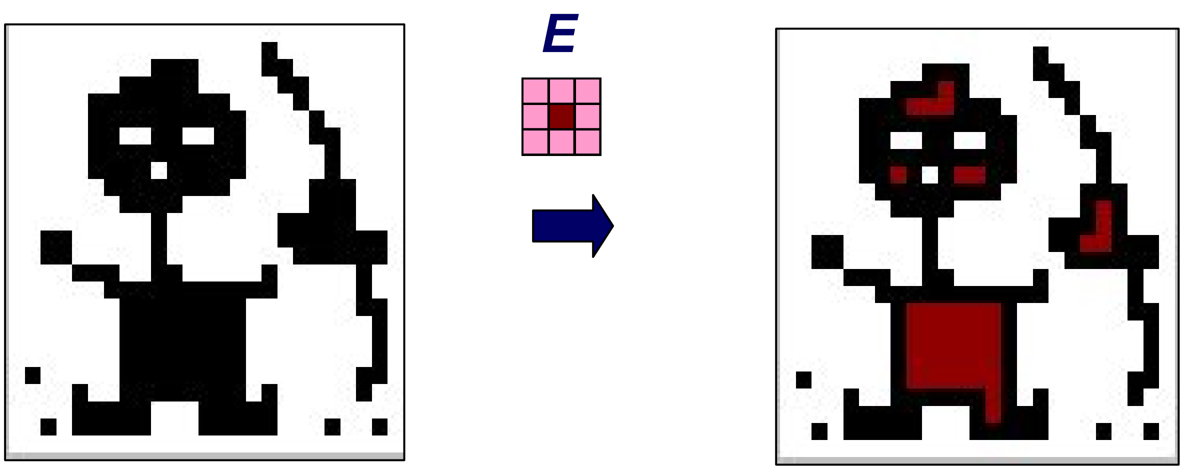
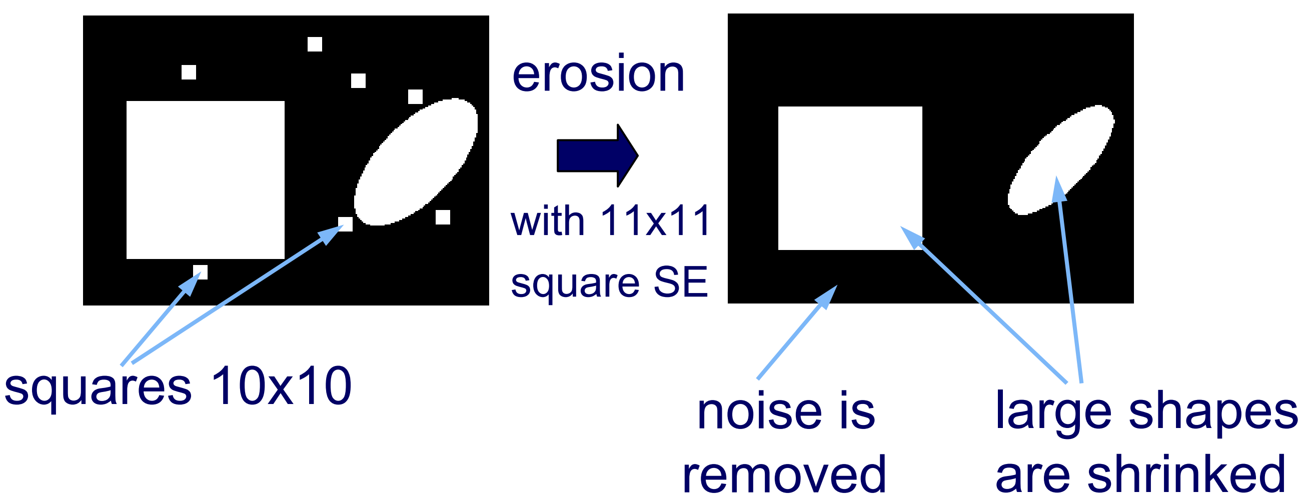


result of erosion

Erosion: properties and use

- Basic property: $\varepsilon_E(X) \subseteq X \subseteq \delta_E(X)$
- Consequences:
  - Eliminate connected components smaller than *E*
  - Eliminate narrow capes
  - Enlarge holes
- Example of application: eliminating irrelevant details (noise)



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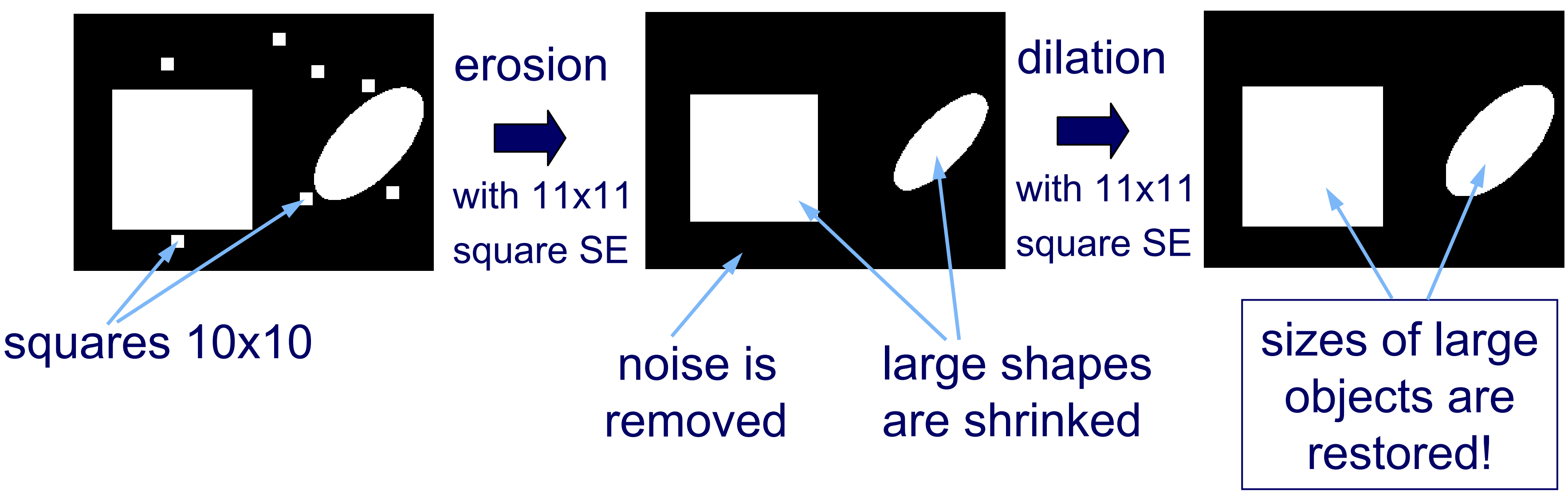
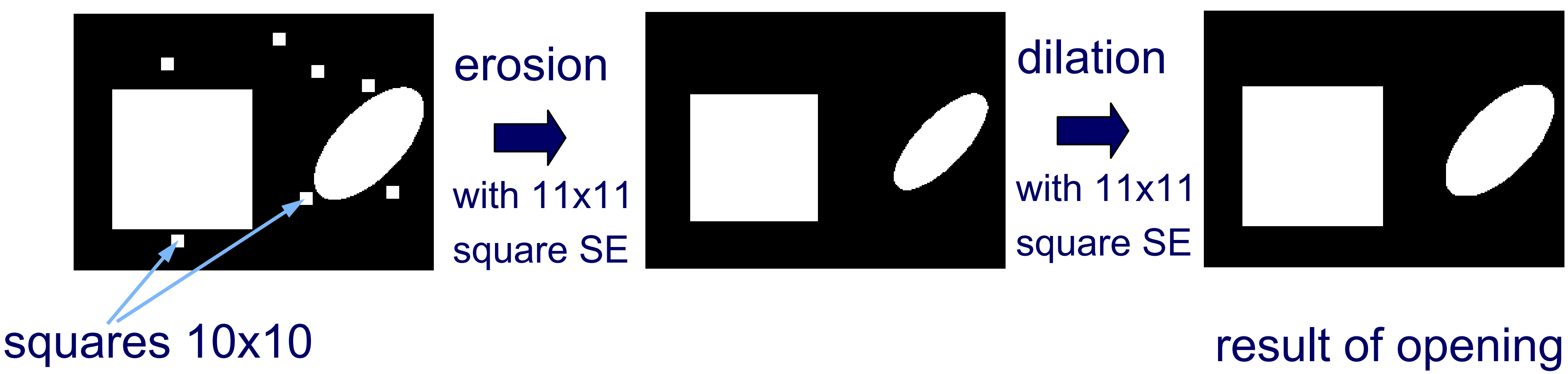


Opening for binary images

- ***Opening*** of a set X by a structuring element E is defined as the erosion of X by E followed by the dilation with the reflected (symmetric with respect to the origin) SE $\tilde{E}$:

$$\gamma_E(X) = \delta_{\tilde{E}}[\varepsilon_E(X)]$$

- Consequences:
  - Objects smaller than E disappear
  - Other objects remain "unchanged"



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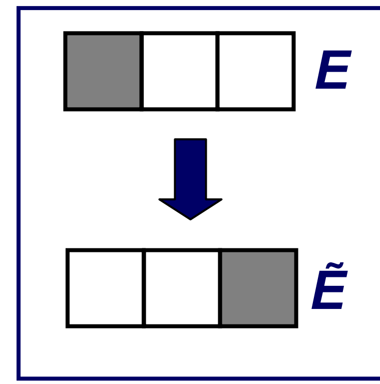

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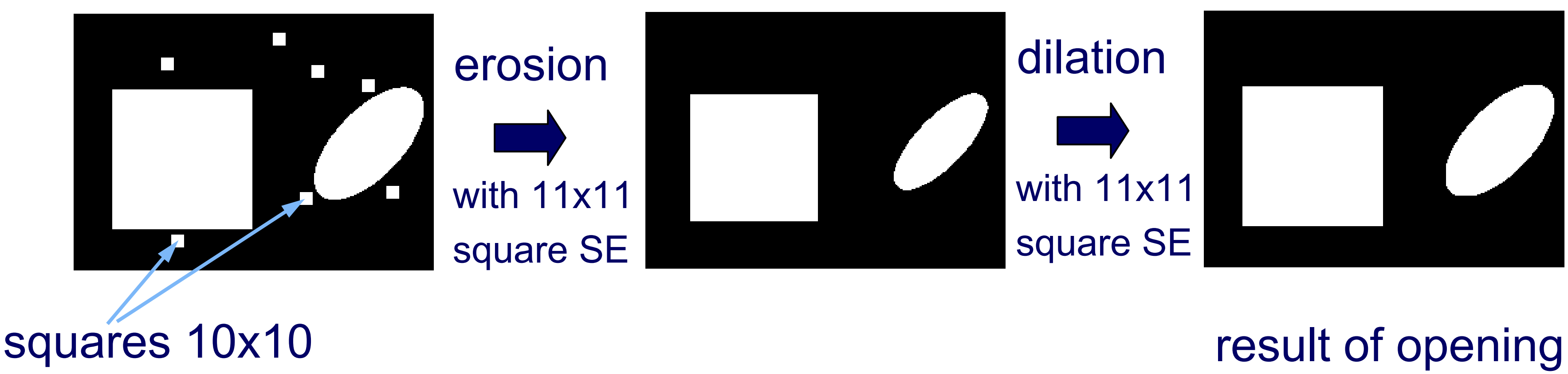

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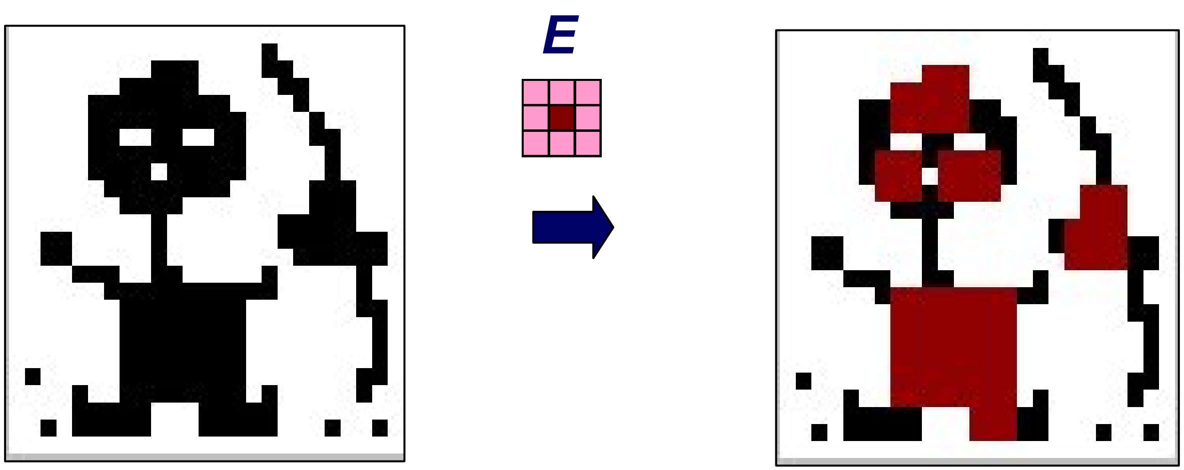


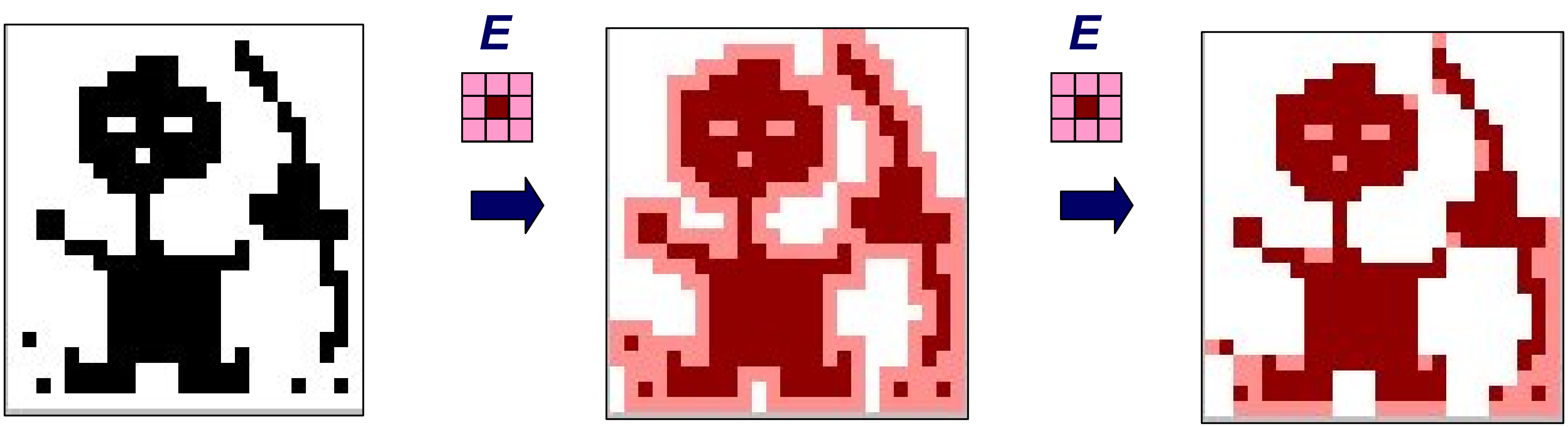
result of opening

Closing for binary images

- ***Closing*** of a set X by a structuring element E is defined as the dilation of X by E followed by the erosion with the reflected SE $\tilde{E}$:

$$\varphi_E(X) = \varepsilon_{\tilde{E}}[\delta_E(X)]$$

- Order: $\gamma_E(X) \subseteq X \subseteq \varphi_E(X)$
- Consequences:
  - Holes smaller than E are eliminated
  - Other objects remain “unchanged”



result of dilation

result of closing

Other MM operators: Top-hat

- ***Top-hat*** transformation of an image X is defined as the difference between the original image X and its opening γ:

$$TH(X) = X - \gamma(X)$$

- Consequences:
  - Objects smaller than E are extracted
  - Other objects disappear

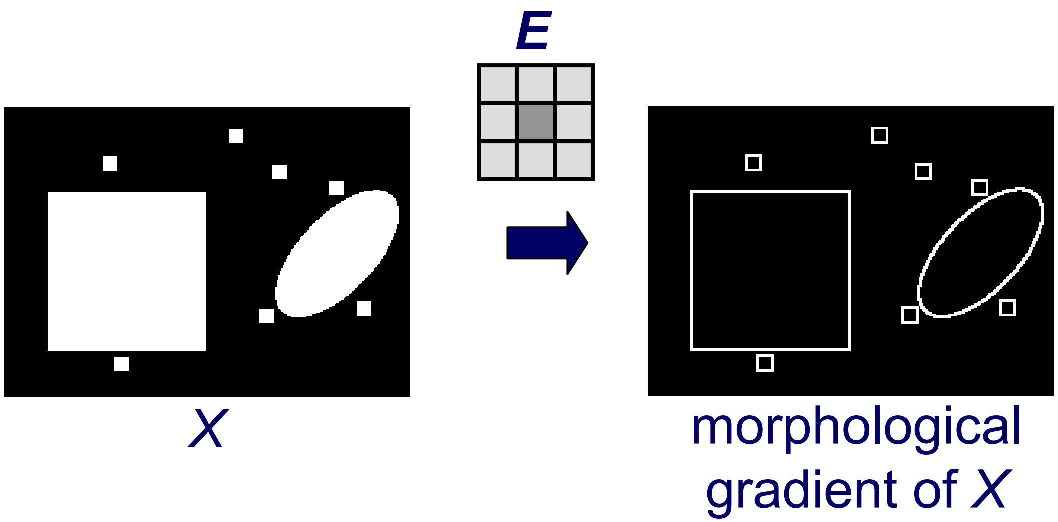
X $\gamma_{11x11}(X)$ $TH(X)$

Morphological gradient

- The basic ***morphological gradient*** of an image X is defined as the arithmetic difference between the dilation and the erosion of X by the elementary SE E :

$$\rho_E(X) = \delta_E(X) - \varepsilon_E(X)$$

- Only symmetric SEs are considered
- Tends to depend less on edge directionality (when compared to first derivatives)



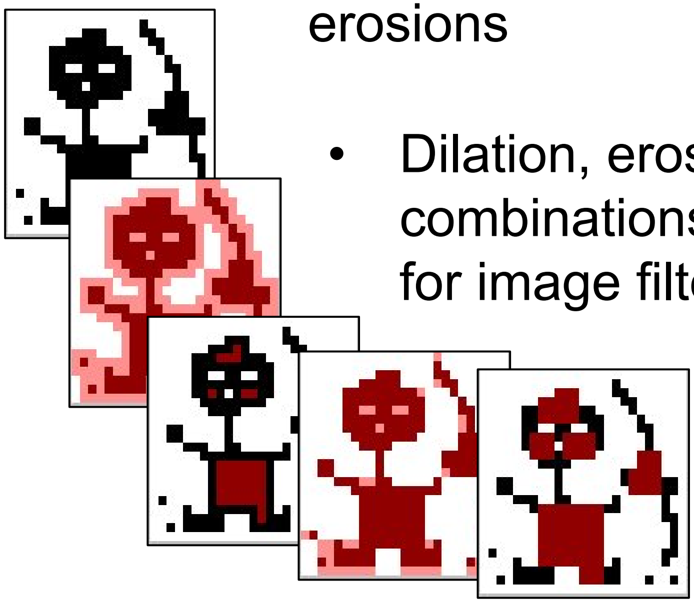
Summary: Basic concepts of MM

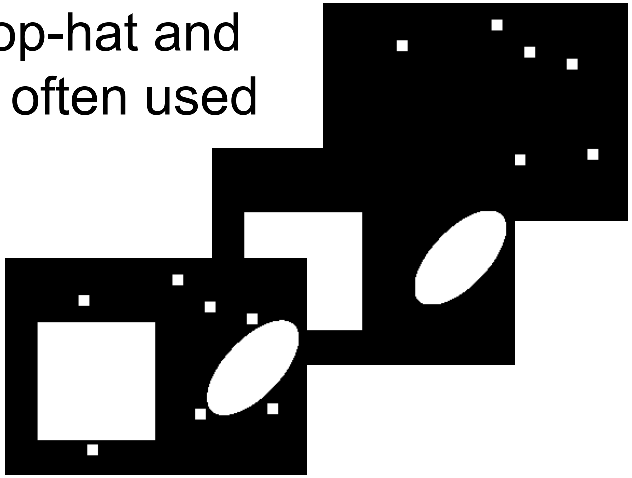
- Mathematical morphology can be defined as a theory for the analysis of spatial structures

- Basic idea of MM: locally compare structures within the image with a reference shape called the Structuring Element

- Dilation and erosion are two fundamental MM operators
  ➔ All other operators are expressed in terms of dilations and erosions

  - Dilation, erosion, opening, closing, top-hat and combinations of these operators are often used for image filtering

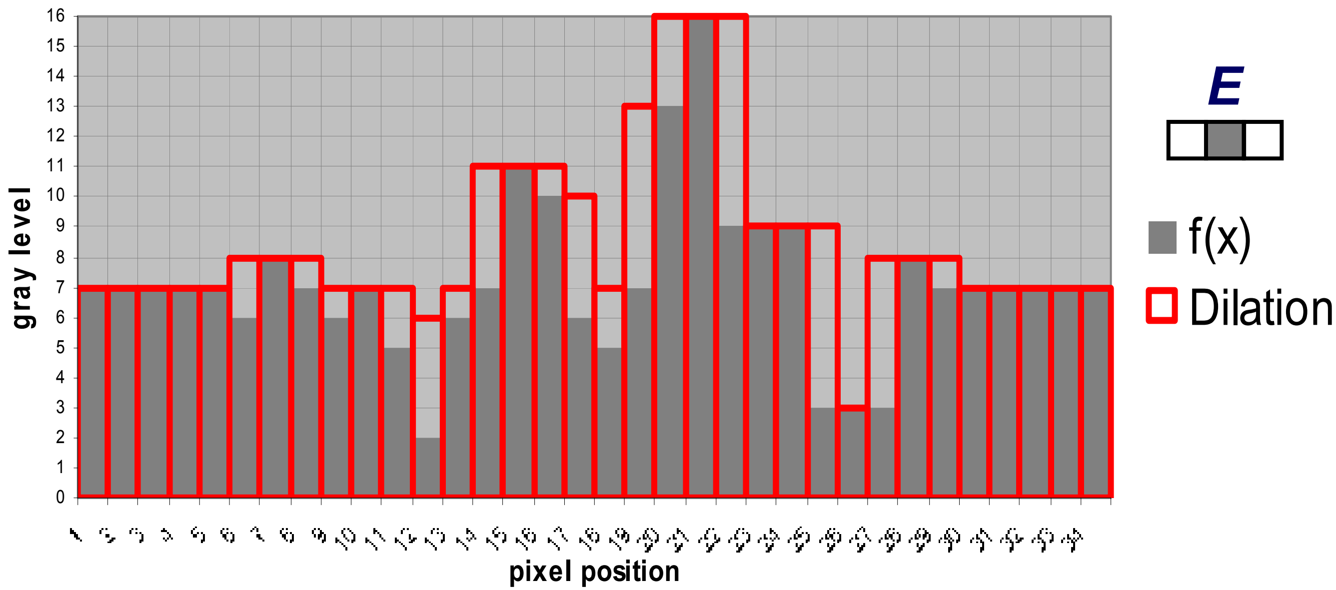




Mathematical morphology for grey-scale images

Dilation for grey-scale images

- ***Dilation:*** replace every pixel by the *maximum* value computed over the neighborhood defined by the structuring element



Dilation for grey-scale images

- ***Dilation:*** replace every pixel by the *maximum* value computed over the neighborhood defined by the structuring element

- Consequence:
  - Features that are *brighter* than their immediate surroundings are *enlarged*
  - Features that are *darker* than their immediate surroundings are *shrinked*
  - Effect driven by the size and shape of the SE

$\delta_5(X)$ ➡

SE: disk of radius = 5

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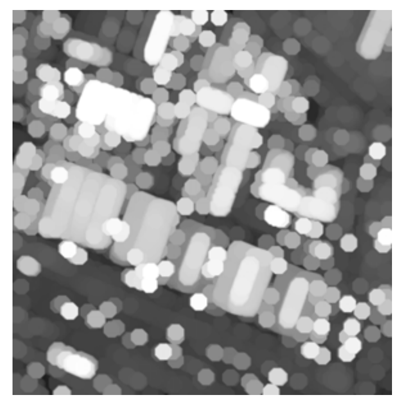
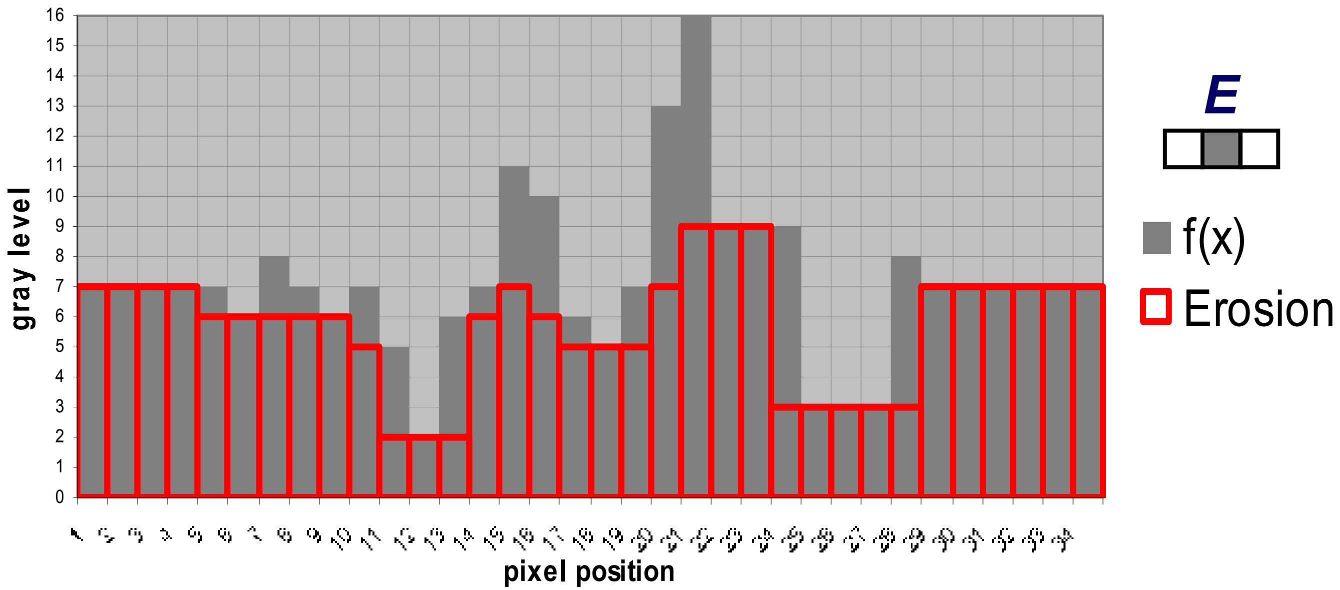
$\delta_{15}(X)$ ➡

SE: disk of radius = 15



Erosion for grey-scale images

- ***Erosion:*** replace every pixel by the *minimum* value computed over the neighborhood defined by the structuring element



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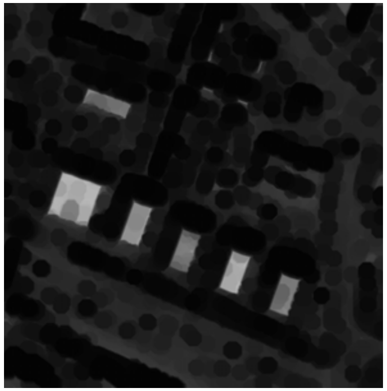
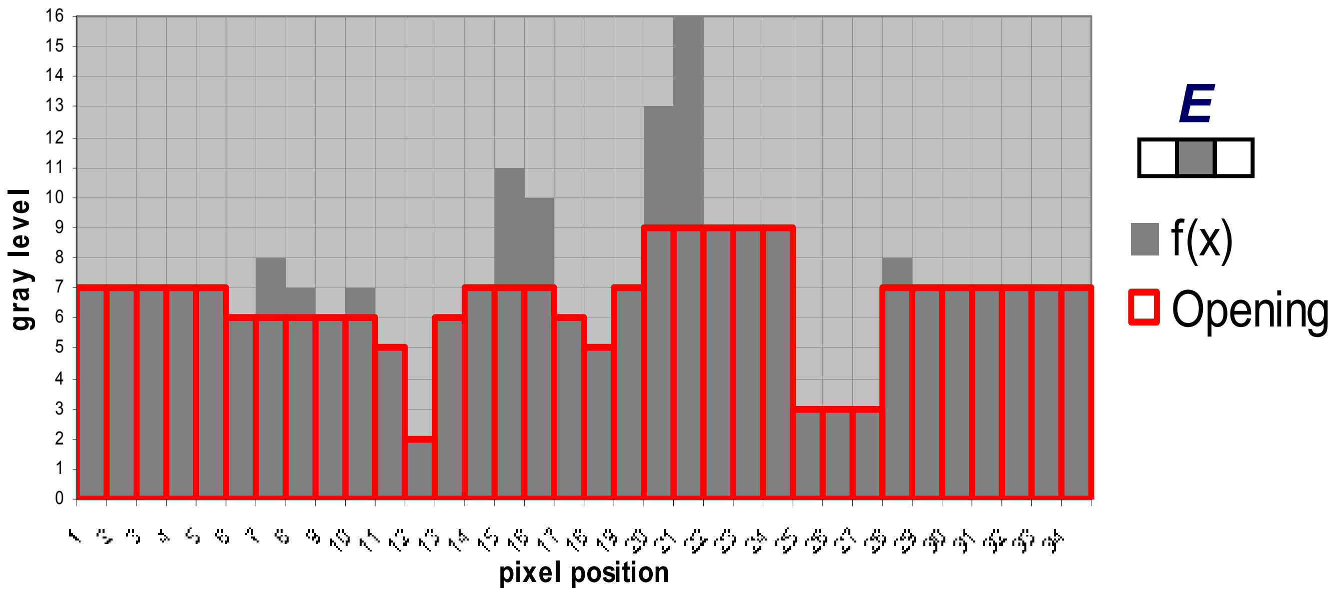


Opening for grey-scale images

- ***Opening:*** erosion followed by a dilation with the symmetrical structuring element



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- Consequence:
  - Features that are ***brighter*** than their immediate surroundings and ***smaller*** than the SE ***disappear***
  - Other features (dark, or bright and large) remain “unchanged”

$\varepsilon_5(X)$

$\delta_5(X)$

$\gamma_5(X)$

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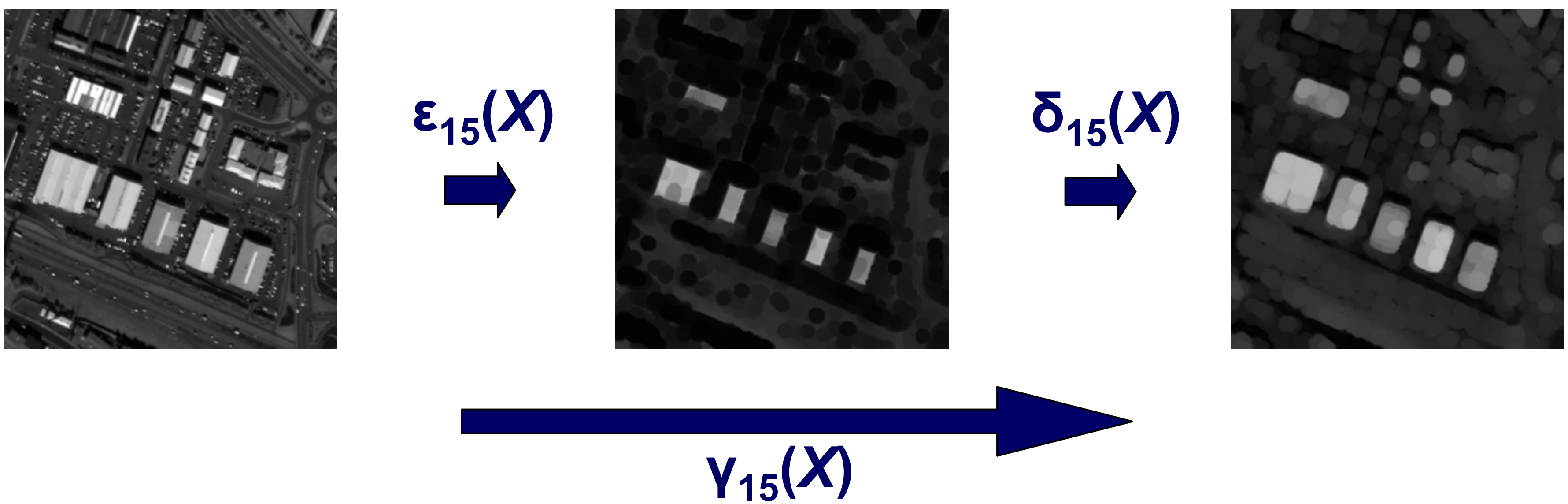

Closing for grey-scale images

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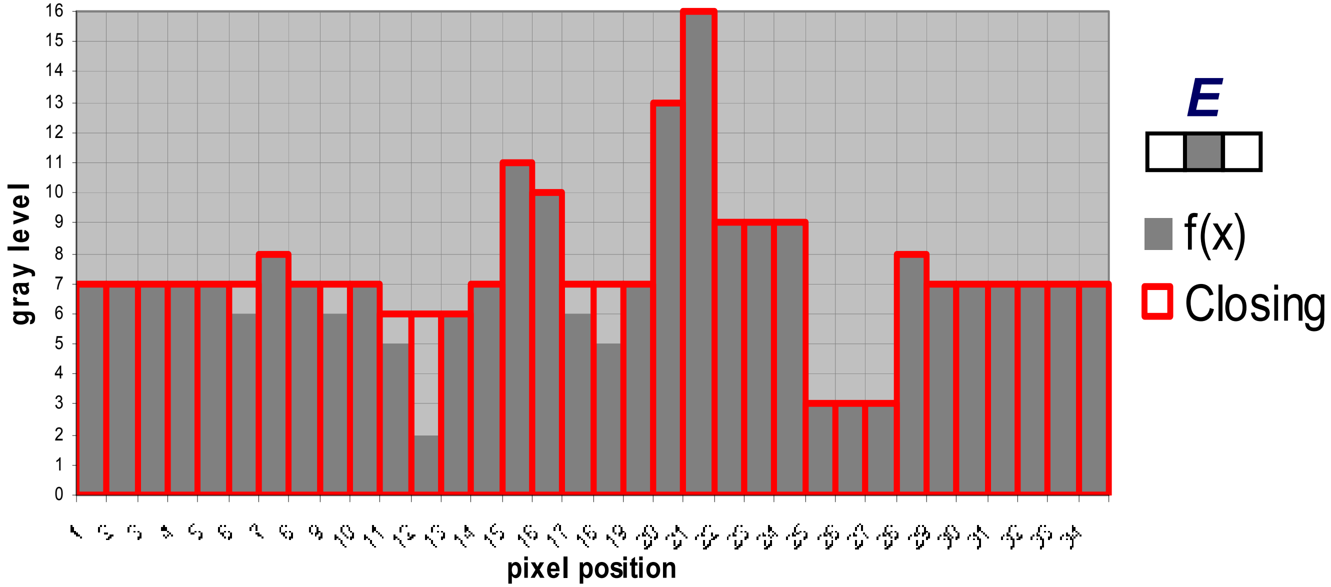

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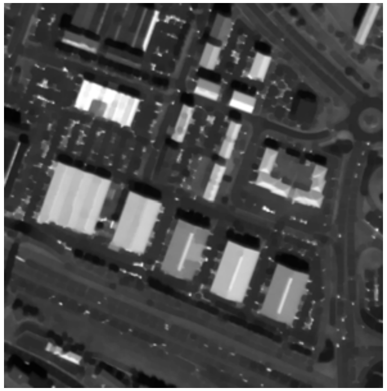
$\delta_5(X)$

$\varepsilon_5(X)$



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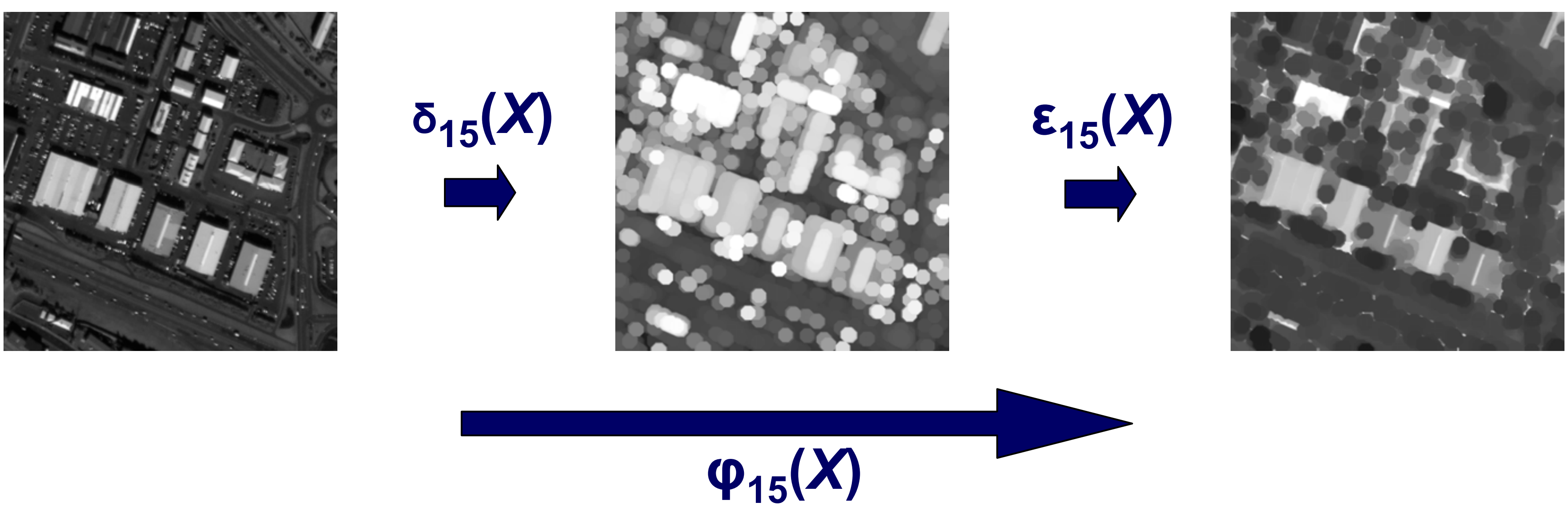


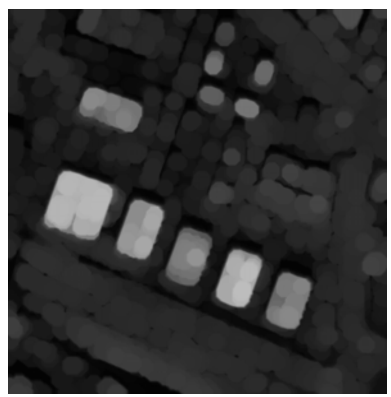
Geodesic reconstruction

- *Connected* operators
- Same properties, with no shape noise

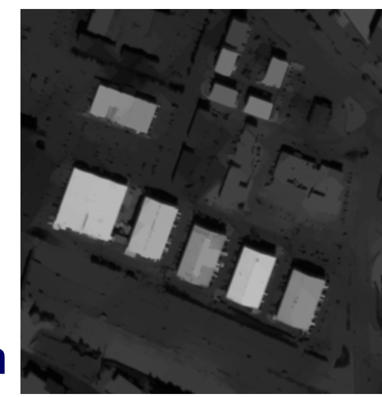
- Opening by reconstruction:
  - Preserves the shape of the objects that are not removed by erosion

Original image: X
Im1 := $\varepsilon_{15}(X)$
Im2 := $\delta_{15}(\text{Im1})$
Im3 := min(Im2, X)
Im1 := Im3
CV

$\gamma_{15}(X)$



opening by reconstruction



To read: P. Soille, Morphological Image Analysis, 2nd ed. Springer-Verlag, 2003.