Biologically Plausible Trajectory Generator

Thierry Viéville

(Presentation thanks to Pierre Kornprobst)
• A Very General Problem

Finding your "way" in geographic map
Making a gesture in a complex environment
Driving without skid or with a power bound
Trajectory Generation Problem

- But also a High-dimensional abstract problem...
State of the Art

• In robotics (two keys)
  Connolly-Grupen (199*) : harmonic potential. Elegant solution of the problem but subject to the curse of dimensionality

• In biology (see e.g. Poucet et al. 2004)
  Seems easy !
Four Key Aspects

• The problem is to be solved:
  
  **Action**: Exploration découvre obstacle +
  generation trajectoire pour aller but
  
  **Global level**: e.g. labyrinth
  
  **Dynamics**: Look and move paradigm (here)
  
  **Degrees of freedom**: A large number of

Very difficult as a Mathematical problem!

Very easily solved by a mice brain!
About Biological Trajectory Generator

- Hypocampal and related structures are involved in trajectory generation

![Diagram of brain structures]

- Entorhinal Cortex
- Dental Gyrus
- CA3 \(\rightarrow\) CA1
- Prefrontal cortex
- Ventral striatum
- Sensor inputs
- Environment loci
- Present location
- Sparse locus map(s)
- Shifts of the present location
- Goal location
- Motor command
About Biological Trajectory Generator

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About Biological Trajectory Generator

- Internal representation: place fields (PF) within locus maps (LM)
- No topography but recruitment on request
- Reactivation of LM when spatial-area changes
- Obstacles PF -> elongated shapes
  Intermediate goals -> isotropic shapes

- Used for [goal oriented + wandering] behavior
- How is this use in/as a sensori-motor loop?
Potential and Trajectory

- Obstacles to avoid are the potential max
- The goal(s) corresponds to its min
- Let's throw a *regular sheet* on this space
- And let it ``roll-down`` along this relief
Potential and Trajectory

• Two caveats with potentials

  Local minima!

  Solutions: Convex profiles, VonBaumgarten curve, ..
  Here we use harmonic potential
  minimize collision probability
  very general (e.g. non holonomic constraints)
  Connolly-Grupen (1994)

Curse of dimensionality!
(exponential complexity with dimension)
Solution: Parametric potential
Potential and Trajectory

Let us consider (here functions are at least twice differentiable):
(a) a system, defined by a state vector $\mathbf{x} \in \mathbb{R}^n$, $n \geq 2$
(b) an initial state, written $\mathbf{x}_0 \in \mathbb{R}^n$,
(c) $r$ constraints defined by scalar inequalities $c_i(\mathbf{x}) > 0$, $i \in \{1..r\}$,
(d) a goal defined by an constraint of the form $c_0(\mathbf{x}) \leq 0$,

We consider a connected domain:
$$\mathcal{U} = \bigcup_{i \in \{0..r\}} \mathcal{U}_i \text{ with } \mathcal{U}_i = \{\mathbf{x}, c_i(\mathbf{x}) > 0\} \text{ with } \mathbf{x}_0 \in \mathcal{U}$$
Potential and Trajectory

Here we define harmonic potentials $V : \mathcal{U} \rightarrow \mathcal{R}$ thus

$$\forall x \in \mathcal{U}, \Delta V(x) = 0 \Rightarrow ||\nabla V(x)|| > 0$$

with

$$\lim_{x \to \partial \mathcal{U}_0} V(x) = -\infty \text{ and } \forall i > 0, \lim_{x \to \partial \mathcal{U}_i} V(x) = +\infty$$

and consider trajectory $\gamma : \mathcal{R}^+ \rightarrow \mathcal{R}^n$ such that:

- starting at the initial point $\gamma(0) = x_0$, with $\gamma'(t) = -\nabla V(\gamma(t))$
- verifying the problem constraints $\forall t \in \mathcal{R}^+, \gamma(t) \in \mathcal{U}$,
- and reaching the goal $\lim_{t \to \infty} c_0(\gamma(t)) = 0$ (asymptotically)
Potential and Trajectory

- **Global goal influence (adaptive gain)**
- **Obstacle local influence**

Suitable harmonic potential are *adelicians*

\[
V(x) = \sum_{i=1}^{r} A_i(x) - A_0(x)
\]

with

\[
\sum_{\partial U_i} \mu_{ij} \Phi(x - v_{ij})
\]

**Only closest points**

Local radial symmetric harmonic

\[
V(x) = \frac{\lambda}{||x||^{n-2}}
\]
Potential and Trajectory

- **Global goal influence (adaptive gain)**
- **Obstacle local influence**

Suitable harmonic potential are adelicions:

\[ V(x) = \sum_{i=1}^{r} A_i(x) - A_0(x) \]

with \[ A_i(x) = \sum_{y_{ij} \in \partial U_i} \mu_{y_{ij}} \Phi(x - y_{ij}) \]

where \( \Phi() \) is the fundamental radial symmetric harmonic function

(for \( n > 2 \), \( \Phi(x) = \frac{\lambda}{||x||^{n-2}} \))

Only closest points
Potential and Trajectory

- Yields a sparse map representation \( O(\text{trajectory-length}) \) ! no direct dim. curse

- Adaptive representation
  - exploration within navigation
  - sensori-motor loop embending
    (including gain control)

- Biological plausibility is well-founded
Labyrinth Experiment

Start  End  Map's loci  Potential field  Trajectory
Various Behaviors

Exploration versus navigation (simple gain adjustment)

$$\gamma'(t) = -G \nabla V(\gamma(t))$$
Learning When Re-run

Improved trajectory

... and ... further exploration
Using Various Data Input

When the map is known

With only the line of sight as input
Wandering Behavior (no goal)
With Intermediate goals
Extension to a 10 d.of. arm

- Considering a high dimensional problem from a trap, begins .. and escape:
Conclusion

• Biological sparse locus maps have inspired a solution against the harmonic potential curse of dim.

• Improved harmonic potential methods show that biological locus maps are sufficient mechanisms to explain exploration / navigation in partially known environments.
Perspectives

- Better link with biological models

- More general behavior generation (gestures, manoeuvres, plans, ..)
Questions?

mailto:Thierry.Vieville@sophia.inria.fr

Software available + on-line demos
http://www-sop.inria.fr/odyssee/imp/trajectory

Detailed report
http://www-sop.inria.fr/rapports/sophia/RR-4539.html

Large-public presentation
http://interstices.info/display.jsp?id=c_14155