Shape Reconstruction

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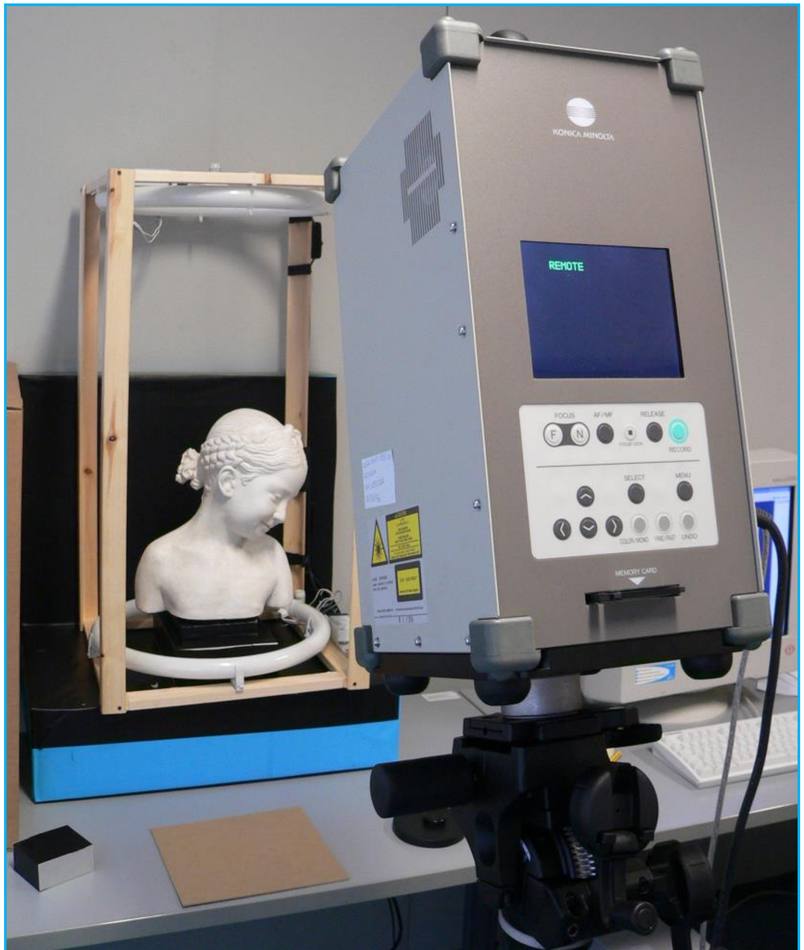
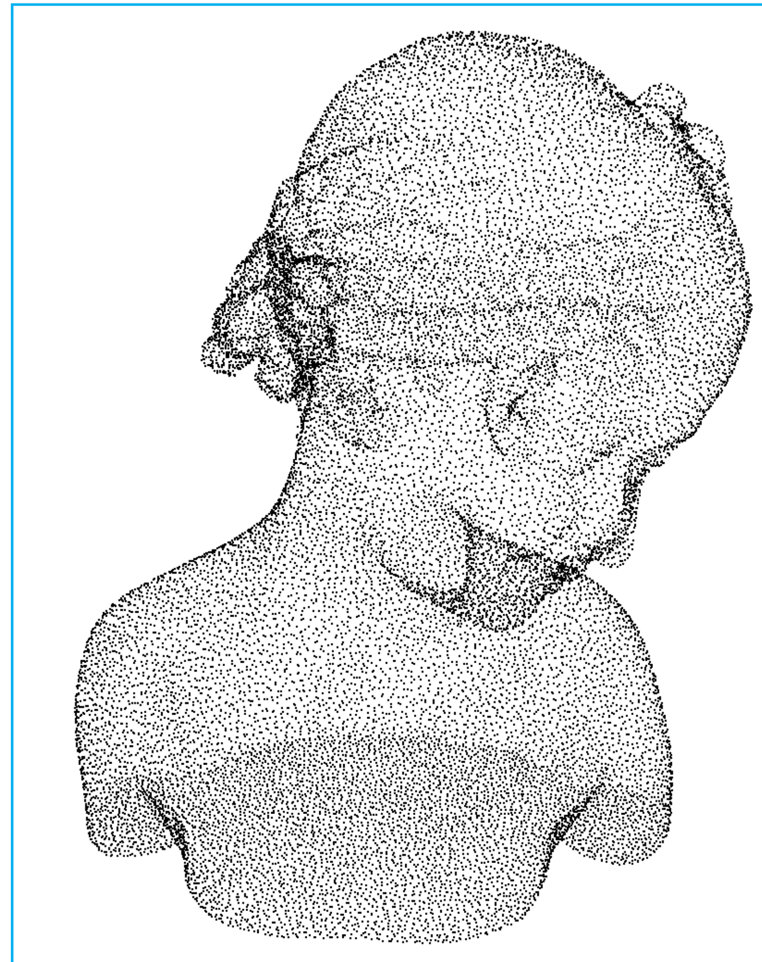
Outline

- Sensors
- Problem statement
- Computational Geometry
  - Convex hull, Voronoi/Delaunay, alpha-shapes

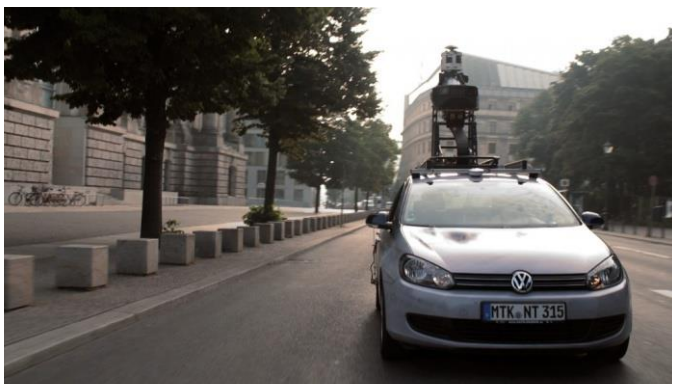
SENSORS

Laser scanning

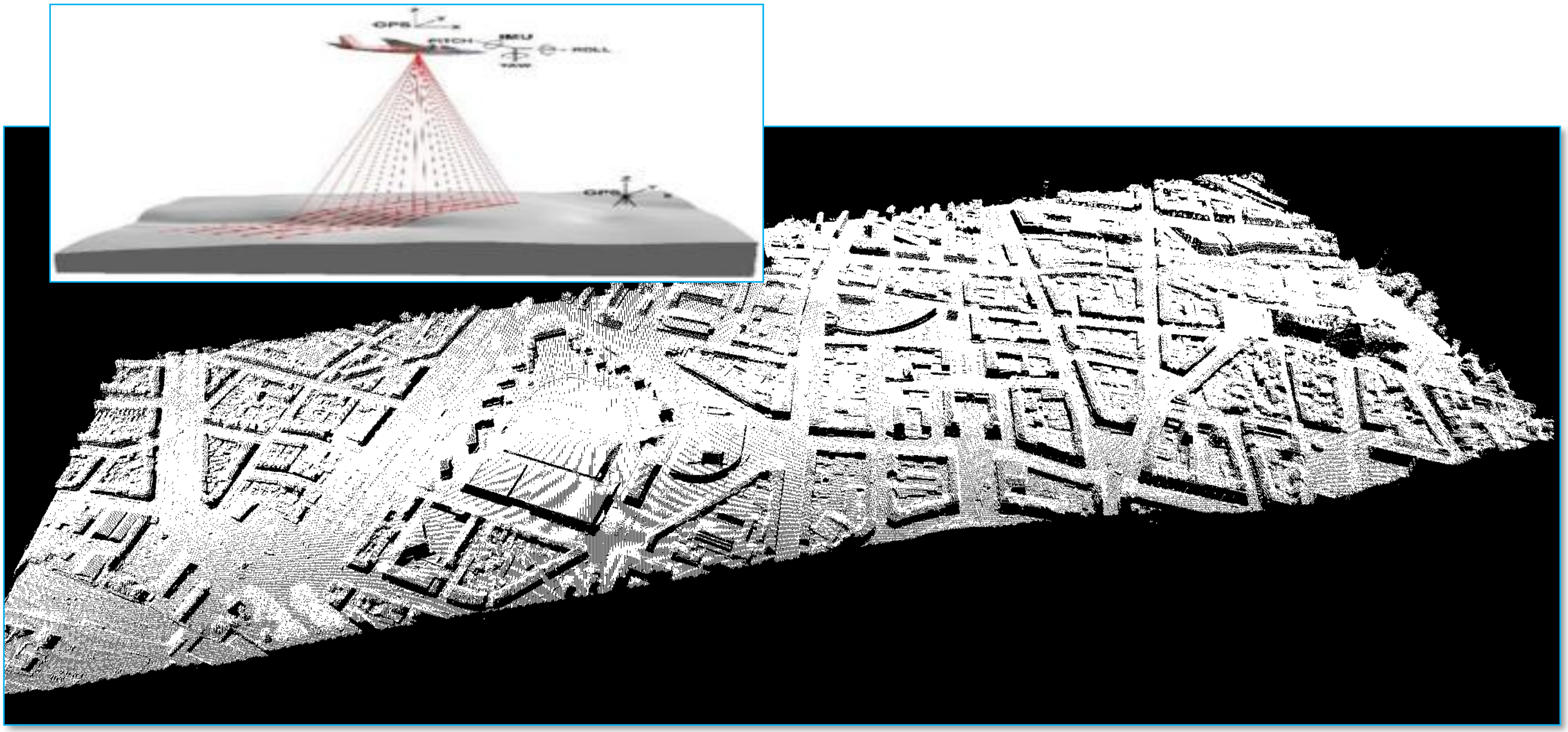




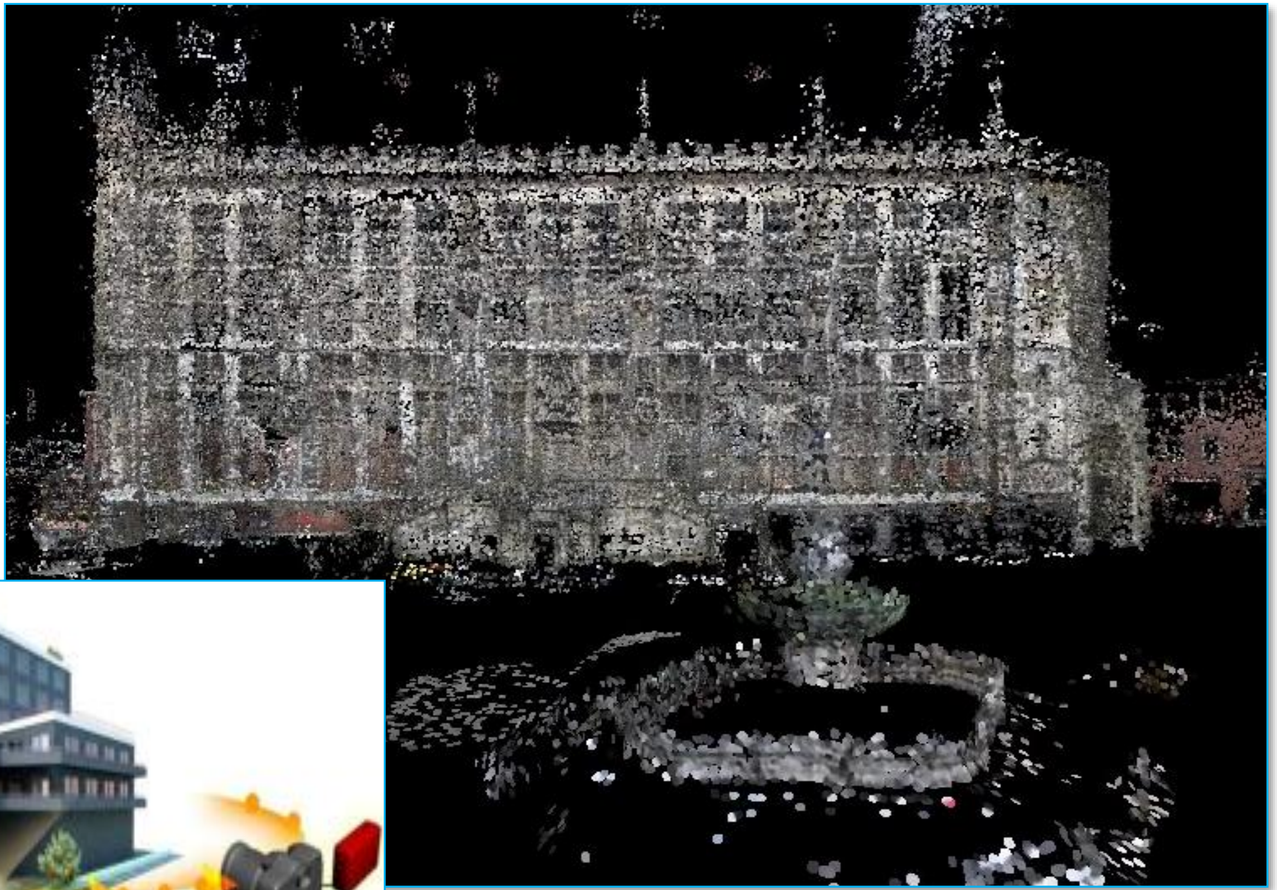
Car-based Laser



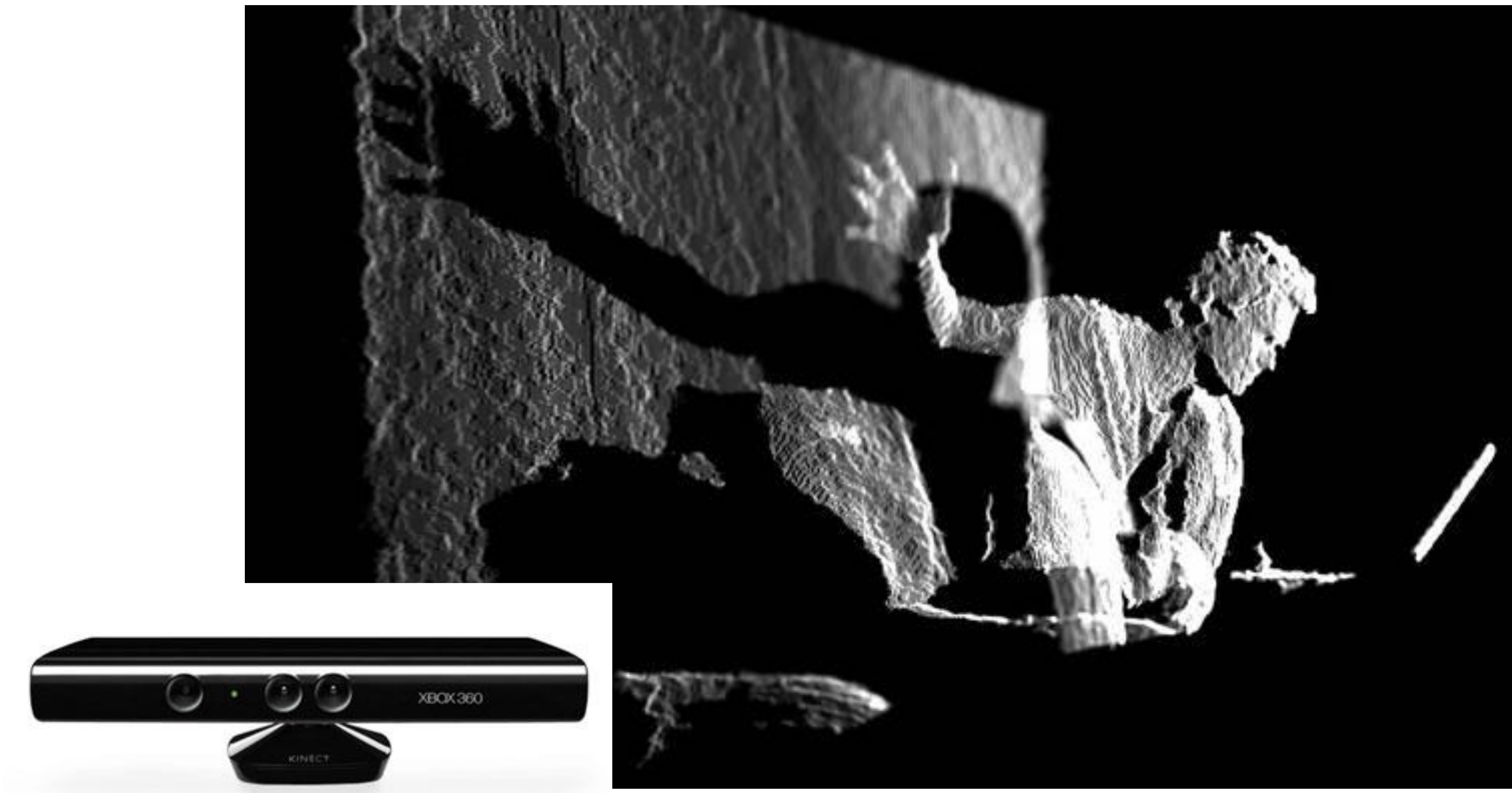
Airborne Lidar



Multi-View Stereo (MVS)



Depth Sensors

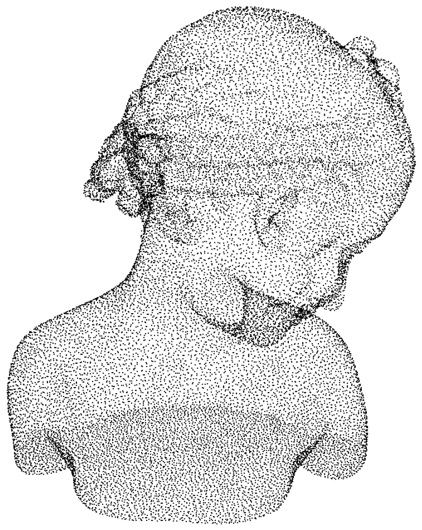


PROBLEM STATEMENT

Reconstruction Problem

Input: point set *P* sampled over a surface *S*:

- Non-uniform sampling
- With holes
- With uncertainty (noise)



point set

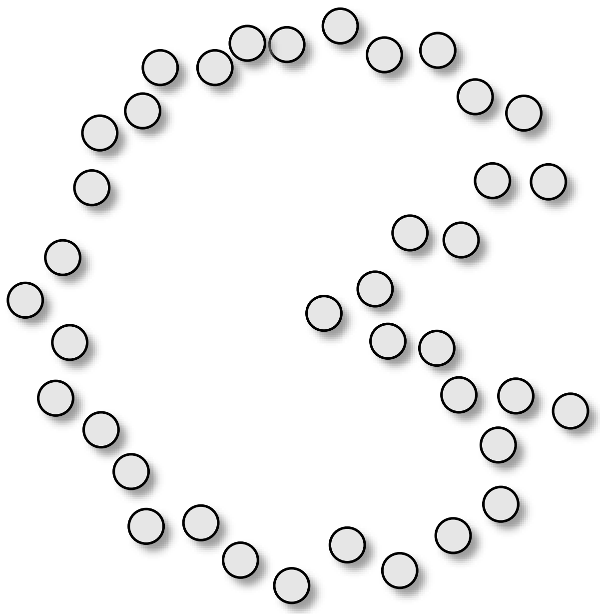
Output: surface

- Approximation of *S* in terms of topology and geometry
- Desired:
  - Watertight
  - Intersection free

reconstruction

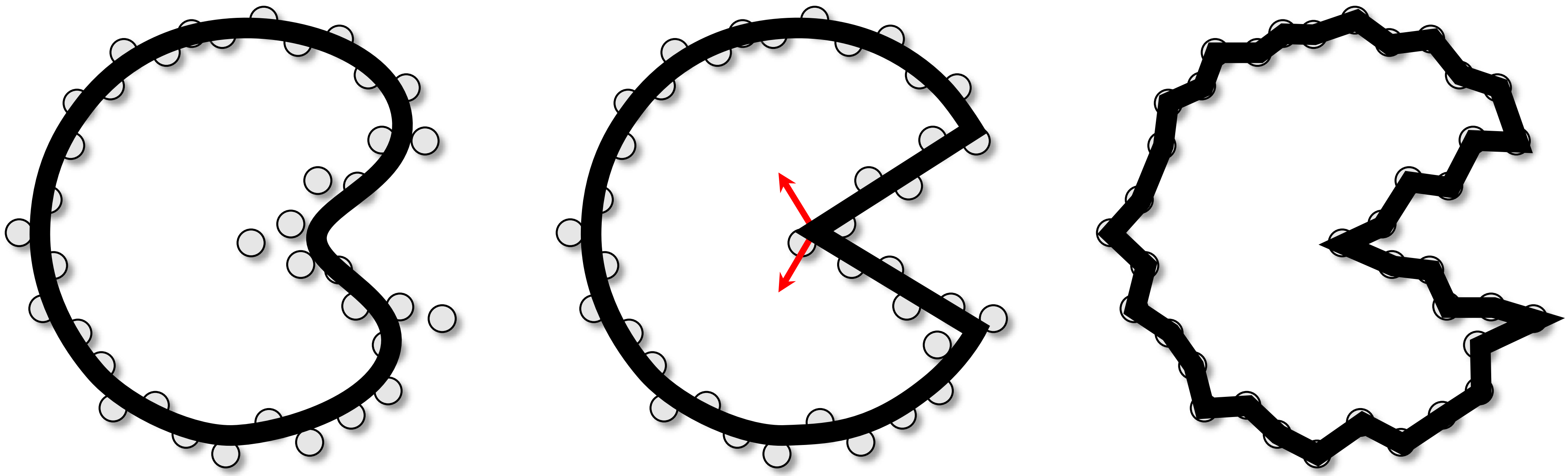
surface

Ill-posed Problem



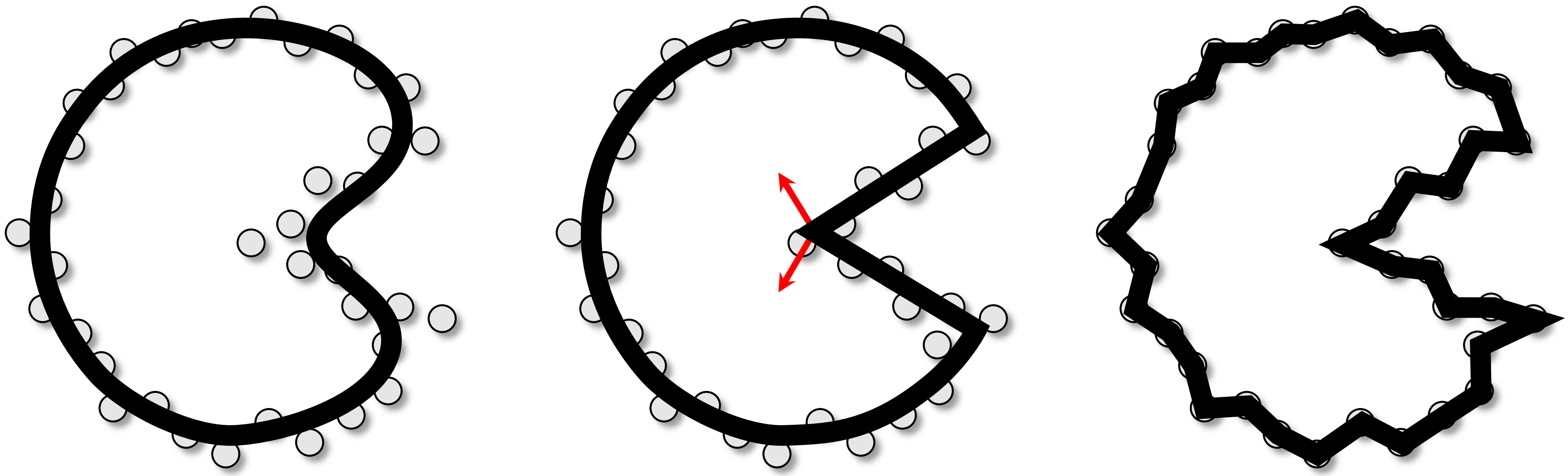
Many candidate surfaces for the reconstruction problem!

Ill-posed Problem



Many candidate surfaces for the
reconstruction problem! How to pick?

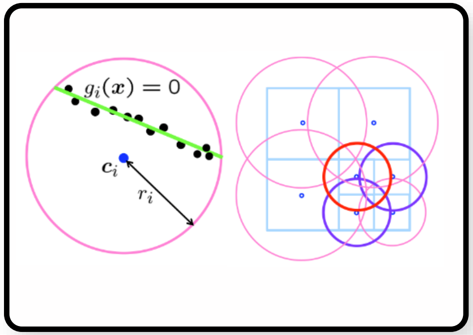
Priors



Smooth Piecewise Smooth "Simple"

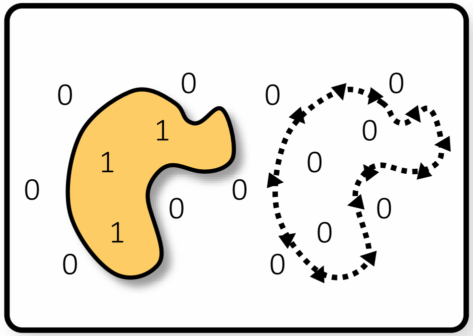
Surface Smoothness Priors

Local Smoothness



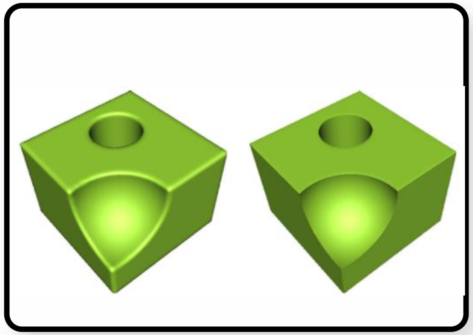
Local fitting
No control away from data
Solution by interpolation

Global Smoothness



Global: linear, eigen, graph cut, ...
Robustness to missing data

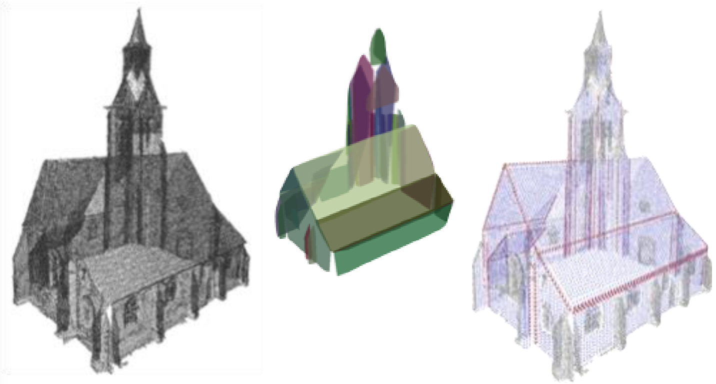
Piecewise Smoothness



Sharp near features
Smooth away from features

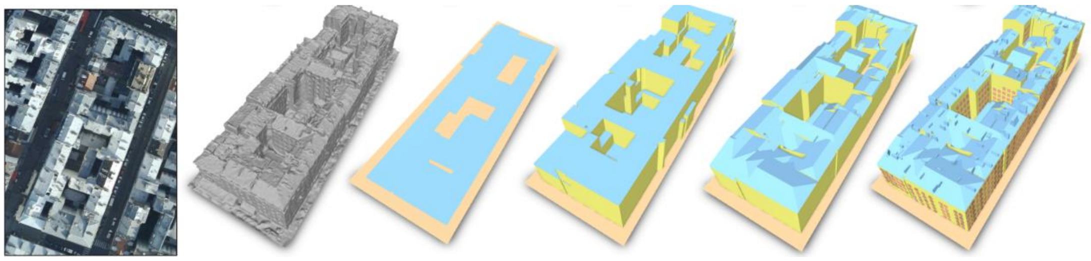
Domain-Specific Priors

Surface Reconstruction
by Point Set Structuring



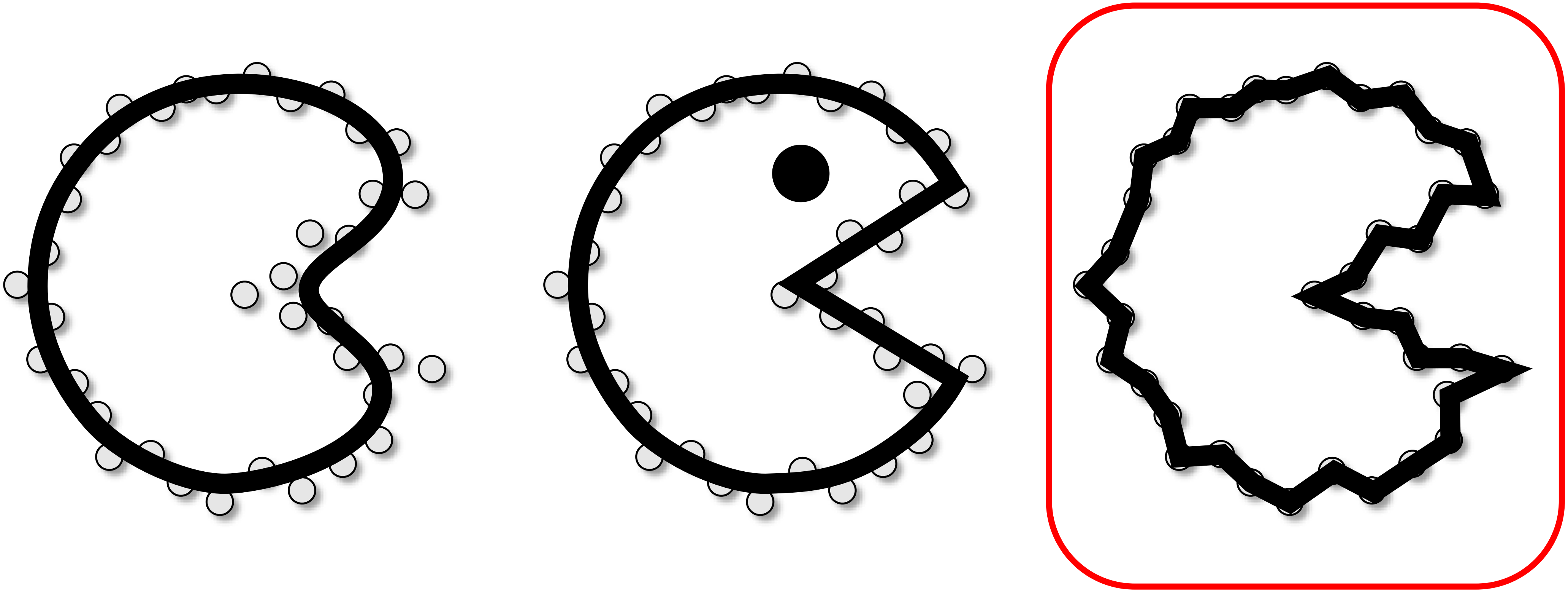
[Lafarge - A. EUROGRAPHICS 2013]

LOD Reconstruction
for Urban Scenes



[Verdie, Lafarge - A. ACM Transactions on Graphics 2015]

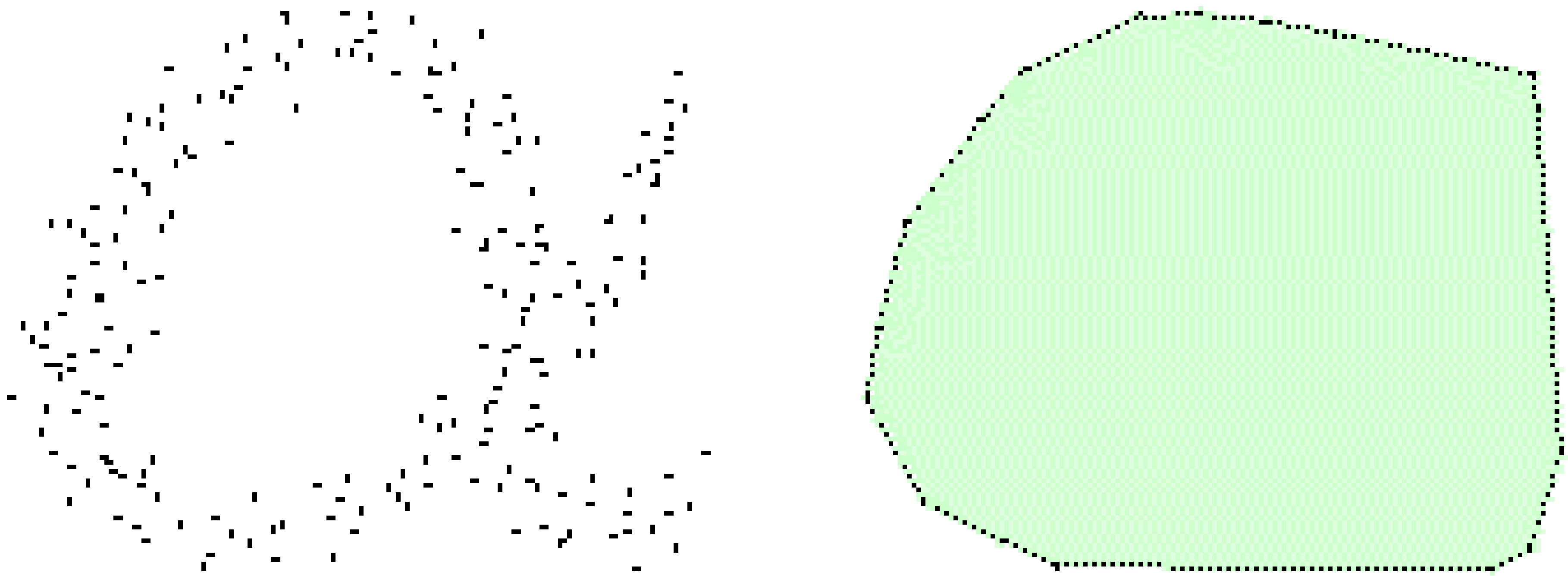
Warm-up



Smooth　　Piecewise Smooth　　“Simple”

CONVEX HULL

Convex Hull

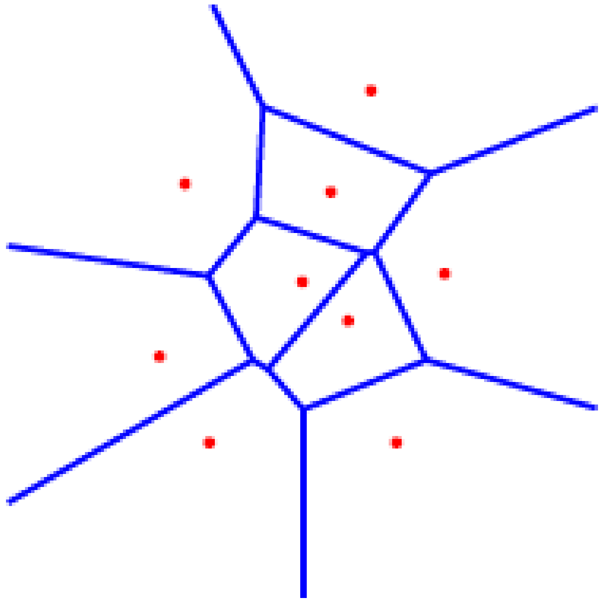


VORONOI / DELAUNAY

Voronoi Diagram

Let $\mathcal{E} = \{\mathbf{p_1}, \ldots, \mathbf{p_n}\}$ be a set of points (so-called sites) in $\mathbb{R}^d$. We associate to each site $\mathbf{p_i}$ its Voronoi region $V(\mathbf{p_i})$ such that:

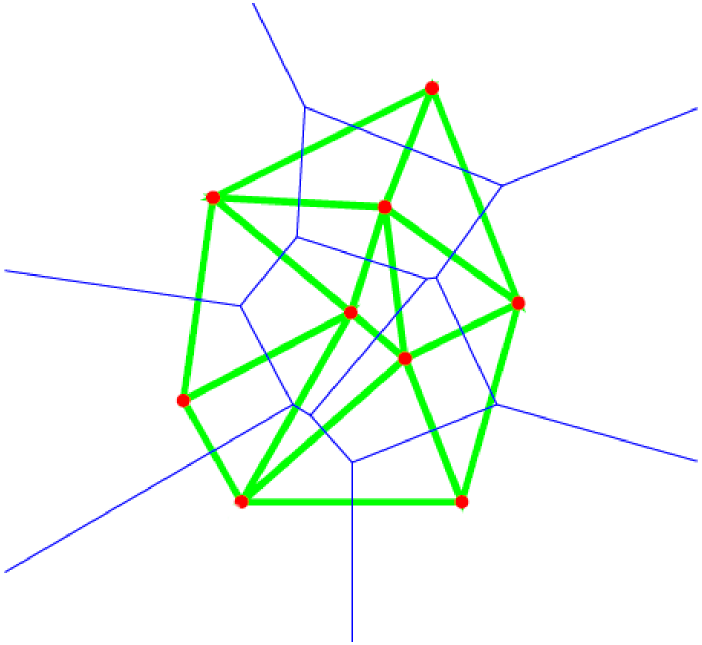
$$V(\mathbf{p_i}) = \{\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x} - \mathbf{p_i}\| \leq \|\mathbf{x} - \mathbf{p_j}\|, \forall j \leq n\}.$$



Delaunay Triangulation

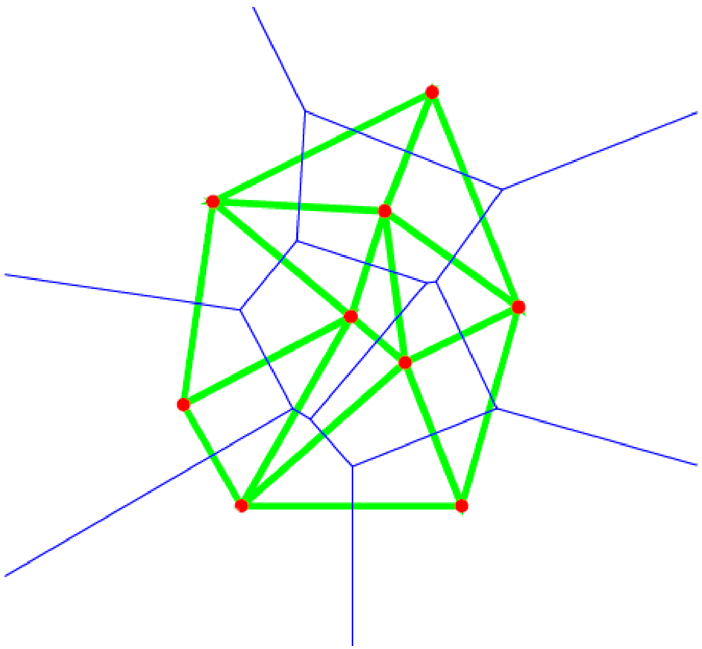
Dual structure of the Voronoi diagram.

The Delaunay triangulation of a set of sites E is a simplicial complex such that k+1 points in E form a Delaunay simplex if their Voronoi cells have nonempty intersection



Delaunay-based

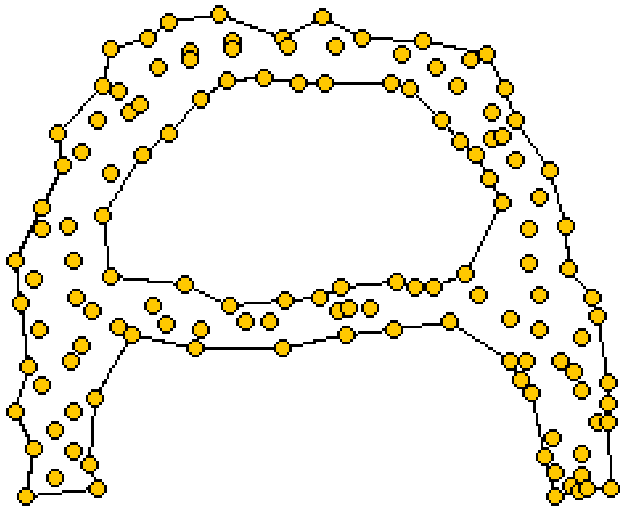
Key idea: assuming dense enough sampling, reconstructed triangles are Delaunay triangles.

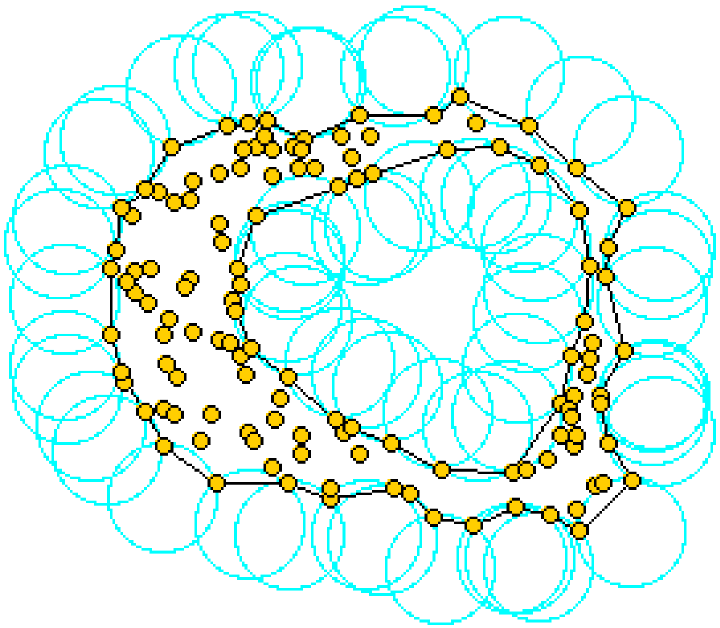


First define

- Medial axis
- Local feature size
- Epsilon-sampling

Alpha-Shapes [Edelsbrunner, Kirkpatrick, Seidel]





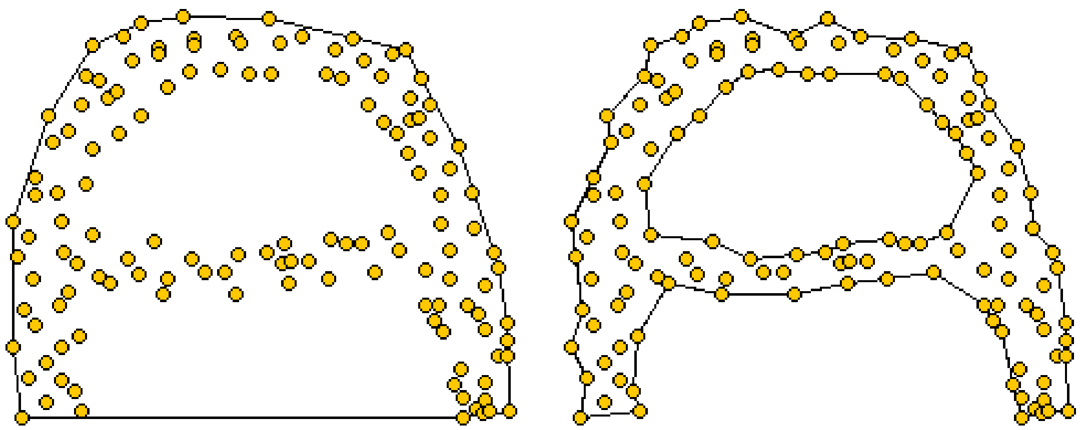
Segments: point pairs that can be touched by an empty disc of radius alpha.

Alpha-Shapes

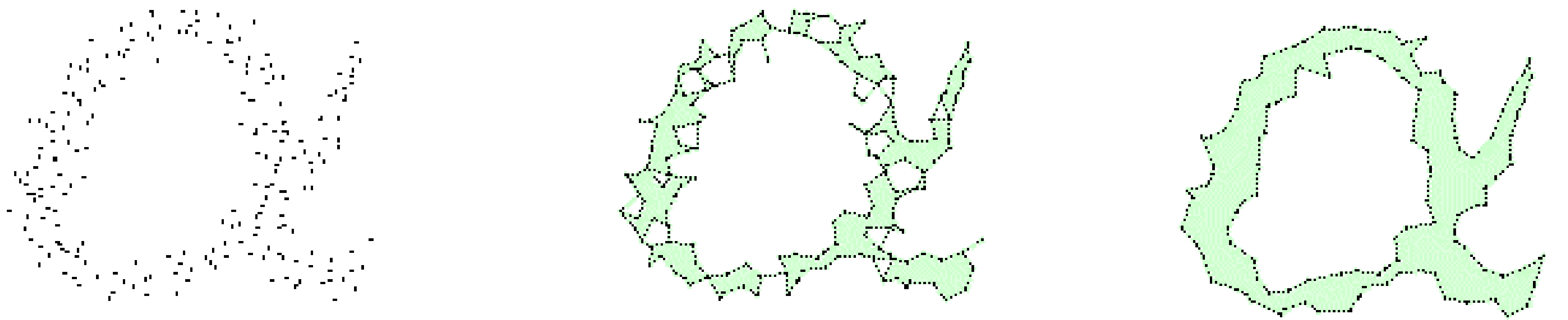
In 2D: family of piecewise linear simple curves constructed from a point set P.

Subcomplex of the Delaunay triangulation of P.

Generalization of the concept of the convex hull.

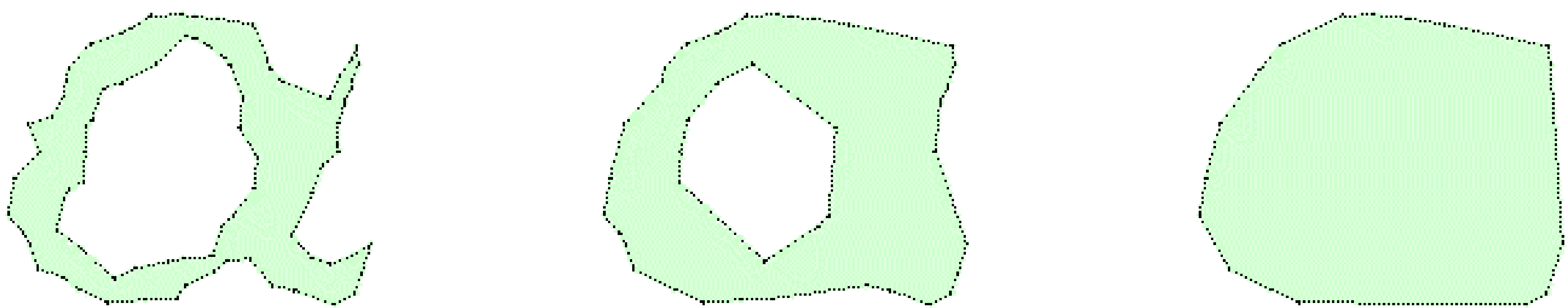


Alpha-Shapes



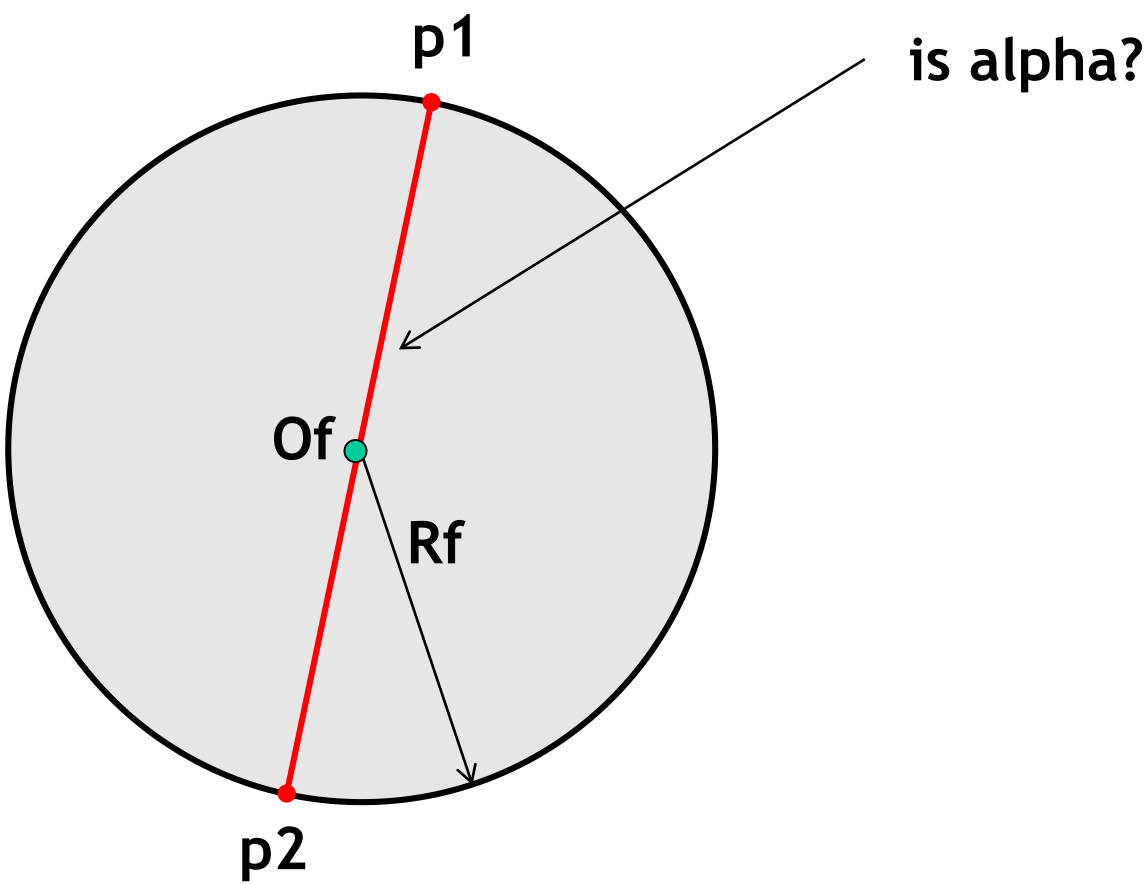
$\alpha = 0$

Alpha controls the desired level of detail.

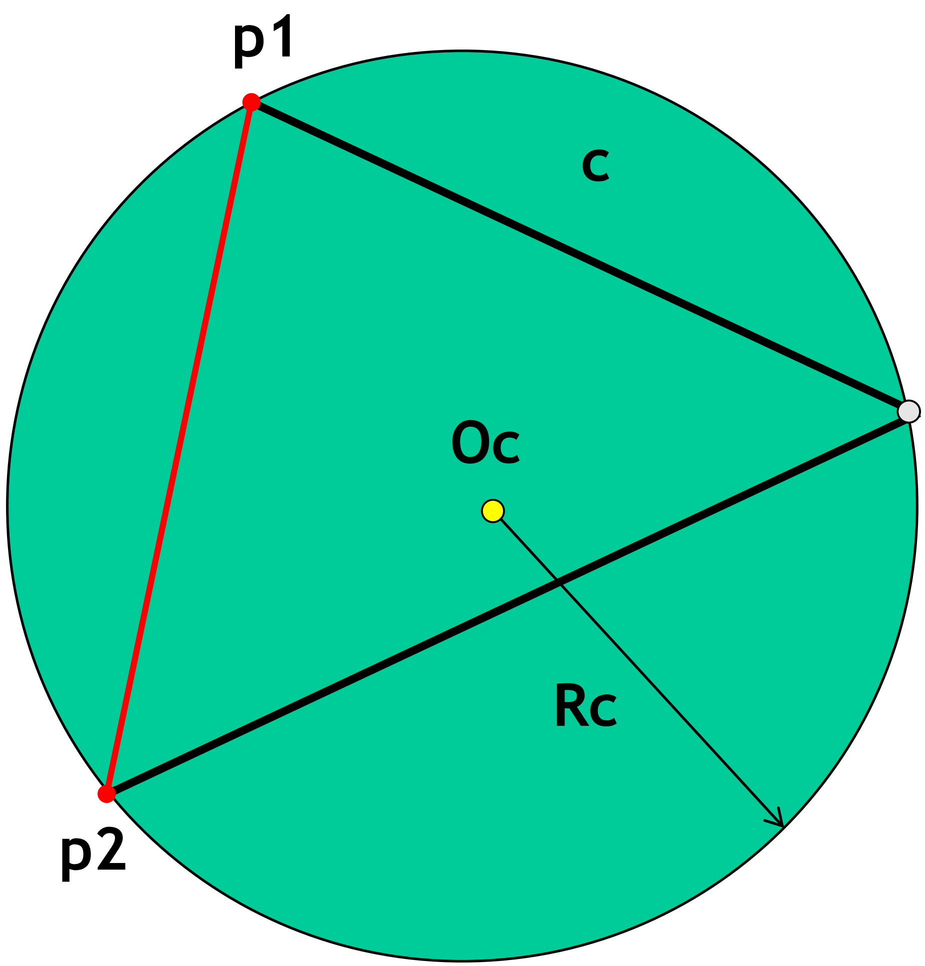


$\alpha = \infty$

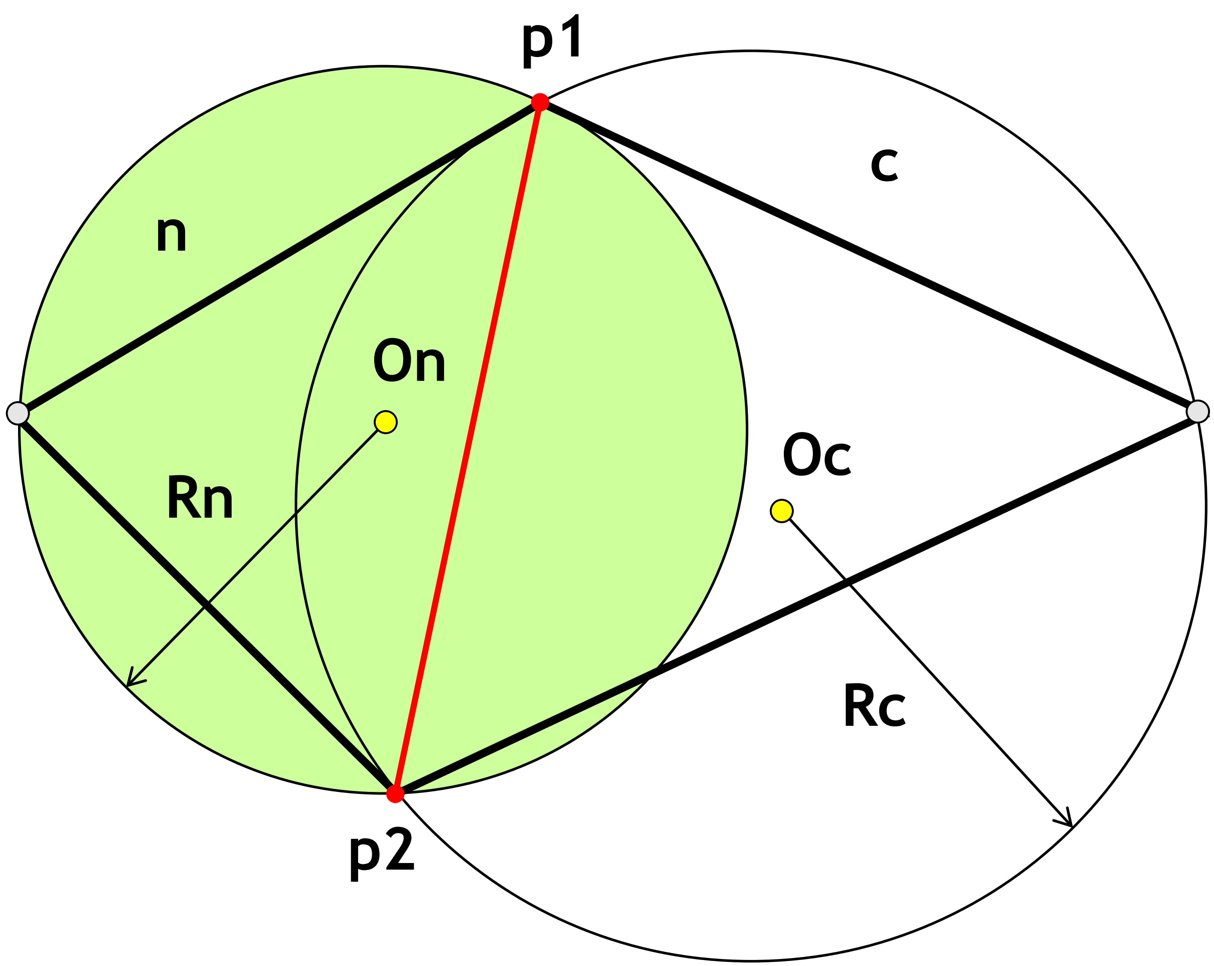
Computing Alpha-Shapes



Computing Alpha-Shapes

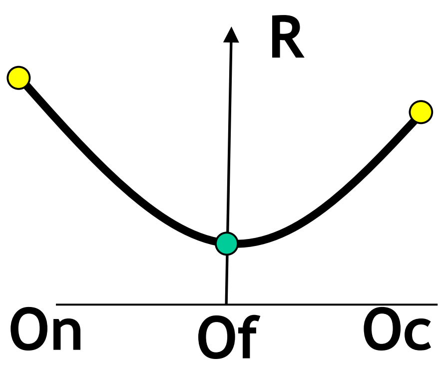


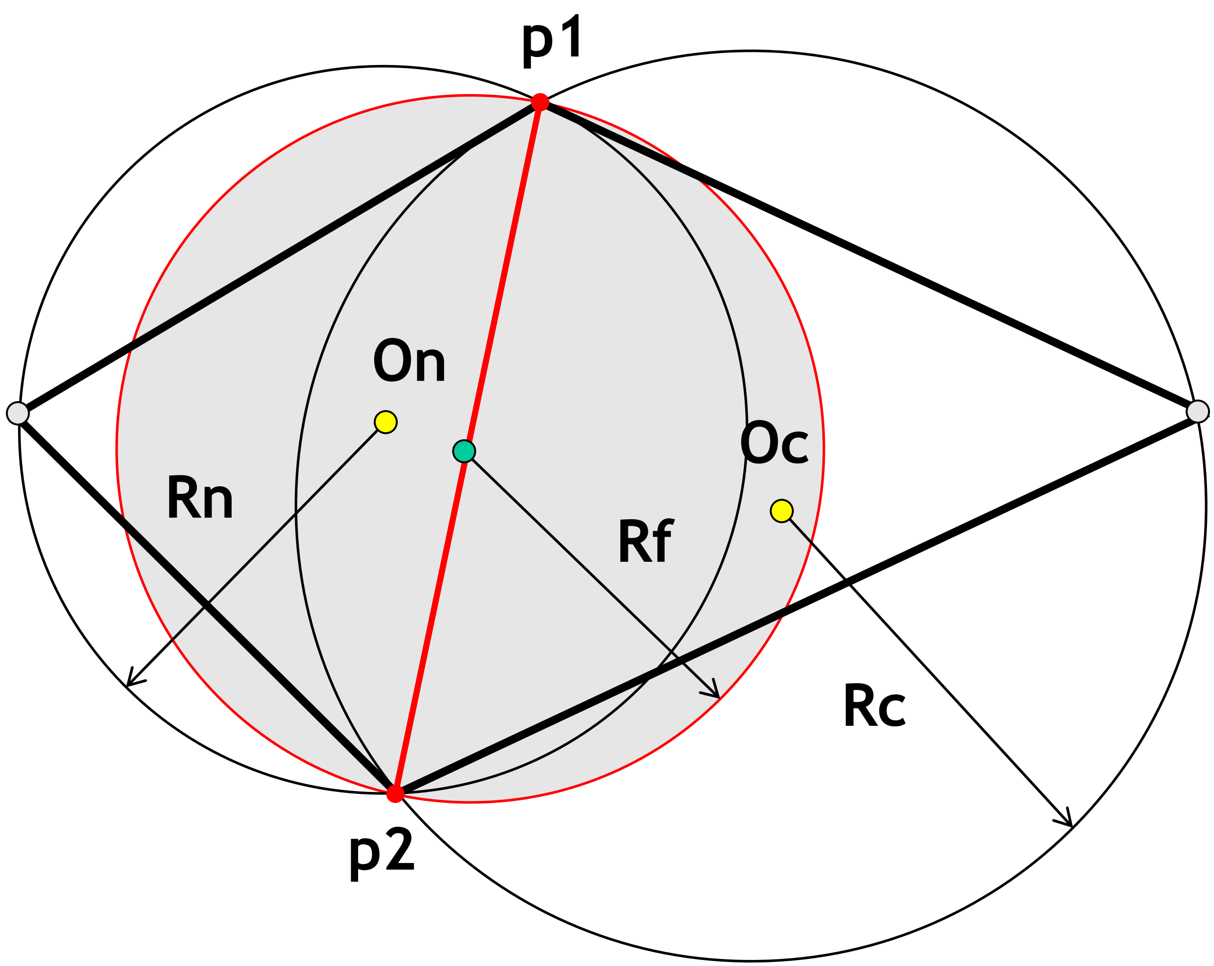
Computing Alpha-Shapes



Computing Alpha-Shapes

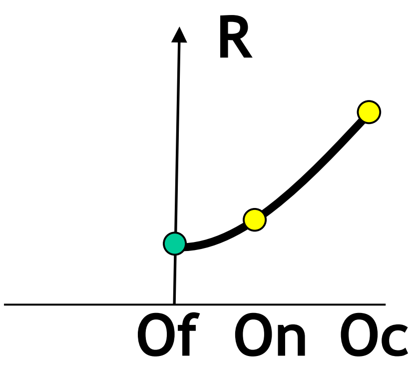
is_alpha if
alpha > Rf
AND
alpha < max(Rc,Rn)

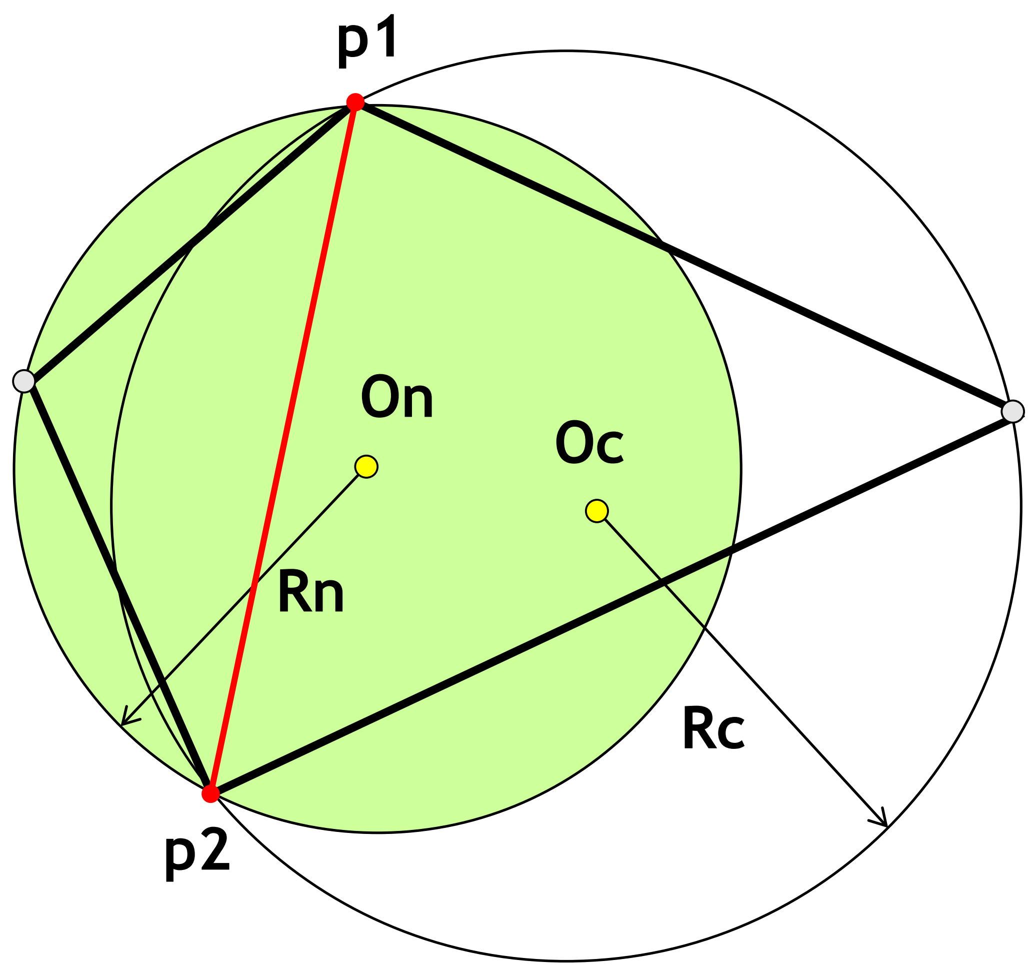




Computing Alpha-Shapes

is_alpha if
alpha >
min(Rc,Rn)
AND
alpha <
max(Rc,Rn)





Computing Alpha-Shapes

is_alpha if
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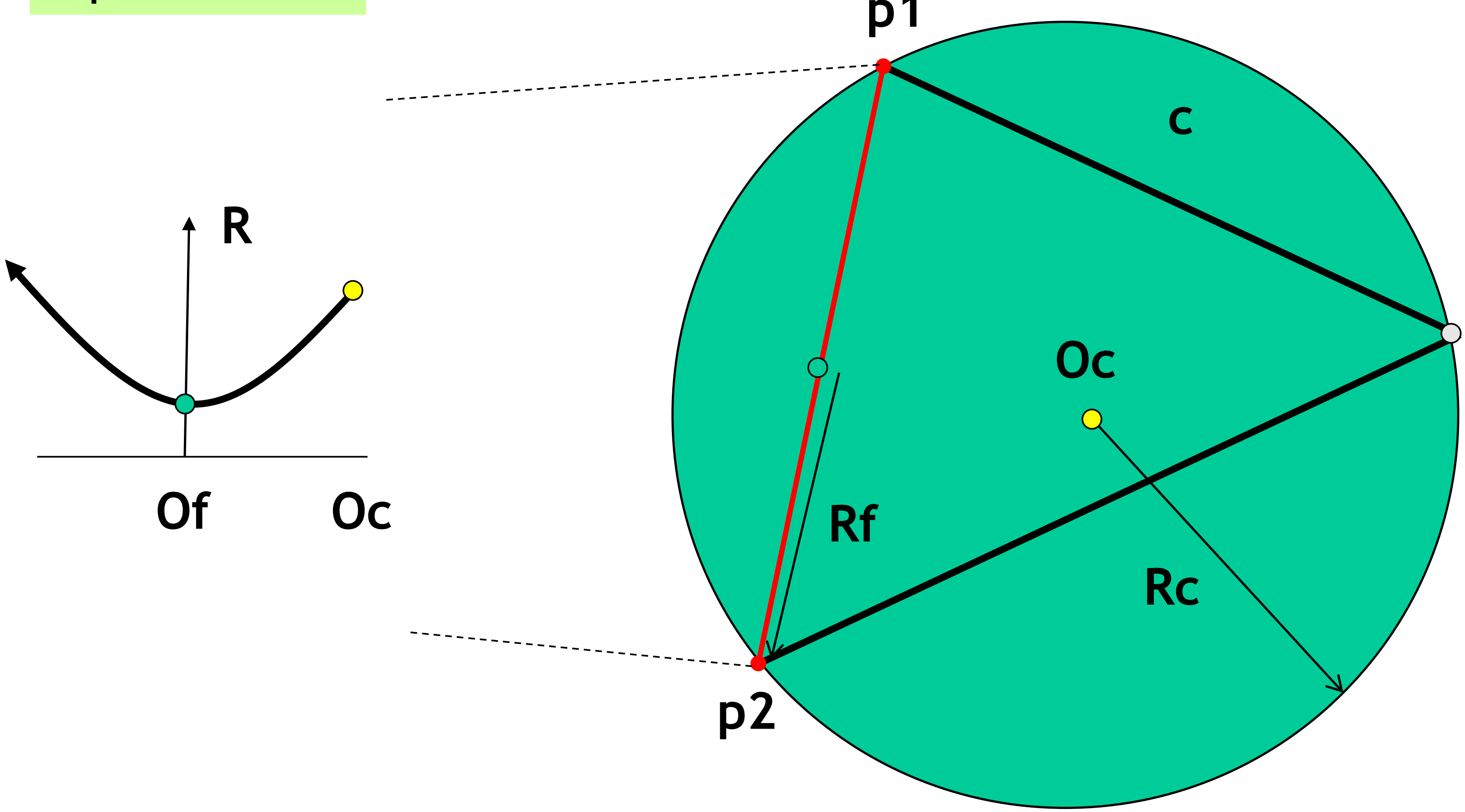
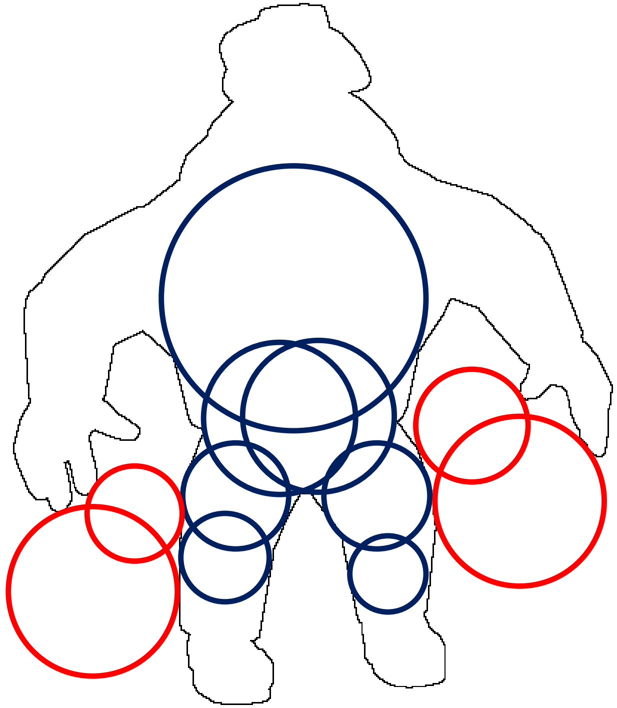


MEDIAL AXIS

Medial Axis

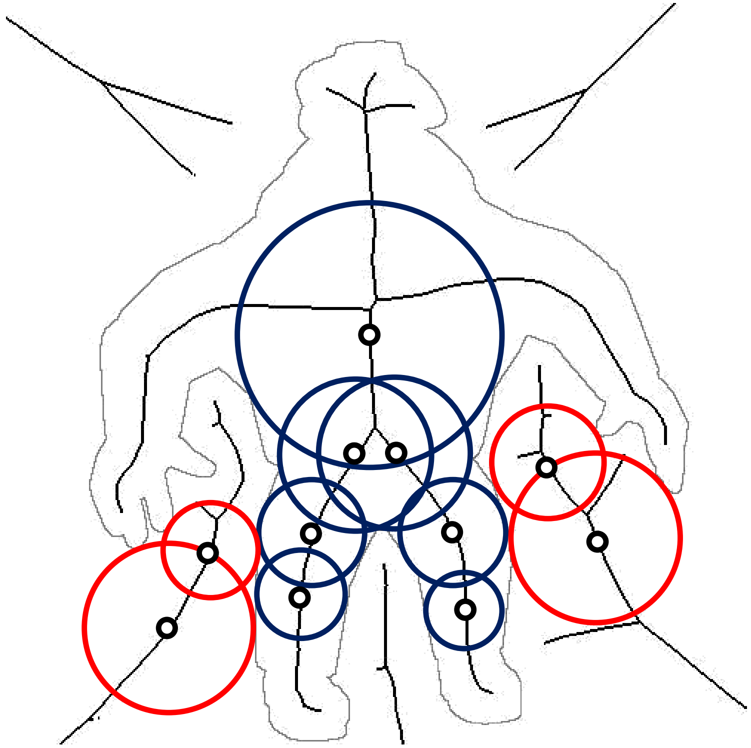
For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.



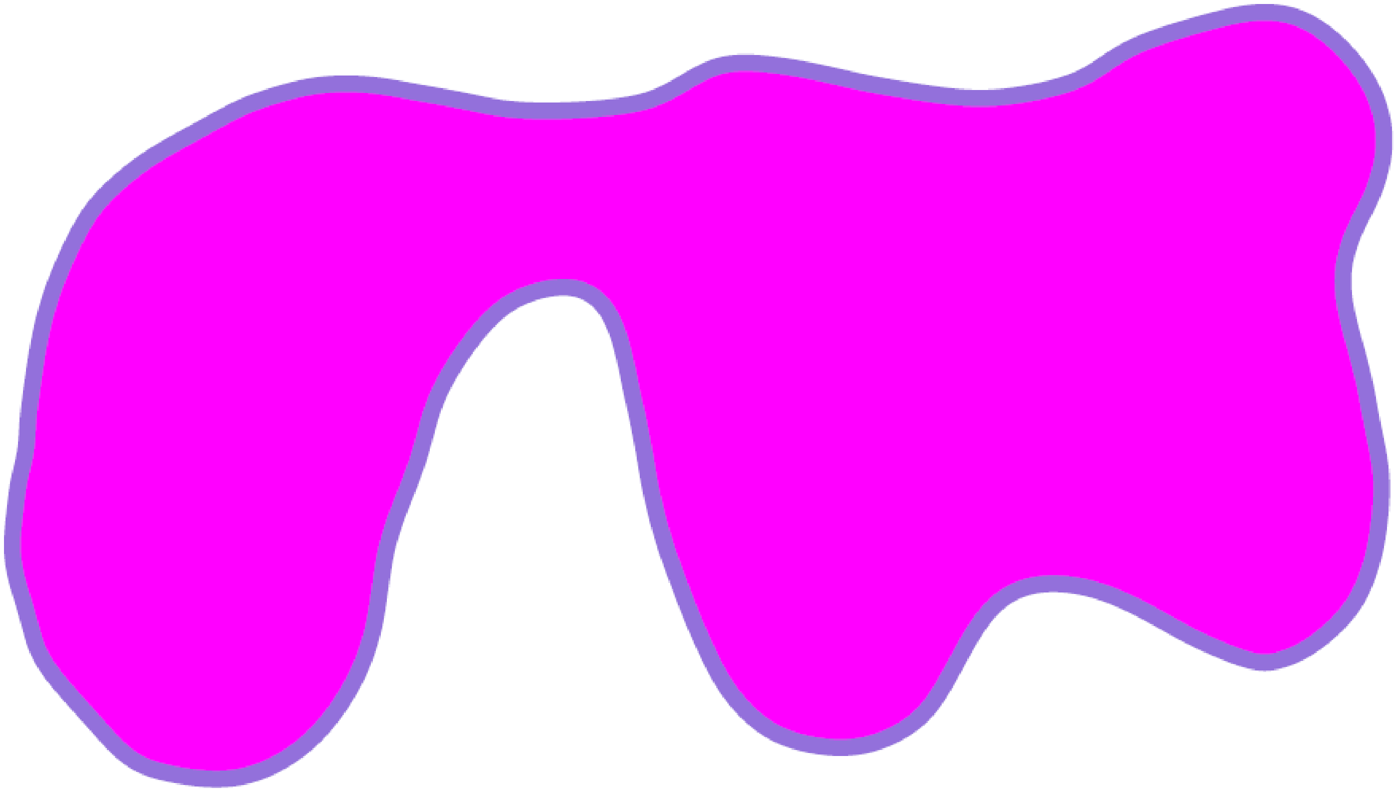
Medial Axis

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

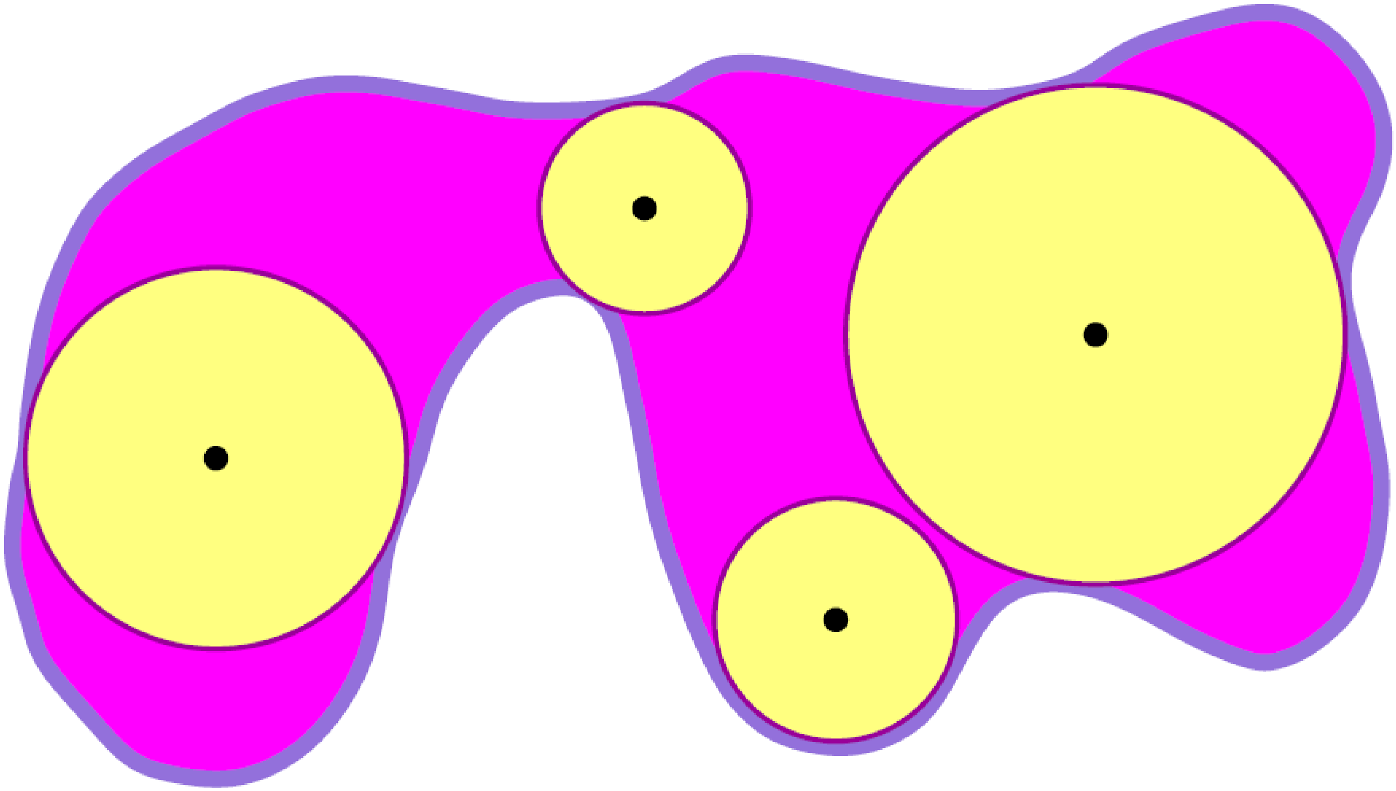
The centers of all such balls make up the *medial axis/skeleton*.



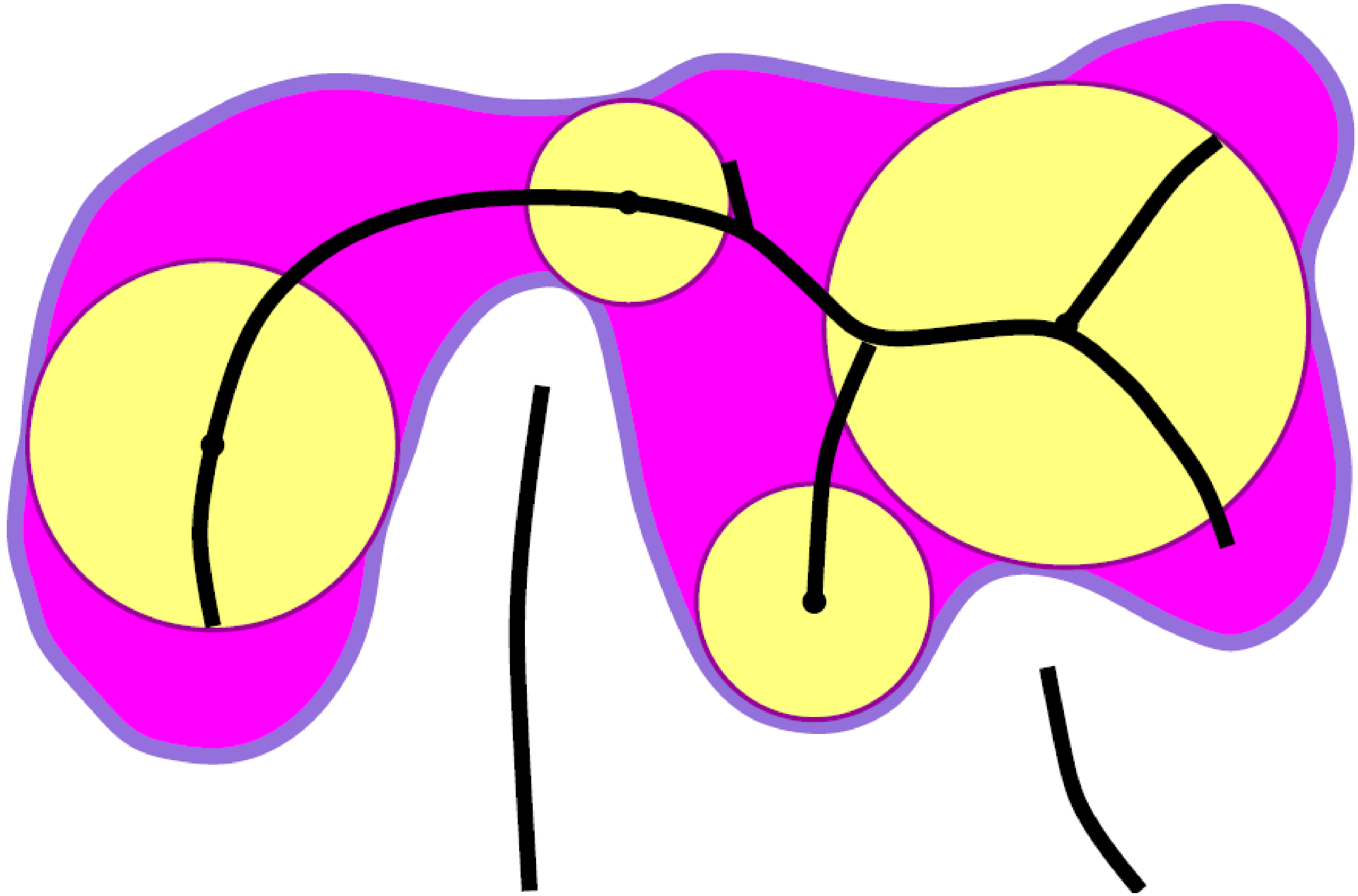
Medial Axis



Medial Axis



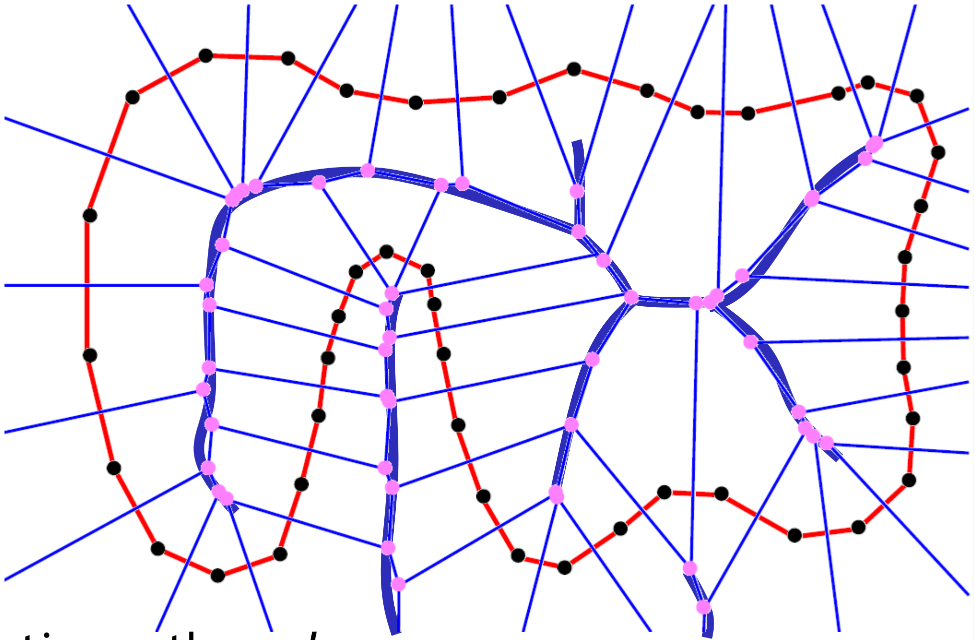
Medial Axis



Medial Axis

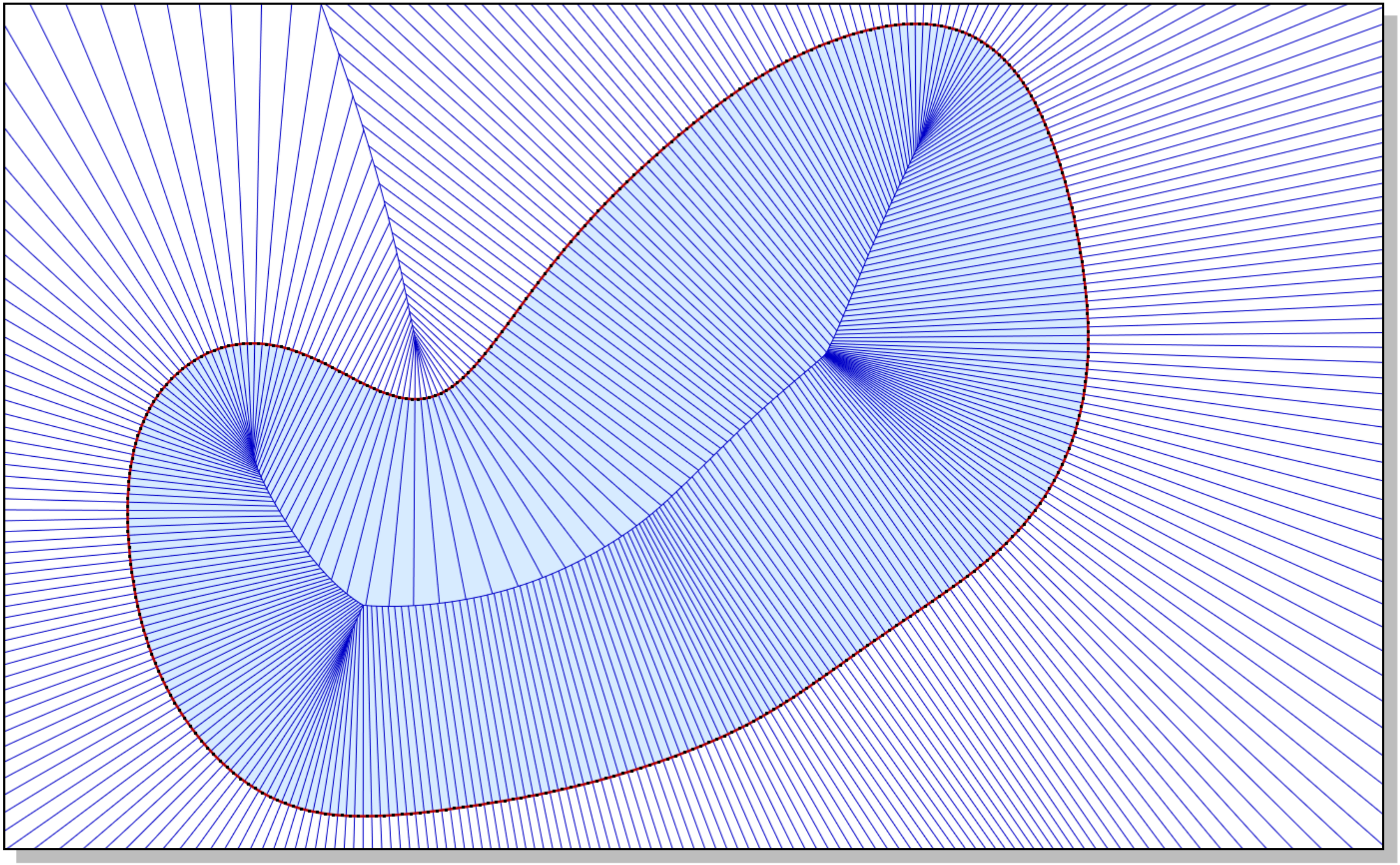
Observation*:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.

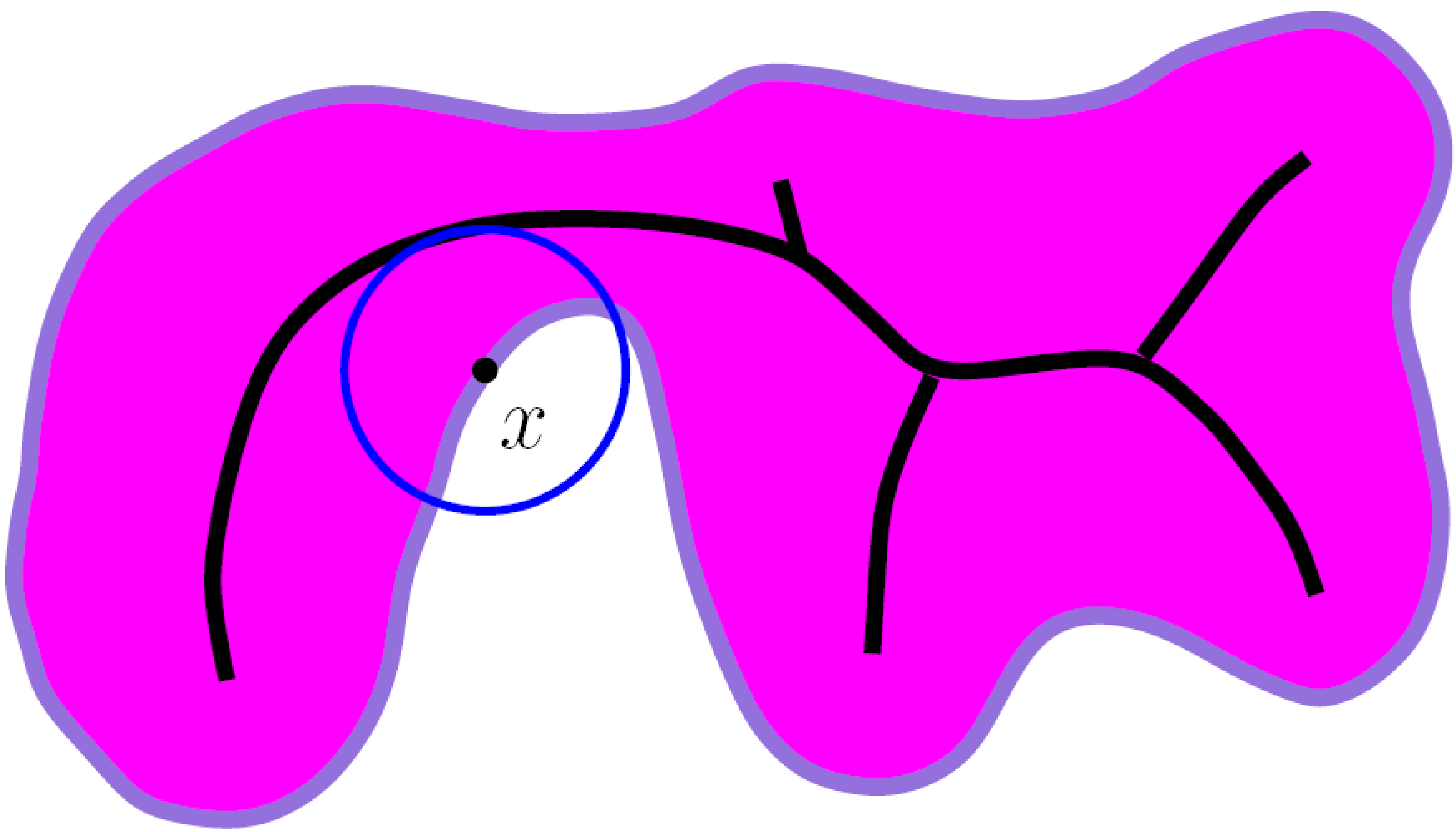


*In 3D, this is only true for a subset of the Voronoi vertices – the *poles*.

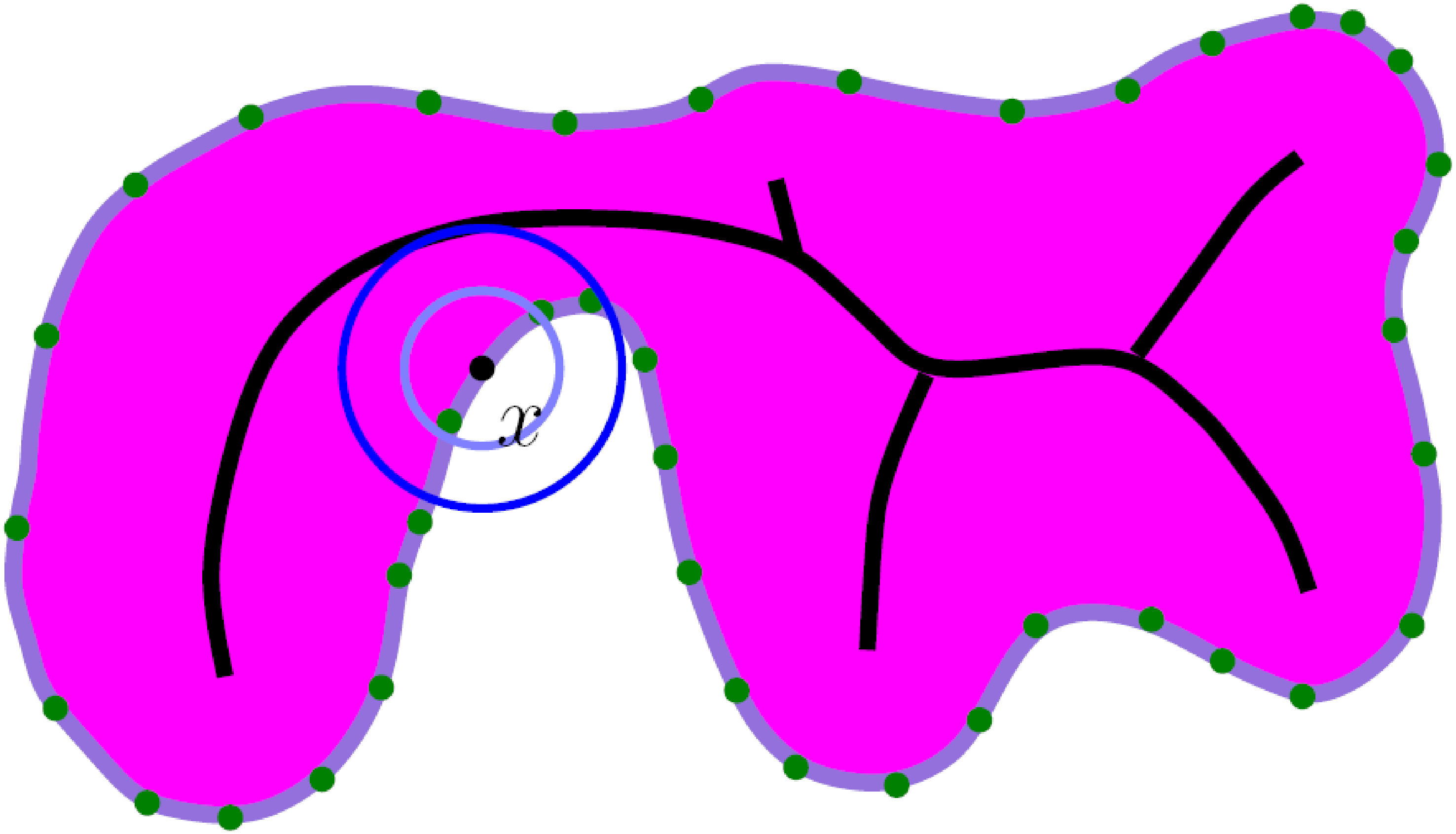
Voronoi & Medial Axis



Local Feature Size



Epsilon-Sampling



Crust [Amenta et al. 1998]

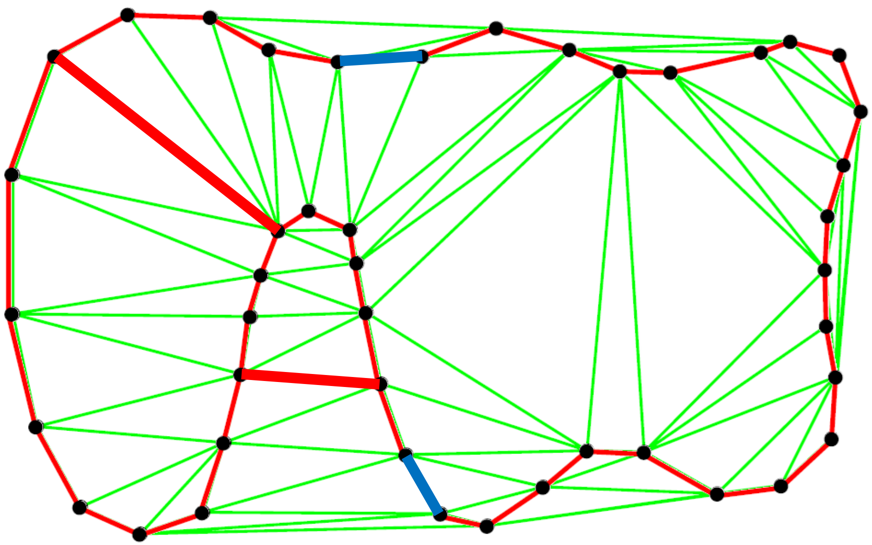
If we consider the Delaunay Triangulation of a point set, the shape boundary can be described as a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the boundary
2. Those traversing the shape.

Discard those that traverse.



Crust [Amenta et al. 1998]

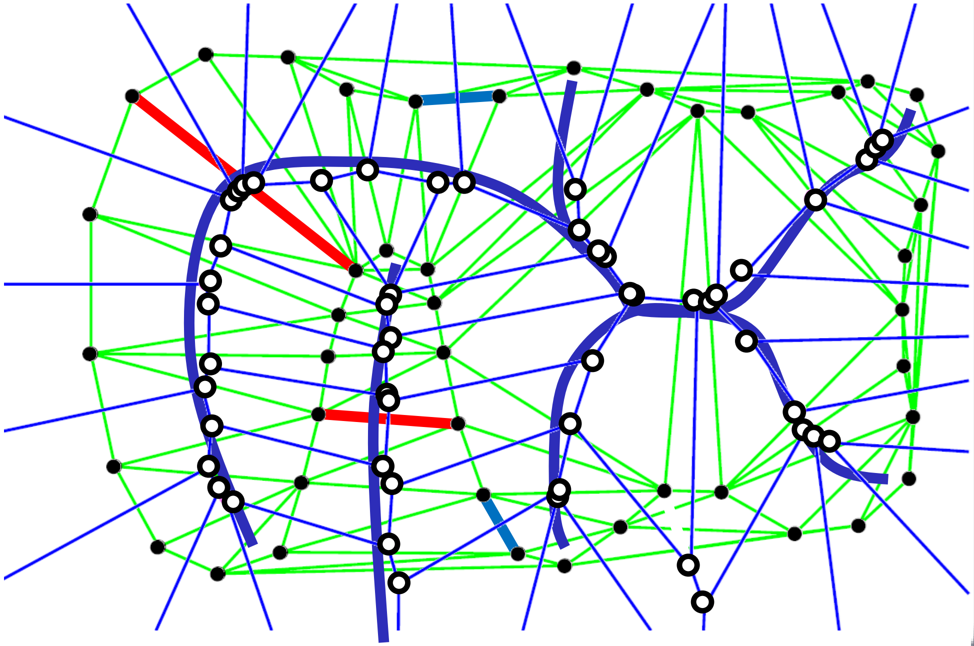
Observation:

Edges that traverse cross the medial axis.

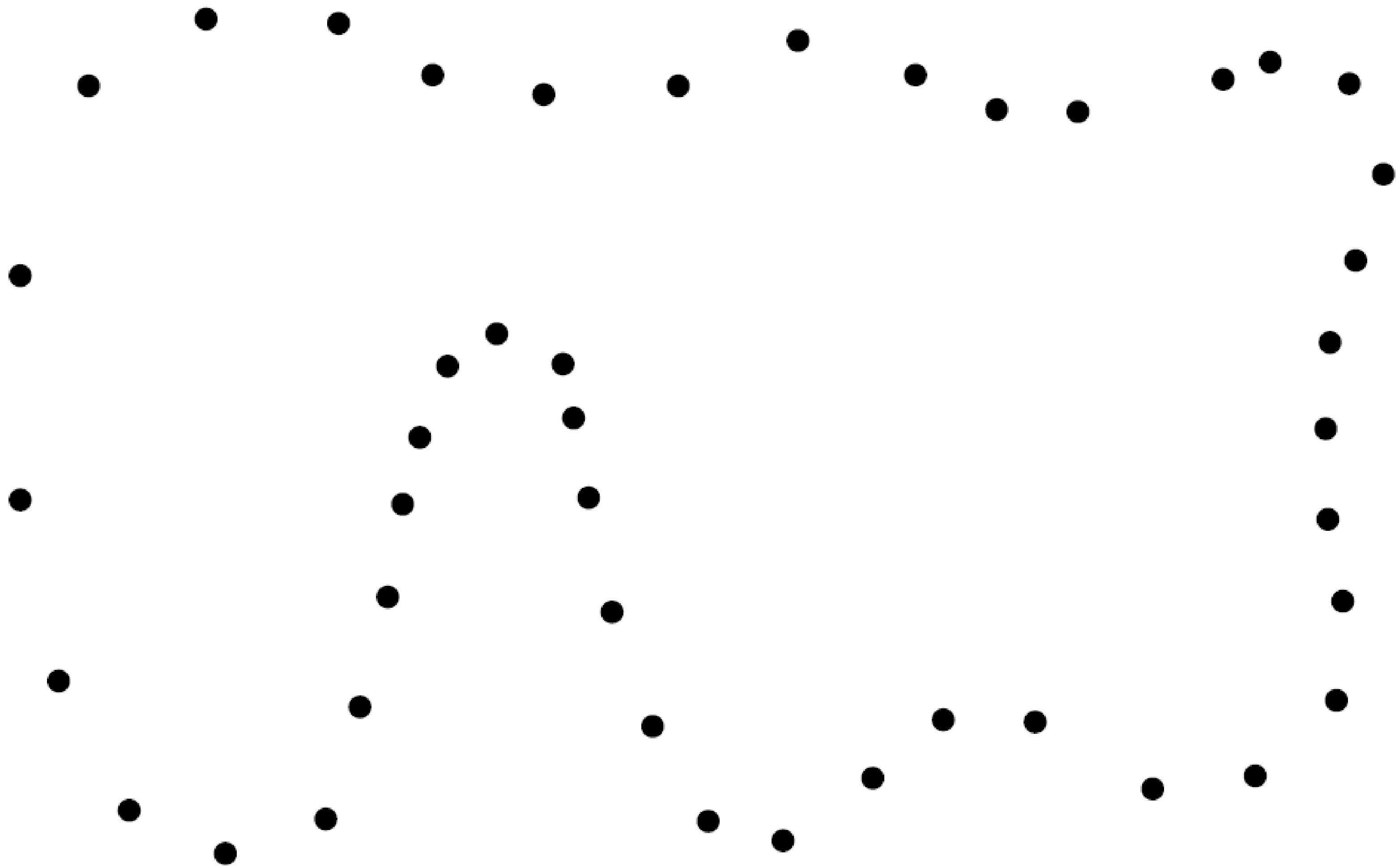
Although we don't know the axis, we can sample it with the Voronoi vertices.

Edges that traverse must

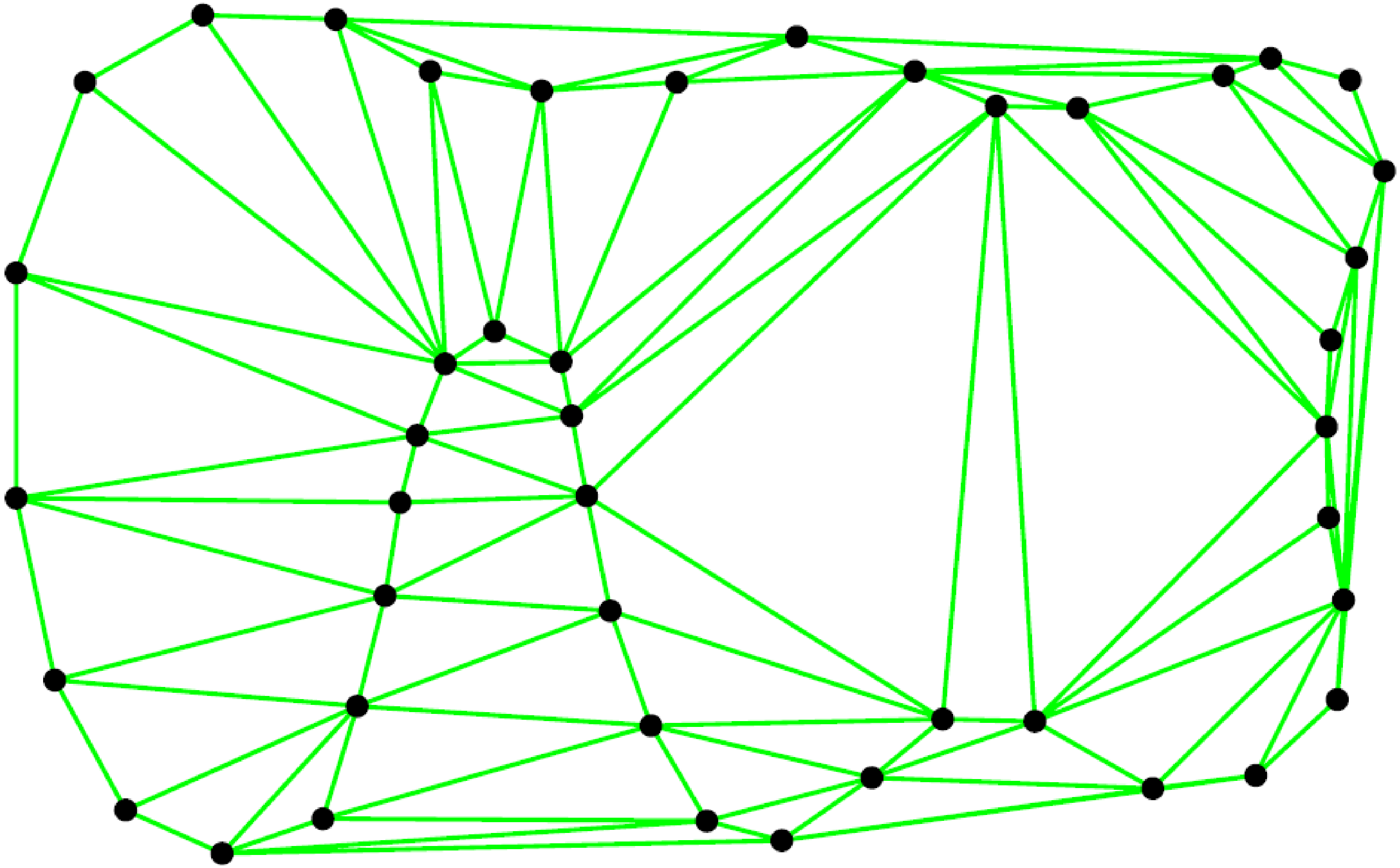
be near the Voronoi vertices.



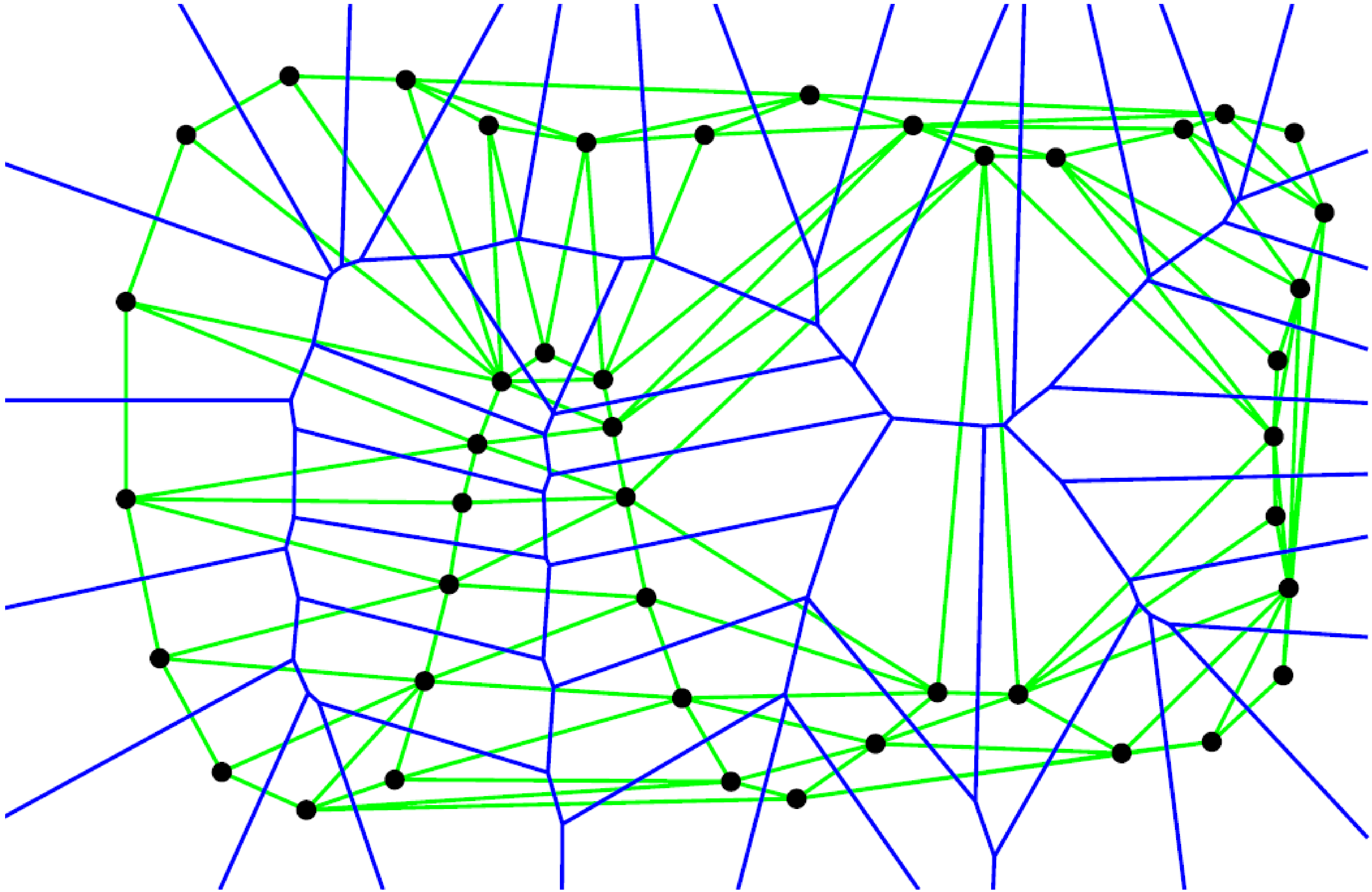
Crust [Amenta et al.]



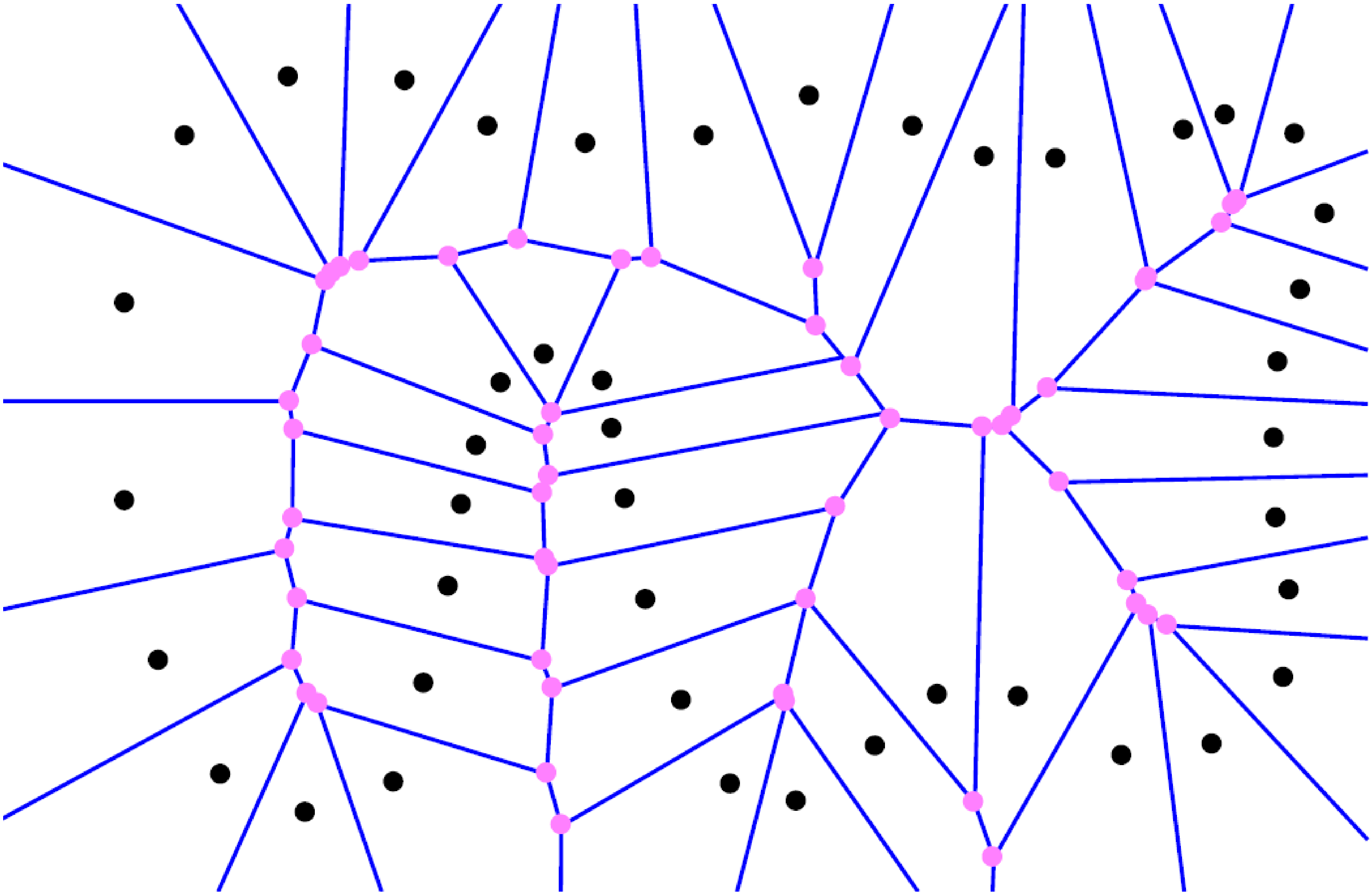
Delaunay Triangulation



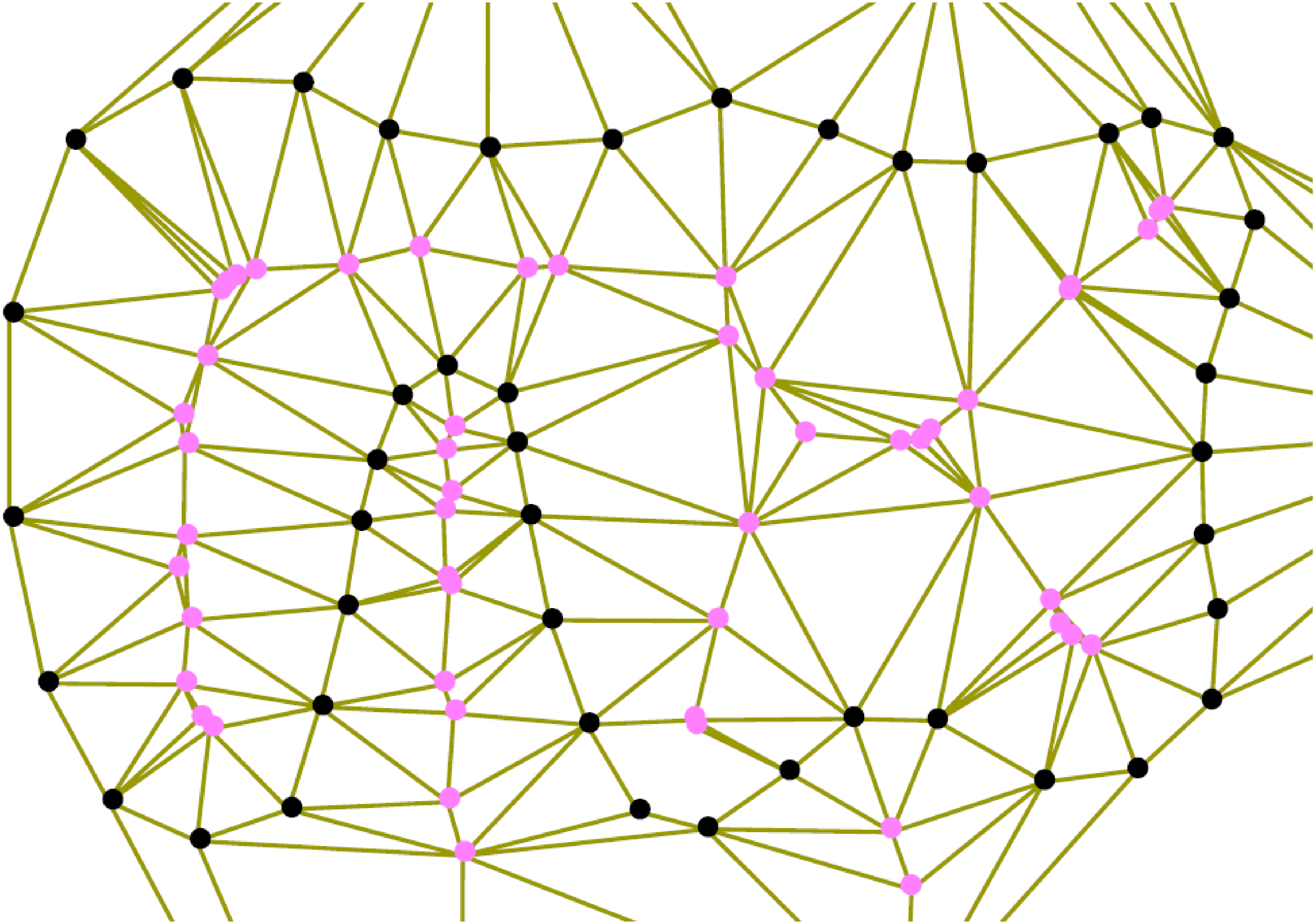
Delaunay Triangulation & Voronoi Diagram



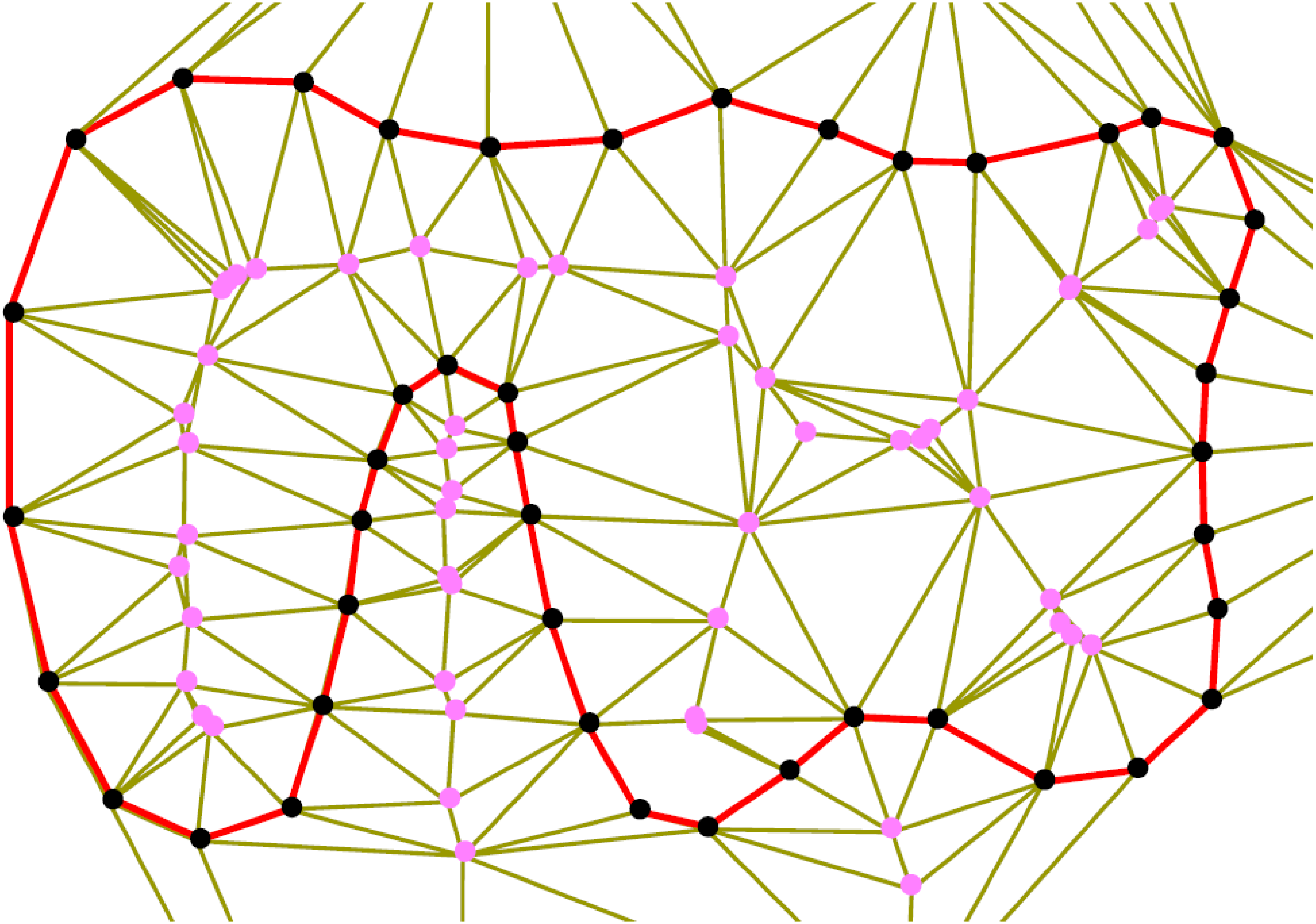
Voronoi Vertices



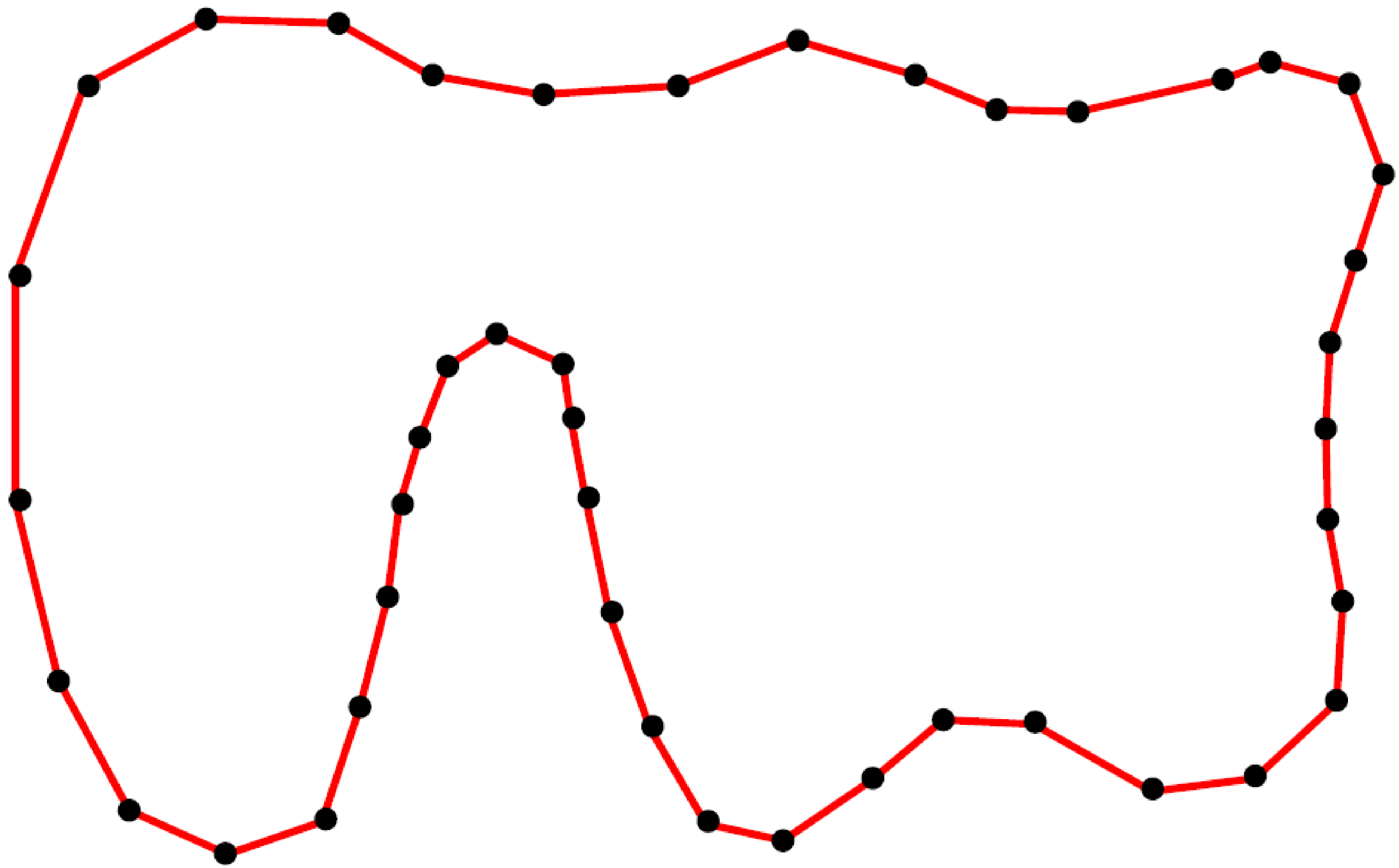
Refined Delaunay Triangulation



Crust



Crust



Crust (variant)

Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is a circle that contains the edge but no Voronoi vertices.