

An Observer-based Longitudinal Control of Car-like Vehicles Platoon Navigating in an Urban Environment

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Abstract—In this paper, we tackle the problem of controlling the longitudinal motion of car-like vehicles platoon navigating in an urban environment that can improve the traffic flow with a minimum number of required communication links. To achieve a higher traffic flow, a constant-spacing policy between successive vehicles is commonly used but this is at a cost of increased communication links as the leader information must broadcast to all the followers. Therefore, we propose a distributed observer-based control law that depends on a hybrid source of neighbors information in which a sensor-based link is used to get the predecessor position while the leader information is acquired through a communication-based link. Then, an observer is designed and integrated into the control law such that the velocity information of the predecessor can be estimated. We start by presenting the platoon model defined in the Curvilinear coordinates with the required transformation between that coordinate and the Cartesian Coordinates so that one can design the control law directly in the Curvilinear coordinates. After that, internal and string stability analysis are conducted. Finally, we provide simulation results, through dynamic vehicular mobility simulator called ICARS, to illustrate the feasibility of the proposed approach and corroborate our theoretical findings.

I. INTRODUCTION

Car sharing system has attracted more attention in recent years to overcome the transportation problems toward an eco-friendly city. However, cars redistribution among all stations is one of the main issues such a system. One of the proposed solutions is to use vehicle platoons guided by professional drivers. A leader vehicle, driven by a human, comes to pick up the vehicles, then join them to a platoon system, and finally drop off the vehicles over the stations so that the vehicles are well distributed.

For platoon motion in the highway environment, a Cartesian frame is often used to describe the platoon motion in order to facilitate the manipulation of geometric transformations. However, in urban environments, the motion has to be represented with respect to a path that the vehicle intends to follow. For path tracking, knowing where the vehicle is in a global frame may not be representative of the driving state of the vehicle. Therefore, the Curvilinear/path coordinates are used instead to represent the vehicle motion [1].

The global architecture of platoon control system consists of two modules includes longitudinal and lateral controllers.

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The former has two sub-modules including Upper-Level Controller (ULC), which provides the desired vehicle linear acceleration that the car has to follow in order to achieve the desired longitudinal states, and Lower-Level Controller (LLC), which generates the required engine throttle/brake to achieve this desired acceleration. In the other hand, the lateral controller generates the required steering angle to achieve the desired lateral states. In this research, we are interested in designing the longitudinal ULC.

The framework of the longitudinal platoon control consists of four components, including Vehicle Dynamics, Information Flow Topology (IFT), Distributed Controller, and Spacing Policy [2].

For the vehicle dynamics, different models are used in the literature, including linear and nonlinear models [3]. Three linear models are frequently used including single integrator model [4], double integrator model [5], third order model [6]. Despite the huge amount of relevant literature to date, few studies in the literature have handled the problem of platooning in the urban environment/path coordinates, e.g. [7]. In these studies, the authors assume a first-order model for the vehicle dynamics which has many defects. Therefore, in our previous work [8], [9], a second order longitudinal dynamic model is used and its corresponding one in the path coordinates is presented such that one can design a controller to achieve control objectives in the path (operational) coordinates explicitly.

For the distributed control, three main control objectives have to be achieved. First, the platoon can navigate in an urban environment (curvilinear path). Second, achieving position and velocity tracking for follower vehicles. Third, guarantee of the string stability, i.e., the error signals will attenuate when propagating downstream the vehicle string. The string instability may result in a rear-end collision. In the literature, several platoon controllers are proposed. The simplest platoon controller is called Adaptive Cruise Control (ACC), which controls only the vehicle gas, brake, and throttle, to maintain a safe distance with the front vehicle. Recently, platoon control is based on a distributed controller known as Cooperative Adaptive Cruise Control (CACC) [10]. The concept of the CACC is the same as ACC, except that in CACC, the vehicles use a communication system to share information with their neighbors to improve the whole system functionalities and performance. In the literature, there are six main approaches in designing a platoon controller, including Linear [11], Optimal [12], \mathcal{H}_∞ -based [13], Sliding Mode [14], Model predictive [15], and Consensus-based controller [6]. In [8], [9], we present a

consensus-based control algorithm that considers the motion of variable velocity platoon.

Several spacing policies are provided in the literature [16]. The constant distance spacing policy is used commonly due to its capability to achieve very high traffic capacity. However, to achieve string stability, more communication links are required to transmit leading vehicle information to all the following vehicles in the platoon.

The Information Flow Topology represents the inter-vehicle communication topology that the vehicles use to acquire the information from its surrounding vehicles. Thanks to the rapid advancement of vehicle-to-vehicle (V2V) communication technology, various IFTs are developed, including Predecessor Following (PF), Predecessor-Leader Following (PLF), Bidirectional (BD), etc. The PLF topology [16] is commonly used in the literature in which the leader communicates with all the vehicles in broadcast, and every other vehicle also considers information from its predecessor to compute the control action. However, for either a low-cost onboard communication module that has limited bandwidth capabilities or for robust communication, high-speed two-channel communication is not feasible/preferred. Therefore, in this study, the proposed control law is designed depending on a hybrid PLF topology in which the leader broadcasts its information to all the vehicles via a communication-based link. For the inter-followers information sharing, the inter-vehicle distance is measured by a distance sensor, e.g. laser, i.e. by a sensor-based link. Therefore, we propose an observer-based control technique. A distributed longitudinal controller designed in the path coordinates is the basis of the proposed control strategy and its role is to achieve position and velocity consensus among the platoon members taking into consideration the nature of the motion in an urban environment. An observer is then introduced in the loop to estimate the predecessor velocity. The merit of using the predecessor velocity in the control is twofold. Firstly, it provides a faster convergence of the platoon with safety. Secondly, it gives a smooth acceleration of the motion of the vehicle, and as a result, fuel saving and passenger comfortability can be achieved.

In the literature, several observer-based consensus control algorithms are introduced for multi-agent systems in [17]–[19] and for platoon systems in [20]. However, these works use the observer to estimate the velocity of either the ego agent or the other vehicles by using the communication links among the agents. Moreover, they do not consider the string stability constraint during the controller design and analysis. In our study, the observer does not require any communication link with other vehicles/agents in the platoon which adds complexity in the design and analysis in order to guarantee both the platoon internal and string stability.

The main contributions of this paper are highlighted as follows.

- A distributed observer-based control algorithm is designed without the need to communicate with the preceding vehicle.
- The proposed algorithm can achieve higher traffic flow

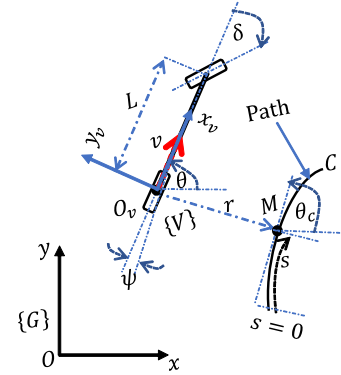


Fig. 1. Kinematic model of car-like vehicle.

(by using constant spacing policy) with a minimum number of required communication links (by sharing position information of the predecessor through distance sensor and estimating the predecessor velocity by an observer).

- Conditions for both internal and string stability are provided.
- The proposed framework is validated via realistic simulation studies are conducted in dynamic vehicular mobility simulator that considers real vehicle dynamics and platooning staff in an urban environment).

The rest of this paper is organized as follows. Section II formulates the platooning problem with kinematic and dynamic models expressed in both body and path coordinates. In Section III, the proposed control law is presented, and both internal and string stability are analyzed. Simulation studies are given in Section IV. Conclusions are made in Section V.

II. PROBLEM STATEMENT

A. Vehicle Kinematic Model

The kinematic model of a car-like vehicle can be simplified to that of a bicycle model [8], see Fig. 1.

1) *Cartesian Coordinates*: Let $\{V\}$, $O_v - x_v y_v$, represents the vehicle body-fixed reference frame with its x -axis in the vehicle's forward direction and its origin at the center of the rear axle, O_v . The configuration of the vehicle, with respect to the world-fixed inertial reference frame, $\{G\}$, $O - xy$, is represented by the generalized coordinates $[x, y, \theta]^T \in \mathbb{R}^3$, where its position is given by $[x, y]^T$ and its orientation in the global frame is represented by θ . δ is the steering angle in the body frame. Since the wheels cannot slip sideways, the vehicle velocity is v in the vehicle x -direction, and zero in the y -direction. Consider the distance L from the rear wheel along the orientation of the vehicle to the front wheel. Then the kinematic model is given by

$$\dot{x} = v \cos(\theta), \quad (1a)$$

$$\dot{y} = v \sin(\theta), \quad (1b)$$

$$\dot{\theta} = \frac{v}{L} \tan(\delta). \quad (1c)$$

2) *Path Coordinates Model*: Let $[s, r, \psi]^T$ represents the curvilinear coordinates. The tracking path, C , defined in the Global Frame, can be represented as a function of several items including path length s (Curvilinear abscissa) at the closest point M to O_v , the angular deviation, $\psi = \theta - \theta_c$, where θ_c is the angle between the path tangent at M and the x -axis, and the lateral deviation, r , which is the signed orthogonal distance from the center of the rear axle, O_v , to the closest point on the path, M . Then, the kinematic model in the path coordinates is represented by

$$\dot{s} = v \frac{\cos(\psi)}{1 - r\kappa(s)}, \quad (2a)$$

$$\dot{r} = v \sin(\psi), \quad (2b)$$

$$\dot{\psi} = v \left(\frac{\tan(\delta)}{L} - \frac{\kappa(s) \cos(\psi)}{1 - r\kappa(s)} \right), \quad (2c)$$

where $\kappa(s)$ is the path curvature M .

B. Car-like Vehicle Longitudinal Dynamic Model

The vehicle longitudinal dynamic behavior is determined by several factors including the engine, drive line, brake system, aerodynamics drag, tire friction, rolling distance, gravitational force, etc [14]. For the vehicle model, we assume:

Assumption 1. *The rigidity and symmetry of vehicle body.*

Assumption 2. *The integration of driving and braking inputs into one control input.*

Assumption 3. *The pitch and yaw motions effects are neglected.*

The general longitudinal dynamic equation is given by

$$M_v a + C_v v + G_v = F_v, \quad (3)$$

where $a = \dot{v}$, M_v , C_v , G_v and F_v denote the vehicle acceleration in the vehicle frame, the vehicle Inertia effect, Coriolis effect, Gravity effect and input force, respectively.

Consider the longitudinal LLC is an inverse model compensation technique, it follows

$$F_v = M_v \mu + C_v v + G_v \quad (4)$$

where μ is the new input signal after system linearization which has to be designed. By applying this LLC, the second order dynamic model can be written in the body coordinates as

$$a = \mu \quad (5)$$

Since we aim at platooning in an urban environment, the relation between the dynamics in the path and Cartesian coordinates will be derived such that the controller can be designed in the path coordinates. The functional block diagram of longitudinal controller with the dynamic model is given in Fig. 2.

To find a relation between the control signals in the path and body coordinates, let's recast (2a) as

$$\dot{s} = Jv, \quad (6)$$

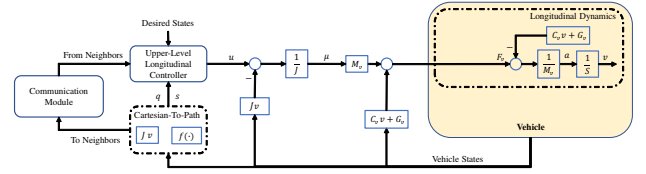


Fig. 2. Block diagram of the vehicle longitudinal dynamic model, where S is the Laplace operator.

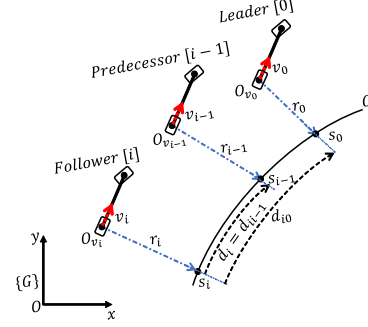


Fig. 3. Platoon of car-like vehicles.

where J is given by

$$J = \frac{\cos(\psi)}{(1 - r\kappa)}. \quad (7)$$

Thus, the vehicle acceleration in the path coordinates, η , is given as

$$\eta = Ja + \dot{J}v, \quad (8)$$

where $\eta = \dot{q} = \ddot{s}$ and $q = \dot{s}$ are the vehicle acceleration and velocity in the path coordinates respectively. Consequently, if the mapping from the control signal (i.e., desired acceleration) in the path coordinates, u , to that in the body coordinates, μ , is given by

$$\mu = \frac{1}{J} (u - \dot{J}v), \quad (9)$$

then, the vehicle longitudinal dynamic model in the path coordinates is given by

$$\dot{s} = q, \quad (10a)$$

$$\dot{q} = \eta = u. \quad (10b)$$

C. Platoon Longitudinal Model

Consider a platoon of $N + 1$ vehicles, as illustrated in Fig. 3, including a leader (with index 0) and N followers (with index i ; $i = 1, 2, \dots, N$). The vehicles can share information by either sensor-based or communication-based link. It is known that the vehicles involved in the car sharing system are often with the same type of small passenger vehicles, or have very close dynamics, hence we can assume

Assumption 4. *All the vehicles have equal dynamics.*

The in-platoon vehicle dynamics in the path coordinates can be given by

$$\dot{s}_i = q_i, \quad (11a)$$

$$\dot{q}_i = \eta_i = u_i, \quad (11b)$$

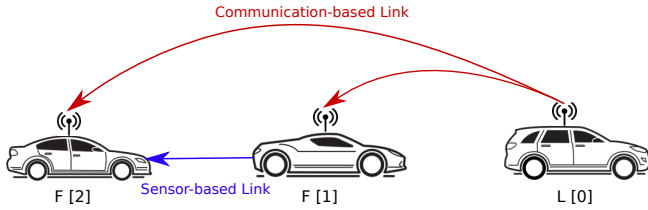


Fig. 4. Information flow topology: Hybrid PLF through communication link between leader and followers and direct sensor-based measurements between followers.

where the transformation between the two coordinates can be implemented using (9).

For the vehicles string, the inter-vehicle distance, $d_i = s_j - s_i$, is the actual Curvilinear distance between vehicle i and its predecessor $j = i - 1$, $d_{i0} = s_0 - s_i$, is the actual Curvilinear distance between vehicle i and leader, $d_{r,i}$ is the desired inter-vehicle distance between vehicles i and j , and $d_{r,i0}$ is the desired inter-vehicle distance between vehicles i and 0. For high traffic capacity, the constant spacing policy method is used, i.e. $d_{r,i}$ is constant.

D. Control Objectives

Under the hybrid IFT given in Fig. 4, the platoon is required to move in an urban environment (Curvilinear path) and to track a variable velocity leader. Each follower i can obtain leader information (i.e., position, velocity, and acceleration s_0, q_0, η_0) and predecessor position information (i.e., s_{i-1}). Under these circumstances, $\forall i = 1, \dots, N$, the following goals have to be achieved:

- estimation of the preceding vehicle velocity to be used in the distributed control law;
- convergence of the velocity estimation, position and velocity tracking errors;
- string stability.

III. CONTROL DESIGN

This section starts by presenting the proposed observer-based control algorithm. After that, the platoon internal stability analysis is presented. Finally, the string stability is analyzed.

A. Observer-based Control Law

The information flow structure among the platoon can be modeled by a graph in which the node dynamics are described by (11). The IFT among followers can be modeled by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{M})$ characterized by the set of nodes $\mathcal{V} = 1, \dots, N$ and set of edge $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The graph topology is described by an adjacency matrix $\mathcal{M} = [m_{ij}]_{N \times N}$, in which the element $m_{ij} = 1$ if vehicle i can obtain information from vehicle j ; otherwise, $m_{ij} = 0$. To include the leader (with index 0) in the network, an augmented digraph $\tilde{\mathcal{G}}$ is used. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ associated with graph \mathcal{G} is given by $l_{ij} = -m_{ij}$ if $i \neq j$, and $l_{ij} = \sum_{k=1}^N m_{ik}$ if $i = j$.

Remark 1. For the Predecessor - Leader Following (PLF) information flow topology, the elements of the adjacency

matrix is given as $m_{ij} = 1$ if $j = i - 1$, and $m_{ij} = 0$ otherwise.

Let us consider a full PLF-based control law given as

$$u_i(t) = \eta_0 + g_{c,2}e_{q,i0}(t) + g_{c,1}e_{s,i0}(t) + g_{o,1}e_{s,i}(t) + g_{o,2}e_{q,i}(t), \quad (12)$$

where $e_{s,i0}(t) = s_0(t) - s_i(t) - d_{r,i0}$, $e_{q,i0}(t) = q_0(t) - q_i(t)$, $e_{s,i}(t) = s_j(t) - s_i(t) - d_{r,i}$, and $e_{q,i}(t) = q_j(t) - q_i(t)$. $G_c = [g_{c,1}, g_{c,2}]^T$, $G_o = [g_{o,1}, g_{o,2}]^T$ are controller parameters to be designed.

Remark 2. In the proposed hybrid PLF, the information required to apply this control law (12) is available except the velocity of the predecessor or the velocity error between the Eco vehicle i and the predecessor velocity, $e_{q,i}$, which has to be estimated using an observer.

For observer design, let us define two states $z_{1,i} = e_{s,i}$ which is known (i.e., measured by the sensor mounted on vehicle i), and $z_{2,i} = e_{q,i}$ which is unknown and we need to estimate. The state equations that relates these two states are given by

$$\begin{aligned} \dot{z}_{1,i} &= z_{2,i}, \\ \dot{z}_{2,i} &= \dot{e}_{q,i}. \end{aligned} \quad (13)$$

The state $z_{2,i}$ can be estimated using the following observer

$$\begin{aligned} \dot{\hat{z}}_{1,i} &= \hat{z}_{2,i} + h_1(z_{1,i} - \hat{z}_{1,i}), \\ \dot{\hat{z}}_{2,i} &= h_2(z_{1,i} - \hat{z}_{1,i}). \end{aligned} \quad (14)$$

Including the observer output in the control law (12), then the i^{th} observer-based control law is proposed as

$$u_i(t) = \eta_0 + g_{c,2}e_{q,i0}(t) + g_{c,1}e_{s,i0}(t) + g_{o,1}\hat{z}_{1,i}(t) + g_{o,2}\hat{z}_{2,i}(t). \quad (15)$$

Let us define the error state vector of vehicle i as $f_i = [f_{1,i}, f_{2,i}]^T$ with $f_{1,i} = e_{s,i0}$ and $f_{2,i} = e_{q,i0}$. Then, the i^{th} closed-loop state space model is given by

$$\dot{f}_i(t) = Af_i(t) + B\epsilon_i(t), \quad (16)$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and

$$\epsilon_i = -g_{c,2}e_{q,i0} - g_{c,1}e_{s,i0} - g_{o,1}\hat{z}_{1,i} - g_{o,2}\hat{z}_{2,i}. \quad (17)$$

Exploiting that $z_{1,i} = e_{s,i} = e_{s,i0} - e_{s,j0}$ and let $\hat{z}_i = [\hat{z}_{1,i}, \hat{z}_{2,i}]^T$, then (14) can be rewritten concisely as

$$\dot{\hat{z}}_i = A - HC\hat{z}_i + HC(f_{1,i} - f_{1,j}), \quad (18)$$

where $H = [h_1, h_2]^T$ and $C = [1, 0, 0]$.

Defining $G_c = \frac{1}{2}KQ_1$, $G_o = \frac{1}{2}KQ_2$, $K = [k_1, k_2]$, and Q_1 and $Q_2 \in \mathbb{R}^{2 \times 2}$, then (17) can be recast as

$$\epsilon_i(t) = -\frac{1}{2}KQ_1f_i(t) - \frac{1}{2}KQ_2\hat{z}_i(t), \quad (19)$$

Let $F = [f_1^T, \dots, f_N^T]^T$, $\hat{Z} = [\hat{z}_1^T, \dots, \hat{z}_N^T]^T$, $\mathcal{E} = [\epsilon_1, \dots, \epsilon_N]^T$, and after some algebraic manipulations, then the system collective dynamics are given by

$$\dot{F} = \left(I_N \otimes A - I_N \otimes \frac{1}{2}BKQ_1 \right) F - \left(I_N \otimes \frac{1}{2}BKQ_2 \right) Z, \quad (20)$$

$$\dot{\hat{Z}} = \left(I_N \otimes (A - HC) \right) \hat{Z} + \left(\mathcal{D} \otimes HC \right) F, \quad (21)$$

where, I_N denotes $(N \times N)$ identity matrix, $\mathcal{D} = \mathcal{L} + \mathcal{P}$, and $\mathcal{P} = \text{diag}\{1, 0, 0\}$.

B. Internal Stability

The closed loop system dynamics represented in (20) and (21) can be recast in the form

$$\dot{X} = A_c X, \quad (22)$$

where $X = [F^T, Z^T]^T$, and

$$A_c = \begin{bmatrix} I_N \otimes (A - \frac{1}{2}BKQ_1) & -I_N \otimes \frac{1}{2}BKQ_2 \\ \mathcal{D} \otimes HC & I_N \otimes (A - HC) \end{bmatrix}. \quad (23)$$

For the internal stability of the proposed platoon observer-based control system let us present the following theorem.

Theorem III.1. *Consider the distributed control of a platoon of connected car-like vehicles with dynamics defined in (2), (3) - (9), and (11), with assumptions 1 - 4, and the observer-based control algorithm proposed in (14) and (15) is applied to it. Then, the system is asymptotically stable as long as the following conditions are met.*

- 1) Find $\Gamma \in \mathbb{R}^{2 \times 2}$ to satisfy

$$(A - HC)\Gamma - \Gamma(A - BK) = -\lambda_i HC, \quad (24)$$

where λ_i is the i^{th} eigenvalue of the matrix \mathcal{D} .

- 2) The gain matrices Q_1 and Q_2 must satisfy

$$\begin{bmatrix} \frac{1}{2}Q_1 & \frac{1}{2}Q_2 \end{bmatrix} = \begin{bmatrix} I_2 \\ \Gamma \end{bmatrix}^{-1}. \quad (25)$$

- 3) Constraints on the gain vector K are

$$k_1 > 0 \text{ and } k_2 > 0. \quad (26)$$

- 4)

$$\lambda_{\max} \left\{ \frac{1}{2}\Gamma BKQ_2 \right\} < \lambda_{\min} \left\{ -(A - HC) \right\}. \quad (27)$$

Proof. For the PLF information flow topology, \mathcal{D} is an upper-triangular matrix and its eigenvalues, λ_i , are real. Therefore, for vehicle i , the matrix A_c can be represented as

$$\bar{A}_c = \begin{bmatrix} A - \frac{1}{2}BKQ_1 & -\frac{1}{2}BKQ_2 \\ \lambda_i HC & A - HC \end{bmatrix}. \quad (28)$$

Exploiting the similarity transformation technique by multiplying the left and right sides of the matrix \bar{A}_c by $\bar{Q} =$

$\begin{bmatrix} I & 0 \\ -\Gamma & I \end{bmatrix}$, and $\bar{Q}^{-1} = \begin{bmatrix} I & 0 \\ \Gamma & I \end{bmatrix}$, respectively, and by conditions (24) and (25), one can obtain

$$\bar{Q}\bar{A}_c\bar{Q}^{-1} = \begin{bmatrix} A - BK & -\frac{1}{2}BKQ_2 \\ 0 & A - HC + \frac{1}{2}\Gamma BKQ_2 \end{bmatrix}, \quad (29)$$

Therefore, the $\text{eig}\{\bar{A}_c\} = \text{eig}\{A - BK\} \cup \text{eig}\{A - HC + \frac{1}{2}\Gamma BKQ_2\}$, where $\text{eig}\{*\}$ denotes the eigenvalues of matrix $*$. For the matrix $(A - HC + \frac{1}{2}\Gamma BKQ_2) < 0$, condition (27) must be satisfied. Moreover, the characteristic equation of the matrix $(A - BK)$ can be expressed as

$$\det(SI_2 - (A - BK)) = S^2 + k_2S + k_1. \quad (30)$$

According to Routh–Hurwitz stability criterion [21], this polynomial (30) is Hurwitz stable under condition (26). This implies that all eigenvalues of \bar{A}_c and consequently A_c have negative real parts. \square

Remark 3. To get a unique solution to Sylvester equation (24), Γ , the matrices $(A - HC)$ and $(A - BK)$ have to be designed such that there are no eigenvalues in common between them.

Remark 4. Adding observer to the control loop increases the complexity of the string stability analysis. Therefore, the next corollary is proposed in order to reduce the number of design parameters which in turn simplifies both the observer-based control design and string stability analysis.

Corollary III.2. *The proposed platoon observer-based control system is asymptotically stable provided that the design parameters are selected as*

$$k_1 = p_c^2, \quad (31a)$$

$$k_2 = 2p_c, \quad (31b)$$

$$h_1 = 2p_o, \quad (31c)$$

$$h_2 = p_o^2, \quad (31d)$$

$$p_o = \gamma p_c, \forall \gamma, p_c, \text{ and } p_o > 0, \quad (31e)$$

$$\gamma > \frac{1}{2} \text{ and } \gamma \neq 1, \quad (31f)$$

where the desired eigenvalues of controller part $(A - BK)$ are assigned at $-p_c$, and that of the observer part, $(A - HC)$, are placed at $-p_o$. γ is a constant that determines the distance between the poles of the controller and the observer.

Proof. By choosing K and H as given in conditions (31a) - (31d), all the poles of $(A - BK)$ and $(A - HC)$ are placed at $-p_c$ and $-p_o$ respectively. Hence, one can choose the eigenvalues to be different by satisfying (31f). Thus, according to Remark 3, solution of (24), Γ , exists, and consequently, condition (25) is met.

Checking stability conditions of (30), and setting the value of k_1 and k_2 , as given in (31a) - (31b), we can satisfy condition (26).

The design of the algorithm parameters as in (31a) - (31e) has another merit that is the number of the design parameters are reduced to γ and p_c .

Substitution (31a) - (31e) into (27), after some algebraic manipulations, the roots of $(A - HC + \frac{1}{2}\Gamma BKQ_2)$ are $(\frac{1}{2} - r \pm \frac{1}{2}\sqrt{4r - 5})$ which are negative definite under a condition for γ given in (31f). \square

C. String Stability

In addition to internal stability, the platoon controller has to guarantee string stability; otherwise, the error signals will amplify when propagating downstream the vehicle string, which may result in rear-end collision.

Definition III.3. L_2 String Stability means that the energy (represented by the L_2 norm) of the position error of vehicle i , $e_{s,i}$, is smaller than the energy of the position error of preceding vehicle $j = i - 1$, $e_{s,j}$ [22]. Since $\|\bar{G}(S) = \frac{E_i(S)}{E_j(S)}\|_\infty = \sup_{e_{s,j} \in L_2} \frac{\|e_{s,i}\|_2}{\|e_{s,j}\|_2}$, the string stability is guaranteed if

$$\|\bar{G}(S)\|_\infty < 1. \quad (32)$$

For simplicity of notation and analysis, let us define the following errors $e_i(t) = s_j(t) - s_i(t) - d_{r,i}$, $\dot{e}_i(t) = q_j(t) - q_i(t)$, $e_{i0}(t) = s_0(t) - s_i(t) - d_{r,i0}$, and $\dot{e}_{i0}(t) = q_0(t) - q_i(t)$.

Theorem III.4. Consider the platoon system presented in Theorem III.1 and the observer-based control algorithm proposed in (15) and its parameterization given in Corollary III.2. Then, the platoon is string stable as long as the conditions (31a) - (31e) are satisfied and the choice of factor γ is constrained by

$$\gamma \geq \sqrt{\frac{9}{12}}. \quad (33)$$

Proof. Let us start with

$$\ddot{e}_i = \eta_j - \eta_i. \quad (34)$$

Substituting (11) and (15) into (34), considering the relations $e_i = e_{i0} - e_{j0}$ and $\dot{e}_i = \dot{e}_{i0} - \dot{e}_{j0}$, and after some algebraic manipulations, we get

$$\ddot{e}_i = -g_{c,2}\dot{e}_i - g_{c,1}e_i - g_{o,1}\hat{z}_{1,i} - g_{o,2}\hat{z}_{2,i} + g_{o,1}\hat{z}_{1,j} + g_{o,2}\hat{z}_{2,j}. \quad (35)$$

The observer model given (18) can be represented in the form of transfer functions as

$$\begin{aligned} \bar{G}_{z_1}(S) &= \frac{\hat{Z}_{1,i}(S)}{E_j(S)} = \frac{h_1S + h_2}{S^2 + h_1S + h_2}, \\ \bar{G}_{z_2}(S) &= \frac{\hat{Z}_{2,i}(S)}{E_j(S)} = \frac{h_2S}{S^2 + h_1S + h_2}. \end{aligned} \quad (36)$$

Writing (35) in the S-domain along with (36), one can obtain

$$\bar{G}(S) = \frac{g_{o,1}\bar{G}_{z_1} + g_{o,2}\bar{G}_{z_2}}{S^2 + g_{c,2}S + g_{c,1} + g_{o,1}\bar{G}_{z_1} + g_{o,2}\bar{G}_{z_2}}. \quad (37)$$

Since $\|\bar{G}(S)\|_\infty = \sup_{w>0} \sqrt{|\bar{G}(jw)|^2} = \sup_{w>0} \sqrt{\frac{\bar{G}_n(w)}{\bar{G}_d(w)}}$, string stability criterion (32) is equivalent to

$$\bar{G}_d(w) - \bar{G}_n(w) > 0, \quad \forall w > 0. \quad (38)$$

Exploiting the definitions of G_c and G_o along with (31a) - (31e), after certain algebraic manipulations, (38) is recast as $\bar{G}_d(w) - \bar{G}_n(w) = 4w^8 + a_6w^6 + a_4w^4 + a_2w^2 + a_0$, (39)

where coefficients $a_{(\cdot)}$ are functions of γ and p_c , and they are given as

$$a_6 = 8\gamma^2 p_c^2, \quad (40a)$$

$$a_4 = -p_c^4(-4\gamma^4 + 16\gamma^3 - 44\gamma^2 + 24\gamma + 3), \quad (40b)$$

$$a_2 = 2p_c^6(2\gamma^4 - 4\gamma^3 + 6\gamma^2 - 3), \quad (40c)$$

$$a_0 = \gamma^2 p_c^8(3\gamma^2 - \gamma + 6). \quad (40d)$$

After some algebraic and numeric manipulations, under condition (33), we find that all the coefficients $a_6 - a_0$ are positive definite independently from the value of p_c , and hence, the string stability constraint (38) is satisfied. \square

IV. RESULTS AND DISCUSSIONS

A. Study Setup

To emulate a realistic setup, the proposed control strategy is tested by implementing the car-like vehicles platoon in a vehicular mobility simulator called ICARS that is developed under the well-known Robot Operating System (ROS). ICARS has features such as real vehicle dynamics and models (engine, transmission, braking systems, etc.), both low and high level measurement units with noises, low level communication module between different parts of the vehicle and an onboard computer, real maps of an urban environment with different road conditions (curvature, bumps, etc.), vehicle communication modules, fully compatible with embedded computer architecture, and Human-Machine Interface to send high-level commands to the vehicle.

We consider a platoon composed of N follower vehicles (of type Robotized Renault ZOE-Q90 LIFE) plus a manually driven leader (of type Renault FLUENCE electric). The physical parameters of the vehicles are given in Table I. The platoon is tested in the Ecole Centrale de Nantes (ECN) campus as an example of urban environment (road with curvatures, bumps, humans, etc). The maps of ECN are shared offline among the vehicles.

The vehicles are initially parked. While the leader moves from its starting position to the destination points, its path is built online and sent via the radio communication module to the followers as the reference path to follow. The radio system receives the vehicle information in the platoon one by one. Thus, an algorithm is developed in the ICARS framework to receive the information from the radio module and then it synchronizes them.

The lateral controller presented in [23] is in charge of producing the steering angle of the vehicle to force the cars to follow the reference path of the leader (by minimizing both the lateral and angular deviations, r_i and ψ_i , respectively).

The longitudinal controller is implemented at a rate of 100 Hz. The control algorithm parameters are selected to guarantee the internal stability and string stability given in Theorem III.1, Corollary III.2 and Theorem III.4. Table II presented the control parameters. For safety and comfortability, the vehicle velocity and acceleration are constrained to certain levels as given in Table II.

TABLE I
VEHICLES PHYSICAL PARAMETERS

Parameter/Vehicle	FLUENCE	ZOE
Mass	1605 kg	1428 kg
Width	1.545 m	1.945 m
Height	1.462 m	1.562 m
Length	4.748 m	4.084 m
Rear-axis to Rear-end	1.140 m	0.657 m
distance between axes	2.701 m	2.588 m
distance between wheels	1.545 m	1.511 m
Wheel radius	0.29 m	0.29 m
Inertia momentum	45.0 kgm ²	28.0 kgm ²

TABLE II
CONTROLLER PARAMETERS

Parameter	Value	Parameter	Value
u_i	$\in [-6, 1] \text{m/s}^2$	v_i	$\in [0, 8] \text{m/s}$
$d_{r,i}$	10 m	γ	6
p_c	1	λ_i	1

B. Simulation Results

In the ICARS, three ZOE following vehicles are used, i.e., $N = 3$. Results of lateral motion are illustrated in Fig. 5a - Fig. 5c. The reference and actual paths of the leader and followers respectively are shown in Fig. 5a. Figures 5b and 5c present the time history of lateral and angular deviations of the followers with respect to the reference path generated from the leader. During the platoon initialization, it is noted a higher deviations due to the time taken by the followers to join the platoon and converge to the leader path. The deviations increase when the leader moves through a curved segment of the road/path. These figures enlighten the tracking capabilities of the lateral controller and the potential of the longitudinal one.

Figures 5d – 5h show the longitudinal motion results. As shown in Fig. 5e, the leader moves with variable velocity depending on the path conditions (by a human driver). This study investigates also the effect of accelerations perturbations as shown in Fig. 5d. The vehicles start at positions make the actual inter-vehicle distances different from the desired, and reach the consensus. At certain places in the path (e.g., road curvature), the position and velocity errors become higher which is desired to allow the follower to adapt its velocity based on the road conditions. Time history of the observer estimation error, $\tilde{z}_{2,i} = e_{q,i,j} - \hat{z}_{2,i}$, is given Fig. 5h which shows a convergence of the estimated predecessor velocity.

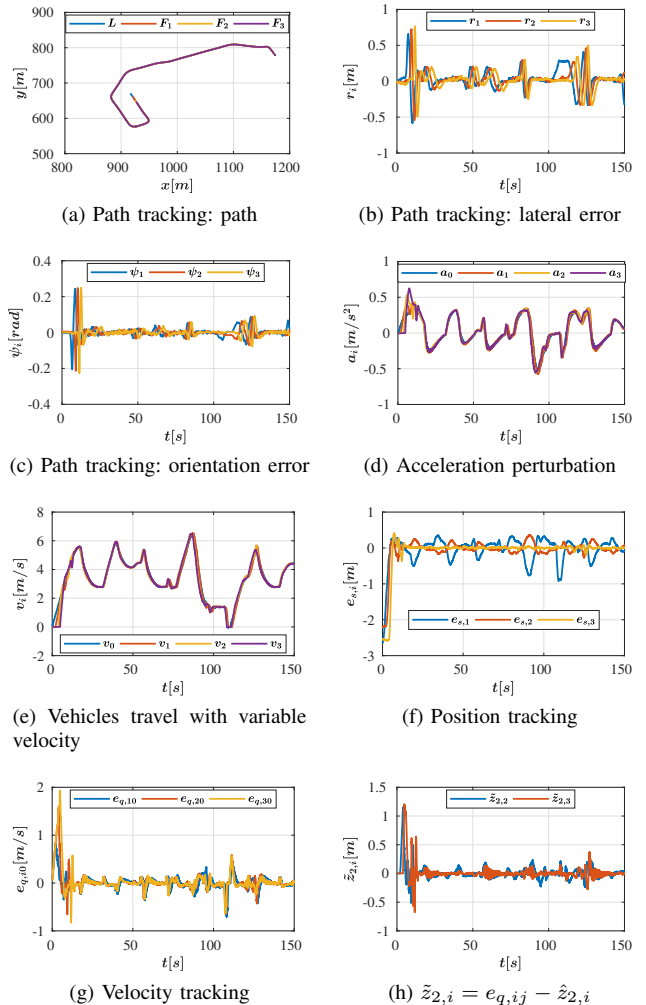


Fig. 5. Platoon simulation results. L : leader actual path, and F_i : follower i actual path. The followers track the path perfectly

To quantify our results, the Root Mean Squared Error (RMSE) of the estimation and tracking errors, calculated after the initialization period is passed, are reported in Table III which are in the acceptable ranges. Furthermore, this table shows that the position tracking errors are attenuated as the RMSE of $e_{s,3} < e_{s,2} < e_{s,1}$, which illustrate the string stability of the platoon. Demonstration of this study is available in [this video](#).

V. CONCLUSIONS

A successful application of an observer-based longitudinal controller for car-like vehicles platoon moving in an urban environment is proposed. Platoon kinematic and dynamic models are introduced in both Curvilinear and Cartesian coordinates with a framework that allows us to design the control law in the Curvilinear coordinates. An observer-based control law is proposed to achieve both consensus and string stability of the platoon vehicles. This control scheme requires a minimum number of communication links meanwhile it can achieve a higher traffic flow by using information from sensor-based and communication-based links. This strategy

TABLE III
RMSE OF TRACKING ERRORS IN SIMULATION.

RMSE	Follower i		
	1	2	3
$e_{s,i}$ [m]	0.2675	0.1141	0.0290
$e_{q,i}$ [m/s]	0.1199	0.0635	0.0480
$\tilde{z}_{2,i}$ [m/s]	-	0.0635	0.0480
r_i [m]	0.1324	0.1080	0.1104
ψ_i [rad]	0.0209	0.0196	0.0198

reduces the traffic in the communication channel which in turn it can enhance the communication robustness, and it enables us to use a low-cost communication module. Conditions for platoon internal stability and string stability are provided. Simulation studies are conducted that enlighten the efficacy of our approach.

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