Modified flatbed tow truck model for stable and safe platooning in presences of lags, communication and sensing delays

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Abstract— **Many ideas have been proposed to reduce traffic congestion problems. One of the proposed ideas is driving in platoon. Constant spacing policy is the most important control policy. It increases traffic density, but it needs very reliable communication channel. Driving with a constant time headway between vehicle is also well known policy and robust control law, but the inter-vehicle distances are very large. We have proposed in [1], [2] a modification for the constant time headway policy. This modification reduces the inter-vehicle distances largely using only one information shared between all vehicles.**

In this work we propose an additional modification of our control law. This modification makes our control law similar, in form, to the classical constant spacing policy, but it only uses the same shared information. This modification improves the stability of the platoon. We proved the robustness of the control law in presence of parasitic actuating lags, sensing and communication delays.

This prove can be also used for proving the stability of classical spacing policy in presence of all previous delays, contrary to what have been proved in some papers in the literatures.

I. INTRODUCTION

Many ideas have been proposed to solve traffic congestions. Platooning using automated car seems to be promising idea. It increase traffic density and safety, at the same time it decrease fuel consumption and driver tiredness [14]. There are many projects on highways platooning, such as the platooning project in the PATH program (Partners for Advanced Transit and Highways) [15], SARTRE Project [6], and CHAUFFEUR 2 project [7]. Nevertheless, research is still going on for highways and urban areas platooning.

It was concluded that for high capacity traffic the constant spacing policy is necessary at the price of inter-vehicle communication [17].

Using communication may cause instability due to transmission delays or data drop. In [8] the effect of communication delays on string stability has been studied. It has been proved that the platoon becomes un stable for any propagation delays in the communicated leader informations. A solution was proposed in [18] by synchronizing all the vehicles to update their controllers at the same time and using the same leader information, it was shown that string stability can be maintained if the delay in preceding vehicle information is small. The effects of clock jitter, which may cause instability, was briefly mentioned. [10] proved string

stability for the leader-predecessor and predecessor-successor framework neglecting information delays between vehicles. The effect of losing the communication is presented in [17]. It has been proved that string stability can be retained, with limited spacing error, by estimating lead vehicle's state during losses.

Another parasitic time delays and lags may be introduced in the physical systems due to actuating and sensing times. This delays may have also significant effects on stability if they are not taken into account. Stability conditions for many control laws, in presence of lags and parasitic delays, can be founded [10], [13], [16], [19]. A detailed study of the effect of delays and lags when using classical time headway policy for homogeneous and heterogeneous platoons is found in [9]. The results show that the time headway policy is more immune, to parasitic sensing and communication delays and actuating lags, than the constant spacing policy. But the large spacings between vehicle make it less important.

In [1], [2] we have proposed a modification of the time headway policy, which reduces the inter-vehicle distances largely to become nearly equal to the desired distance. These works were generalized to urban platoons [3], [4]. In lateral control, we used sliding mode control to ensure stability and robustness. Safety of platoon, when using this control law, was briefly studied in [1], [4] and deeply treated during critical scenarios in [5] . These scenarios include leader and followers hard braking taking into account even in case of communication loss.

In this paper, we continue our previous work. We concentrate on controlling identical tourist cars on nearly flat highways. We propose a modification to our control law. This modification enhances the robustness of the control and increase the immunity to parasitic actuating lags, sensing and even larges communication delays.

This paper is organized as follows: in section II we present a model for the vehicle with and without taking the lags and delays into account, In addition we will give a model for the platoon. The control law will be given in section III. String stability is proved in section IV. Then in section V, we show simulation results. Conclusion and perspective are done in the final section.

II. MODELING

A. Longitudinal Model of the Vehicle

We take a simplified longitudinal dynamic model [2], [9]:

$$\ddot{x} = \dot{v} = W \tag{1}$$

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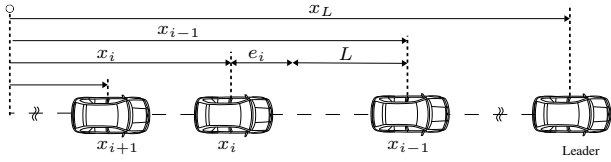[2] G. GARCIA is with Ecole Centrale de Nantes (ECN), Nantes, France

Fig. 1.   A platoon

where x: Position of the vehicle, W: is the control input.

B. Vehicle model taking into account parasitic time delays and lags

The model given in (1) is ideal modem and is not sufficient in reality. Using it may lead to unstable control due to presence of parasitic delays and lags. Lags make the net engine torque not immediately equal to the desired torque computed by the controller. Another source of instability is the delay in the communicated data. This delay is due to heavy communications or data drops.

A system model taking into account actuating lags and sensing delays is found in [9]. We extend this model to take into account communication delay, this give us the following vehicle model:

$$\tau_i \; \ddot{v}_i(t) + \dot{v}_i(t) = u(t - \Delta_i, \tau_{c_i}) \qquad (2)$$

where τ_i is the combination of the all the lags taken as a *lumped* lag, Δ_i is the combination of the all the sensing time delays taken as a *lumped* delay, τ_{c_i} is communication delay.

C. Platoon Model

The platoon is a set of vehicles moving together at the same speed and keeping a desired distance L between each two consecutive vehicles.

The spacing error of the i-th vehicle, assuming a point mass model for all vehicles, is defined as follow:

$$e_i = \Delta X_i - L \qquad (3)$$

where:

- $\Delta X_i = x_{i-1} - x_i$: real spacing between car number i and its predecessor, car number $i-1$.
- x_i: position of i-th vehicle.
- L: desired inter-vehicle distance.
- $\dot{e}_i = \dot{x}_{i-1} - \dot{x}_i = v_{i-1} - v_i$:the kinematic evolution of the spacing error
- $v_i = \dot{x}_i$ represents the speed of the i-th vehicle.

The longitudinal model of the platoon, shown in fig. 2 is called flatbed tow track model [1]. It is a set of vehicles virtually connected by one-directional spring-damper systems, and a virtual truck which is set to drive at a speed V, the value of V being known to all vehicles of the platoon. In this paper, we proposed to add new virtual spring between each vehicle and the virtual truck. This enhanced the stability and made our control law similar to constant spacing policy. The main difference is that in our case all the vehicles receive only the speed of the virtual truck V then each vehicle compute the position of the virtual truck X_V by integration.
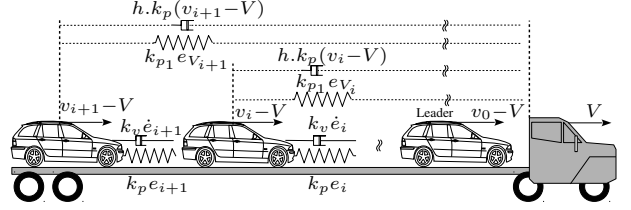

Fig. 2.   Enhanced flatbed tow truck model

III. CONTROL LAW AND SPACING ERROR DYNAMICS

A. Control Objectives

The main objectives of the control law are:

1) Make the inter-vehicle distance equal to L so $e_i \to 0$.
2) All vehicles must move at the same speed so $v_i \to v_L$.
3) Stable platoon (String stability).
4) Increase traffic density.
5) Safety (collision free).
6) Stability and safety in case of communication losses.
7) Stability and safety even in presence of sensor time delay, actuator lags and communication delays.

Objectives from 1 to 6 are deeply studied in [1]–[5]. In this work we deal with Objectives 7.

B. Longitudinal Control

Introducing the virtual truck in the new longitudinal model enable us to deal with relative speed instead of the absolute speed, this enhances the performance of the longitudinal control by reducing the distance required to ensure string stability. This model is a modification of the classical time headway policy by subtracting a new term V form all speeds.

Spacing error becomes [2]:

$$\delta_i = e_i - h \left(v_i - V \right) = e_i + h \; \dot{e}_{V_i} \qquad i = 1...N \quad (4)$$

We add new term $\frac{\lambda_1}{h} e_{V_i}$ to our control law given in [2]. The new term is proportional to the distance between the i-th vehicle and the truck:

$$W_i = \frac{\dot{e}_i + \lambda \; \delta_i + \lambda_1 \; e_{V_i}}{h}, \quad i = 1...N \qquad (5)$$

Where $e_{V_i} = X_{V_i} - x_i - i \; L$,

N: is the total number of vehicles in the platoon.

V: is a common speed value shared by all vehicles of the platoon, it must be the same value for all the vehicles at the same sampling time [1], [2].

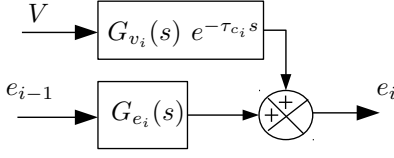X_{V_i}: is the position of the virtual camion, it can be computed by accumulating V.

Fig. 3. Spacing error model taking delays and lags into account

C. Longitudinal Control With Delays and Lags:

The control law of the platoon when taking into account delays and lags becomes the following:

$$W_i(t, \Delta_i, \tau_{c_i}) = \frac{\dot{e}_i(t - \Delta_i) + \lambda \delta_i(t, \Delta_i, \tau_{c_i}) + \lambda_1 e_{V_i}(t, \Delta_i, \tau_{c_i})}{h} \quad (6)$$

Where:

$$\delta_i(t, \Delta_i, \tau_{c_i}) = e_i(t - \Delta_i) - h \left(v_i(t - \Delta_i) - V(t - (\Delta_i + \tau_{c_i})) \right) \quad (7)$$

$$e_{V_i}(t, \Delta_i, \tau_{c_i}) = X_V(t - (\Delta_i + \tau_{c_i})) - x_i(t - \Delta_i) - i\, L \quad (8)$$

With no loss of generality, we assume that $v_N(0) = v_i(0) = ... = v_0(0)$, $a_N(0) = a_i(0) = ... = a_0(0)$, $\delta_N(0) = \delta_i(0) = ... = \delta_0(0)$ $(0 \leq i \leq N)$ at the initial conditions.

We define $\Delta_{c_i} = \tau_{c_{i-1}} - \tau_{c_i}$ the propagation delay, form vehicle i to vehicle $i - 1$, of the leader's transmitted data.

For homogeneous platoon we have :

$\Delta_i = \Delta_{i-1} = ... = \Delta, \tau_i = \tau_{i-1} = ... = \tau, \ \tau_{c_{i-1}} - \tau_{c_i} = \Delta_{c_i} = ... = \Delta_{c_1} = \Delta_c$ so $\tau_{c_i} = i\,\Delta_c$. Hence, $G_{e_i} = G_{e_i}(s) = G_{e_{i-1}} = ... = G_{e_1} = G_e$, $G_{V_i} = G_{V_{i-1}} = ... = G_{V_1} = G_V$.

Using (6), (7) and (2) and then by calculating Laplace transformation taking into account the previous assumptions we get:

$$e_i(s) = G_e(s)e_{i-1}(s) + G_V(s)e^{-\tau_{c_i}s}V(s), \quad i = 2...N \quad (9)$$

Where

$$G_e(s) = \frac{(s + \lambda)\ e^{-\Delta\ s}}{h\tau s^3 + h\ s^2 + ((1 + h\lambda)s + \lambda + \lambda_1)e^{-\Delta\ s}} \quad (10)$$

$$G_V(s) = \frac{(\lambda\ h\ s + \lambda_1)\ e^{-\Delta\ s}(e^{-\Delta_c\ s} - 1)}{s(h\tau s^3 + h\ s^2 + ((1 + h\lambda)s + \lambda + \lambda_1)e^{-\Delta\ s})} \quad (11)$$

Equation (9) shows that the error of the i-th vehicle is not just a function of e_{i-1} but it is also a function of the shared speed $V(s)$ as shown in fig. 3, this is due to presence of communication delay.

It is very important to compute the dynamics of e_1. This dynamics has an important effect on the stability and the safety of the platoon. By using (6), (7) and (2) and by adding and subtracting $(\tau h\ddot{v}_0 + h\dot{v}_0 + \lambda hv_0 + \lambda_1 x_0)$ we get the dynamics of e_1 as a function of the speed of the leader v_0 and V :

$$\tau h\dddot{e}_1(t) + h\ddot{e}_1(t) + (1 + \lambda h)\dot{e}_1(t - \Delta) + \lambda e_1(t - \Delta) =$$

$$\tau h\ddot{v}_0(t) + h\dot{v}_0(t) + \lambda hv_0(t - \Delta) - \lambda hV(t - (\Delta + \tau_{c_1}))$$

$$+\lambda_1 x_0(t - \Delta) - \lambda_1 X_V(t - (\Delta + \tau_{c_1}))$$

$$\quad (12)$$

We compute Laplace transformation:

$$e_1(s) = F_e v_0(s) - F_V V(s) \quad (13)$$

$$F_e = \frac{\tau hs^3 + h\ s^2 + (\lambda hs + \lambda_1)e^{-\Delta\ s}}{s(\tau hs^3 + h\ s^2 + ((1 + \lambda h)s + \lambda + \lambda_1)e^{-\Delta\ s})} \quad (14)$$

$$F_V = \frac{(\lambda hs + \lambda_1)e^{-(\Delta + \Delta_c)s}}{s(\tau hs^3 + h\ s^2 + ((1 + \lambda h)s + \lambda + \lambda_1)e^{-\Delta\ s})} \quad (15)$$

IV. STABILITY

A. String Stability of Longitudinal Control

The general string stability definition in the time domain is given in [15], in essence, it means all the states are bounded if the initial states (position, speed and acceleration errors) are bounded and summable.

In [12] we find a sufficient condition for string stability:

$$\|e_i\|_\infty \leq \|e_{i-1}\|_\infty \quad (16)$$

which means that the spacing error must not increase as it propagates through the platoon. To verify this condition, the spacing error propagation transfer function is defined by:

$$G_i(s) = \frac{e_i(s)}{e_{i-1}(s)} \quad (17)$$

A sufficient condition for string stability in the frequency domain is derived:

$$\|G_i(s)\|_\infty \leq 1 \quad and \quad g_i(t) > 0 \quad i = 1, 2..N \quad (18)$$

where $g_i(t)$ is error propagation impulse response of the i-th vehicle.

We proved the stability of the platoon in two steps: firstly by finding stability conditions taking into account only parasitic sensing time delay and lags, Then we add the communications delays and we checked stability.

B. System Stability With Parasitic Time Delay and Lags:

We neglect communication delays. All the equation and the condition which will be found here will be also used when taking into account the communication delay.

Neglecting communication delays makes $G_V(s) = 0$ and we get:

$$e_i(s) = G_e(s)e_{i-1}(s) \qquad (19)$$

In this case we can use (18) to check the stability so we have to verify that $\|G_e\| < 1$.

We have:

$$\|G_e(\omega)\| = \sqrt{\frac{a}{a + \mu + \lambda_1^2 + 2\lambda\lambda_1}} \qquad (20)$$

A sufficient condition to ensure the stability is $\mu \geq 0$. This gives a group of conditions that verify the stability of the platoon in presence of lags and sensor delays:

$$\left\{ \begin{array}{ll} \lambda \leq \frac{h - 2(\Delta + \tau) + 2\lambda_1\tau\Delta}{2(h(\Delta + \tau) - \Delta\tau)} & \& \quad \frac{\lambda_1}{\lambda} < \frac{h}{2} \quad \& \\[2mm] \lambda \geq \frac{\lambda_1\tau - 1}{h - \tau} & \& \quad h \geq 2(\Delta + \tau) \end{array} \right\} \qquad (21)$$

The last condition is to ensure that $\lambda \geq 0$.

C. System Stability with Communication Delays:

Stability can be verified easily using condition (18) when the error is only a function of the previous error. When the errors become a function of additional variables we have to check the maximum limits of the state variables (spacing, speed and acceleration errors). The system is stable if state variables in the platoon are always bounded [15].

Using (9) we can get progressively the relation between $e_i(s)$ and $e_1(s)$:

$$e_i(s) = G_e^{i-1} e_1 + G_V \ e^{-i\Delta_c \ s} \frac{1 - (G_e \ e^{-\Delta_c s})^{i-2}}{1 - G_e \ e^{-\Delta_c s}} \ V(s) \qquad (22)$$

So we have:

$$\|e_i\| \leq \underbrace{\|G_e\|^{i-1} \|e_1\|}_{\xi_1} + \underbrace{\|G_v\| \left\| \frac{1 - (G_e \ e^{-\Delta_c s})^{i-2}}{1 - G_e \ e^{-\Delta_c s}} \right\| \|V\|}_{\xi_2} \qquad (23)$$

In the following we study the limits of spacing error of the vehicle i when $i \longrightarrow \infty$.

The first term ξ_1 is bounded ($\forall \omega$ and $i \longrightarrow \infty$) if $\|G_e\| \leq 1$ and $\|e_1\|$ is bounded.

The conditions that keep $\|G_e\| \leq 1$ are already given in (21).

From (13) we can prove that $\|e_1\|$ is also bounded because the norm of $\|F_e\|$ and $\|F_V\|$ converge toward zero for high frequencies. For low frequencies e_1 becomes equal to $\lambda h(V - v_i) + \lambda_1(X_V - x_0)$, this can be bounded if we choose V correctly. For all other frequencies, the nominator of $\|F_e\|$ and $\|F_V\|$ is always larger or equal to $\sqrt{\omega^2(\lambda + \lambda_1)^2}$ (we already proved that $\mu \geq 0$). This means that the nominators are larger than zero $\forall \omega \neq 0$; So $\exists \ \alpha_1 \leq \infty$ and $\alpha_2 \leq \infty$: $\|F_e\| \leq \alpha_1$ and $\|F_V\| \leq \alpha_1 \ \forall \omega \neq 0$. Then the first term always converge toward zero $\forall \omega$ when $i \longrightarrow \infty$.

For the second term ξ_2 we have $0 < \|1 - G_e \ e^{-\Delta_c s}\| \leq 2$, $0 < \|1 - (G_e \ e^{-\Delta_c s})^i\| \leq 2$. We can also prove that $\|G_V\|$ is always limited for all ω; so ξ_2 is limited for all frequencies even when $\omega = 0$ hence the platoon is stable for limited communications delays.

So we can conclude that the platoon is stable in presence of lags, sensing delays and even communication delays. The conditions of stability in presence of lags and sensing delays are given in (21). While the maximum acceptable communication delay $\Delta_{c_{max}}$ can be defined by safety conditions.

V. SIMULATIONS

Simulation has been done using Matlab. A large platoon, consisted of 60 vehicles, is created. In reality, the platoons are much more smaller, but we use this big platoon just to verify that the error is not increasing even for the vehicle with a big index ($i \longrightarrow \infty$). The desired inter-vehicle distance $L = 10 \ m$. The leader accelerates from stationary state to reach a speed of $140 \ km/h$ and then it make emergency stop. We take parasitic sensing delay equal to $\Delta = 200 \ ms$, the actuating lags equal $\tau = 200 \ ms$ and a communication delay between each consecutive vehicles equal to $\Delta = 50 \ ms$. We take $h = 2, \lambda = 0.7, \lambda_1 = 0.2$. To ensure safety, the maximum acceptable acceleration/deceleration to keep safety is $\mp 4.5 \ m.s^{-2}$. For clarity, we only show one speed from each ten consecutive vehicles.

We can see in fig.4 that the platoon is stable because the errors are not increasing through the platoon. In addition, we can see that the spacings between vehicles are always larger than zero so the platoon is safe. Previously in [2] we chose $L = 5m$, we notice here that we have doubled the desired inter-vehicle distance to accommodate the errors generated from lags and delays. We tested the system with the worst cases (acceleration from zero to maximum speed with maximum acceleration and then we applied the emergency stopping) to verify the stability and safety in its limits. In practice we add additional safety distance in the desired distance to ensure more safety.

VI. DISCUSSION

- The new modification improves the performance of our control law, without requiring new data from other vehicle. Each car can compute the current position of the truck using the shared speed V. So X_V is always the same for all vehicles.

- In case of communication loss, all vehicles switch to autonomous stable mode by making $V \rightarrow 0$ and $X_v \rightarrow x_i$ (for the i-th vehicle). This enable the vehicles to switch to classical time headway policy.
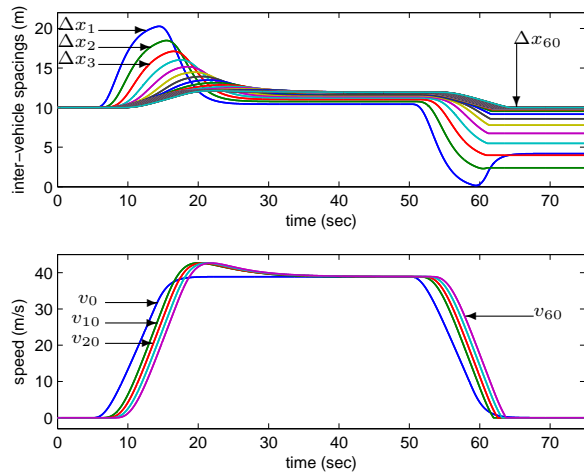
Fig. 4. Inter-vehicle spacings in presence of lags, sensing and communication delays

- No need to transmit the value of X_V because it is computed in each vehicle by integrating V. Losing the communication will prevent the vehicles to have the same value for X_V; so it is necessary to update X_V for all the vehicles after each communication loss.
- The error of integration will have no effect on the stability because this error will be the same for all the vehicles. But the most important condition is to keep $X_v - x_0$ limited.

We can see that this control law with our control law proposed in our previous works [2] and the classical time headway represent integrated frame work for controlling the platoon with decreasing communication rate respectively and we can switch from one law to another smoothly in case of communication difficulties.

VII. CONCLUSION

In this paper we have addressed the control of platoons on highways. The longitudinal dynamics is modeled using modified flatbed tow truck model. We proved the robustness of this control law to lags, parasitic delays and even for communication delays. Sufficient stability conditions was given in (21). In the future work, passenger comfort and the safety of the platoon will be studied.

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