Altruistic Distributed Target Allocation for Stable Navigation in Formation of Multi-robot System

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Abstract: This paper deals with keeping the formation of a group of mobile robots. A set of virtual targets (points) form a virtual structure of the same shape as the desired formation. Hence, to join and to keep this formation, each robot has only to track one of these targets. The objective of the paper is to propose a cooperative strategy between the robots in order to rapidly join the virtual structure: instead of assigning ahead the targets to the robots, this strategy consists of making each one able to negotiate the closest target. If the latter is desired by a lot of robots, it is left to the robot which meets more difficulties to find an other target. Negotiation is based on a minimalist communication of relative cost coefficients between the robots. Simulation and experimental results validate the proposed contributions.

Keywords: Motion control, Navigation in formation, Control architectures, Cooperative multi-robot system, Virtual structure.

1. INTRODUCTION

Controlling and coordinating Multi-robot systems MRS are an attractive research subject thanks to their large application fields (spatial exploration, platooning, rescue, etc.). In this paper, we are particularly interested in the navigation in formation task with a reactive manner. Works given in the literature converge to three principle approaches: hierarchical approach, behavior based, and the virtual structure strategy. In the first approach, one or many robots are considered as leaders while the other robots are the followers. Generally, the leader tracks a predefined trajectory while the followers track its transformed coordinates (Léchevin et al. (2006)), (Gustavi and Hu (2008)). This approach is simple to perform. However, it is noticed that a leader failure leads to stop the whole system. In behavior based approach (Antonelli et al. (2010)), (Balch and Arkin (1999)), all the robots are homogeneous. It means that perception and control are equitably distributed on the robots. This method is then much more tolerant to failure (Parker (1996)) than the hierarchical approach. Behavior based implies that each one has a set of weighted behaviors (basic tasks) to achieve. The resulting behavior of the group emerges from the basic ones without an explicit model of the overall cooperative behavior. However, this approach is upbraided for the way to choose the applied control to each robot. In fact, according to perception information, control system switches between behaviors (competitive approach (Brooks (1985))), or merges several controllers (motor schema (Arkin (1986))). This naturally makes hard studying the stability of the overall control. Virtual structure approach considers the formation as a single virtual body. The shape of the latter is the desired formation shape, and its motion is translated into the desired motion of each vehicle (Do (2007)), (Li et al. (2005)). The virtual structure is generally tackled through potential field methods (Ogren et al. (2002)), (Mastellone et al. (2007)): thus, all members of the formation track assigned nodes which move into the desired configuration. Each node applies an attractive field to the corresponding robot whereas obstacles and other robots apply repulsive field to avoid collision. The weakness of virtual structure is that potential applications are limited especially when the formation shape needs to be frequently reconfigured.

To overcome drawbacks of these strategies, it was proposed to combine virtual structure and behavior based in (Benzerrouk et al. (2010)). The achieved task (attaining and maintaining a desired formation while avoiding collision) is divided into two basic tasks (behaviors): attraction to a dynamic target, and obstacle avoidance. These behaviors do not use potential fields which allows possible reconfiguration of the formation.

In this paper, a particular attention is given to the cooperative strategy between the robots. Hence, only the part of maintaining the formation is treated and obstacle avoidance will not be detailed. In fact, in the literature, it is noticed that the formation emerges because each robot tracks its target. These targets are assigned ahead to the robots: In (Balch and Arkin (1999)), relative positions of the robots in the formation are based on an identification number ID. Each robot obtains then the relative position corresponding to its ID. The same strategy is followed in (Lewis and Tan (1997)). Even in recent works, every robot tracks a target already assigned to it (Ghommam et al. (2008)), (Lalish et al. (2006)), (Ren and Beard (2004)) and optimizing the allocation of these targets was not studied.

This preliminary assignation allows to avoid conflicts (to not choose the same target) between the robots in a simple way. However, if these robots optimize the way of sharing the targets according to their initial positions, formation can be attained faster and some deadlock situations can be avoided. In fact, by assuming homogeneous robots (they have the same

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constraints (maximal velocities, accelerations, etc.)), reducing covered distances leads to reducing time to attain the formation.

In this paper, we are then interested in the target negotiation between the robots. It is proposed that each one negotiates, in a distributed manner, the closest target with the others in order to reduce the total distance completed and thus, time to attain the formation. If one target is desired by many robots, it is given up to the one which seems having higher costs for the other targets. A form of altruism is then observed between the robots.

The idea of the dynamic allocation of the targets is inspired from the auction sales activity. The latter is used in the literature for the task allocation to MRS (Dias et al. (2006)) such as exploration (Kalra et al. (2005)), visiting different locations (Tovey et al. (2005)), and box pushing (Brian and Mataric' (2002)). Three main auction mechanisms were developed. Combinatorial auctions (Berhault et al. (2003)) treat all the possible combination of the tasks. Hence, they give optimal results. However, time computation becomes easily heavy when the tasks and robots number increase. Moreover, this computation requires a central unit with a total knowledge of the environment. This is inconsistent with our desired distributed architecture of control. Repeated parallel auctions (Dias (2004)), treat each task separately of the other tasks. Auctions are repeated every time interval to test if one task can be improved if it is allocated to an other robot. In sequential mechanisms (Tovey et al. (2005)), each robot auctions each task individually taking into account its previous state. Therefore, there is no need to a central coordination. However, robots have to communicate their costs to determine the winner of each task.

As in (Nanjanath and Gini (2010)), our algorithm is close to a combination of the two last methods since each robot wins a target or gives it up to another by computing and comparing (itself) costs of these targets. Allocation of the targets can occur every ΔT to adapt the robots to the formation changes. Only a minimalist communication is needed with the proposed algorithm.

The remainder of the paper is organized as follow: in next section (2), the task of navigation in formation is defined and the dynamic allocation of the targets algorithm is detailed. Section 3 reminds the proposed control law insuring that each robot joins the formation. Section 4 gives simulation and experimental results. Finally, conclusion and some prospects are given in section 5.

2. NAVIGATION IN FORMATION USING VIRTUAL STRUCTURE

2.1 The virtual structure principle

Before discussing cooperative strategy, the adopted virtual structure principle is reminded. Consider N robots with the objective of reaching and maintaining them in a given formation. The proposed virtual structure that must be followed by the group of robots is defined as follow:

- Define one point which is called the main dynamic target (cf. Figure 1),
- Define the virtual structure to follow by defining N_T nodes (virtual targets) to obtain the desired geometry. Each node i is called a secondary target and is defined according to a specific distance D_i and angle Φ_i with respect to the main target. Secondary targets defined by this way have then the same orientation θ_T. However, each target i will have its linear velocity v_{T_i}. The number of these targets N_T must be $N_T \geq N$.

An exemple to get a triangular formation is given in figure 1.

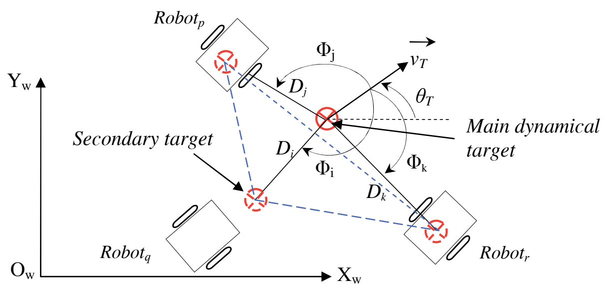


Fig. 1. Keeping a triangular formation by defining a virtual geometrical structure.

2.2 Cooperative strategy between the robots: dynamic allocation of the targets

The idea is that robots cooperate in order to reduce time of attaining the formation. As already discussed, each robot chooses the closest target to track. However, this may create conflicts when many robots choose the same target. To avoid this conflict, a hierarchy between them was adopted in (Benzerrouk et al. (2010)). Hence, the desired target is given up to the robot of a higher rank. However, this hierarchy is not justified especially if the robots have the same characteristics. In this paper, it is then proposed that each robot computes a coefficient per target to describe its interest for this one. Computed every time interval ΔT, this coefficient informs if this target is very close or very far from the robot comparing to the other targets. It is called *Relative Cost Coefficient* (RCC) and is noted δ. Comparing RCCs of the same target allows to each robot to decide if it takes this target or gives it up to an other.

In what follows, the RCC of a robot i for the target j is noted δ_{ij}. It is computed as

$$\delta_{ij} = \frac{d_{S_{ij}}}{\sum_{k=1}^{N_T} d_{S_{ik}}} = \frac{d_{S_{ij}}}{d_{S_{ij}} + \sum_{k=1,k\neq j}^{N_T} d_{S_{ik}}} \tag{1}$$

where $d_{S_{ij}}$ is the distance between the robot i and the target j. For a robot i, the set of RCCs for all the targets is put in a vector Δ_i.

It is clear that (cf. Equation 1) $0 \leq \delta_{ij} \leq 1$

Moreover, δ_{ij} is as close to 0 as

$$d_{S_{ij}} \ll \sum_{k=1,k\neq j}^{N_T} d_{S_{ik}} \tag{2}$$

Thus, every robot prefers the target with the smallest RCC because it is the closest one. It is then noticed that the same result would be obtained by simply comparing the distances to the different targets and directly choosing the closest one. However, the main objective of the RCC is to negotiate the desired target with the others. Hence, if two robots i and k ask for the same target j (they are in conflict for this target), distances $d_{S_{ij}}$ and $d_{S_{kj}}$ are not sufficient to know which robot has to obtain it in order to attain faster the formation.

Therefore, to negotiate their targets, robots act according to the following proposition:

Proposition 1. *If many robots are in conflict for one target, then this target is left to the robot having the smallest RCC for this target.*

In fact, according to (2), the strategy of this proposition is to compare the situation of the robots according to the existing targets and to give up the desired one to the furthest robot from the other targets. The proposed distributed strategy for dynamic allocation of the targets which allows an altruism between the robots is given in algorithm 1.

Require: Vectors Δ_i, $i = 1..N$.
Set of robots $I = \{1..N\}$.
Ensure: Choice of the virtual target to follow.

1: **while** (Target not chosen) **do**
2: choose the target j corresponding to the smallest RCC $\Delta_i(j)$;
3: **if** $\Delta_i(j) < \min_k(\Delta_k(j)), \forall k \neq i, k \in I$ **then**
4: go to line 12;
5: **else**
6: remove k corresponding to $\min_k(\Delta_k(j))$ from I;
7: choose an other target l such that $\Delta_i(j) < \Delta_i(l) < \Delta_i(m), \forall m \neq j$;
8: $j = l$;
9: go to line 3;
10: **end if**;
11: **end while**;
12: go toward the chosen target ;

Algorithm 1: Distributed virtual target assignment $(N_T \geq N)$.

The proposed algorithm is distributed on all the robots. It requires that each robot i communicates only its vector Δ_i to the other ones. It is also proposed that a vector Δ_i includes the subscript i indicating the robot identifier. Identifiers of the robots are randomly chosen and do not indicate any hierarchy for the target assignment.

According to this algorithm, every robot is able to deduce if the desired target will be really available or it will be taken by an other one having a less corresponding RCC. Negotiation and allocation of the target is then done in a distributed manner.

It is noticed that the required communication process is very basic and can be summarized in algorithm 2. The time interval δ_t to wait (line 2, algorithm 2) allows to avoid collision between network packets.

1: receive the vectors Δ_k such that $k = 1..i-1$;
2: wait a time δt, then send the vector Δ_i;
3: receive the vectors Δ_k such that $k = i+1..N$;

Algorithm 2: Sequential communication process of the robot i with the other robots.

Even if most of conflicts in target assignment are solved thanks to algorithm 1, some points need to be discussed:

- if the robot i has the same RCC value for a target j as an other robot k (which means $\Delta_i(j) = \Delta_k(j)$), then i can search the second possible target l for itself such that $\Delta_i(j) < \Delta_i(l) < \Delta_i(m), \forall m \neq l$, and the second possible target n for the robot k with $\Delta_k(j) < \Delta_k(n) < \Delta_k(m), \forall m \neq n$ (robot k has naturally the same reasoning). The robot i keeps the target j if $(\Delta_i(l) > \Delta_k(n))$, because it means that k will find the target n with a cheaper RCC. Otherwise, if $(\Delta_i(l) < \Delta_k(n))$, the robot i gives up the target j because l seems cheaper for it than the target n for the robot k.
- If the robots i and k have also the same RCC for their next targets $(\Delta_i(l) = \Delta_k(n))$ (targets l and n are as defined above). In this case, the target j can be indifferently taken by i or k. However, to avoid that both the robot choose the target j, or both leave it, it is proposed that the robot with the higher subscript obtains it. This convention cannot be considered as a hierarchy between the robots since they choose their targets with the same RCCs.
- Finally, this distributed reasoning can be easily applied if more than two robots negotiate the same target. Note that according to algorithm 2, communication is done once at the beginning of negotiations. It is then not affected (by becoming tedious) when many robots are in conflict for a target.

3. THE APPLIED ROBOT CONTROL

3.1 Attraction to a Dynamic Target Controller

To remind the attraction to a Dynamic Target Controller which allows to keep the formation, consider a robot i with (x_i, y_i, θ_i) pose. This robot has to track its secondary dynamic target. To simplify notations in the following, the same subscript of the robot is given to its target. The latter is then noted $T_i(x_{T_i}, y_{T_i}, \theta_T)$ (cf. Figure 2) and the variation of its position can be described by

$$\begin{cases} \dot{x}_{T_i} = v_{T_i}.cos(\theta_T) \\ \dot{y}_{T_i} = v_{T_i}.sin(\theta_T) \end{cases} \quad (3)$$

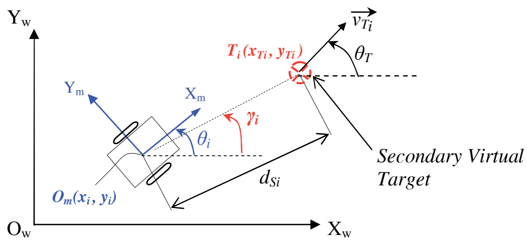


Fig. 2. Attraction to a dynamic target.

Let's also introduce the used robot model (cf. Figure 2). Experimental results are made on Khepera robots, which are unicycle mobile robots. Their kinematic model can be described by the well-known equations (cf. Equation 4).

$$\begin{cases} \dot{x}_i = v_i.cos(\theta_i) \\ \dot{y}_i = v_i.sin(\theta_i) \\ \dot{\theta}_i = \omega_i \end{cases} \quad (4)$$

where θ_i, v_i and ω_i are respectively the robot orientation, the linear and angular velocities.

The set-point angle that the robot must follow, to reach its dynamic target, is given by

$$\theta_{S_{ati}} = \arcsin(b\sin(\theta_T - \gamma_i)) + \gamma_i \quad (5)$$

Where $b = \frac{v_{T_i}}{v_i}$. γ_i is the angle that the robot would have if it was directed to its target (cf. Figure 2). This set-point has been obtained by keeping γ_i constant. More details and proofs are available in (Benzerrouk et al. (2010)).

3.2 The used control law

The used control law, allows to each robot i to converge to its set-point.

$$v_i = v_{max} - (v_{max} - v_{T_i})e^{-(d_{S_i}^2/\sigma^2)} \tag{6a}$$

$$\omega_i = \omega_{S_{ati}} + k\tilde{\theta}_i \tag{6b}$$

where

- v_i and ω_i are linear and angular velocities of the robot respectively. It is also noted $\omega_{S_i} = \dot{\theta}_{S_{ati}}$ where $\dot{\theta}_{S_{ati}}$ is the angular variation of $\theta_{S_{ati}}$.
- v_{max} is the maximum linear speed of the robot,
- σ, k are positive constants,
- $\tilde{\theta}_i$ is the error orientation so that $\tilde{\theta}_i = \theta_{S_{ati}} - \theta_i$ which gives $\dot{\tilde{\theta}}_i = \omega_{S_{ati}} - \omega_i$.

Lyapunov based stability allowed to prove the convergence of the robot to its target (Benzerrouk et al. (2010)). Convergence of the whole multi-robot system to the set-point virtual structure can then be derived by studying the following Lyapunov function

$$V = \sum_{k=1}^{N} V_k \tag{7}$$

where V_k is the Lyapunov function associated to the robot k. This function was defined as

$$V_k = \frac{1}{2}\tilde{\theta}_k^2 \tag{8}$$

It has been proved that $\dot{V}_k < 0$ (when $\tilde{\theta}_k \neq 0$)(Benzerrouk et al. (2010)). Therefore, it can be easily deduced that

$$\dot{V} = \sum_{k=1}^{N} \dot{V}_k < 0 \tag{9}$$

The global system is then asymptotically stable. Moreover, it can be noticed that the applied angular velocity ω_i allows exponential convergence of the error orientation to 0.

However, this theoretical convergence is applied to non-holonomic mobile robots. It means that stability will be insured only if the angular set-point is reachable by the robot while considering its kinematic constraints (maximal velocities, maximal accelerations).

In (Benzerrouk et al. (2010)), it has been proved that the robot converges to its target only if

$$v_i \geq v_{T_i} \Leftrightarrow b \leq 1 \tag{10}$$

According to equation (6a), it is noticed that the linear velocity of the robot verifies the condition given by inequation (10), and takes into account its maximal linear velocity. However, it is noted that linear velocity of the secondary targets depends on their relative position in the virtual structure. The choice of D_i and angle Φ_i affect then v_{T_i}.

Moreover, the variation of the angular set-point $\dot{\theta}_{S_{ati}}$ has to stay reachable by the robot. Indeed, the angular velocity supported by the robot has a maximal value noted ω_{max}. Defining the bounaries of $\dot{\theta}_{S_{ati}}$ so that $|\omega_i| \leq \omega_{max}$ and those of D_i and Φ_i so that $v_{T_i} < v_{max}$ will be discussed in a future work.

4. SIMULATION AND EXPERIMENTAL RESULTS

To show the relevance of the proposed algorithm for the dynamic allocation of the targets, it is proposed to simulate a group of 5 robots reaching a formation ($N = 5$). Thus, the MRS is simulated with different initial positions IPs according to the virtual structure.

For every IP, the simulation is made twice: one with a prior assignation of the targets (target T_i to robot R_i), and one with the proposed algorithm where the robots use the RCC to obtain their targets. In the two cases, time to reach the formation is measured in order to evaluate the proposed algorithm performance. **Note that subscripts of robots and targets are fixed once for all the simulations.** The formation is considered reached if the distance $d_{S_{ij}}$ separating **every** robot R_i from the chosen target T_j is such that $d_{S_{ij}} \leq r_0$; where r_0 is the radius of a small virtual circle in the neighborhood of the targets. Comparing the two approaches, it can be seen that negotiation of the targets using RCC generally offers a better time of convergence. It is noticed that the difference depends on the initial position of the robots. Only the initial positions where the robots are already close to their prior assigned targets allows a better result (which is not the most general case).

In fact, for a prior assignation case (without negotiation), robots may be in the other side of the virtual structure compared to their assigned targets. They have then to avoid each other and to uselessly navigate to far targets. To illustrate this problem, An example of initial positions is given in figure 3(a). In the case of prior assignation, robot R_i has to join target T_i. To keep obvious the order of the targets in the figures, a straight trajectory is given to the virtual structure (see figure 3(b)) (a circular trajectory is used in experimental results). Taking the exemple of robot R_5, it can be seen that it has to go until target T_5 when it does not negotiate the closest one (cf. Figure 3(b)). However, by using RCC algorithm (cf. Figure 3(c)), it obtains the target T_2 which is much closer. Meanwhile, R_5 did not choose T_3 even if it was the closest one (cf. Figure 3(a)). In fact, it gives T_3 up to the robot R_3 which was behind it at the beginning of the simulation. In the same manner, R_1 gives up the closest target (T_5) to R_4 and takes T_1.

Table 1. Time to reach the formation for different initial positions IPs (s).

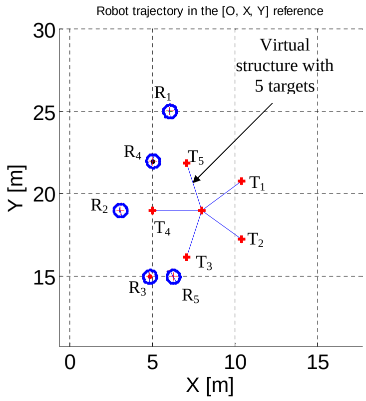
	Without negotiation of the targets	negotiation of the targets
IPs_1	9.9	8.2
IPs_2	12.2	9.3
IPs_3	20.8	15.9
IPs_4	21.8	20

4.1 Experimental results: a formation of 3 robots

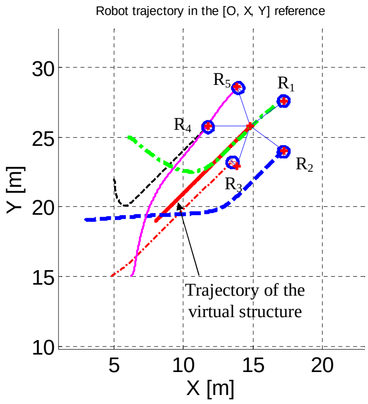
Experimentations are implemented on Khepera III robots. As first tests, only perception of the MRS is still centralized. Hence, navigation is achieved on a platform equipped with a camera giving positions and orientations of the robots (cf. Figure 4). These one have to join and to maintain a triangular virtual structure. The latter has a circular trajectory such that it stays reachable. First, it has a clockwise motion (cf. Figure 5(a)). Every robot calculates then the RCC for the targets.

Results are given in table 2.

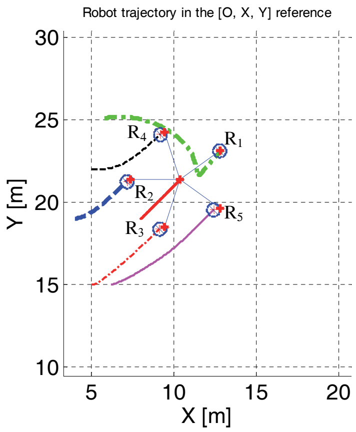
For the robot R_1, the smallest RCC corresponds to T_3. This one is not desired by any other robot since the RCC of R_2 and R_3 for this target is not the smallest one comparing to the other targets. However, R_2 and R_3 ask both for T_2 through their RCC.



(a) Initial position of the robots



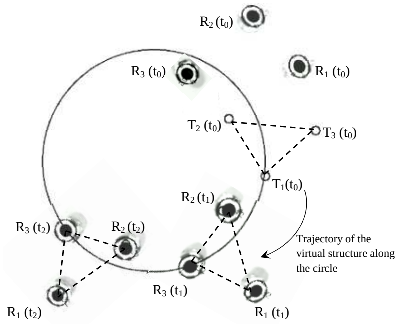
(b) A prior assignation of the targets



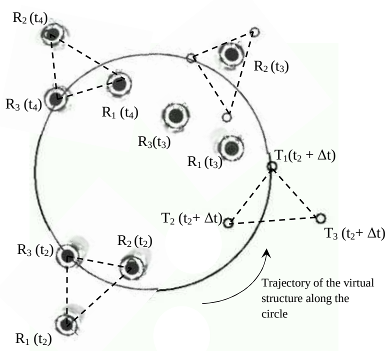
(c) Negotiation of the targets using RCC algorithm

Fig. 3. Trajectory of the robots reaching the formation: prior assignation versus negotiation of the targets.

Fig. 4. Khepera III robot.



(a) $t_0 \to t_2$: clockwise motion of the virtual structure



(b) $t_2 \to t_4$: switching to counter-clockwise motion

Fig. 5. Real trajectory of the robots. Distributed allocation (a) and reallocation (b) of the targets. Notation: $T_i(t_j)$ Target i at moment j, $R_i(t_j)$ Robot i at moment j.

Since R_2 has the smallest one, R_3 has to search for an other. It takes the remained target T_1.

Table 2. RCCs at moment (t_0) (truncated values to 2 decimal digits).

	T_1	T_2	T_3
R_1	0.41	0.32	0.25
R_2	0.39	0.23	0.33
R_3	0.39	0.24	0.41

At moment $t_2 + \Delta t$, a jump of the virtual structure state is produced (cf. Figure 5(b)). Also, the dynamic of the virtual structure is changed so that its motion becomes counter-clockwise. The robots recalculate the RCC for each target. The RCC are given in table 3. This table shows that all the robots prefer target T_2. R_1 obtains it because it has the smallest corresponding RCC. R_2 and R_3 search then for the target with the RCC immediately higher than the RCC of T_2. Again, both are interested by T_1. The latter is obtained by R_3 because its RCC is smaller. R_2 takes the remained target T_3. It can be seen that R_2 and R_3 give up T_2 to R_1 (altruism). Distances between the robots and their targets are given in figure 6. They decrease until 0 which confirm that the formation is reached and maintained. When the virtual structure dynamic is changed, robots are far from their targets which explain the observed jumps. The same observations are noticed on the global Lyapunov function (cf. Figure 7).

5. CONCLUSIONS AND FUTURE WORKS

In this paper, the formation of a multi-robot system, based on the virtual structure strategy, was studied. A cooperative protocol between the robots was proposed in order to rapidly attain the formation. Instead of a prior assignation of their

Table 3. RCCs at moment $(t_2 + \Delta t)$(truncated values to 2 decimal digits).

	T_1	T_2	T_3
R_1	0.36	0.21	0.38
R_2	0.37	0.22	0.40
R_3	0.34	0.26	0.40

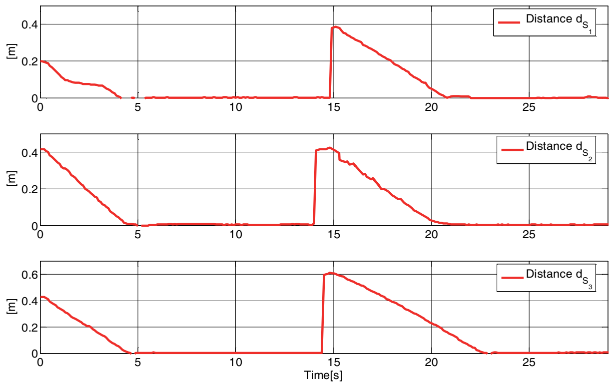


Fig. 6. Variation of the distance d_{S_i} between the robot i and the chosen target $(i = 1..3)$.

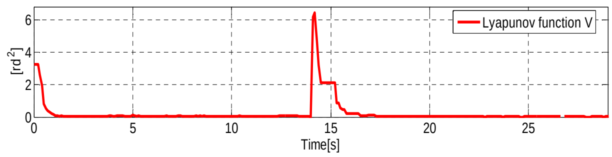


Fig. 7. Evolution of the global Lyapunov function V.

places in the formation (virtual targets), it is proposed that each robot negotiates its one with the others by communicating relative cost coefficients for each target. An altruism is then observed between the robots where collective gain (to reach quickly the formation) is preferred to the robot own gain (to choose the closer target). Moreover, simulation results show the performance offered by the proposed algorithm. Time to reach the formation was improved comparing to a prior assignation. Even if the obtained results are not the most optimal ones, this work addresses a challenging subject (fully distributed cooperation).

In the proposed experimental results, dynamic negotiation of the targets was enabled at the beginning of the experimentation and when switching to the virtual structure dynamic. In order to be done in a completely reactive way, discussing and justifying its frequency (ΔT) will be addressed in future works.

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