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Adaptive control of four-wheel-steering off-road mobile robots: Application to path tracking and heading control in presence of sliding

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***Abstract*— In this paper, automatic path tracking of a four-wheel-steering vehicle in presence of sliding is addressed. The attractive feature of such a steering system is that, despite of sliding phenomena, both lateral and angular deviations can be explicitly controlled. Indeed, previous research has demonstrated that high-precision path tracking on a low grip terrain can be achieved with two-wheel-steering vehicles. However, in this case, only the lateral deviation is kept satisfactorily close to zero, the angular deviation is non null in order to compensate for sliding effects. In this paper, previous adaptive control laws are extended to the case of four-wheel-steering mobile robots with the aim to servo both lateral and angular deviations. Relying on an extended kinematic model, a backstepping control approach, which considers successively front and rear steering control, has been designed. Real world experiments have been carried out on a low adherent terrain with a four-wheel-steering vehicle equipped with a single RTK-GPS. This demonstrates the capabilities of the proposed control law and its robustness in real all-terrain conditions.**

I. INTRODUCTION

Accurate automatic guidance of an off-road vehicle at relatively high speed is still challenging, since it requires to take into account the numerous dynamic phenomena, usually disregarded in classical approaches. As pointed out for example in [13], the low grip conditions usually met in such a context damage particularly the accuracy of classical path tracking control laws designed from rolling without sliding assumptions (as developed in [10] or in [12]). This can be seriously penalizing from the application point of view, especially in agriculture, where autofarming emerges as a promising solution [2], but where high-precision path tracking is required whatever the grip conditions encountered.

In previous work [8], high accuracy guidance of a farm two-wheel-steering vehicle has been achieved, despite sliding phenomena. Nevertheless, a crab angle between the vehicle heading and the tangent to the reference path is systematically observed, since such a behavior is necessary to compensate for the sliding forces acting on the uncontrolled rear wheels. The actuation of these latter theoretically enables the explicit control of both lateral and angular deviations, and appears to be very interesting from an agricultural point of view: an angular error, which is moreover varying according to the sliding conditions, leads to an unsatisfactory heading of the mounted implement, and may cause indiscriminate placement of field inputs (seeds, fertilizers, pesticides). As a consequence, some recent marketed farm tractors, such as Claas Xerion or JCB Fastract, propose manually steered rear wheels, in order to compensate for the crabway motion on a slope. The automatic control of the crab angle for such vehicles would nevertheless provide an increased accuracy and would offer extended possibilities in soil exploitation.

In the literature, four-wheel-steering vehicles have been studied with the aim, on one hand to increase the manoeuvrability of vehicles operating in confined space (e.g. handlers, loaders, self-propelled sprayers [9]) and, on the other hand to design active security devices for high speed road vehicles (see for instance [6]). In the latter case, the objective is to modify the vehicle lateral dynamics thanks to the active rear wheels, in order to reduce side slip angles and vehicle yaw rate. Several control approaches have been proposed to meet this aim. Generally, they are based on gain-scheduled feedforward control techniques: the rear steering angle, whose range never exceeds one or two degrees, is computed as a function of the front steering angle, the yaw rate, and predefined gains related to the vehicle speed. These techniques have already been implemented on marketed cars, but do not address the variability of grip conditions pointed out in an off-road context. Path tracking algorithms proposed for such vehicles, mainly concerned with automatic parking applications, also disregard grip conditions: for instance, in [5], a horizontal plane and rolling without sliding are assumed, in order for the vehicle models to exhibit the flatness property. This feature is then used to plan a suitable parking trajectory and servo the four-wheel-steering vehicle along it.

On the contrary, the objective of this paper is to take advantage of the rear steering actuation to compensate for sliding effects during path tracking on a natural environment. Based on a backstepping approach and adaptive techniques, the proposed control law on one hand ensures accurate path following, despite sliding phenomena, and on the other hand explicitly controls the angular deviation whatever grip conditions. This paper is organized as follows. First, an extended kinematic model accounting for sliding effects is developed, and on-line estimation of grip conditions is addressed. Then, path tracking is considered using a backstepping approach. In a first step, the rear steering angle is assumed to be measured and the extended model is then turned into chained form, so that a front steering control law can be designed with the sole aim to servo vehicle lateral deviation. Then, a rear steering control law is designed in order to ensure also the convergence of the vehicle angular deviation to some set point. Next, the experimental mobile robot is presented, as well as the available measurements. Finally, the capabilities of the proposed control law are investigated through full-scale experiments on a slippery ground.

II. Extended kinematic model

A. Model formalism

Since vehicles considered in this paper are expected to move on a natural terrain, the low grip conditions must be accounted in the modeling, in order to enable, in next section, the design of accurate path following control laws. Dynamical models incorporating tire/ground interaction forces, as described in [4] or in [1], do not appear tractable when vehicles are moving off-road: numerous parameters have then to be known. This is a concern, especially when grip conditions are variable. Alternatively, it is here proposed to extend classical four-wheel-steering kinematic models, such as [10], since they propose a suitable structure to address control design.

Consequently, each two front and rear wheels are considered equivalent to two virtual wheels located at mid-distance between the actual ones, as depicted on figure 1. In order to account for sliding phenomena, two additional parameters - homogeneous with sideslip angles in a dynamic model - are added to the classical representation. In the same way than in [8], these two angles denoted respectively β_F and β_R for the front and rear axle, represent the difference between the theoretical direction of the linear velocity vector at wheel centers, described by the wheel plane, and their actual direction. These angles are assumed to be entirely representative of the sliding influence on vehicle dynamics.

The notations used in the paper are listed below and depicted on figure 1.

- F and R are respectively the center of the front and rear virtual wheels. R is the point to be controlled.
- L is the vehicle wheelbase.
- θ_v is the orientation of vehicle centerline with respect to an absolute frame $[O, X_O, Y_O)$.
- V_r is the vehicle linear velocity at point R, assumed to be strictly positive and manually controlled.
- δ_F and δ_R are the front and rear steering angles. They constitute the two control variables.
- β_F and β_R are the front and rear side slip angles.
- M is the point on the reference path Γ which is the closest to R. M is assumed to be unique, see hypothesis (2) below.
- s is the curvilinear abscissa of point M along Γ.
- $c(s)$ is the curvature of path Γ at point M.
- $\theta_\Gamma(s)$ is the orientation of the tangent to Γ at point M with respect to the absolute frame $[O, X_O, Y_O)$.
- $\tilde{\theta} = \theta_v - \theta_\Gamma$ is the vehicle angular deviation with respect to Γ.
- y is the vehicle lateral deviation at point R with respect to Γ.

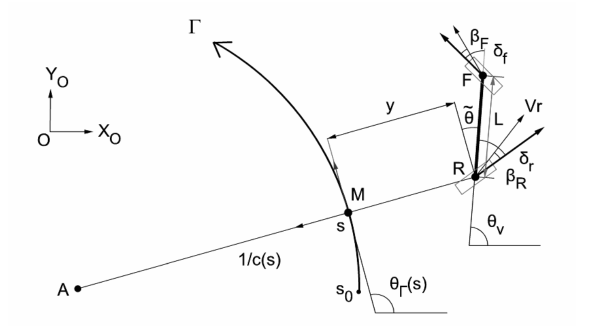


Fig. 1. Path tracking parameters and variables

As the control objective is to follow reference path Γ, the equations of motion have to be derived with respect to this path. It can be established see [10], that:

$$\left\{\begin{array}{lcl} \dot{s} & = & V_r \frac{\cos(\tilde{\theta}+\delta_R-\beta_R)}{1-c(s)\,y} \\ \dot{y} & = & V_r \sin(\tilde{\theta}+\delta_R-\beta_R) \\ \dot{\tilde{\theta}} & = & V_r\left[\cos(\delta_R-\beta_R)\lambda_1-\lambda_2\right] \end{array}\right. \tag{1}$$

with: $\lambda_1 = \frac{\tan(\delta_F-\beta_F)-\tan(\delta_R-\beta_R)}{L}$, $\lambda_2 = \frac{c\cos(\tilde{\theta}+\delta_R-\beta_R)}{1-c(s)\,y}$

It can be noticed that this model becomes singular when $y = \frac{1}{c(s)}$, i.e. when points A and R (depicted on figure 1) are superposed. This problem is not encountered in practice since, on one hand actual path curvatures are quite small, and on the other hand, the vehicle remains close to Γ when properly initialized. The lateral deviation is thereby always smaller than the radius of curvature of Γ. As a result, the assumption (2) can be made and will be used in the sequel.

$$|y| < \frac{1}{|c(s)|} \quad \Rightarrow \quad 1-c(s)\,y > 0 \tag{2}$$

B. Known data and grip estimation

Model (1) accurately describes the vehicle motion in presence of sliding as soon as the two additional parameters β_F and β_R are known. Therefore, the estimation of these two variables appears to be of crucial importance. As pointed out for example in [11] in the case of dynamical representations, the direct evaluation of side slip angles appears to be hardly feasible at a reasonable cost. Their estimation classically requires the use of huge measurement systems (such as expensive inertial measurement units...), and needs some preliminary assumptions with respect to adherence conditions. The variability of the soil conditions encountered in natural environment, as well as their on-line modifications, do not permit to apply directly observer algorithms used in on-road context.

An observer is here proposed to achieve sideslip angles indirect estimation, relying on the sole exteroceptive measurements $\bar{X} = \left[\begin{array}{cc} \bar{y} & \bar{\tilde{\theta}} \end{array}\right]^T$ (respectively the measured lateral and angular deviations). This observer is based on the duality between observation and control. As proposed in [7], β_F and β_R are considered as control variables to be designed in order to ensure the convergence of the extended model outputs to the measured variables. More precisely, model (1) without curvilinear abscissa equation, is rewritten as a non-linear state representation:

$$\dot{\hat{X}} = f(\hat{X}, u) \tag{3}$$

where f is derived from (1):

$$f(\hat{X}, u) = \begin{cases} V_r \sin(\hat{\tilde{\theta}} + \delta_R + u_2) \\ V_r \left[\frac{\cos(\delta_R + u_2)[\tan(\delta_F + u_1) - \tan(\delta_R + u_2)]}{L} - \right. \\ \left. c(s) \frac{\cos(\hat{\tilde{\theta}} + \delta_R + u_2)}{1 - c(s)\hat{y}} \right] \end{cases} \tag{4}$$

$\hat{X} = [\ \hat{y} \quad \hat{\tilde{\theta}}\ ]^T$ is the observed state and $u = [\ u_1 \quad u_2\ ]^T = -[\ \beta_F \quad \beta_R\ ]^T$ are the side slip angles to be estimated, considered as the control variables of system (3). As side slip angles do not exceed few degrees in practice, let us linearize this state equation with respect to "control vector" u in the vicinity of zero (i.e. no sliding). It leads to:

$$\dot{\hat{X}} = f(\hat{X}, 0) + B(\hat{X})u \tag{5}$$

with B denoting the derivative of f with respect to u, evaluated at $u = 0$:

$$B(\hat{X}) = \begin{bmatrix} 0 & V_r \cos(\hat{\tilde{\theta}} + \delta_R) \\ \frac{V_r \cos \delta_R}{L \cos^2 \delta_F} & V_r \frac{c(s) \sin(\hat{\tilde{\theta}} + \delta_R)}{1 - c(s)\hat{y}} - \frac{V_r}{L \cos \delta_R} \\ & -V_r \sin \delta_R \frac{\tan \delta_F - \tan \delta_R}{L} \end{bmatrix} \tag{6}$$

The matrix B is invertible when $\hat{\tilde{\theta}} + \delta_R \neq \frac{\pi}{2}[\pi]$, $V_r \neq 0$ and $\delta_r \neq \frac{\pi}{2}[\pi]$. These conditions are met in practical path following conditions. Using this formalism and hypothesis, the observation of sliding parameters is achieved thanks to the following observer equation:

$$u = B(\hat{X})^{-1} \left\{ G \cdot e - f(\hat{X}, 0) + \dot{\bar{X}}^M \right\} \tag{7}$$

where $e = \hat{X} - \bar{X}$, G is an Hurwitz matrix, which constitutes the observer gain, and $\dot{\bar{X}}^M$ is the numeric derivative of the measured state. This expression of control vector u leads to the following error dynamics:

$$\dot{e} = G \cdot e \tag{8}$$

which ensures the convergence of the observed state to the measured one. The gain matrix G allows to decrease the impact of sensor noise. Observer equation (7) provides then an estimation of the side slip angles introduced into the bicycle model depicted on figure 1 and ensures that this extended model fits with the measured behavior of the vehicle. Therefore, it constitutes a relevant basis for mobile robot control design.

III. Control law design

A. Motivations

The control objective is to perform an accurate path tracking with respect to lateral and angular deviations (respectively y and $\tilde{\theta}$), compensating for the effects of low grip conditions. As the velocity is viewed as a measured parameter (manually controlled), system inputs are the front and rear steering angles (δ_F and δ_R). The expression of extended kinematic model (1) accounting for sliding effects is still consistent with classical models of wheeled mobile robots. As a consequence, according to [10], it can be turned into a linear model named chained form without any approximation. Nevertheless, in the case of two steering axles, such a transformation requires the integration of δ_R into the state vector, rewriting model (1) as model (9):

$$\dot{x} = f(x) + g_1(x)w_1 + g_2(x)w_2 \tag{9}$$

with

$$x = \begin{bmatrix} y \\ \tilde{\theta} \\ \delta_R - \beta_R \end{bmatrix}, g_1 = \begin{bmatrix} 0 \\ \frac{V_r \cos(\delta_R - \beta_R)}{L} \\ 0 \end{bmatrix}, g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$f = \begin{bmatrix} V_r \sin(\tilde{\theta} + \delta_R - \beta_R) \\ -V_r \frac{\sin(\delta_R - \beta_R)}{L} - V_r \frac{c(s) \cos(\tilde{\theta} + \delta_R - \beta_R)}{1 - c(s)y} \\ 0 \end{bmatrix} \tag{10}$$

and $[w_1 \ \ w_2]^T = [\tan(\delta_F - \beta_F) \ \ \dot{\delta}_R - \dot{\beta}_R]^T$.
Using this representation, a model with two chains can be obtained and control laws can be designed for δ_F and $\dot{\delta}_R$ thanks to linear control theory. Unfortunately, this imposes to use $\dot{\delta}_R$ as the control variable for the rear axle. An integration is then required, which may generate instability in practice, due to the inevitable delays present on steering actuators.

B. Backstepping approach

An alternative based on backstepping method is then proposed, allowing the design of control laws for steering angles instead of their derivative. In a first step, let us consider the rear steering angle as a measured parameter in model (1). From this hypothesis, it results that $\delta_R - \beta_R$ can be considered as a unique rear side slip angle denoted β_{R2}. Model (1) is then consistent with a front steering mobile robot in presence of sliding such as considered in [8], with a unique control variable δ_F. Following the same methodology than in this reference, the state and control transformations (11) can be introduced.

$$\begin{array}{lll} [s, y, \tilde{\theta}] & \rightarrow & [a_1, a_2, a_3] = [s, y, (1 - c\, y) \tan(\tilde{\theta} + \beta_{R2})] \\ [V_r, \delta_F] & \rightarrow & [m_1, m_2] = [\frac{V_r \cos(\tilde{\theta} + \beta_{R2})}{1 - c(s)\, y}, \frac{da_3}{dt}] \end{array} \tag{11}$$

It leads to the following chained system:

$$\begin{cases} \dot{a}_1 = \frac{d\,a_1}{dt} = m_1 \\ \dot{a}_2 = \frac{d\,a_2}{dt} = a_3 m_1 \\ \dot{a}_3 = \frac{d\,a_3}{dt} = m_2 \end{cases} \tag{12}$$

which can be also expressed with derivatives with respect to the curvilinear abscissa:

$$\begin{cases} a'_2 = \frac{da_2}{da_1} = a_3 \\ a'_3 = \frac{da_3}{da_1} = m_3 = \frac{m_2}{m_1} \end{cases} \tag{13}$$

In order to control the lateral deviation, a judicious choice for m_3 is (14), since it leads to a second order differential equation, ensuring the convergence of $a_2 = y$ to zero.

$$m_3 = \frac{m_2}{m_1} = -K_d a_3 - K_p a_2 \quad (K_d, K_p > 0) \tag{14}$$

Injecting (14) into (11) and considering β_{R2} as slow-varying with respect to the dynamic imposed by the two gains K_p and K_d, the control law for the front axle can finally be derived as:

$$\begin{aligned}\delta_F &= \arctan\Big\{ \tan(\delta_R - \beta_R) \\ &\quad + \tfrac{L}{\cos(\delta_R-\beta_R)}\big(\tfrac{c(s)\cos\tilde{\theta}_2}{\alpha} + \tfrac{A\cos^3\tilde{\theta}_2}{\alpha^2}\big)\Big\} + \beta_F\end{aligned} \tag{15}$$

with:

$$\left\{\begin{array}{lcl}\tilde{\theta}_2 &=& \tilde{\theta} + \delta_R - \beta_R \\ \alpha &=& 1 - c(s)y \\ A &=& -\frac{K_d^2\,y}{4} - K_d\,\alpha\tan\tilde{\theta}_2 + c(s)\alpha\tan^2\tilde{\theta}_2\end{array}\right. \tag{16}$$

The gains (K_p, K_d) allow to specify a settling distance instead of a settling time. In the sequel, it is chosen $K_p = \frac{K_d^2}{4}$ in order to obtain a critical damping $\xi = 1$. With control law (15), the lateral deviation is satisfactorily servoed to zero, as well as $\tilde{\theta}_2$. Convergence of this latter variable implies that $\tilde{\theta}$ converges to $\beta_R - \delta_R$, and not to some desired set point, as expected.

Actual control of $\tilde{\theta}$ can now be addressed using the rear steering variable. This constitutes the second step of the approach. Reporting control law (15) into the third equation in model (1) leads to the following angular deviation dynamic with respect to curvilinear abscissa:

$$\tilde{\theta}' = (-\frac{K_d^2\,y}{4\,\alpha} - K_d\tan\tilde{\theta}_2 + c(s)\tan^2\tilde{\theta}_2)\,\cos^2\tilde{\theta}_2 \tag{17}$$

As above mentioned, control law (15) imposes that $\tilde{\theta}_2$ stays close to zero. As a result, the term $\cos^2\tilde{\theta}_2$ can be considered as equal to 1, so that:

$$\tilde{\theta}' = -\frac{K_d^2\,y}{4\,\alpha} - K_d\tan\tilde{\theta}_2 + c(s)\tan^2\tilde{\theta}_2 \tag{18}$$

In view of (18), two cases must be distinguished, according to the curvature value.

1) Straight line following: ($c(s)$=0): in that case, the angular deviation dynamics (18) can be simplified as:

$$\tilde{\theta}' = -\frac{K_d^2\,y}{4} - K_d\tan\tilde{\theta}_2 \tag{19}$$

Then, the error dynamic $\tilde{\theta}' = K_{d2}\,(\tilde{\theta}_{ref} - \tilde{\theta})$ with $K_{d2} > 0$ can easily be imposed by proposing the following rear steering law:

$$\delta_R = \beta_R - \tilde{\theta} + \arctan\Big(\frac{-K_d\,y}{4} - \frac{K_{d2}\,(\tilde{\theta}_{ref} - \tilde{\theta})}{K_d}\Big) \tag{20}$$

This ensures the convergence of $\tilde{\theta}$ to $\tilde{\theta}_{ref}$

2) Curve line following ($c(s)$ $\neq$0): using the notation $X = \tan\tilde{\theta}_2$, equation (18) can be rewritten as:

$$-\tilde{\theta}' - \frac{K_d^2\,y}{4\,\alpha} - K_d\,X + c(s)\,X^2 = 0 \tag{21}$$

Once more, the objective is to impose $\tilde{\theta}' = K_{d2}\,(\tilde{\theta}_{ref} - \tilde{\theta})$. If it was achieved, then the discriminant of equation (21) would be:

$$\Delta = \frac{K_d^2}{\alpha} - 4\,c(s)\,K_{d2}\,(\tilde{\theta} - \tilde{\theta}_{ref}) \tag{22}$$

As α is assumed to be always strictly positive, see hypothesis (2), the condition $\Delta > 0$ leads to:

$$\left\{\begin{array}{lcll}(\tilde{\theta} - \tilde{\theta}_{ref}) &<& \frac{K_d^2}{4\,c(s)\,K_{d2}\,\alpha} & if\ c(s) > 0 \\ (\tilde{\theta} - \tilde{\theta}_{ref}) &>& \frac{K_d^2}{4\,c(s)\,K_{d2}\,\alpha} & if\ c(s) < 0\end{array}\right. \tag{23}$$

The choice for (K_d, K_{d2}) and the limit values of $c(s)$ and y lead, in the worse case, to a $\pm 30°$ bound on $(\tilde{\theta} - \tilde{\theta}_{ref})$, which is always satisfied in practice.

Since Δ has been shown to be strictly positive, two solutions can be derived. Considering the actuators range of variation, only one of the solutions can be applied. As a result, the rear control law achieving the expected convergence can be written as following:

$$\delta_R = \beta_R - \tilde{\theta} + \arctan\left\{\frac{K_d - \sqrt{\frac{K_d^2}{\alpha} - 4\,c(s)\,K_{d2}\,(\tilde{\theta} - \tilde{\theta}_{ref})}}{2c(s)}\right\} \tag{24}$$

Expressions (20) and (24) constitute the rear steering law for respectively straight and curve line following. The continuity of these expressions, when $c(s)$ tends to zero, can be established by standard but tedious computations.

C. Stability of the backstepping controller

The stability of the whole non-linear control strategy, composed of control law (15) for the front steering angle and (20) if $c(s) = 0$ or (24) if $c(s) \neq 0$ for the rear steering angle, can be checked using Lyapunov theory. Consider Lyapunov function candidate, with $\epsilon = \tilde{\theta}_{ref} - \tilde{\theta}$:

$$V = \tfrac{1}{2}\left\{y^2 + (\alpha\tan\tilde{\theta}_2)^2 + \epsilon^2\right\} \tag{25}$$

The derivative of the positive function V with respect to curvilinear abscissa (homogeneous with the time derivative considering a non-null velocity) leads, after calculations, to the following expressions (whatever the curvature value):

$$\tfrac{dV}{ds} = -K_d\alpha^2\tan^2\tilde{\theta}_2 - K_{d2}(\epsilon\cos\tilde{\theta}_2)^2 \tag{26}$$

which is always negative. The stability of the mobile robot trajectory tracking and the convergence of both ϵ and $\tilde{\theta}_2$ to zero is then ensured. As a result, injecting the asymptotic value of $\tilde{\theta}_2$ into equation (17) establishes that the lateral deviation y also converges to zero. This finally demonstrates the stability of path tracking control in presence of sliding, with respect to lateral and angular deviations, with front and rear control laws (15) and (20) if $c(s) = 0$ or (15) and (24) if $c(s) \neq 0$.

IV. EXPERIMENTAL RESULTS

The experimental platform is the all-terrain four-wheel steering vehicle depicted on figure 2. The vehicle weight and maximum speed are respectively 600 kg and 18 km/h, and it can climb slopes up to 45°. The only exteroceptive sensor on-boarded is a RTK-GPS receiver, whose antenna has been located straight up the point R (see figure 1). It supplies an absolute position with a 2cm accuracy, at a 10Hz sampling frequency, and allows to estimate the vehicle heading thanks to a Kalman filter.

Fig. 2. Experimental platform

The path to be followed is recorded by a preliminary run achieved in manual driving. In this paper, two types of path have been recorded: a straight line achieved on a 15% sloping ground (mobile robot running perpendicularly to the slope as depicted on figure 2) and a curved path depicted on figure 3 achieved on a flat ground. In both cases, the terrain was an irregular wet grass ground, where the vehicle is inevitably prone to slide as it will be experimentally checked in the sequel. In the forthcoming experimental results, the vehicle speed is 1.8 m/s (6.5 km/h). The control gains (K_p, K_d) are set to $(0.16, 0.8)$ in order to impose a 11m settling distance for the convergence of the lateral deviation. Finally, $K_{d2} = 1.1$ has been chosen in order to impose a 3m settling distance for the convergence of the angular deviation.

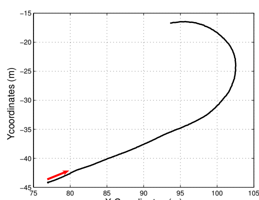
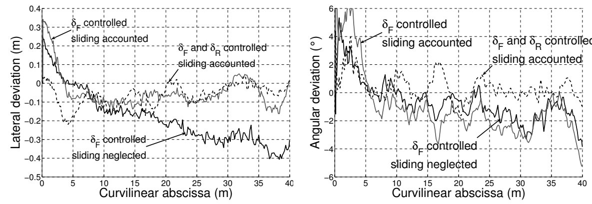


Fig. 3. Path to be followed on a flat and slippery ground

A. Result during straight line on sloping ground

Several straight line following on a slope have been performed using different control laws during a straight line following on a slope. Firstly, a classical control law neglecting for sliding effects has been applied only on the front steering wheels (results are reported in black plain line on figure 4). The expression of this one-axle control law can be derived from (15) by setting sliding parameters to zero ($(\beta_F, \beta_R) = (0, 0)$). It can be seen that, with this classical approach, the tracking error as well as the angular deviation cannot reach the desired zero value because of sliding effects (the lateral deviation converges close to -30cm, while the angular deviation reaches -2°). A second test has been performed, still using only the front steering wheels, but accounting for sliding (control law (15)) (results are depicted in gray plain line). The same angular deviation than before can be observed (the asymptotic value -2° allows to compensate for rear side slip angle), but an acceptable tracking error within ± 10cm is obtained. Finally, path tracking results when using both front and rear steering control laws, with a null desired angular deviation, are shown in black dashed line. With this control strategy, both lateral and angular deviations are able to reach null values.

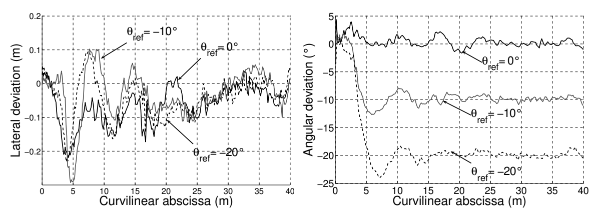


(a) Lateral deviation (b) Angular deviation

Fig. 4. Validation of algorithm in slope

These first results permit to point out the benefit of rear steering control in order to achieve accurate path tracking in sliding conditions. The proposed algorithm indeed permits to compensate for sliding effects in order to preserve an almost null tracking error, but also to ensure the convergence of the angular deviation to a set point. In slope, sliding phenomena can then be compensated without admitting a crab angle.

Beyond this improvement, the proposed control laws for front and rear axles, allow to specify any desired value for the robot heading with respect to the reference path orientation. Using the same reference path than in the previous tests, three path tracking have been performed with the proposed algorithm using different values of desired angular deviation. The tracking results are compared on figure 5: the references $\tilde{\theta}_{ref} = 0°$, $\tilde{\theta}_{ref} = -10°$ and $\tilde{\theta}_{ref} = -20°$ are shown respectively in black plain line, gray plain line and black dashed line.



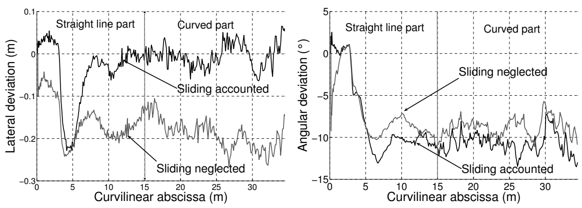
(a) Lateral deviation (b) Angular deviation

Fig. 5. Path tracking in slope with different $\tilde{\theta}_{ref}$

As it can be seen, after a settling time, the tracking error is not affected by the value of $\tilde{\theta}_{ref}$ since lateral deviations reported on figure 5(a) present the same evolution whatever the reference. In the meanwhile, after a settling distance, the angular deviations (depicted on figure 5(b)) reach the desired value for $\tilde{\theta}_{ref}$. This demonstrates the capabilities of the proposed algorithm to control both lateral and angular deviations almost independently, despite sliding phenomena.

B. Result during curved path following

The last result proposed in this paper is devoted to curved path following, and in particular the transition between rear control law expressions when $c(s)$ becomes non null. Path tracking of the reference path depicted on figure 3 has been achieved at a velocity of 6.5 km/h on a wet grass ground. The front and rear steering control laws proposed in this paper have been used and desired angular deviation of $\tilde{\theta}_{ref} = -10°$ has been chosen to highlight angular deviation control. Path following has first been run disregarding sliding phenomena (β_F and β_R set to zero). The result is reported in gray plain line on figure 6. The second tracking has been achieved with sliding accounted. Results are reported in black plain line.



(a) Lateral deviation (b) Angular deviation

Fig. 6. Path tracking on a curved path $\tilde{\theta}_{ref}$

As it can be noticed, when sliding are accounted, the path tracking error (after a settling distance) stays very close to zero: it remains within a $\pm$10cm range in the straight line part (before curvilinear abscissa 15m) as well as in the curved part (after 15m). In the same way, the angular deviation also converges and remains close to the chosen reference -10° (within a $\pm 2°$ variation range) whatever the curvature value. On the contrary, both lateral and angular deviations are not able to converge to their desired value when sliding are neglected. Finally, it can be seen that both error signals (y and $\tilde{\theta}$) are not affected by the transition in the curvature ($c(s) = 0$ for s <15m and $c(s)$ reaching to 0.12m^{-1} for s >15m). Therefore, these experimental tests show the capability of the proposed approach to control accurately both lateral and angular deviations in low grip conditions whatever the shape of desired path.

V. Conclusions and future work

This paper addresses path tracking problem for an off-road four-wheel-steering vehicle moving on a slippery ground. The adaptive control algorithm proposed in that paper aims at achieving a high accurate guidance with respect to both lateral and angular errors whatever the grip conditions and the shape of path to be followed. To achieve this objective, an extended kinematic model accounting for sliding effects via two additional side slip angles is defined. Thanks to an observer estimating these two sliding parameters, this model is able to be representative of the mobile robot dynamics despite harsh perturbations due to low grip conditions. As a result, it constitutes a suitable basis in order to derive control laws to tackle path tracking problem in natural environment. As classical control design (based on a complete chained system transformation) leads to instability due to low level actuator delays, an alternative control law based on a backstepping approach, using an incomplete linear form, is proposed. It permits to design two control laws for front and rear steering wheels.

As a result, lateral and angular dynamics with respect to a reference path can be controlled almost independently with a high accuracy (around 5cm for lateral deviation and one degree for vehicle orientation during full-scale experiments) whatever the ground (grip conditions and geometry) and whatever the shape of path to be followed.

Due to material limitations (mobile robot capabilities and sensor sample time), the velocity was limited to 8 km/h during experiments. Nevertheless, the theoretical validity of the approach is preserved at faster evolutions (tested in simulation). However, the increase in velocity can lead to tracking error overshoots during curvature transient phases. This point can be addressed by predictive algorithms, considering future path curvature (as achieved on bigger vehicles). The increase in velocity with respect to both material and control points is under development. The extension of this work to the case of a vehicle with a trailer is currently also investigated. Since we are concerned with agricultural applications, the case of large trailers or on-loaded implements constitutes a challenging problem.

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