

Omnidirectional Visual Servoing From Polar Lines

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Abstract—Motivated by the growing interest for omnidirectional sensors on robotic applications and particularly on vision-based control, we present a new framework to handle in a visual servoing scheme the projection of line features into the image plane of a central catadioptric camera. As it is well known, the projection of a 3D line in the image plane of a central catadioptric camera is a conic curve. We propose to use the polar line of the image center with respect to this conic curve to define the input of the vision-based control scheme. The visual observations obtained from the polar lines lead to a minimal representation of projected lines. An efficient control scheme based only on two image features can then be designed. Simulation and experimental results confirm the validity of our approach.

I. INTRODUCTION

Vision-based servoing schemes are flexible and effective methods to control robot motion from camera observations [13]. They are traditionally classified into three groups, namely position-based, image-based and hybrid-based control [10], [13], [15]. These three schemes make assumptions on the link between the initial, current and desired images since they require correspondences between the visual features extracted from the initial image with those obtained from the current and desired ones. These features are then tracked during the camera (and/or the object) motion. If these steps fail the visually based robotic task can not be achieved [7]. Typical cases of failure arise when matching joint image features is impossible (for example when there is no common features in the initial and desired images) or when some parts of the visual features get out of the field of view during the servoing. Some methods have been investigated to resolve this deficiency based on path planning [17], switching control [8], zoom adjustment [20], geometrical and topological considerations [9]. However, such strategies are sometimes delicate to adapt to generic setup. Conventional cameras suffer thus from restricted field of view. There is thus significant motivation for increasing the field of view of the cameras [5]. Many applications in vision-based robotics, such as mobile robot localization [6] and navigation [24], can benefit from panoramic fields of view provided by omnidirectional cameras. In the literature, there have been several methods proposed for increasing the field of view of camera systems [5]. One effective way is to combine mirrors with conventional imaging system. The obtained sensors are referred to as catadioptric imaging systems. The resulting imaging systems have been termed central catadioptric when a single projection center describes the world-image map-

ping. From a theoretical and practical view point, a single center of projection is a desirable property for an imaging system [2]. Baker and Nayar in [2] derive the entire class of catadioptric systems with a single viewpoint. Clearly, visual servoing applications can also benefit from such sensors since they naturally overcome the visibility constraint. Vision-based control of robotic arms, single mobile robot or formation of mobile robots appear thus in the literature with omnidirectional cameras (refer for example to [4], [19], [23], [18]).

This paper is mainly concerned with the use of projected lines extracted from central catadioptric images as input of a visual servoing control loop. When dealing with real environments (indoor or urban) or industrial workpiece, line features are a natural choice. Nevertheless, most of the effort in visual servoing are devoted to points [13], only few works have investigated the use of lines in visual servoing with traditional cameras (refer for example to [1], [10], [14]). The interaction matrix plays a central role to design vision-based control law. It links the variations of image observations to the camera velocity. The analytical form of the interaction matrix is available for some image features (points, circles, lines, ...) in the case of conventional cameras [10]. Barreto *et al.* in [4] studied the central catadioptric interaction matrix for a set of image points. In [12], the hybrid control scheme has been extended to the entire class of central catadioptric camera. In [18] a generic analytical form of the central catadioptric interaction matrix for the image of 3D straight lines was derived. This framework was exploited to design control laws for positioning task of a six degrees of freedom manipulator or for trajectory following task for a mobile robot. However, in this framework the input of the control scheme has been chosen as the five parameters of the conic curve resulting from the projection into the image plane of a 3D straight line whereas the conic has only two degrees of freedom (since it is essentially the image of a line, although distorted by the mirror).

This paper is concerned with the latter issue. Indeed, we propose to use a minimal representation (*i.e* a two parameters representation) of the straight line projection into the catadioptric image in order to design an efficient vision-based control scheme. Namely, the visual observations are obtained from the polar line of the image center with respect to the conic. A minimal and generic analytical form of the central catadioptric interaction matrix for the image of 3D straight lines is then derived from this new representation and it is finally exploited

to design control laws for a positioning task of a six degrees of freedom manipulator or for a trajectory following task for a mobile robot.

The remainder of this paper is organized as follows. In Section II, following the description of the central catadioptric camera model, line projection in the image plane is studied. This is achieved using the unifying theory for central panoramic systems introduced in [11]. The essential point of this contribution is that, the use of polar lines allows us to consider the conic curves in the physical omnidirectional image as straight line in an equivalent virtual perspective camera knowing only an estimated image center (*i.e* mirror parameters and focal length are not required). Thus, one can make use of any line representation in perspective image and hence, choose a minimal one. In Section III the image-based control law we have used, is briefly presented. In Section IV, we derive a minimal and generic analytical form of the interaction matrix for projected lines (conics) using polar lines. Simulated and experimental results are presented in Section V and VI.

II. CENTRAL CATADIOPTRIC IMAGING MODEL

A vision system has a single viewpoint if all rays joining a world point and its projection in the image plane pass through a single point called principal projection center. Conventional perspective camera is a typical example of single viewpoint vision sensor. The well known pin-hole model assumes that the mapping of world points into points in the image plane is linear in homogeneous coordinates. There are single viewpoint systems whose geometry can not be modeled using the conventional pin-hole model. Baker and Nayar in [2] derived the entire class of catadioptric systems with a single viewpoint. They show that a central catadioptric system with a wide field of view can be built by combining an hyperbolic, elliptical or planar mirror with a perspective camera and a parabolic mirror with an orthographic camera. However the mapping between world points and points in the image plane is no longer linear. In [11], Geyer and Daniilidis introduced a unifying model for all central catadioptric imaging system where the conventional perspective camera appears as a particular case. To do so, they proposed to use a virtual unitary sphere as a calculus artefact. We now recall this unified model.

A. Camera model

Let \mathcal{F}_c and \mathcal{F}_m be the frames attached to the conventional camera and to the mirror respectively. In the sequel, we suppose that \mathcal{F}_c and \mathcal{F}_m are related by a translation along the Z -axis (\mathcal{F}_c and \mathcal{F}_m have the same orientation as depicted in Figure 2). The origins C and M of \mathcal{F}_c and \mathcal{F}_m will be termed optical center and principal projection center respectively. The optical center C has coordinates $[0 \ 0 \ -\xi]^T$ with respect to \mathcal{F}_m and the image plane $Z = \psi - 2\xi$ is orthogonal to the Z -axis where ξ and ψ describe the type of sensor and the shape of the mirror, and are function of mirror shape parameters (refer to [3]). Consider the virtual unitary sphere centered in M as shown in Fig.2 and let \mathcal{X} be a 3D point

with coordinates $\mathbf{X} = [X \ Y \ Z]^T$ with respect to \mathcal{F}_m . The world point \mathcal{X} is projected in the image plane into the point of homogeneous coordinates $\underline{\mathbf{x}}_i = [x_i \ y_i \ 1]^T$.

$$\underline{\mathbf{x}}_i = \mathbf{K} \begin{bmatrix} X & Y \\ Z + \xi \|\mathbf{X}\| & Z + \xi \|\mathbf{X}\| \\ 1 & 1 \end{bmatrix}^T \quad (1)$$

The matrix \mathbf{K} can be written as $\mathbf{K} = \mathbf{K}_c \mathbf{M}$ where the upper triangular matrix \mathbf{K}_c contains the conventional camera intrinsic parameters, and the diagonal matrix \mathbf{M} contains the mirror intrinsic parameters:

$$\mathbf{M} = \begin{bmatrix} \psi - \xi & 0 & 0 \\ 0 & \psi - \xi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K}_c = \begin{bmatrix} f_u & \alpha_{uv} & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that, setting $\xi = 0$, the general projection model becomes the well known perspective projection model.

B. Projection of lines

Let \mathcal{L} be a 3D straight line in space lying on plane Π which contains the principal projection center M (see Figure 2). The binormalized Euclidean Plücker coordinates [1] of the 3D line are defined as $\mathcal{L} : [\bar{\mathbf{n}}^T \ \bar{\mathbf{u}}^T \ h]^T$ with $\bar{\mathbf{n}}^T \bar{\mathbf{u}} = 0$ and h is the distance of the 3D line \mathcal{L} to the origin of the corresponding definition frame. The unit vectors $\bar{\mathbf{n}} = [n_x, n_y, n_z]^T$ and $\bar{\mathbf{u}} = [u_x, u_y, u_z]^T$ are respectively the orthogonal vector to the interpretation plane Π and the orientation of the 3D line \mathcal{L} and are expressed in the mirror frame \mathcal{F}_m . If the 3D line is projected in the perspective camera then the unit vector $\bar{\mathbf{n}}$ contains the coefficients of the 2D line equation in the image plane. Indeed, any world points $\mathbf{X} = [X, Y, Z]^T \in \Pi$ lying on \mathcal{L} verifies:

$$n_x X + n_y Y + n_z Z = 0 \quad (2)$$

and its perspective projection $\mathbf{x} = [x, y]^T$ verifies:

$$n_x x + n_y y + n_z = 0$$

Let \mathcal{S} be the intersection between the interpretation plane Π and the spherical mirror surface. \mathcal{S} represents the line projection in the mirror surface. Note that all 3D lines of Π are projected onto \mathcal{S} . The projection \mathcal{S} of \mathcal{L} in the catadioptric image plane is then obtained using a conventional imaging system. It can be shown using (1) and (2) (or following [3], [18]) that 3D points lying on \mathcal{L} are mapped into points in the image \mathbf{x} which verify:

$$\underline{\mathbf{x}}_i^T \mathbf{K}^{-T} \mathbf{\Omega} \mathbf{K}^{-1} \underline{\mathbf{x}}_i = 0 \quad (3)$$

with :

$$\mathbf{\Omega} = \begin{bmatrix} n_x^2 - \xi^2(1 - n_y^2) & n_x n_y(1 - \xi^2) & n_x n_z \\ n_x n_y(1 - \xi^2) & n_y^2 - \xi^2(1 - n_x^2) & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{bmatrix}$$

$\mathbf{\Omega}$ is defined by five coefficients. Nevertheless, the catadioptric image of a 3D line has only two degrees of freedom. In the sequel, we show how we can get a minimal representation using polar lines.

In [3], Barreto *et al* consider the geometric properties of the polar lines in order to calibrate central catadioptric cameras.

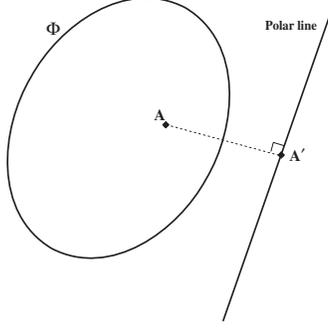


Fig. 1. Polar line

We will show that polar lines can also be advantageously used for visual servoing purposes. Let us first define the polar line of a point with respect to a conic curve.

Definition 1: Let a 2D conic curve Φ , a point A in the same definition plane of Φ and its conjugate \hat{A} with respect to conic Φ (refer to Figure 1). The straight line through \hat{A} which is perpendicular to the line of the points AA' is called the polar line of A with respect to the conic Φ , and it has for coordinates $\overline{AA'} \propto \Phi A$.

Let $\mathbf{l}_i = \Omega_i \mathbf{O}_i$ be the polar line of the optical center with respect to the conic Ω_i where $\mathbf{O}_i = [u_0 \ v_0 \ 1]^T$ and u_0 and v_0 are the coordinates of the optical center. The equation of the polar line is given by:

$$\mathbf{l}_i^T \underline{\mathbf{x}}_i = 0 \quad \text{or} \quad l_x x_i + l_y y_i + l_z = 0 \quad (4)$$

When the pixel elements of the image plane of the conventional camera are square ($f_u = f_v = f_c$) and the skew factor equals zero ($\alpha_{uv} = 0$), the vector \mathbf{l}_i is given, up to a scale factor, by:

$$\mathbf{l}_i \propto \Omega_i \mathbf{O}_i = \mathbf{K}^{-T} \Omega \mathbf{K}^{-1} \mathbf{O}_i = \mathbf{K}^{-T} [n_x \ n_y \ n_z]^T \quad (5)$$

Note that in order to compute \mathbf{l}_i , we need only to know the optical center \mathbf{O}_i . It can easily be obtained using two lines defined by the intersection of three conics [3].

Proposition 1: Consider a virtual perspective camera defined by the frame $\mathcal{F}_v = \mathcal{F}_m$ (see figure 2) such that the internal parameters of the virtual camera chosen equal to the internal parameters of the catadioptric camera (*i.e.* $\mathbf{K}_v = \mathbf{K}_c \mathbf{M}$). Then the perspective projection in the virtual image of the 3D line \mathcal{L} is defined by the intersection between the virtual image plane Π_v and the interpretation plane Π as defined below. It is a line in the virtual image plane given by:

$$ax_{vi} + by_{vi} + c = 0, \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} \propto \mathbf{K}_v^{-T} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \quad (6)$$

Corollary 1: The polar line \mathbf{l}_i computed from the physical conic curve projection of \mathcal{L} in the omnidirectional image is the perspective projection of \mathcal{L} into the virtual camera image plane.

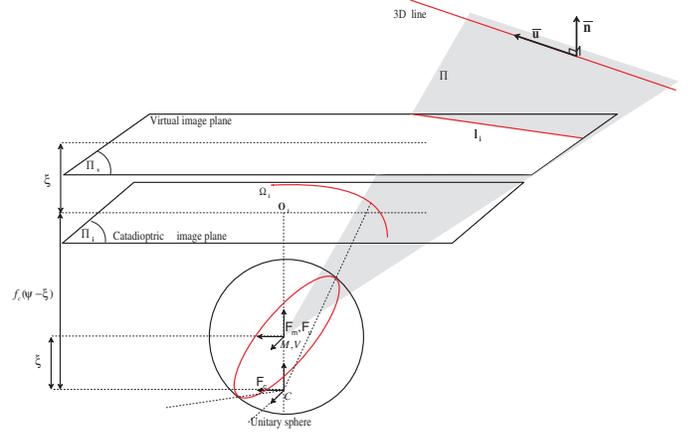


Fig. 2. 3D line in virtual image plane

This corollary is fundamental since it allows us to represent the physical projection of a 3D line in a catadioptric camera by a simple (polar) line in a virtual perspective camera rather than a conic. Indeed, equations (5) and (6) define the same line in the image space since equation (5) can be rewritten as:

$$\mathbf{l}_i \propto \mathbf{K}^{-T} [n_x \ n_y \ n_z]^T \quad (7)$$

Knowing the optical center \mathbf{O}_i , it is thus possible to use the linear pin-hole model for the projection of a 3D line instead of the non linear central catadioptric projection model. Hence, one can choose any image line representation. For instance, equation (4) can be rewritten, using the parametric representation (ρ, θ) of a line, as:

$$\cos(\theta)x_i + \sin(\theta)y_i + \rho = 0 \quad (8)$$

where:

$$\theta = \arctan 2(l_y, l_x) \quad \text{and} \quad \rho = -\frac{l_z}{\sqrt{l_x^2 + l_y^2}} \quad (9)$$

This representation is minimal since the polar line can be defined by two parameters ρ and θ . Consequently, in the sequel \mathbf{l}_i will be represented in the image frame by $\hat{\mathbf{l}}_i = [\rho \ \theta]^T$.

This representation is not unique since $\hat{\mathbf{l}}_{i1} = [\rho, \theta + 2k\pi]^T$ and $\hat{\mathbf{l}}_{i2} = [-\rho, \theta + (2k+1)\pi]^T$ represent the same line. We overcome this ambiguity by considering an oriented projective geometry [22]. When sampling the catadioptric image of a 3D line, the fitted conic is defined up to sign. This sign can be fixed using the gray level gradient perpendicular to the conic. This method has been used in [10] when servoing image line features with classical cameras.

III. CONTROL LAW

In order to control the movements of a robot from visual features, one defines a task function as [21]:

$$\mathbf{e} = \hat{\mathbf{L}}^+(\mathbf{s} - \mathbf{s}^*) \quad (10)$$

where:

- \mathbf{s} is composed of the current features extracted from the catadioptric image.
- \mathbf{s}^* is the desired value of \mathbf{s} .
- $\hat{\mathbf{L}}^+$ is the pseudo-inverse of the model of the interaction matrix \mathbf{L} .

The interaction matrix \mathbf{L} is defined by

$$\dot{\mathbf{s}} = \mathbf{L}\tau \quad (11)$$

and it links the variation of the visual features \mathbf{s} to the screw vector $\tau = [\nu \ \omega]^T$ of the catadioptric camera, where ν and ω are the instantaneous linear and angular velocities respectively.

The time derivative of the task function (10) is:

$$\dot{\mathbf{e}} = \frac{d\hat{\mathbf{L}}^+}{dt}(\mathbf{s} - \mathbf{s}^*) + \hat{\mathbf{L}}^+\dot{\mathbf{s}} = (\Theta(\mathbf{s} - \mathbf{s}^*) + \hat{\mathbf{L}}^+\mathbf{L})\tau \quad (12)$$

where $\Theta(\mathbf{s} - \mathbf{s}^*)$ is a 6-dimensional square matrix and equals zero when $\mathbf{s} = \mathbf{s}^*$. If we wish an exponential decay of the task function \mathbf{e} towards 0, the control law is given by:

$$\tau = -\lambda\mathbf{e} = -\lambda\hat{\mathbf{L}}^+(\mathbf{s} - \mathbf{s}^*) \quad (13)$$

From equations (12) and (13), the closed-loop system is $\dot{\mathbf{e}} = -\lambda(\Theta(\mathbf{s} - \mathbf{s}^*) + \hat{\mathbf{L}}^+\mathbf{L})\mathbf{e}$. It is well known that this system is locally asymptotically stable in the neighborhood of \mathbf{s}^* if and only if $\hat{\mathbf{L}}^+\mathbf{L}$ is a positive definite matrix. To compute the control law (13), it is necessary to provide an approximated interaction matrix $\hat{\mathbf{L}}^+$. In the next section, we derive a generic analytical form of the interaction matrix for polar line features.

IV. INTERACTION MATRIX OF CENTRAL CATADIOPTIC CAMERA FOR POLAR LINES

We consider now that the conics in the real image are mapped into lines in the virtual image. We can thus use any visual servoing method from lines we like. Here, we choose the reference method proposed in [10] and related to the minimal (ρ, θ) representation.

Consider the vector $\mathbf{s} = [\mathbf{s}_1^T \dots \mathbf{s}_n^T]^T$ where \mathbf{s}_k is a 2-dimensional vector containing the parameters ρ and θ of each projected 3D line in the virtual image:

$$\mathbf{s}_k = [\rho_k \ \theta_k]^T \quad (14)$$

Remind that the conic curve resulting from the projection of a 3D line in the image plane of the catadioptric camera is defined by 5 parameters as given in equation (??), and as shown in [18], only 2 degrees of freedom of a robot can be controlled. By using the polar line of the image center with respect to the conic, the observation vector \mathbf{s}_k is minimal and without redundancy. Now, we will derive explicitly the interaction matrix associated to the (ρ, θ) representation in the image rather than the interaction matrix associated to the (ρ_c, θ_c) representation in the camera frame as it is done in [10]. In this way, the image signal is directly servoed instead of the reconstructed camera signal obtained after the plane to plane colineation \mathbf{K} . Indeed, in the original approach [10], the \mathbf{K} -matrix appears in the servoed error, while it appears here only in the interaction matrix. Thus only the transient phase is affected by errors on \mathbf{K} and not the convergence point.

Let us write the equation (11) as:

$$\dot{\mathbf{s}}_k = \mathbf{L}_{snk}\mathbf{L}_{nk}\tau = \mathbf{L}_k\tau \quad (15)$$

where:

$$\mathbf{L}_k = \mathbf{L}_{snk}\mathbf{L}_{nk} \quad (16)$$

Knowing that $\dot{\mathbf{n}}_k = \mathbf{L}_{nk}\tau$, we deduce $\mathbf{L}_{snk} = \frac{\partial \mathbf{s}_k}{\partial \mathbf{n}_k}$. \mathbf{L}_{snk} is the interaction matrix between visual observation motion and the normal vector variation, and \mathbf{L}_{nk} links the normal variation to the catadioptric camera motions. It can be shown that [1]:

$$\dot{\mathbf{n}} = \frac{1}{h}(\mathbf{I}_3 - \mathbf{n}\mathbf{n}^T)[\bar{\mathbf{u}}]_{\times}\nu + [\bar{\mathbf{n}}]_{\times}\omega$$

The interaction matrix between the normal vector and the camera motion is thus:

$$\mathbf{L}_{nk} = \begin{bmatrix} \frac{1}{h_k}(\mathbf{I}_3 - \bar{\mathbf{n}}_k\bar{\mathbf{n}}_k^T)[\bar{\mathbf{u}}_k]_{\times} & [\bar{\mathbf{n}}_k]_{\times} \end{bmatrix} \quad (17)$$

The interaction matrix \mathbf{L}_{snk} is obtained by computing the partial derivative of (14) with respect to the normal vector $\bar{\mathbf{n}}_k$:

$$\mathbf{L}_{snk} = \sqrt{\gamma_k^2 + 1} \begin{bmatrix} -\sin \theta_k & \cos \theta_k & 0 \\ u_0 - \rho_k \cos \theta_k & v_0 - \rho_k \sin \theta_k & -f \end{bmatrix} \quad (18)$$

where $f = f_c(\psi - \xi)$ is the combined focal of the mirror and the conventional camera and

$$\gamma_k = \frac{1}{f}(u_0 \cos \theta_k + v_0 \sin \theta_k - \rho_k)$$

The global interaction matrix \mathbf{L}_k is obtained by combining equations (17) and (18):

$$\mathbf{L}_k = \begin{bmatrix} \mathbf{A} & \mathbf{B} \end{bmatrix} \quad (19)$$

where

$$\mathbf{A} = \frac{\sqrt{\gamma_k^2 + 1}}{\gamma_k h_k} \begin{bmatrix} -\eta_k \cos \theta_k & & \\ \beta_k \cos \theta_k u_{xk} + (\beta_k \sin \theta_k + f\gamma_k)u_{yk} & & \\ -\eta_k \sin \theta_k & & -\eta_k \gamma_k \\ -(\delta_k \cos \theta_k + f\gamma_k)u_{xk} - \delta_k \sin \theta_k u_{yk} & \gamma_k(\beta_k u_{xk} - \delta_k u_{yk}) & \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} \gamma_k \cos \theta_k & \gamma_k \sin \theta_k \\ f \sin \theta_k - \gamma_k \beta_k & -f \cos \theta_k + \gamma_k \delta_k \\ -1 & \\ u_0 \sin \theta_k - v_0 \cos \theta_k & \end{bmatrix}$$

with

$$\begin{aligned} \eta_k &= (u_{xk} \cos \theta_k + u_{yk} \sin \theta_k) \\ \beta_k &= (\rho_k \sin \theta_k - v_0) \\ \delta_k &= (\rho_k \cos \theta_k - u_0) \end{aligned}$$

The rank of the interaction matrix \mathbf{L}_k is 2. To control the 6 *dof* of the robot arm, 3 lines at least are thus necessary. The global interaction matrix for the observation vector \mathbf{s} is $\mathbf{L} = [\mathbf{L}_1^T \dots \mathbf{L}_n^T]^T$ where \mathbf{L} is $2n \times 6$ -dimensional matrix and for 3 lines \mathbf{L} is square. As can be seen on equation (19), only the 3D parameters u_x/h and u_y/h have to be introduced in the interaction matrix. As usual when visual information is used in image-based control, these parameters only act on

the translation velocities. As previously explained, a chosen estimation of the interaction matrix is used to design the control law. The value of \mathbf{L} at the desired position is a typical choice. In this case, the 3D parameters have to be estimated only for the desired position.

Note that the interaction matrix of the observation vector $\tilde{\mathbf{s}} = g(\mathbf{K}, \mathbf{s})$ defined in the camera frame (*i.e.* before the plane-to-plane collineation \mathbf{K}) is easily obtained by replacing in \mathbf{L}_k the focal length f by 1 and the coordinates u_0 and v_0 of the image center by 0:

$$\tilde{\mathbf{L}}_k = \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \end{bmatrix} \quad (20)$$

where

$$\tilde{\mathbf{A}} = \frac{1}{h_k} \begin{bmatrix} \lambda_{\theta_k} \cos \tilde{\theta}_k & \lambda_{\theta_k} \sin \tilde{\theta}_k & -\lambda_{\tilde{\rho}_k} \tilde{\rho}_k \\ \lambda_{\rho_k} \cos \tilde{\theta}_k & \lambda_{\rho_k} \sin \tilde{\theta}_k & -\lambda_{\rho_k} \tilde{\rho}_k \end{bmatrix}$$

and

$$\tilde{\mathbf{B}} = \begin{bmatrix} -\tilde{\rho}_k \cos \tilde{\theta}_k & -\tilde{\rho}_k \sin \tilde{\theta}_k & -1 \\ (\tilde{\rho}_k^2 + 1) \sin \tilde{\theta}_k & -(\tilde{\rho}_k^2 + 1) \cos \tilde{\theta}_k & 0 \end{bmatrix}$$

with

$$\lambda_{\theta_k} = \frac{\sqrt{\tilde{\rho}_k^2 + 1}}{\tilde{\rho}_k} (\cos \tilde{\theta}_k u_{xk} + \sin \tilde{\theta}_k u_{yk})$$

$$\lambda_{\rho_k} = \sqrt{\tilde{\rho}_k^2 + 1} (\sin \tilde{\theta}_k u_{xk} - \cos \tilde{\theta}_k u_{yk})$$

As expected, the interaction matrix $\tilde{\mathbf{L}}$ (valid for all central imaging system) has the same form that the one obtained when considering a conventional camera [10].

V. SIMULATION RESULTS

In this section, we present simulation results of central catadioptric visual servoing of a 6 *dof* robotic arm using polar line features as visual measurements.

In this simulation, we used a para-catadioptric camera (a parabolic mirror combined with an orthographic camera). Similar results are obtained using an hyper-catadioptric camera (combining an hyperbolic mirror with a perspective camera). From an initial position, the catadioptric camera mounted onto the robot arm has to reach the desired position. This means that the initial observation vector \mathbf{s} containing the polar line parameters of each observed 3D line, reaches the desired observation vector \mathbf{s}^* . To be close to a real setup, image noise has been added when extracting the conic curve. The real conic of the projected 3D line in the catadioptric image plane is sampled with a step of 10 pixels. Then, a uniform distribution random noise with a variance of 10 pixels is added to the sampled points in the direction of the normal vector to the real conic curve (sampled points are represented by crosses in figures 3(a) and 3(b)). The obtained points are fitted to get the coefficients of the conic curve. The image center is then estimated using two lines defined by the intersection of three conics [3]. Polar lines are computed using this estimated image center. The interaction matrix \mathbf{L} is computed using erroneous intrinsic camera parameters (estimated image center and $\pm 10\%$ on the focal length). The value of \mathbf{L} at the desired position is used to calculate the control law (equation (13)).

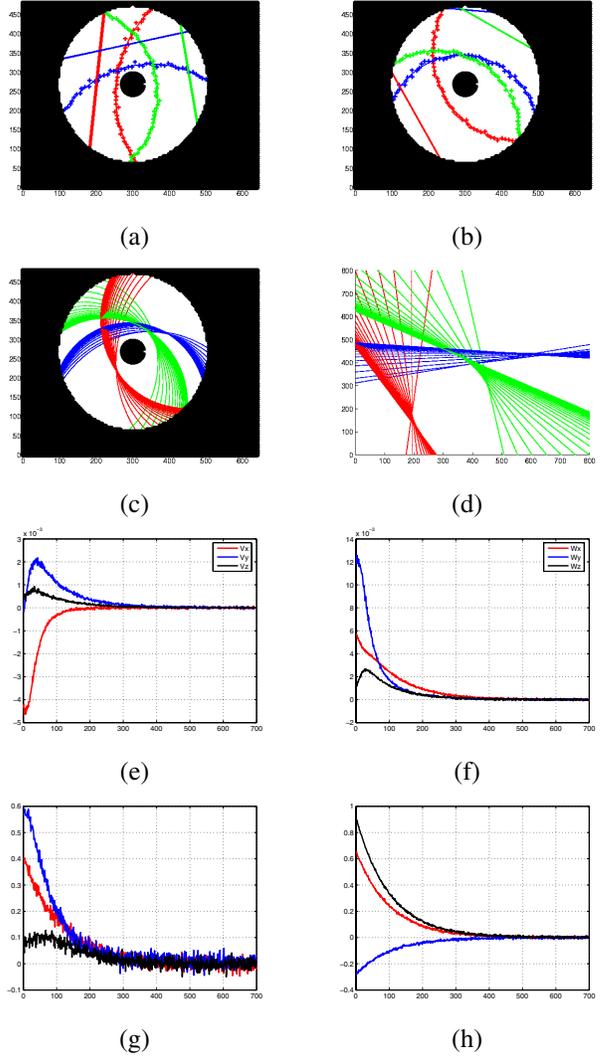


Fig. 3. Para-catadioptric camera: (a) initial image, (b) desired image, (c) trajectory of the conics in the image plane, (d) trajectory of the polar lines in the virtual image plane, (e) translation velocities [m/s], (f) rotational velocities [rad/s], (g) $(\rho - \rho^*)$ vector errors, (h) $(\theta - \theta^*)$ vector errors

The images corresponding to the initial and desired positions are shown by figures 3(a) and 3(b). These figures show also the polar lines corresponding to conics. Figures 3(c) and 3(d) show respectively the trajectories of the conics and their associated polar lines in the image plane when a para-catadioptric camera is used. These trajectories confirm that the positioning task is correctly realized. The translational and rotational velocities of the para-catadioptric camera are given in figures 3(e) and 3(f). As shown in figures 3(g) and 3(h), the observation vector of the error between the current and desired observation vectors are well regulated to zeros.

VI. EXPERIMENTAL RESULTS

The robotic system is composed of a Pioneer 3 mobile robot (unicycle robot) and of a para-catadioptric camera mounted in order to coarsely align the mobile robot rotation axis and the camera optical axis (see Figure 4(a)). The task to achieve

consists in driving the robot parallel to a given 3-D straight line. The reference 3-D straight line is shown in Figure 4(a). The circle in the image corresponding to the projection of the reference straight line is tracked thanks to a modified version of the software described in [16].

The image of circle and its polar line corresponding to the initial and desired cameras positions are given in Figures 4 (c) and (d). Angular velocity is given in Figure 4(b). The orientation error of the polar line in the image is linear with respect to the angular velocity of the mobile robot, and it converges to zero. This implies that the line following task is correctly realized.

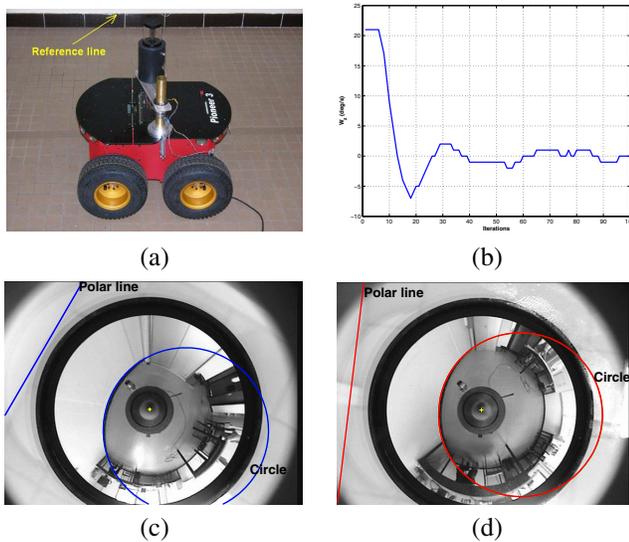


Fig. 4. (a) Mobile robot Pioneer 3 equipped with an omnidirectional camera, (b) rotational velocities ω_z [rad/s], (c) initial image, (d) desired image

VII. CONCLUSION

In this paper, we have proposed to use a minimal representation (*i.e.* a two parameters representation) of the projection of a 3D straight line (conic) into the image plane of a central catadioptric camera in order to design an efficient vision-based control scheme. The visual observations have been obtained from the polar line of the image center with respect to the conic which allows to turn the physical omnidirectional camera (of any kind: parabolic, hyperbolic, spherical) observing conic curves into a virtual perspective camera with unlimited field of view observing straight lines knowing only the optical center. A minimal and generic analytical form of the central catadioptric interaction matrix for the image of 3D straight lines has been derived from this new representation. The obtained interaction matrix has been exploited to design image-based control laws. In future work, the robustness and stability analysis with respect to the 3D parameters and calibration errors must be studied.

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