Free Boundary Problems 2012

A GENERAL PHASE TRANSITION MODEL FOR VEHICULAR TRAFFIC

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Outline of the talk

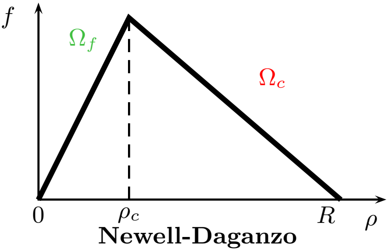
First order macroscopic models

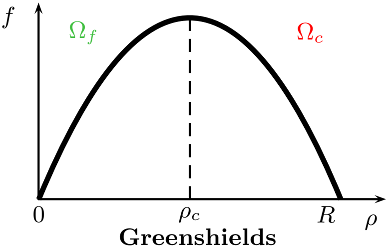
Lighthill-Whitham '55, Richards '56, Greenshields '35:

- Traffic state: density $\rho(t,x)$ of vehicles at time t and location x
- Non-linear transport equation: scalar one dimensional conservation law

$$\partial_t \rho + \partial_x f(\rho) = 0, \quad f(\rho) = \rho v(\rho)$$

- Empirical flux function: the fundamental diagram



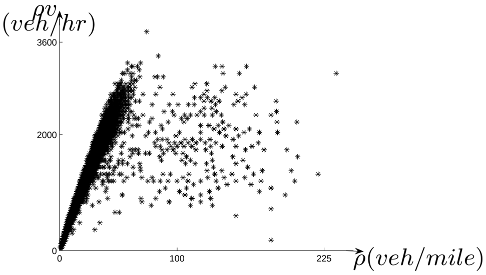


with R the maximal or *jam* density, and ρ_c the critical density:

- flux is increasing for $\rho \leq \rho_c$: free-flow phase
- flux is decreasing for $\rho \geq \rho_c$: congested phase

Motivation for higher order models

- Traffic satisfies "mass" conservation. What about other fundamental conservation principles from fluid dynamics: conservation of momentum, conservation of energy?
- Experimental observations of fundamental diagrams are more complex than postulated by first order traffic models



Viale Muro Torto, Roma

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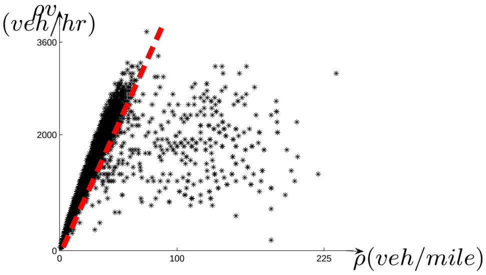

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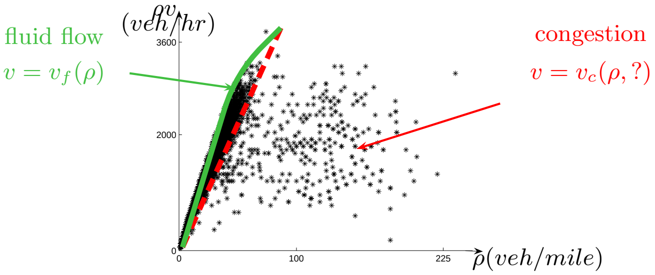

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Second order models

- **Payne '71:**

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t v + v \partial_x v = -\dfrac{c_0^2}{\rho} \partial_x \rho + \dfrac{v_*(\rho) - v}{\tau} \end{cases}$$

Critics (Del Castillo et al. '94, Daganzo '95):

- drivers should have only positive speeds;
- anisotropy: drivers should react only to stimuli from the front.

Second order models

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- **Aw-Rascle '00:**

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho w) + \partial_x(\rho v w) = 0 \end{cases} \qquad v = v(\rho, w)$$

$w = v + p(\rho)$ Lagrangian marker, $p = p(\rho)$ "pressure"

Second order models

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- **Colombo '02:**

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t q + \partial_x((q - Q)v) = 0 \end{cases} \qquad v = v(\rho, q)$$

q "momentum", Q road parameter

Outline of the talk

Macroscopic traffic flow models

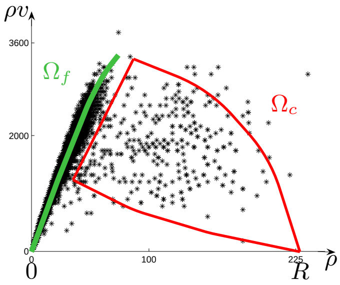
Traffic flow models with phase transitions

Analytical study

Finite volume numerical schemes

Extension to road networks

Phase transition (Colombo '02)



Fluid flow: $(\rho, q) \in \Omega_f$

$$\begin{cases} \partial_t \rho + \partial_x(\rho v_f) = 0 \\ q = \rho V \end{cases}$$

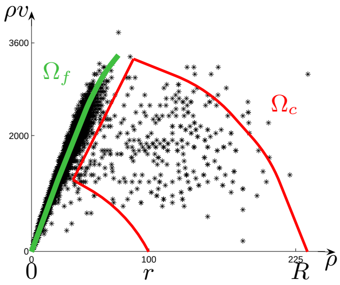
$$v_f(\rho) = \left(1 - \frac{\rho}{R}\right) V$$

Congestion: $(\rho, q) \in \Omega_c$

$$\begin{cases} \partial_t \rho + \partial_x(\rho v_c) = 0 \\ \partial_t q + \partial_x((q - Q) v_c) = 0 \end{cases}$$

$$v_c(\rho, q) = \left(1 - \frac{\rho}{R}\right) \frac{q}{\rho}$$

Aw-Rascle model with phase transition



Fluid flow: $(\rho, y) \in \Omega_f$

$$\begin{cases} \partial_t \rho + \partial_x(\rho v_f) = 0 \\ y = \rho V \end{cases}$$

$$v_f(\rho) = \left(1 - \frac{\rho}{R}\right) V$$

Congestion: $(\rho, y) \in \Omega_c$

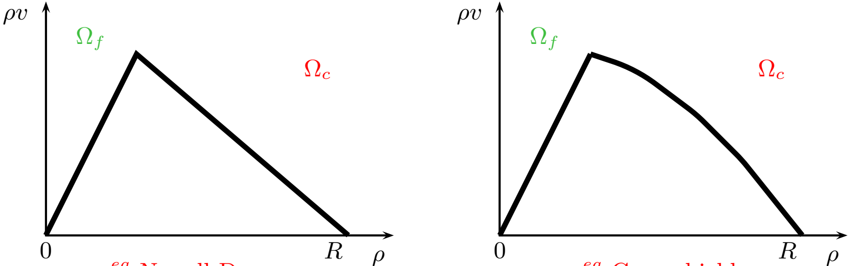
$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t y + \partial_x(yv) = 0 \end{cases}$$

$$y = \rho(v + p(\rho)), \quad p(\rho) = V_{ref} \ln \frac{\rho}{R}$$

(Goatin '06)

General view point

congestion = perturbation from equilibrium



v_c^{eq} Newell-Daganzo $\qquad$ v_c^{eq} Greenshields

Fluid flow: in Ω_f

$$\partial_t \rho + \partial_x(\rho v_f) = 0$$

$$v_f(\rho) = V$$

Congestion: in Ω_c

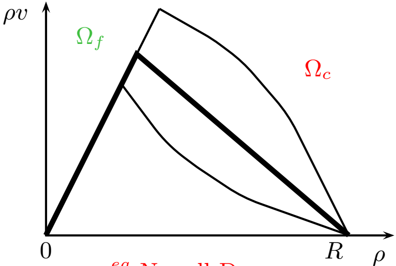
$$\begin{cases} \partial_t \rho + \partial_x(\rho v_c) = 0 \\ \end{cases}$$

$$v_c(\rho, q) = v_c^{eq}(\rho)$$

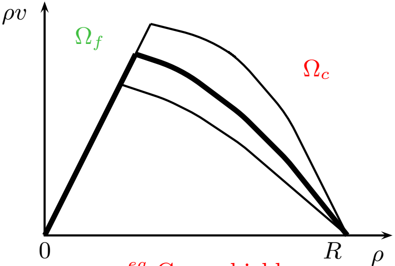
(Blandin-Work-Goatin-Piccoli-Bayen '11)

General view point

congestion = perturbation from equilibrium



v_c^{eq} Newell-Daganzo



v_c^{eq} Greenshields

Fluid flow: in Ω_f

$$\partial_t \rho + \partial_x(\rho v_f) = 0$$

$$v_f(\rho) = V$$

Congestion: in Ω_c

$$\begin{cases} \partial_t \rho + \partial_x(\rho v_c) = 0 \\ \partial_t q + \partial_x(q v_c) = 0 \end{cases}$$

$$v_c(\rho, q) = v_c^{eq}(\rho)(1+q)$$

(Blandin-Work-Goatin-Piccoli-Bayen '11)

Outline of the talk

Analysis of congestion phase

Eigenvalues	$\lambda_1(\rho,q) = v_c^{eq}(\rho)\,(1+q) + q\,v_c^{eq}(\rho) + \rho\,(1+q)\partial_\rho\, v_c^{eq}(\rho)$	$\lambda_2(\rho,q) = v_c^{eq}(\rho)(1+q)$
Eigenvectors	$r_1 = \begin{pmatrix} \rho \\ q \end{pmatrix}$	$r_2 = \begin{pmatrix} v_c^{eq}(\rho) \\ -(1+q)\,\partial_\rho\, v_c^{eq}(\rho) \end{pmatrix}$
Nature of the Lax-curves	$\nabla\lambda_1.r_1 = \rho^2\,(1+q)\partial^2_{\rho\rho}\, v_c^{eq}(\rho) + 2\,\rho\,(1+2\,q)\,\partial_\rho\, v_c^{eq}(\rho) + 2\,q\,v_c^{eq}(\rho)$	$\nabla\lambda_2.r_2 = 0$
Riemann invariants	$v_c^{eq}(\rho)\,(1+q)$	q/ρ

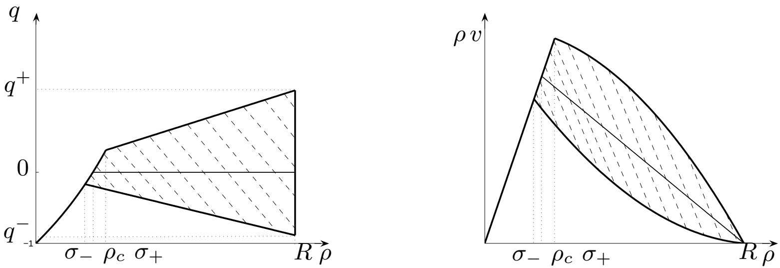
- First family of Lax-curves is not genuinely-non-linear (in flux-density coordinates curves GNL equivalent to all Lax-curves have same concavity)
- Second family of Lax-curves is linearly degenerate (information propagates at a constant speed)
- System is Temple class (shock and rarefaction curves coincide)

Domain definition

Definition of free-flow and congestion phases as invariant domains of dynamics

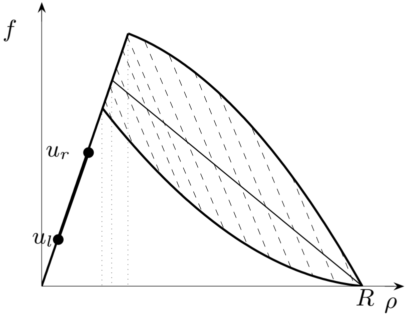
$$\Omega_f = \{(\rho, q) \mid (\rho, q) \in [0, R] \times [0, +\infty[\ , v_c(\rho, q) = V \ , \ 0 \leq \rho \leq \sigma_+\}$$

$$\Omega_c = \left\{(\rho, q) \mid (\rho, q) \in [0, R] \times [0, +\infty[\ , v_c(\rho, q) < V \ , \ \frac{q^-}{R} \leq \frac{q}{\rho} \leq \frac{q^+}{R}\right\}$$



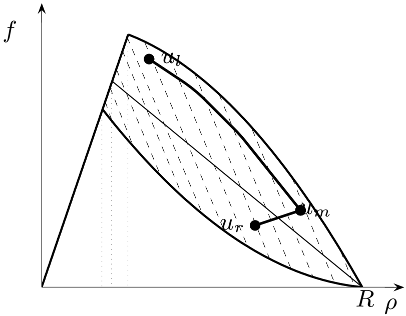
Model parameters: free-flow speed V, jam density R, critical density ρ_c, upper and lower bound for perturbation q^- and q^+

Riemann solver



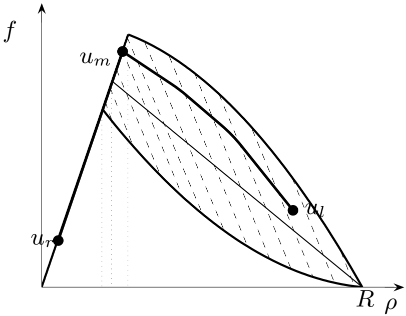
- free-flow to free-flow: contact discontinuity

Riemann solver



- free-flow to free-flow: contact discontinuity
- congestion to congestion: shock or rarefaction + contact discontinuity

Riemann solver



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- congestion to congestion: shock or rarefaction + contact discontinuity
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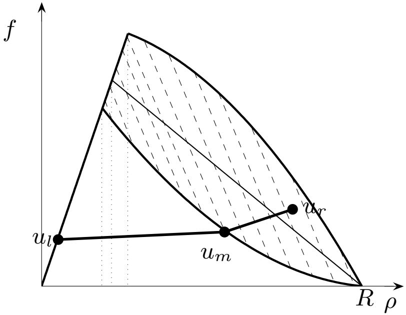

- free-flow to free-flow: contact discontinuity
- congestion to congestion: shock or rarefaction + contact discontinuity
- congestion to free-flow: shock or rarefaction + contact discontinuity
- free-flow to congestion: phase transition + (rarefaction) + contact discontinuity

Cauchy problem - admissible solutions

Consider weak solutions $\mathbf{u} = (\rho, q)$ of

$$\begin{cases} \partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) = 0 \\ \mathbf{u}(0, x) = \mathbf{u}_0(x) \end{cases}$$

where

$$\begin{cases} \mathbf{f}(\mathbf{u}) = \mathbf{f}_f(\mathbf{u}) = \begin{pmatrix} \rho v_f(\rho) \\ q v_f(\rho) \end{pmatrix} & \text{if} \quad (\rho, q) \in \Omega_f \\ \mathbf{f}(\mathbf{u}) = \mathbf{f}_c(\mathbf{u}) = \begin{pmatrix} \rho v_c(\rho, q) \\ q v_c(\rho, q) \end{pmatrix} & \text{if} \quad (\rho, q) \in \Omega_c \end{cases}$$

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- $\forall \varphi$ in $C_c^1(\mathbb{R}^2)$ with compact support contained in $\mathbf{u}^{-1}(\Omega_f)$

$$\iint (\mathbf{u}\partial_t \varphi + \mathbf{f}_f(\mathbf{u})\partial_x \varphi)\, dx dt + \int \mathbf{u}_0(x)\, \varphi(0,x) dx = 0$$

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Cauchy problem - admissible solutions

- Mass conservation at phase transitions:
  phase transition speed Λ must satisfy Rankine-Hugoniot conditions

$$\Lambda(\rho_+ - \rho_-) = F_+ - F_-$$

with

$$F_- = \begin{cases} \rho_- \, v_f(\rho_-) & \text{if } \rho_- \in \Omega_f \\ \rho_- \, v_c(\rho_-, q_-) & \text{if } \rho_- \in \Omega_c \end{cases}$$

$$F_+ = \begin{cases} \rho_+ \, v_f(\rho_+) & \text{if } \rho_+ \in \Omega_f \\ \rho_+ \, v_c(\rho_+, q_+) & \text{if } \rho_+ \in \Omega_c \end{cases}$$

Cauchy problem - well posedness

Theorem (Colombo-Goatin-Priuli '06)

$\forall M > 0$,there exists a semigroup $S : \mathbb{R}^+ \times \mathcal{D} \mapsto \mathcal{D}$ s.t.

- $\mathcal{D} \supseteq \{\mathbf{u} \in \mathbf{L^1} : \mathrm{TV}(\mathbf{u}) \leq M\}$;
- $\|S_{t_1}\mathbf{u}_1 - S_{t_2}\mathbf{u}_2\|_{\mathbf{L^1}} \leq L(M) \cdot \left(\|\mathbf{u}_1 - \mathbf{u}_2\|_{\mathbf{L^1}} + |t_1 - t_2|\right) \quad \forall \mathbf{u}_1, \mathbf{u}_2 \in \mathcal{D}$.

Sketch of proof

- Existence:
  - Construction of sequence of approximate solutions by wave-front tracking method (piecewise constant approximations: Dafermos '72, DiPerna '76, Bressan '92, Risebro '93)
  - Proof of convergence of the sequence of approximate solutions using BV compactness result (Helly's theorem)
  - Show that limit is a weak solution to the Cauchy problem
- Uniqueness: shift differentials

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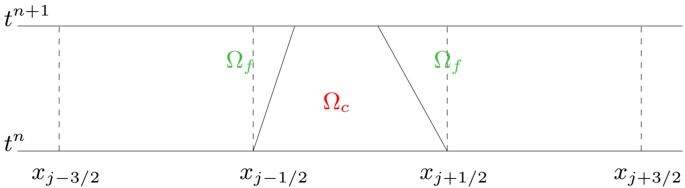
Finite volume numerical schemes

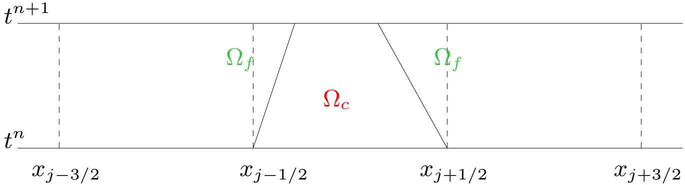


Problem:

$\Omega_f \cup \Omega_c$ is not convex $\longrightarrow$ Godunov method doesn't work in general

$$\mathbf{u}_j^n \in \Omega_c, \mathbf{u}_{j+1}^n \in \Omega_f \quad \not\Rightarrow \quad \mathbf{u}_j^{n+1} \in \Omega_f \cup \Omega_c$$

Finite volume numerical schemes



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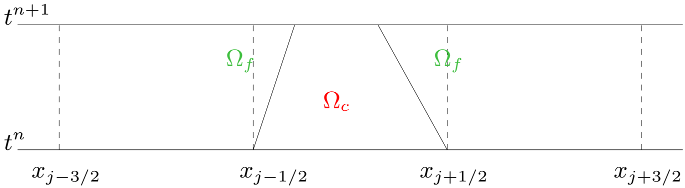
Solutions

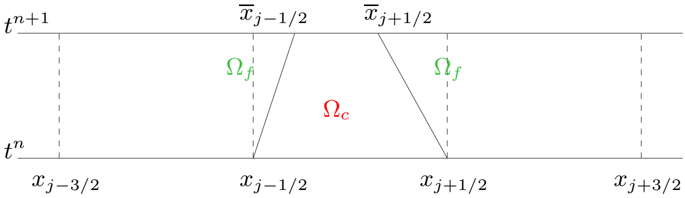
- **moving meshes for phase transitions:** Zhong - Hou - LeFloch '96;
- **transport-equilibrium method:** Chalons '07.

Godunov method



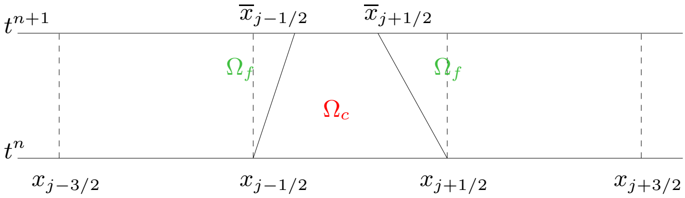
$$\mathbf{u}_j^{n+1} = \frac{1}{\Delta x} \int_{x_{j-1/2}}^{x_{j+1/2}} \mathbf{v}(\Delta t, x)dx$$

Modified Godunov method (Chalons-Goatin '08)



$$\overline{\mathbf{u}}_j^{n+1} = \frac{1}{\overline{\Delta x}_j} \int_{\overline{x}_{j-1/2}}^{\overline{x}_{j+1/2}} \mathbf{v}(\Delta t, x)dx$$

Modified Godunov method (Chalons-Goatin '08)



$$\overline{\mathbf{u}}_j^{n+1} = \frac{1}{\Delta x_j} \int_{\overline{x}_{j-1/2}}^{\overline{x}_{j+1/2}} \mathbf{v}(\Delta t, x) dx$$

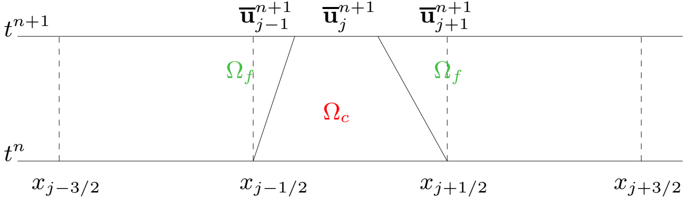
Green's formula:

$$\overline{\mathbf{u}}_j^{n+1} = \frac{\overline{\Delta x}}{\overline{\Delta x}_j} \mathbf{u}_j^n - \frac{\Delta t}{\overline{\Delta x}_j} (\overline{\mathbf{f}}_{j+1/2}^{n,-} - \overline{\mathbf{f}}_{j-1/2}^{n,+})$$

with numerical flux

$$\overline{\mathbf{f}}_{j+1/2}^{n,\pm} = \mathbf{f}(\mathbf{v_r}(\sigma_{j+1/2}^{\pm}; \mathbf{u}_j^n, \mathbf{u}_{j+1}^n)) - \sigma_{j+1/2} \mathbf{v_r}(\sigma_{j+1/2}^{\pm}; \mathbf{v}_j^n, \mathbf{v}_{j+1}^n)$$

Random sampling



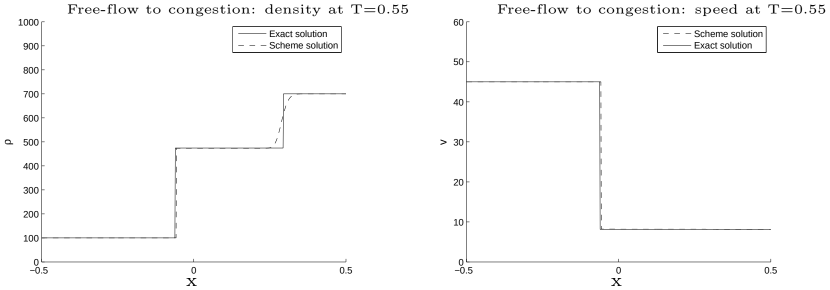
(a_n) **equi-distributed random sequence** in $]0,1[$ (ex. Van der Corput)

$$\mathbf{u}_j^{n+1} = \begin{cases} \overline{\mathbf{u}}_{j-1}^{n+1} & \text{si} \quad a_{n+1} \in \left]0, \frac{\Delta t}{\Delta x}\sigma_{j-1/2}^{+}\right[\\ \overline{\mathbf{u}}_{j}^{n+1} & \text{si} \quad a_{n+1} \in \left[\frac{\Delta t}{\Delta x}\sigma_{j-1/2}^{+}, 1 + \frac{\Delta t}{\Delta x}\sigma_{j+1/2}^{-}\right[\\ \overline{\mathbf{u}}_{j+1}^{n+1} & \text{si} \quad a_{n+1} \in \left[1 + \frac{\Delta t}{\Delta x}\sigma_{j+1/2}^{-}, 1\right[\end{cases}$$

$\sigma_{j+1/2}$ = phase transition speed at $x_{j+1/2}$
$\sigma_{j+1/2}^{+} = \max\{\sigma_{j+1/2}, 0\}$, $\sigma_{j+1/2}^{-} = \min\{\sigma_{j+1/2}, 0\}$

Benchmark tests

Newell-Daganzo with $V = 45$, $R = 1000$, $\rho_c = 220$, $\sigma_- = 190$, $\sigma_+ = 270$:

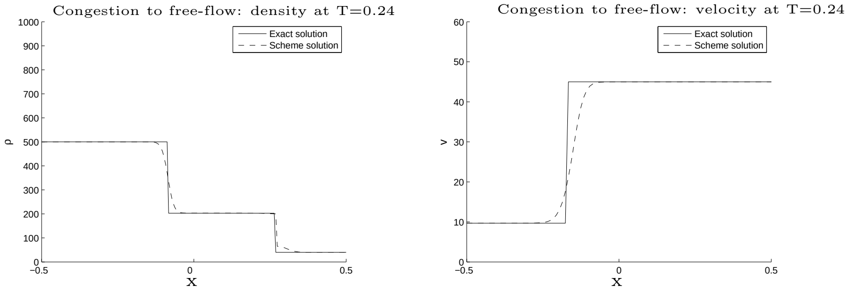


Initial data: $\mathbf{u}_l = (100, 0) \in \Omega_f$, $\mathbf{u}_r = (700, 0.5) \in \Omega_c$ above equilibrium.
Gives: phase transition + 2-contact discontinuity linked by
$\mathbf{u}_m = (474, -0.42) \in \Omega_c$.

(Blandin-Work-Goatin-Piccoli-Bayen '11)

Benchmark tests

Newell-Daganzo with $V = 45$, $R = 1000$, $\rho_c = 220$, $\sigma_- = 190$, $\sigma_+ = 270$:



Initial data: $\mathbf{u}_l = (500, 0.3) \in \Omega_c$ under equilibrium, $\mathbf{u}_r = (40, 0) \in \Omega_f$.
Gives: shock + 1-contact discontinuity linked by metastable state
$\mathbf{u}_m = (199, -0.12) \in \Omega_c$.

(Blandin-Work-Goatin-Piccoli-Bayen '11)

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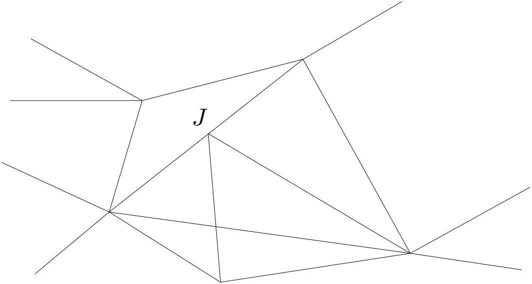
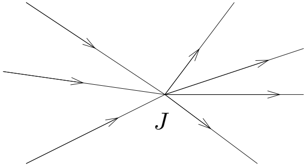
Road networks with phase transitions



$\mathcal{I}$: unidirectional roads $I_i =]a_i, b_i[$, $i = 1, \ldots, N$
$\mathcal{J}$: junctions

Riemann problem at J

$$\begin{cases} \partial_t \mathbf{u}_k + \partial_x \mathbf{f}(\mathbf{u}_k) = 0, \\ \mathbf{u}_k(0,x) = \mathbf{u}_{k,0} \end{cases}$$

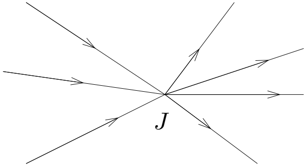


Riemann solver $\mathcal{RS}_J : (\mathbf{u}_{1,0}, \dots, \mathbf{u}_{n+m,0}) \longmapsto (\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_{n+m})$ s.t.

- conservation of cars: $\sum_{i=1}^{n} \mathbf{f}_1(\hat{\mathbf{u}}_i) = \sum_{j=n+1}^{n+m} \mathbf{f}_1(\hat{\mathbf{u}}_j)$;
- waves with negative speed in incoming roads;
- waves with positive speed in outgoing roads.

Riemann problem at J

$$\begin{cases} \partial_t \mathbf{u}_k + \partial_x \mathbf{f}(\mathbf{u}_k) = 0, \\ \mathbf{u}_k(0,x) = \mathbf{u}_{k,0} \end{cases}$$



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- waves with negative speed in incoming roads;
- waves with positive speed in outgoing roads.

Consistency condition:

$$\mathcal{RS}_J\big(\mathcal{RS}_J(\mathbf{u}_{1,0}, \dots, \mathbf{u}_{n+m,0})\big) = \mathcal{RS}_J(\mathbf{u}_{1,0}, \dots, \mathbf{u}_{n+m,0}) \qquad \textbf{(CC)}$$

Dynamics at junctions

(A) prescribe a fixed distribution of traffic in outgoing roads

$$A = \{a_{ji}\} \in \mathbb{R}^{m\times n} : \ 0 < a_{ji} < 1, \sum_{j=n+1}^{n+m} a_{ji} = 1$$

(B) maximize the flux through the junction

(Coclite-Garavello-Piccoli '05)

Wave-front tracking

- piecewise constant approximate solutions $\mathbf{u}_\nu$
- bound on the total variation

Assumption $(\mathbf{H})$

$\exists\, \bar v > 0$ s.t. $\mathbf{u}_\nu = (\mathbf{u}_{1,\nu}, \ldots, \mathbf{u}_{N,\nu})$ takes values in the set
$\widetilde{\Omega} = \Omega_f \cup (\Omega_c \cap \{(\rho, q) \in \Omega_c : v_c(\rho, q) \geq \bar v\})$.

bound on $\mathrm{TV}_f = \mathrm{TV}(\mathbf{f}_1(\mathbf{u}_\nu)) \Rightarrow$ bound on $\mathrm{TV}(\rho_\nu)$ and $\mathrm{TV}(q_\nu)$

Existence theorem (Colombo-Goatin-Piccoli '09)

If $\mathrm{TV}(\mathbf{u}_0) \leq C$ and $\mathbf{u}_\nu$ satisfies $(\mathbf{H})$:

- $\mathbf{u}_\nu \to \mathbf{u}$ in $\mathbf{L^1}_{loc}$;
- $\mathbf{u}$ is solution on I, $\forall I$;
- consistency: $\mathcal{RS}_J(\mathbf{u}_J(t)) = \mathbf{u}_J(t)$, a.e. $t > 0$, $\forall J$.

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- Fabio Priuli (Università di Padova)
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Thank you for your attention!