Erratum to ‘A geometrical take on invariants of low-dimensional manifolds found by integration’

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In the proof of theorem 3 of [2] it is stated that the manifold constructed in figure 4 is diffeomorphic to $M_g^{S(3\mathrm{D})} \cup C_g \times S^1$. This is in general not correct, it should be $M_g^{S(3\mathrm{D})} \cup T_{C_g}$, with T_{C_g} a mapping torus. The statement of Theorem 3 is nonetheless true. The reason for this is that

$$\int_{T_{C_g}} f \mathrm{dvol} = 0,$$

with f locally defined in terms of the metric. This can be seen as follows: consider T_{C_g} and deform part of it such that it is isometric to $[a, b] \times C_g$. Now introduce a second copy of $T_{C_g}$and cut in the parts isometric to $[a, b] \times C_g$. The two disjoint parts are now both diffeomorphic to $[c, d] \times C_g$. We can now deform the parts isometric to $[a, b] \times C_g$ and glue then together such that we get the trivial mapping torus $C_g \times S^1$. See figure 1 for a sketch. From this construction we can conclude that

$$2 \int_{T_{C_g}} f \mathrm{dvol} = \int_{C_g \times S^1} f \mathrm{dvol} = 0.$$

The result now follows as in [2].

As stated in the discussion on page 2181 of [2]

$$\int_M f \mathrm{dvol}$$

is zero for manifolds M of the type $L \times S^1$ (and also for mapping tori of arbitrary dimension by adapting the argument above). It is moreover claimed that cut and paste techniques are insufficient to provide results to theorems 2 and 3 of [2] in higher dimensions. This is not correct, due to a result given in [1]. To explain this, we briefly recalll some notions from [1]. Let M be a closed manifold and

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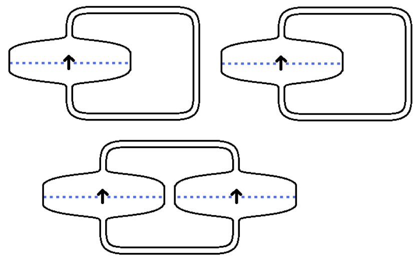

Figure 1: Two non-trivial mapping tori that are reassembled into a trivial mapping torus.

$N \subset M$ a closed submanifold of codimension 1 with trivial normal bundle. If one cuts M open along N one obtains a manifold M' with boundary $\partial M' = N + N$. Pasting the boundary together in a different manner gives a new closed manifold $\tilde{M}$. $\tilde{M}$ is said to have been obtained by cutting and pasting M (Schneiden und Kleben in German or SK for short).

We shall assume that a topological invariant $t \in \mathbb{R}$ for n-dimensional manifolds is compatible with disjoint unions, that is if $M = M_1 + M_2$ then $t(M) = T(M_1) + t(M_2)$. Such t is called an SK-invariant if whenever M_1 and M_2 are compact n-manifolds with diffeomorphic boundaries and $\phi, \psi : \partial M_1 \to \partial M_2$ orientation preserving diffeomorphisms, then

$$t(M_1 \cup_\phi -M_2) = t(M_1 \cup_\psi -M_2).$$

Here $-M_2$ means M_2 with reversed orientation and $M_1 \cup_\phi -M_2$ means M_1 pasted to M_2 along the boundary by ϕ and smoothed.[1]

Corollary 1.4 of [1] now states that any SK-invariant for smooth manifolds is a linear combination of the Euler characteristic and the signature in the oriented case.

By the generalisation of the constructions above and in [2] the invariants found by integration are SK-invariants and thus linear combinations of the Euler characteristic and the signature.

Acknowledgements. In addition to our previous acknowledgements, the authors express their profound gratitude to Shmuel Weinberger for discussion and pointing out [1].

[1] U. Karras, M. Kreck, W.D. Neumann and E. Ossa. *Cutting and Pasting of Manifolds; SK-Groups.* Number 1 in Methematics Lecture Series. Publish or Perish, Boston, 1973.

[2] M.H.M.J. Wintraecken and G. Vegter. *A geometrical take on invariants of low-dimensional manifolds found by integration.*

[1] There is an analogous definition in the non-oriented case.