Analysis and Design of Message Ferry Routes in Sensor Networks using Polling Models

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***Abstract*—We consider a Ferry based Wireless Local Area Network (FWLAN), in which information is forwarded from a base station to sensors, or gathered from sensors to a base station using a moving Ferry. The sensors are scattered in a large area and do not have direct radio connectivity with the base station. The ferry thus serves as a relay that enables communication between the sensors and the base station. Our goal in this paper is to design optimal routes of the Ferry moving along which it distributes/collects the messages. Our analysis and optimization results build heavily on the theory of polling systems which we extend here in order to handle the case of continuous location of the demand. We derive optimal trajectories for various scenarios: uplink, downlink and their combination. We extend some of these results to the case of several base stations and several ferries.**

I. Introduction

Message Ferry is a mobile relay station that serves as “postman“ to deliver to static or dynamic wireless nodes messages (or packets) and to collect messages from them. Mobile BS have been proposed in the context of mobile Ad Hoc Networks [18], of Vehicular Ad-Hoc Networks (Vanets) [15] and of wireless (static) sensor networks [16]. In the UmassDiesel project, computers have been installed in 30 out of 40 buses and these then serve as Message Ferry to deliver messages to throw boxes (see http://prisms.cs.umass.edu/diesel/).

We are concerned with a message ferry that serves as a mobile access point in a local area network which we call FWLAN (Ferry Wireless LAN); the ferry delivers and collects messages from quasi-static nodes on some geographic area Δ. By quasi-static we mean that the sensors could be mobile but on a time scale much slower than that of the cyclic rounds of the ferry. We are interested in designing optimal ferry routes. If the ferry has to reach physically each sensor that has information to transmit or to receive, then the problem can be mapped to the classical vehicle routing problem, where a central point receives service requests from points on the plane. One or several service vehicles are then sent for handling the requests and one has to design efficient or optimal vehicle's route that pass through all the points, see [4], [14], [13]. There is however a fundamental difference between these problems. In the vehicle routing problem, the routes need to pass **through** all the points in space that require service. In contrast, the routes of the message ferry need only to pass in the transmission and reception range of the nodes. Furthermore, assuming a fixed transmission power, the range can be increased at the cost of decreasing throughput. There is another major difference: during uplink service, the ferry will not even be aware of the intended users before the start of new cycle, it is aware of an active user only when it reaches a point in the transmission range of the user.

As a first step we considered a cellular type architecture in [9]: the ferry can accept or dispatch messages only when it arrives to one of a finite number of predetermined stops, each stop is assigned with an area (it can viewed as cell corresponding to the stop) of Δ. This architecture was analyzed using theory of polling systems with finite number of queues ([3], [5], [17]). Performance measures like average virtual workload in the system under stationarity, stability factor were obtained and optimal routes were derived.

In this paper we drop the cellular structure of [9] and allow the shuttle to stop at any point on its route. Indeed, we assume that the shuttle stops whenever it finds itself at a point on its path that is closest to an user that has some packet to send or to receive. This architecture might be advantageous, as it allows the ferry to communicate with the all users at the smallest distance possible, for a given ferry route.

The new system can only be analyzed using a polling system with a continuum of queues. For such polling systems, much is known about the stability [3], [8], [2], but little is known about the performance measures, e.g., average virtual workload in the system under stationarity. The only relevant results available correspond to sym-

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metric exhaustive polling systems which are useful for studying only the case of uplink traffic under symmetric conditions. We derive expressions for the performance of the so called “globally gated“ discipline, whose restriction to finitely many queues was studied in [7], and which allows us to analyze the downlink case. We derive explicit expressions for the expected waiting times and workload without assuming any symmetric structure. In addition, we propose expressions for continuum polling with mixed service disciplines which allow us to handle combined up and downlink traffic. We extend some of these results and of the design of optimal route to the case of several base stations and several shuttles.

In designing the ferry's routes and stops, we aim at minimizing the expected waiting times at all nodes in the Pareto sense[1]. Our theoretical results in some cases, allow us to obtain optimal policies in a stronger sense.

II. System Model and Notations

We consider a geographical area Δ in which static users are scattered. We assume that the network is sparse and there is no direct global connectivity. In order to receive messages from the users or to send messages to them, a ferry called “message ferry“ or “message shuttle“ moves around and serves as a postman. In order to route the data to and from outside the area, the shuttle has to pass through a BS that serves as a gateway. The ferry goes to the BS once in every cycle to collect and deposit the information. The ferry “serves“ the nodes in a cyclic manner. We use throughout terms from queuing theory; by “serves“ a message we mean that the ferry transmits it if the connection is downlink (i.e. the message is destined to a node), or receives it, if it is an uplink message.

Ferry's Route : The ferry moves in a closed cyclic path $\mathcal{C}$ repeatedly with constant speed c_1. Each point q in the cyclic path is assigned with a set of points $I(q) \subset \Delta$ that are closest to it. That is, in most of the situations, set $I(q)$ consists of all points $x \in \Delta$ such that the projection of x to the trajectory $\mathcal{C}$ is q. For example, if we consider Δ to be an annular ring with the Ferry moving along a co-centric circle, then the Ferry will stop at a point on the circle if there is an active node (i.e. a node to or from which there is a message to transmit) located at the same angle. However, there can be situations in which the sets $I(q)$ have to be defined differently and those situations are explained in appropriate locations.

[1]Let W be the set of vectors of expected waiting times achieved by a class of design policies. A vector of $w_1 \in W$ obtained by some dominates another vector w_2 if all entries of w_1 are smaller than or equal to those of w_2 with at least one entry being strictly smaller. A minimum vector in W in the Pareto sense is one that is not dominated by any other vector in W.

The shuttle continues with its journey after serving the user. Every time it stops and restarts it spends an extra time c_2 for acceleration. We map the ferry route to a line for analysis and assume that BS is located at 0. The indexing in this paper is done in a circular manner.

Arrival process: We consider traffic generated at the nodes which we call “uplink“, and traffic that arrives to the nodes called “downlink“. We shall use for both cases the term “arrival“. Uplink/Downlink traffic arrives according to an independent marked point processes $\{T_n, M_n\}$, where T_n is the arrival time of the nth point and $M_n = [X_n, \eta_n]$ are the corresponding i.i.d. marks:

- T_n is a Poisson point process with parameter λ;
- X_n is the location of an arrival with distribution P_X;
- η_n is the size of the message whose distribution can depend upon X_n, it has finite first and second moments $\eta_b(x), \eta_b^{(2)}(x)$ everywhere.

We assume that at any time there can be at maximum one user waiting at a point. This for example is the case when P_X is a continuous distribution.

We use the following notations throughout the paper. The points in the area Δ are represented by x (if it is a sample point) or X (if it is a random position). The points on the cyclic path $\mathcal{C}$ are represented by q or Q.

Radio channel and service time: The Ferry uses a wireless link to serve the users. It can receive/transmit the messages from/to the nodes at a distance of d from it at a rate $\kappa(d)$ for some decreasing function $\kappa(.)$. Further every time the ferry stops, it requires an extra time c_2 for accelerating. Thus the total time required for serving an upload/download message of size η, when the user is located at $x \in \Delta$ and is associated with $q(x) \in \mathcal{C}$ is equal to its size divided by the service rate,

$$\frac{\eta}{\kappa(||q(x) - x||)} + c_2.$$

The point $x \in \Delta$ can for example be associated with the closest point on the circular path $\mathcal{C}$, i.e. in this case

$$q(x) := \arg\min_{q \in \mathcal{C}} ||q - x||.$$

Throughout the paper $||.||$ represents either the area (length) of the two (one) dimensional region and or the distance between two points.

Pareto Optimality: All the users are served as the ferry moves on $\mathcal{C}$. Any arrival at a point $x \in \Delta$ has to wait for time $W(q(x))$ and the expected values of these waiting times $E[W](q)$ in general can depend upon the point $q \in \mathcal{C}$ standing at which the ferry serves them. We are interested in designing an optimal route (among some given class of trajectories) which minimizes integral of the weighted expected waiting times at all the points in the cyclic path $\mathcal{C}$, i.e.

$$\int_0^{|\mathcal{C}|} E[W](q)\zeta(q)dq$$

with $\{\zeta(q)\}$ appropriate positive weights, as that would give a Pareto optimal solution for the problem of minimizing the expected waiting times for all the users.

We need to choose appropriate weights $\{\zeta(q)\}$. If $\zeta(q)$ is chosen to be the arrival density to the points that are handled by q, i.e. $\lambda P_{I(q)}$ then the integral corresponds to the total expected number of active sensors (by Little's Theorem). If we further multiply pointwise the integrand by the expected service times, i.e., if $\zeta(q) = \lambda P_{I(q)} b(q)$, then we obtain the total expected virtual workload in the system[2]. Virtual workload has been obtained in literature for finitely many queues ([7], [5], [17], [3]) and is used in [9]. These expressions are however not available in case of continuous polling systems except for the case of symmetric system with exhaustive service. The exhaustive service can only model the uplink service (more details in coming sections) and hence the existing results are not adequate to analyze this problem for all important cases. We derive the virtual workload expressions for many more interesting cases in Appendix A, so as to obtain a Pareto optimal ferry route for all scenarios.

III. Ferry moving in a annular ring

We study a ferry moving in a annular ring using the theory of continuous polling systems of Appendix A. We discuss design of optimal (which minimizes the virtual workload) ferry paths $\mathcal{C}$ and optimal partitioning of the area into line segments $\{I_q; q \in \mathcal{C}\}$. Solving this problem in complete generality will be a very difficult task. Hence we instead obtain optimal ferry path among a special class of ferry paths. Prior to discussing the optimality issues one first needs to map the FWLAN to a continuous polling. This task is taken up immediately. Later, in the first subsection of this section, we consider optimal paths among paths which are represented by a single circle placed inside the annular area. In the subsection following that, we consider a closed path consisting of N circular paths placed optimally in the area or equivalently we consider N ferries each moving in its own circular path.

The ferry moves inside an annular ring

$$\Delta := \{x \in \mathcal{R}^2 : h_1^2 \leq |x| \leq h_2^2\}$$

[2]**Virtual Workload :** The total service time required by all the waiting users ([6]).

of Figures 1, 2. Ferry moves in a predefined closed cyclic path $\mathcal{C}$ and each point q in the cyclic path is assigned with a line segment $I(q) \subset \Delta$, which is either obtained using the nearest point basis (in subsection III-A) or is computed in some optimal sense (in sub section III-B). Thus the ferry stops at a point q in the cyclic path if there is an active user in line segment $I(q)$ when it touches the point. We begin the study of this FWLAN by mapping it to a polling system of Appendix A.

<u>Server and path of the polling system :</u> The ferry represents the server of the polling system. The ferry stops at a point q in its path only when there is a user with (downlink/uplink) request anywhere on the strip $I(q)$. Thus the entire strip $I(q)$ is modeled as a point on the server's path, in an equivalent continuous polling system.

<u>Service times:</u> The time c_2 for acceleration is required only when the ferry stops and hence is added to the equivalent service time. An arrival (η, X) is associated with the point $q(X) := q$ of the ferry route if X lies in corresponding strip, i.e., if $X \in I(q)$. Thus service time of an arrival (η, X) is given by :

$$B(X, \eta) = B(q(X)) = \frac{\eta}{\kappa(|X - q(X)|)} + c_2.$$

The expected service time (and the corresponding second moment) for the arrivals corresponding to a point $q \in \mathcal{C}$,

$$\begin{aligned}
b(q) &= E[B(X,\eta)|q(X) = q] \\
&= E_X\left[\left.\frac{\eta}{\kappa(|q - X|)}\right| X \in I(q)\right] + c_2. \\
b^{(2)}(q) &= E[B(X,\eta)^2|q(X) = q] \\
&= E_X\left[\left.\frac{\eta^2}{\kappa(|q - X|)^2}\right| X \in I(q)\right] \\
&\qquad -c_2^2 + 2c_2 b(q).
\end{aligned}$$

<u>Service types:</u> At the beginning of every cycle, the ferry collects the downlink information from the BS and distributes it across all the intended users as it traverses through its one cycle. Thus the downlink service corresponds to globally gated service (see Appendix A for details). While it is traveling via its circular path, it also collects the uplink information from all the active uplink nodes as and when it encounters one. Thus the uplink service corresponds to exhaustive service (see details of exhaustive service in Appendix A). Thus the FWLAN supporting both the uplink and downlink services is modeled by a mixed-service continuous polling system, in which the uplink requests represent the exhaustive service users while the downlink requests correspond to the globally gated service users. The respective probabilities

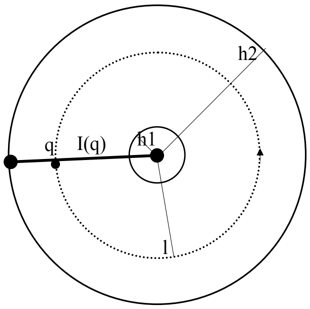

Fig. 1. One Ferry in an annular ring

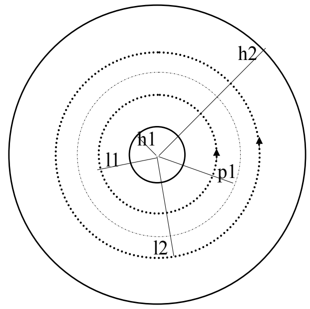

Fig. 2. Two Ferries in an annular ring

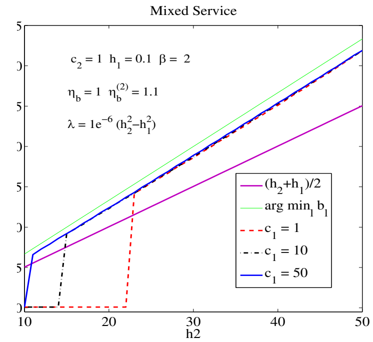

Fig. 3. Optimal orbit radius for a ferry moving continuously in a annular ring

that an arrival is globally gated or exhaustive is given by,

$$\begin{aligned} p_{ggs} &= Prob(\text{Downlink Arrival}) \text{ and} \\ p_{es} &= Prob(\text{Uplink Arrival}). \end{aligned}$$

Position of arrival in the 'polling system' : The position of arrival in FWLAN is given by P_X over Δ. Every arrival in the strip $I(q)$ marks the arrival at point q of $\mathcal{C}$ in the equivalent polling system. The probability distribution, $P_Q(A) := P_X(\cup_{q \in A} I(q))$ for any Borel set $A \subset \mathcal{C}$, represents this arrival distribution. For example, if $\mathcal{C} = \{q : ||q|| = l\}$ for some $h_1 \leq l \leq h_2$, $P_X \sim \mathcal{U}(\Delta)$ is uniform distribution and $I(q) = \{x : \angle x = \angle q\}$ (with $\angle x$ representing the angle made by the line joining 0, x with the x axis), then P_Q will be uniform distribution on $\mathcal{C}$.

The results of Appendix A can now be applied and the virtual workload of the ferry with uplink and or downlink arrivals can be calculated using (5) for any given cyclic path $\mathcal{C}$ and the corresponding line segments $\{I_q\}_{q \in \mathcal{C}}$.

A. *Ferry moving in one circular path*

The ferry moves on one concentric circle of radius l in the annular ring (Figure 1), i.e.,

$$\mathcal{C}_l = \{q \in \mathcal{R}^2 : ||q|| = l\}.$$

In this case, in view of the fact that κ is a decreasing function of the distance d, the best possible thing is to associate every arrival (η, X) with the nearest point of the ferry route, i.e., with the point

$$q(X) := \arg\min_{q \in \mathcal{C}_l} |q - X|$$

of ferry route. Hence the line segments will be angular segments (see figure 1),

$$I(q) = \{x \in \Delta : \angle(x) = \angle(q)\} \text{ for all } q \in \mathcal{C}_l.$$

We would now like to find the optimal radius of this circle and we are interested in optimizing the virtual workload (5)

$$l^* = \arg\min_{l \in [h_1, h_2]} V_{mix}(l).$$

To simplify the computations, in this section we assume:

- uniform arrivals in annular area, $P_X \sim \mathcal{U}(\Delta)$;
- the distribution of η is independent of location of arrival and the type of arrival (uplink or downlink), i.e., $E[\eta|X] \equiv \eta_b$ and $E[\eta^2|X] \equiv \eta_b^{(2)}$ and for both the arrivals;
- the rate function resulting from the losses in wireless medium considers only the direct path for attenuation and is calculated assuming a receive transmit antennae difference of 1, i.e.,
$$\kappa(d) = (1 + d^2)^{-\alpha} \tag{1}$$
where $\beta = 2\alpha$ is the path loss factor;
- The uplink and downlink arrivals occur with equal probabilities $p_{ggs} = p_{es} = 1/2$.

Since the line segments $\{I(q)\}$ in this case are angular segments for calculating the moments of the equivalent expected service times of a polling system $\{b(q;l)\}_{q \in \mathcal{C}_l}$ one will require the radius $R = ||X||$ of the arrival. Note that the moment $b(q;l)$ represents the average service time of the user belonging to point q of $\mathcal{C}_l$. Under the assumptions of this section, radius R has density

$$f_R(r) = \frac{2r}{h_2^2 - h_1^2} \tag{2}$$

and further it is easy to see that the the moments of the overall service time will be independent of the position $q \in \mathcal{C}_l$ but depend upon the ferry path radius l. The service moments are for all $q \in \mathcal{C}_l$ are:

$$\begin{aligned} b_l &= b(q;l) \\ &= \eta_b \int_{h_1}^{h_2} \left(1+(r-l)^2\right)^\alpha \frac{2rdr}{h_2^2-h_1^2} + c_2, \\ b_l^{(2)} &= b^{(2)}(q;l) \\ &= \eta_b^{(2)} \int_{h_1}^{h_2} \left(1+(r-l)^2\right)^{2\alpha} \frac{2rdr}{h_2^2-h_1^2} - c_2^2 + 2c_2 b_l. \end{aligned}$$

For example for $\alpha = 1$ the above can be computed as,

$$\begin{aligned} b_l &= \eta_b \left(\frac{1}{2}(h_1^2+h_2^2+2) + l^2 - \frac{4l(h_1^2+h_1h_2+h_2^2)}{3(h_1+h_2)} \right) \\ &\quad + c_2 \\ b_l^{(2)} &= \frac{\eta_b^{(2)}}{15(h_2^2-h_1^2)} \Bigg(-5h_1^6 + 24h_1^5 l - 15h_1^4(3l^2+1) \\ &\quad + 40h_1^3(l^3+l) - 15h_1^2(l^2+1)^2 h_2^2 \Big(5h_2^4 - 24h_2^3 l \\ &\quad + 15h_2^2(3l^2+1) - 40h_2(l^3+l) + 15(l^2+1)^2 \Big) \Bigg) \\ &\quad - c_2^2 + 2c_2 b_l. \end{aligned}$$

Under the simplified assumptions of this section the workload for mixed service (5) simplifies to:

$$V_{mix,l} = \frac{\lambda b_l \lambda b_l^{(2)}}{2(1-\lambda b_l)} + \frac{3\lambda b_l |\mathcal{C}_l| c_1^{-1}}{4(1-\lambda b_l)}. \quad (3)$$

From (3), it is easily seen that $V_{mix,l}$ depends upon l only via $|\mathcal{C}_l|$ and the moments $b_l, b_l^{(2)}$ and hence one can easily obtain some asymptotic characteristics of l^* like :

- if the propagation coefficient $\beta = 2\alpha$ tends to zero l^* tends to h^1, i.e., the optimal path for the ferry is the inner circle (as then the moments $b_l, b_l^{(2)}$ depend lesser and lesser on l).
- as the speed of the ferry c_1 tends to zero l^* again tends to h^1 (the second term in (3) becomes more and more significant).
- as the speed of the ferry increases to infinity, optimal radius will be determined only by the moments of the service time. There are more arrivals in the upper half annular ring than the lower annular ring (see equation (2)) and hence the service times are improved on average if the ferry route is in the upper annular ring. Thus the optimal radius will be larger than $(h_1+h_2)/2$.

Mixed service numerical example : In figure 3 we consider a FWLAN which supports both uplink and downlink services. The parameters of the FWLAN are mentioned in the figure itself and we plot the optimal radius as a function of the outer radius h_2 of the annular ring. We notice again that the optimal radius tends towards the inner circle either as the speed of the ferry decreases or as h_2 reduces. We also plot the optimizer of the expected service time,

$$l_b^* := \arg \min_{l \in [h_1, h_2]} b_l,$$

in the same figure and note that the optimal radius is close to this optimizer whenever either the annular ring is large or whenever the speed of the ferry is large. This can also be understood by studying the expression for virtual workload (3). Hence in these scenarios one can obtain the optimal radius as (for $\beta = 2$)

$$l^* \approx l_b^* = \frac{2(h_1^2+h_2^2+h_1h_2)}{3(h_1+h_2)}.$$

B. *N servers moving simultaneously in the annular area*

When the annular rings span over larger area, ferry moving in one circular path may not be optimal. Hence we would like to consider a ferry moving in N circular paths before completing the area. To make things simpler we assume that there will be N ferries moving simultaneously (but independent of each other) with the same speed c_1 respectively on circular paths of radii $l_1, l_2, \cdots, l_N$ (see figure 2). Without loss of generality we also assume that the BS is placed on the entire segment $\{x : \angle x = 0\}$, i.e., any of the ferries communicate with the BS instantaneously the moment they touch the angle 0. Throughout this paper we are neglecting the time taken for communication between the BS and the ferry(ies). Let

$$\mathcal{C}_n := \{q : ||q|| = l_n\}$$

and let $B(x,p) := \{y : ||y-x|| < p\}$ represent the open ball while $\bar{B}(x,p)$ represent its closure. The first ferry caters the users in the annular ring $\bar{B}(0,p_1) - B(0,h_1)$ while the last one covers the annular ring $\bar{B}(0,h_2) - B(0,p_{N-1})$. Any intermediate n^{th} ferry covers the area $\bar{B}(0,p_n) - B(0,p_{n-1})$. Let $p_0 := h_1$ and $p_N := h_2$. In this subsection we choose optimally

$$\mathbf{p} := [p_1, p_2, \cdots, p_{N-1}]^T \text{and } \mathbf{l} := [l_1, l_2, \cdots, l_N]^T,$$

which in turn define the line segments

$$I_n(q) = \{x \in \bar{B}(0,p_n) - B(0,p_{n-1}) : \angle q = \angle x\}.$$

We map each ferry to a separate continuous polling system. As before, the moments of the service times for the n^{th} ferry is

$$\begin{aligned} b_{n,\mathbf{p},\mathbf{l}} &= \eta_b \int_{p_{n-1}}^{p_n} \left(1+(r-l)^2\right)^\alpha \frac{2rdr}{p_n^2-p_{n-1}^2} + c_2 \\ b_{n,\mathbf{p},\mathbf{l}}^{(2)} &= \eta_b^{(2)} \int_{p_{n-1}}^{p_n} \left(1+(r-l)^2\right)^{2\alpha} \frac{2rdr}{p_n^2-p_{n-1}^2} \\ &\quad - c_2^2 + 2c_2 b_{n,\mathbf{p},\mathbf{l}}. \end{aligned}$$

Using these moments one can estimate the N virtual workloads $\{V_{mix,n}(\mathbf{p},\mathbf{l})\}$ as a function of the vectors $\mathbf{p}$, $\mathbf{l}$. We now propose to chose optimal ferry paths by :

$$(\mathbf{p}^*,\mathbf{l}^*) = \arg\min_{\mathbf{p},\mathbf{l}} \sum_{n=1}^{N} \frac{p_n^2 - p_{n-1}^2}{h_2^2 - h_1^2} V_{mix,n}(\mathbf{p},\mathbf{l}). \qquad (4)$$

In Table I we consider an example with 2 servers moving in the annular area and supporting uplink service. That is, this system does not support downlink service and this example is constructed in this way to study various possible scenarios. We obtain the optimal $\mathbf{l}^*, p^*$ for different values of the outer radius h_2 and the common speed of the ferries c_1 while the rest of the parameters are kept constant. We see again that the optimal radii of the ferry routes move towards the inner circle as the speed of the vehicle reduces and this effect is pronounced as the the radius of the outer circle decreases. This effect is also seen in the case of single server and can be explained using the virtual workload (5) of Appendix A:

- V depends upon the radius l via the moments of the effective service times $b^{(1)}$, $b^{(2)}$ and the circumference $|\mathcal{C}|$. Virtual workload V_{mix} (5) is directly proportional to all three factors.
- The influence of l on the two moments depends upon the path loss factor β, if β is close to zero the moments are almost independent of l while the optimal l will be above the center of the annular ring for practical path loss factors (as there will be more arrivals in upper ring than the lower one). Thus, for practical values of path loss factors the two moments reduce as l increases from the inner radius.
- However the circumference $|\mathcal{C}|$ has a contrast effect, it increases linearly with l.
- For large values of speed c_1, the influence of $|\mathcal{C}|$ on V_{mix} reduces and hence the optimal radii will be above the center of the annular rings.
- For smaller values of speed c_1, we see the reverse effect.
- The area of the annular ring also influences, as with larger areas the average service moments will be larger and hence the influence of the moments on V_{mix} will be larger than that of the circumference $|\mathcal{C}|$.

IV. FWLAN WITH TWO BASE STATIONS

In this section we consider a slightly more sophisticated FWLAN which is equipped with two base stations. These base stations have also to be placed in optimal locations along with the design of optimal ferry routes. In this section, we consider downlink service alone and the extension to the uplink will be attempted in the future. Before we proceed with FWLAN examples with two base stations, we note here that many of the two base station examples can result in a special case of service called elevator polling (see [1] which describes the scheme for discrete polling system and Appendix A which briefly describes an elevator service based continuous polling system). This polling system will have 'fair' behavior : the expected waiting time of an user in general depends upon the point standing at which the user will be served, however in elevator type services it is equal for all the users (this is true even if the arrivals are not uniform). This 'fair' behavior is a welcome feature and it might be advantageous to consider the paths and the BS locations that can result in this behavior. In Appendix A we derive the expressions for the expected waiting time for a continuous polling system with elevator gating which are applied to some interesting FWLAN examples with two base stations.

A. *Ferry moving in a straight line in Rectangular area*

We consider a rectangular area $\Delta = [-l, l] \times [-d, d]$. A ferry moves in the horizontal straight line as shown in Figure 4. There are two base stations, each one of them is located at the end of the ferry path. The ferry say starts its cycle by collecting all the downlink data from the BS in the left moves towards right with speed c_1, distributes the data to all the tagged users. Once reaching the BS in the right it collects all the data from the BS and then moves towards left. This procedure repeats. Both the base stations have access to all the data. We analyze this FWLAN using the elevator polling system.

Let the base stations be placed respectively at $(p_1, 0)$, $(p_2, 0)$. Let $\mathbf{p} := (p_1, p_2)$. In this case the ferry path is given by $\mathcal{C}_{\mathbf{p}} = \{(q, 0) : p_1 \le q \le p_2\}$. The partitions are calculated based on the nearest distance criterion.

$$I(q) = I(q,0) = \begin{cases} \{q\} & \times[-d,d] \quad p_1 < q < p_2 \\ [-l, p_1] & \times[-d,d] \quad q = p_1 \\ [p_2, l] & \times[-d,d] \quad q = p_2 \end{cases}$$

Note in this case that, both the ends of the ferry path cover not just a line segment but a rectangular strip. We make simplifying assumptions as in section III. Under these assumptions

$$\begin{aligned} b(q;\mathbf{p}) &= \\ c_2 + &\begin{cases} \eta_b \int_{-d}^{d} (1+y^2)^{\alpha} \frac{dy}{2d} & p_1 < q < p_2 \\ \eta_b \int_{-l}^{p_1} \int_{-d}^{d} (1+y^2+(p_1-x)^2)^{\alpha} \frac{dxdy}{4ld} & q = p_1 \\ \eta_b \int_{p_2}^{l} \int_{-d}^{d} (1+y^2+(p_2-x)^2)^{\alpha} \frac{dxdy}{4ld} & q = p_2, \end{cases} \\ b^{(2)}(q;\mathbf{p}) &= 2b(q;\mathbf{p})c_2 - c_2^2 \\ + &\begin{cases} \eta_b^{(2)} \int_{-d}^{d} (1+y^2)^{2\alpha} \frac{dy}{2d} & p_1 < q < p_2 \\ \eta_b^{(2)} \int_{-l}^{p_1} \int_{-d}^{d} (1+y^2+(p_1-x)^2)^{2\alpha} \frac{dxdy}{4ld} & q = p_1 \\ \eta_b^{(2)} \int_{p_2}^{l} \int_{-d}^{d} (1+y^2+(p_2-x)^2)^{2\alpha} \frac{dxdy}{4ld} & q = p_2. \end{cases} \end{aligned}$$

h_2	$\beta = 2\alpha = 2$, $c_1 = 100$ (l_1^*, p^*, l_2^*)	$\beta = 2\alpha = 2$, $c_1 = 10$ (l_1^*, p^*, l_2^*)	$\beta = 2\alpha = 2$, $c_1 = 1$ (l_1^*, p^*, l_2^*)
20	16.29, 17.56, 18.78	16.17, 17.59, 18.67	16.02, 17.61, 18.55
25	17.71, 20.21, 22.66	17.62, 20.30, 22.59	17.49, 20.39, 22.51
30	19.20, 22.90, 26.58	19.14, 23.06, 26.54	19.05, 23.27, 26.52
35	20.74, 25.61, 30.52	20.73 25.85, 30.51	20.68, 26.22, 30.56

TABLE I
EXAMPLE : 2 SERVERS MOVING IN ANNULAR AREA WITH $h_1 = 15$, $c_2 = 10$, $\eta_b = 1$, $\eta_b^{(2)} = 1.1$, $\lambda = 1e^{-7}$

Note here that even under symmetric assumptions the service moments are not equal at all points, thus resulting in an asymmetrical polling system. With the above the overall service moments are given by,

$$\begin{aligned} b_{\mathbf{p}} &= 2d(l - p_2)b(p_1; \mathbf{p}) + 2d(p_1 + l)b(p_2; \mathbf{p}) \\ &\quad + 2d(p_2 - p_1)b(p_1+; \mathbf{p}) \\ b_{\mathbf{p}}^{(2)} &= 2d(l - p_2)b^{(2)}(p_1) + 2d(p_1 + l)b^{(2)}(p_2) \\ &\quad + 2d(p_2 - p_1)b^{(2)}(p_1+; \mathbf{p}). \end{aligned}$$

In the above by $b(p_1+; \mathbf{p})$ we meant the expected value at any point with its x component just greater than p_1.

Thus the service moments can be different at different points in the ferry path. However, because of the elevator service, the expected weighting times are equal at all the points (see Appendix A). Thus, it will be Pareto optimal to optimize the common expected weighting time itself which from (6) of Appendix A equals

$$\frac{\lambda b_{\mathbf{p}}^{(2)}}{2(1 - \lambda b_{\mathbf{p}})} + (p_2 - p_1)c_1^{-1}.$$

Further in this symmetric case it is easy to see that $p_1^* = -p_2^*$. Thus,

$$p_2^* = \arg \min_{0 < p_2 \le d} \frac{\lambda b_{-p_2, p_2}^{(2)}}{2(1 - \lambda b_{-p_2, p_2})} + 2p_2 c_1^{-1}.$$

Note here that optimal choice of p_2^* optimally places the two base stations as well as optimally designs the ferry path. In this example, it is easy to see that : (1) As the speed of the ferry c_1 increases, the p_2^* moves towards l, i.e., if the ferry can move at large speeds, it is optimal to place the base stations at both the edges of the area and allow the ferry to traverse the entire width of Δ. This conclusion is also true whenever the path loss factor β is large. (2) If the ferry moves at moderate speeds and or the path loss factor is small p_2^* reduces. Thus for small path loss factors, the ferry can move in shorter line segments and cover the entire area optimally.

We simplified the problem greatly by considering that the ferry moves in a horizontal line placed at the center of the area. This simplification is good as long as the arrivals are uniform and the file sizes are distributed the same way at all points. For asymmetrical arrivals this choice of ferry route may not be good. In this case it might be better to consider a general straight line passing through points (p_1, p_2) and (p_3, p_4) as ferry path. The base stations are placed once again at both ends of the ferry path. One can estimate the expected waiting time in terms of the parameter $\mathbf{p} := (p_1, p_2, p_3, p_4)$ in a similar way and choose $\mathbf{p}^*$ optimally. This $\mathbf{p}^*$ places the base stations optimally and designs the ferry routes optimally in more generality. If the breadth of Δ, $2d$, is large it might be better to consider zig zag ferry paths (Figure 5) with two base stations.

B. *FWLAN in Annular area*

In figure 6 we consider annular area with 2 base stations placed inside the area. Two ferries move inside this area in a circular path of radius l, one of them covers the upper arc (spanning over angle θ as in Figure 6) joining the two base stations while the second one covers the lower arc. As in the previous section, each of the ferries move from one base station to the other and hence change their directions in alternate cycles. This system can again be modeled by 2 independent elevator polling systems. The optimal base station placement and ferry route design can be achieved by optimal computing (θ, l) so as to minimize the expected waiting time.

V. CONCLUSIONS

We obtained analytical expressions for the expected virtual workload in a Ferry based Wireless LAN, which allowed us to optimize the Ferry trajectories. Our analysis could handle uplink as well as downlink traffic, it further could handle the combined uplink and downlink traffic. Minimizing the average virtual workload results in a Pareto minimization of the expected waiting times in the system. In order to obtain the expressions for the workload in the system we had to extend the fundamental theory of continuous polling systems. We have further presented some extensions to the case of several base stations and or several ferries. We observe that by using two base stations, one can achieve fairness : the expected waiting times are independent of the positions of arrivals.

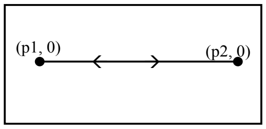

Fig. 4. One Ferry in Rectangular area with 2 BS

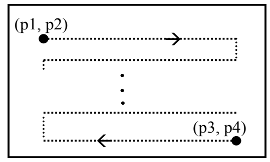

Fig. 5. More general paths for the ferry with two base stations

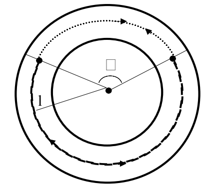

Fig. 6. Two Ferries in Annular area with 2 BS

APPENDIX A

In a polling system with continuum of queues, the server moves continuously and stops at point q only when it finds a user with request. Not much theory is available for calculating the expected virtual workload of a continuous polling system. In [10], [11] we both summarize the relevant existing results as well as *derive new results related to continuous polling systems,* which are useful for analyzing the FWLAN. In this appendix we summarize the same.

The server is moving continuously on a circle $\mathcal{C}$ of radius R with speed c_1. The arrival process is modeled by a Poisson process with intensity λ and every arrival is associated with two marks, the position $Q \in \mathcal{C}$ distributed as P_Q and the service times B. Let $b(q)$ $b^{(2)}(q)$ represent the conditional first two moments of the service time B conditioned on the event that the position of arrival is at point q.

Types of Services : In a *globally gated service*, the server closes a fictitious gate each time it arrives at a fixed point in the circle and tags all the users that arrived before the closure of this gate. In the current cycle, i.e., till the time it retouches the fixed point, it moves along the circular path and serves only the tagged users as and when it encounters one. On the contrary in a *gated/exhaustive service* polling system, the users are served more immediately after their birth: they are served the first time that the server encounters them. A *mixed service* polling system is a system in which the servers attends some of the users with globally gated service (with probability p_{ggs}) and the rest with exhaustive service (probability $p_{es} = 1 - p_{ggs}$).

Virtual Workload : We obtain the virtual workload of this mixed system as a limit of a 'corresponding' discrete polling system (see [10], [11] for details) and it equals.

$$\begin{aligned} V_{mix} &= \\ & \frac{\lambda^2 b b^{(2)}}{2(1-\lambda b)} + \frac{\lambda b |\mathcal{C}| c_1^{-1}}{2} + p_{ggs} \lambda c_1^{-1} E[Q b_{ggs}(Q)] \\ & + \frac{\lambda^2 |\mathcal{C}| c_1^{-1}}{2(1-\lambda b)} \left(b^2 + (p_{ggs} b_{ggs})^2 + p_{ggs} p_{es} b_{ggs} b_{es} \right) \quad (5) \end{aligned}$$

where $|\mathcal{C}| = 2\pi R$,

$$\begin{aligned} b_{ggs} &:= E_Q \left[b(Q) 1_{\{\text{Q globally gated service}\}} \right], \\ b_{es} &:= E_Q \left[b(Q) 1_{\{\text{Q Exhaustive service}\}} \right] \text{ and} \\ b &= p_{ggs} b_{ggs} + p_{es} b_{es} \end{aligned}$$

respectively represent the globally gated, the exhaustive and the overall average service times and $b^{(2)}$ is the second moment of the overall service time.

Elevator Service : A polling system with *elevator service* is similar to globally gated one except that the server moves in the opposite direction every time it reaches the point at which the global gate closes. The server moves in counter clockwise direction in alternate, say odd numbered cycles and in the reverse direction in the even numbered cycles. In [10], [11] it is shown for elevator service that the expected waiting time at any point $q \in \mathcal{C}$ is independent of its position q. The common expected waiting time and the virtual workload for this system (obtained in [10]) are derived to be,

$$E[W](q) = \frac{\lambda b^{(2)}}{2(1-\lambda b)} + |\mathcal{C}| c_1^{-1} \quad \text{and} \qquad (6)$$

$$V_{elevator} = \lambda b \left(\frac{\lambda b^{(2)}}{2(1-\lambda b)} + |\mathcal{C}| c_1^{-1} \right). \qquad (7)$$

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