Estimation of uncertain models of activated sludge process with interval observers

M.Z. Hadj-Sadok
INRIA Comore
BP 93, 06902
Sophia Antipolis, France
mzakaria@sophia.inria.fr
Fax: +33 4 92 38 78 58

J.L. Gouzé
INRIA Comore
BP 93, 06902
Sophia Antipolis, France
gouze@sophia.inria.fr
Fax: +33 4 92 38 78 58

Abstract This paper deals with the designs of observers (software sensors) for uncertain models of wastewater treatment. We assume that the model (4 state variables) is such that the growth rate is unknown and other parameters (e.g. the organic concentration in the influent) are uncertain, i.e. we only know their upper and lower bounds. Given the measurements of the dissolved oxygen concentration, we build interval observers, giving dynamic bounds containing the variables to estimate. The width of these bounds are related to the width of the uncertainty bounds of the parameters. In some cases, we show that we can estimate exactly the unknown variables despite the uncertainties on the model.

Key-words uncertain models, interval observers, wastewater treatment process.

1 Introduction

The control of biological wastewater treatment processes has recently aroused much interest. The primary motivation is linked to stricter and stricter constraints fixed by the future legislation regarding removal of polluting wastes below an acceptable level before the release in waterbodies. However, the lack of on-line instruments has limited the application of control theory to these processes. A large amount of additional information can be obtained through available measurements with the help of state observers. These observers ("software sensors") are algorithms designed to reconstruct the unmeasured states from the measured ones under a priori knowledge of the model. During this last decade, many methods for the design of non linear estimators were developed, such as the approach based on output injection [1], [2], the high gain approach [3], [4] and the approach based on conservation law applied to bioprocesses [5].

The efficiency of such observers strongly depends on the accuracy of the models. However, the process dynamics of biological treatments are often poorly understood and the models are therefore uncertain. Moreover, some inputs or parameters could be influenced by metabolic modifications and, therefore, can fluctuate with time [5]. The goal of our work is to study estimation problems taking into account the uncertain nature of process inputs and model parameters.

Generally, and even if the exact values of some variables in biological models are unknown (parameter fluctuations, influence of external factors, ...), we can assess their upper and lower bounds. Including this partial knowledge, we will design a software sensor called an *interval observer* providing with certainty the time-varying bounds of unmeasured variables. The approach presented in this work is based on an *interval analysis*: given an interval of variation of the uncertain variable, the objective is to reconstruct dynamically guaranteed intervals for the missing states.

Unlike the previous studies developed in [6], [7] and [8], the design of the interval observers is not based on the assumption that measurements of substrate are available but rather relies on dissolved oxygen measurements. This choice is motivated by the fact that dissolved oxygen sensors are more developed and thus, more reliable [9]. The purpose is to estimate the intervals of variation of substrate and the biomass concentrations.

The paper is organized as follows: in the next section, we will derive a dynamical model of the activated sludge process. Then, an asymptotic state observer independent of the unknown part of the model (growth rate) will be presented in section 3. Section 4.1 is devoted to asymptotic interval observers in the case where the substrate concentration in the input stream is uncertain. In section 4.2, we add another constraint by assuming that the oxygen mass transfer coefficient is not well known. The yield coefficient and the substrate in the input stream are supposed to be unknown in section 4.3. In each case, we will design an interval observer, giving upper and lower bounds on the missing variables.

2 Mathematical model

The main component of the activated sludge process is an aerobic biological reactor in which the substrate is removed by suspended micro-organisms. This basin is aerated to provide the dissolved oxygen necessary for the growth of bacterial populations. It is also supposed to be continuously stirred, involving an homogeneous distribution of dissolved oxygen and nutrient concentrations within the reactor. The mixing of the liquid medium in the reactor can also be maintained by a mechanical aeration. During their growth, the bacteria tend to agglomerate into flocs, which are called activated sludge. As indicated in FIG. 1, this aerator is closely tied to a sedimentation tank, in which the liquid is clarified. We assume that the total solid part is separated from the liquid by the sedimentation of the flocs in the settler. A portion of the sludge collected in the sedimentation tank is recycled to the aeration basin, while the rest is regularly wasted to other sludge treatment units. The recycling provides a continuous sludge inoculation and extends the mean sludge residence time for the adaptation of microorganisms to the available nutrients.

For the last decades, the increase in basic understanding of mechanisms involved in biological wastewater treatment has resulted in the development of advanced mathematical models [10] [11] [12], [13]. Even if the model of the IAWPRC Task Group [14] seems to be the reference for activated sludge process, its complexity limits the practical application for design and process control purposes. To simplify the understanding of the application of the interval observers, the model presented below will only describe the organic material removal, the basic model is developed in [15]. For our study, we consider the cell growth of one bacterial population on a limiting organic substrate and dissolved oxygen such as the main biological reaction in the aeration tank. The bioreactions in the solid-liquid separator are neglected. We suppose that, due to the gravity, all solid components will settle and concentrate at the bottom of the settling tank whereas the sedimentation of soluble organic matter is not significant. Some more elaborated models of sedimentation tanks are presented in [16] [17] [18].

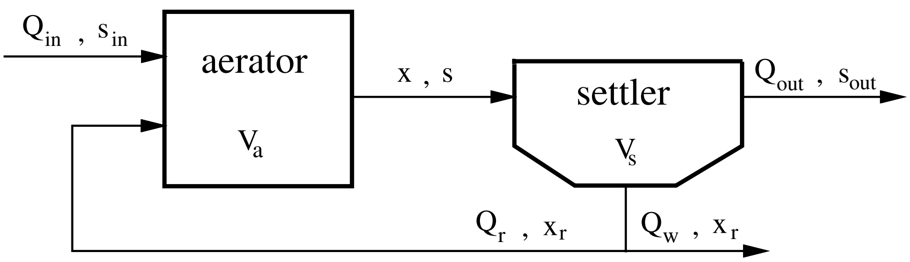

Figure 1: Schematic diagram of the activated sludge process

The dynamical model has to describe three basic phenomena. First, the reaction kinetics in the aerator linked to microbial growth, substrate degradation and oxygen uptake and described by the terms including the growth rate function ($\mu(\cdot)$). Second, the transport dynamics of inflow and outflow in both tanks described by the terms including the dilution rate ($D(t)$). Finally, the gas-liquid transfer of dissolved oxygen described by the term including the oxygen mass transfer coefficient (K_La). We suppose that the tanks volumes are constant. The mass balance of the various constituents gives the following set of equations (the derivative of e.g. y with respect to time is indicated as $\dot{y}$):

$$
\left\{
\begin{array}{lll}
\dot{x} = \mu(\cdot)x - (1+r)D(t)x + rD(t)x_r & ; & x(0) = x_0 \\
\dot{s} = -\dfrac{\mu(\cdot)x}{Y_s} - (1+r)D(t)s + D(t)s_{in}(t) & ; & s(0) = s_0 \\
\dot{c} = -\dfrac{\mu(\cdot)x}{Y_c} - (1+r)D(t)c + D(t)c_{in}(t) + K_La\ (c_s - c) & ; & c(0) = c_0 \\
\dot{x_r} = v(1+r)D(t)x - v(w+r)D(t)x_r & ; & x_r(0) = x_{r0}
\end{array}
\right. \tag{1}
$$

with :

$$
D(t) = \frac{Q_{in}}{V_a} \quad ; \quad r = \frac{Q_r}{Q_{in}} \quad ; \quad w = \frac{Q_w}{Q_{in}} \quad ; \quad v = \frac{V_a}{V_s}
$$

where x, s, c and x_r are the state variables representing the biomass, the substrate, the dissolved oxygen and the recycled biomass concentrations, respectively. Q_{in}, Q_{out}, Q_r and Q_w are the influent, effluent, recycle and waste flow rates, respectively. V_a and V_s represent the aerator and settler volumes. s_{in} and c_{in} correspond to the substrate and the dissolved oxygen concentrations in the feed stream. Y_s and Y_c refer to the yield coefficients of the growth of biomass on substrate and the growth of biomass on dissolved oxygen, respectively. K_La is the oxygen mass transfer coefficient and c_s is the saturation constant. $\mu(\cdot)$ is the specific growth rate.
Generally, the key parameter $\mu(\cdot)$ is a very complex time-varying function of state process and several other factors (pH, temperature, ...) and its analytical modeling is often uncertain. Numerous growth functions are proposed in [5]. However a basic qualitative knowledge is available. A well known biological fact is that the growth capacity of micro-organisms population is intrinsically limited irrespectively of the environmental conditions. So the specific growth rate can be bounded as:

$$
0 \leq \mu(\cdot) \leq \mu_{max} \tag{2}
$$

Moreover, it is obvious that a lack of substrate or dissolved oxygen in the aeration tank prevents the growth of micro-organisms. This means that:

$$
\mu(\cdot) = 0 \quad \text{if} \quad s = 0 \text{ or } c = 0 \tag{3}
$$

In this study, the activated sludge process is considered under the following hypotheses :

- H1 : the biomass concentration x, the substrate concentration s and the recycled biomass concentration x_r are not measured.
- H2 : the dissolved oxygen concentration c is the only measurable state variable of the plant.
- H3 : the coefficients (Y_c, c_s) and the process parameters (r, w, v) are constant and known.
- H4 : the specific growth rate $\mu(\cdot)$ is unknown and verifies (2) and (3).
- H5 : the dissolved oxygen concentration in the feed stream c_{in} is known.
- H6 : the dilution rate $D(t)$ is known. We suppose that there exist positive bounds $\underline{d}$ and $\bar{d}$ such that $\underline{d} \leq D(t) \leq \bar{d} \quad \forall\, t \geq 0$.

Concerning the input s_{in}, the oxygen mass transfer coefficient K_La and the yield coefficient Y_s, we will establish specific hypotheses through the following sections. The objective of this work is to cope with the uncertainty of these variables.

The mathematical analysis of the model equations shows that the state variables are positive and bounded because the inputs are bounded (cf. the BIBS analysis in [5], page 52).

3 Asymptotic observer

Biological wastewater treatment plant control has long been limited by the lack of cheap and reliable online instruments. Recently, more and more researches have been devoted to the application of software sensors in this area for state estimation ([19], [5], [4]) and parameter estimation of the reaction kinetics ([20], [5], [3]). A detailed experimental application of on-line state estimation in bioreactors is described in [21].
Hereafter, we suppose that we have accurate measurements of the dissolved oxygen concentration in the aeration tank and the goal of the study is to estimate the substrate, the biomass and the recycled sludge in different cases.
In this section, we assume that all the process inputs and model parameters are well known. The purpose is to reconstruct the values of the unmeasured variables.
Generally, it is very hard to have a full knowledge of bioprocess kinetics due to the metabolic variations and the influence of many physico-chemical factors. In this part we will asymptotically estimate the missing states under the constraint that the growth rate is unknown, using techniques inspired from the work of Bastin and Dochain [5]. The idea is to eliminate the unknown part of the model which is the growth function $\mu(\cdot)$ by using a linear change of variable. It is interesting to note that the general dynamic model illustrated in [5] assumes that the dilution matrix is diagonal. This is not the case of the model (1), due to the recycled flow. We define the following linear combination of the state variables as it is done in [22]:

$$Z = \begin{bmatrix} x + Y_c c \\ x_r \\ s - Y_c c / Y_s \end{bmatrix} = X + K\ c \tag{4}$$

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad ; \quad X = \begin{bmatrix} x \\ x_r \\ s \end{bmatrix} \quad ; \quad K = \begin{bmatrix} Y_c \\ 0 \\ -Y_c / Y_s \end{bmatrix}$$

The dynamics of these new variables are independent of the growth rate $\mu(\cdot)$:

$$\dot{Z} = D(t) A\ Z + B(c) \quad ; \quad Z_0 = \begin{bmatrix} x_0 + Y_c c_0 \\ x_{r0} \\ s_0 - Y_c c_0 / Y_s \end{bmatrix}$$

$$A = \begin{bmatrix} -(1+r) & r & 0 \\ v(1+r) & -v(w+r) & 0 \\ 0 & 0 & -(1+r) \end{bmatrix}$$

$$B(c) = \begin{bmatrix} Y_c\big(Dc_{in} + K_L a(c_s - c)\big) \\ -Y_c v(1+r) Dc \\ Ds_{in} - \big(Dc_{in} + K_L a(c_s - c)\big) Y_c / Y_s \end{bmatrix}$$

From the measurements of c, we can estimate x, x_r and s by this asymptotic observer:

$$\begin{cases} \dot{\hat{Z}} = D(t) A\ \hat{Z} + B(c) \quad ; \quad \hat{Z}(0) = \hat{X}_0 + K\ c_0 \\ \hat{X} = \hat{Z} - K\ c \end{cases} \tag{5}$$

$\hat{X}_0$ given initial estimates on X.

The dynamics of the errors $E = X - \hat{X}$ are described by the equation:

$$\dot{E} = D(t) A\ E \quad ; \quad E(0) = X_0 - \hat{X}_0$$

To study these dynamics, we can simplify the model equations by rescaling the time by $D(t)$ to obtain a system with the same phase portrait. Indeed, from hypothesis H6, we can choose

$\tau = \int_0^t D(u)du = F(t)$ as the new scale of time. Since $D(t) \geq \underline{d}$, F is invertible ($t = F^{-1}(\tau)$). Then we have:

$$\begin{aligned}
\frac{dE(\tau)}{d\tau} &= \frac{dE(t)}{dt}\frac{dt(\tau)}{d\tau} \\
&= D(t)A\ E(t)\ \frac{1}{D(F^{-1}(\tau))} \\
&= A\ E(F^{-1}(\tau))
\end{aligned}$$

If we denote $E(\tau) = E(F^{-1}(\tau))$ then the dynamic of observation error in the new time basis is described by the following system:

$$\frac{dE(\tau)}{d\tau} = A\ E(\tau) \quad ; \quad E(0) = X_0 - \hat{X}_0$$

A is a Hurwitz matrix with the following negative eigenvalues: $-(1+r)$, $-a - \sqrt{a^2 - vw(1+r)}$ and $-a + \sqrt{a^2 - vw(1+r)}$ where $a = [(1+r) + v(w+r)]/2$. Thus, zero is an asymptotically stable equilibrium. The observer (5) converges at a fixed rate (dependent on $D(t) \geq \underline{d}$).

4 Interval observer

4.1 Unknown substrate influent concentration

In this section, we suppose that the substrate concentration in the feed stream s_{in} is unknown. The oxygen mass transfer coefficient K_La and the yield coefficient Y_s are known.
We assume that only time-varying bounds on s_{in} are known ($s_{in}^-(t) \leq s_{in}(t) \leq s_{in}^+(t) \quad \forall\ t \geq 0$). We also suppose that a priori bounds (even loose) on initial values x_0, x_{r0} and s_0 are known. Generally, it is difficult to have accurate data on the influent concentration at the input of wastewater treatment plant. However, we can have an idea on the variation of this concentration. For example, a typical diurnal load pattern of a medium size treatment plant is given in [23]. Moreover, in [24], Eckenfelder has indicated minimum and maximum values of loading conditions for an oxidation ditch plant (variant of activated sludge process).

The objective of this study is to derive an observer which allows the on-line estimation of upper and lower bounds of the missing states. This estimator has to fulfill two basic conditions. First, the observed interval has to encompass the unmeasured variables even when the uncertainty on the input is large. Second, the observed interval has to converge towards a bounded interval with a reasonable accuracy, even if the initial bounds are loose. To achieve such a goal, the design of these observers is based on an adequate choice of the known bounds of the uncertain variables as estimator inputs and on the specific structure of the matrix A in (5) which defines a cooperative system [25] (cf. Appendix).

As above, we define the linear combination (4) to eliminate the growth kinetics from the dynamics of the new variables. From the measurements of c and the knowledge of K_La, c_{in}, s_{in}^+ and s_{in}^-, we can estimate time-varying bounds of x, x_r and s by the following interval observer, giving an interval of estimation $[\hat{X}^-(t), \hat{X}^+(t)]$ dynamically related to the interval of uncertainty $[s_{in}^-(t), s_{in}^+(t)]$:

$$\left\{\begin{array}{ll}
\dot{\hat{Z}}^+ = D(t)A\ \hat{Z}^+ + B^+(c) \quad ; & \hat{Z}^+(0) = X_0^+ + K\ c_0 \\
\dot{\hat{Z}}^- = D(t)A\ \hat{Z}^- + B^-(c) \quad ; & \hat{Z}^-(0) = X_0^- + K\ c_0 \\
\hat{X}^+ = \hat{Z}^+ - K\ c & \\
\hat{X}^- = \hat{Z}^- - K\ c &
\end{array}\right. \tag{6}$$

$$B^*(c) = \begin{bmatrix} Y_c\big(Dc_{in} + K_L a(c_s - c)\big) \\ -Y_c v(1+r)Dc \\ Ds_{in}^* - \big(Dc_{in} + K_L a(c_s - c)\big)Y_c/Y_s \end{bmatrix}$$

The symbol * denotes + and $-$, respectively.

Moreover, we impose given initial estimates X_0^+ and X_0^- on upper and lower bounds of X respectively. We can choose loose bounds to allow a large margin for the initial interval. Thus, the following inequalities are verified:

$$X_0^- \leq X_0 \leq X_0^+ \tag{7}$$

Let us prove that the observed intervals bound the actual values x, x_r and s. We define the upper error $E^+ = \hat{X}^+ - X$ and the lower error $E^- = X - \hat{X}^-$. The dynamical equations of the observation errors are written as follows:

$$\left| \begin{array}{lcl} \dot{E^+} = D(t)\Big(A\ E^+ + B_e^+(t)\Big) & ; & E^+(0) = X_0^+ - X_0 \\ \dot{E^-} = D(t)\Big(A\ E^- + B_e^-(t)\Big) & ; & E^-(0) = X_0 - X_0^- \end{array} \right.$$

where: $B_e^+(t) = \begin{bmatrix} 0 \\ 0 \\ s_{in}^+(t) - s_{in}(t) \end{bmatrix}$; $B_e^-(t) = \begin{bmatrix} 0 \\ 0 \\ s_{in}(t) - s_{in}^-(t) \end{bmatrix}$

Lemma 1 *If $E^+(0)$ and $E^-(0)$ are nonnegative, then $E^+(t)$ and $E^-(t)$ are nonnegative for all positive time t.*

Proof. Since A is off-diagonal positive, the two systems in E^+ and E^- are cooperative systems [25]. Moreover, the inputs $B_e^+(t)$ and $B_e^-(t)$ are positive for $t \geq 0$ ($s_{in}^-(t) \leq s_{in}(t) \leq s_{in} + (t)$). Let us remark that $E^+(0)$ and $E^-(0)$ are nonnegative, because of the initial estimates (7). Then from property 3 of the appendix, we deduce that the errors $E^+(t)$ and $E^-(t)$ remain nonnegative for all $t > 0$.

Remark. In this linear case, we can also prove this result by checking directly that the vector field is repulsive at the boundaries ($E_i^+ = 0$ or $E_i^- = 0$; $i = 1, 2, 3$).

So we can conclude that if $X_0^- \leq X_0 \leq X_0^+$ then $X^-(t) \leq X(t) \leq X^+(t) \quad \forall\, t \geq 0$.

We now prove that the observed interval is still bounded. Let us study the convergence of the total error $E = E^+ + E^- = X^+ - X^-$ between the upper and the lower bound. We have:

$$\dot{E} = D(t)\Big(A\ E + B_m(t)\Big) \quad ; \quad E(0) = \hat{X}_0^+ - \hat{X}_0^- \tag{8}$$

with $B_m(t) = \begin{bmatrix} 0 \\ 0 \\ s_{in}^+(t) - s_{in}^-(t) \end{bmatrix}$

Due to physical reasons, the input uncertainty is bounded. As indicated at the end of section 3, we can scale the time ($\tau = \int_0^t D(u)du$) and eliminate $D(t)$ so that we obtain a linear time-invariant system forced by $B_m(\tau)$; since the input is bounded and the matrix A is stable, the state variables are bounded [26]. We deduce that the width of the observed interval is bounded since the input uncertainty is bounded.

Let us study the influence of the uncertainty of the process input on the observed intervals. We can state a more precise result in the asymptotic case. We suppose that there exists a positive

constant M such that $\left(s_{in}^+(t) - s_{in}^-(t)\right) \leq M \quad \forall\, t \geq 0$.
Let us consider the new system:

$$\dot{\tilde{E}} = D(t)\Big(A\ \tilde{E} + \tilde{B}_m\Big) \quad ; \quad \tilde{E}(0) = E(0) \tag{9}$$

with $\tilde{B}_m = \begin{bmatrix} 0 \\ 0 \\ M \end{bmatrix}$

D is bounded so that as above, the system (9) can be normalized in time to eliminate $D(t)$. We can deduce that (9) has an asymptotic stable equilibrium (A is stable):

$$\tilde{E}^* = -A^{-1}\tilde{B}_m = \begin{bmatrix} 0 \\ 0 \\ M/(1+r) \end{bmatrix} \tag{10}$$

Let us define $\bar{E} = \tilde{E} - E$ then:

$$\dot{\bar{E}} = D(t)\Big(A\ \bar{E} + \Big(\tilde{B}_m - B_m(t)\Big)\Big) \quad ; \quad \bar{E}(0) = 0$$

Through the same reasoning on cooperative systems with positive inputs, we can prove that $\bar{E}$ is nonnegative since A is off-diagonal positive and $B_m(t) \leq \tilde{B}_m\ \forall\ t \geq 0$. Hence, $E(t) \leq \tilde{E}(t)$ for all positive time t. In particular, the limit set (Cf. [26]) of E is upper bounded by $\tilde{E}^*$ [25]. In other words, when t tends to infinity, $E(t) \leq \tilde{E}^*$. According to (10), this means that after some transients, the interval observer will estimate the two biomass variables without errors resulting from the input uncertainty because the two first components of $\tilde{E}^*$ are zero. Moreover, after a transient period, the width of the observed interval for substrate concentration can vary but is still bounded by a fixed term proportional to the maximum uncertainty of the input. We can state the following proposition:

Proposition 1 *For $s_{in}(t)$ uncertain but belonging to $[s_{in}^-(t), s_{in}^+(t)]$, and for initial bounds $[X_0^-, X_0^+]$ of X_0, the observer gives an interval $[\hat{X}^-(t), \hat{X}^+(t)]$ containing $X(t)$ for all $t \geq 0$. The observation bounds (x^+, x^-) and (x_r^+, x_r^-) will converge asymptotically to the actual values x and x_r respectively, independently of the uncertainty on $s_{in}(t)$. The interval observation of s is bounded, and converges towards an interval depending on the uncertainty on $s_{in}(t)$. The rate of convergence is determined by the operating conditions of the system.*

So, compared to the estimation results given by substrate measurements [6], the information obtained by dissolved oxygen have improved the observer performances. Indeed, the width of the observed intervals for the biomass concentrations has been reduced to zero after some transients. We are able to exactly reconstruct the values of x and x_r, despite the uncertainty on $s_{in}(t)$.

4.2 Unknown substrate influent concentration and mass transfer coefficient

The factor K_La is linked to the aeration process. This parameter is the product of the mass transfer coefficient by the inter-facial area per unit volume. Hence, it is difficult to have accurate estimates of this coefficient. Moreover K_La can be a time-varying parameter. For example, this coefficient may be subject to fluctuations caused by air flow rate as indicated in [27]. In this section, we suppose that the substrate concentration s_{in} and the oxygen mass transfer coefficient K_La in the feed stream are unknown. In this part, K_La can vary with time. We assume that time-varying bounds on s_{in} and K_La are known ($s_{in}^-(t) \leq s_{in}(t) \leq s_{in}^+(t)$ and $K_La^-(t) \leq K_La(t) \leq K_La^+(t) \quad \forall\ t \geq 0$). The yield coefficient Y_s is known.

Based on the same structure as the observer (6), we derive the following interval observer by an adequate choice of the bounds of the uncertain inputs:

$$\left\{\begin{array}{l}\dot{\hat{Z}}^{+}=D(t)A\ \hat{Z}^{+}+B^{+}(c)\quad ;\quad \hat{Z}^{+}(0)=X_0^{+}+K\ c_0\\[2ex] \dot{\hat{Z}}^{-}=D(t)A\ \hat{Z}^{-}+B^{-}(c)\quad ;\quad \hat{Z}^{-}(0)=X_0^{-}+K\ c_0\\[2ex] \hat{X}^{+}=\hat{Z}^{+}-K\ c\\[2ex] \hat{X}^{-}=\hat{Z}^{-}-K\ c\end{array}\right. \tag{11}$$

$$B^{*}(c)=\left[\begin{array}{c}Y_c\big(Dc_{in}+K_La^{*}(c_s-c)\big)\\ -Y_cv(1+r)Dc\\ Ds_{in}^{*}-\big(Dc_{in}+K_La^{\star}(c_s-c)\big)Y_c/Y_s\end{array}\right]$$

The pair of symbols $(*,\star)$ denote $(+,-)$ and $(-,+)$, respectively.

As in previous section, we can prove that if we choose initial bounds sufficiently loose such that $X_0^{-}\leq X_0\leq X_0^{+}$, then we obtain the following inequalities:

$$X^{-}(t)\leq X(t)\leq X^{+}(t)\quad \forall\, t\geq 0$$

We now study the convergence of the total error $E=E^{+}+E^{-}=X^{+}-X^{-}$ between the upper and the lower bounds. We have:

$$\dot{E}=D(t)\Big(A\ E+B_m(t)\Big)\quad ;\quad E(0)=\hat{X}_0^{+}-\hat{X}_0^{-} \tag{12}$$

with $B_m(t)=\left[\begin{array}{c}Y_c\big(K_La^{+}(t)-K_La^{-}(t)\big)(c_s-c)/D(t)\\[2ex] 0\\[2ex] \big(s_{in}^{+}(t)-s_{in}^{-}(t)\big)+\big(K_La^{+}(t)-K_La^{-}(t)\big)(c_s-c)Y_c/\big(Y_sD(t)\big)\end{array}\right]$

By the same reasoning developed in previous section, we can prove that E is bounded [26] and state asymptotic results. There exist positive constants M and N such that $\big(s_{in}^{+}(t)-s_{in}^{-}(t)\big)\leq M$ and $\big(K_La^{+}(t)-K_La^{-}(t)\big)\leq N\quad \forall\, t\geq 0$. The new system:

$$\dot{\tilde{E}}=D(t)\Big(A\ \tilde{E}+\tilde{B}_m\Big)\ ;\ \tilde{E}(0)=E(0)$$

with $\tilde{B}_m=\left[\begin{array}{c}Y_cNc_s/\underline{d}\\ 0\\ M+Nc_sY_c/(Y_s\underline{d})\end{array}\right]$

has an asymptotic stable equilibrium:

$$\tilde{E}^{*}=-A^{-1}\tilde{B}_m=\left[\begin{array}{c}(w+r)Nc_s/(w(1+r)\underline{d})\\ N/(w\underline{d})\\ (Y_sM+Y_cNc_s)/(Y_s(1+r)\underline{d})\end{array}\right]$$

As in previous section, we can show that for large t, $E(t)$ is upper bounded by $\tilde{E}^{*}$ [25] [26]. We can state the following proposition:

Proposition 2 *For $s_{in}(t)$ and $K_La(t)$ uncertain but belonging to $[s_{in}^{-}(t),s_{in}^{+}(t)]$ and $[K_La^{-}(t),K_La^{+}(t)]$ respectively, and for initial bounds $[X_0^{-},X_0^{+}]$ of X_0, the observer gives an interval $[\hat{X}^{-}(t),\hat{X}^{+}(t)]$ containing $X(t)$ for all $t\geq 0$. This interval is bounded and converges towards an interval depending only on the uncertainty on $K_La(t)$ for the bounds estimation of x and x_r and depending on the uncertainty on $s_{in}(t)$ and $K_La(t)$ for the bounds estimation of s. The rate of convergence is determined by the operating conditions of the system.*

4.3 Unknown substrate influent concentration and yield coefficient

The interval observer presented in Section 4.1 converged to the actual values of biomass concentrations even if the substrate concentration in the influent was uncertain. In this section, we assume in addition that one parameter of the model (1) is uncertain. The estimators developed in previous sections assume that the yield coefficient Y_s is well known but in practice, this is not always the case. As often, this parameter cannot be accurately determined [28]. We suppose that the yield coefficient is bounded below by Y_s^- and above by Y_s^+. The coefficients Y_s^- and Y_s^+ are known.

We define the linear combination (4). From the measurements of c and the knowledge of K_La, s_{in}^+, s_{in}^-, Y_s^+ and Y_s^-, an interval observer analogous to (6), is written:

$$\begin{cases} \dot{\hat{Z}}^+ = D(t)A\ \hat{Z}^+ + B^+(c) \quad ; \quad \hat{Z}^+(0) = X_0^+ + K^+c_0 \\ \dot{\hat{Z}}^- = D(t)A\ \hat{Z}^- + B^-(c) \quad ; \quad \hat{Z}^-(0) = X_0^- + K^-c_0 \\ \hat{X}^+ = \hat{Z}^+ - K^-c \\ \hat{X}^- = \hat{Z}^- - K^+c \end{cases} \tag{13}$$

$$B^*(c) = \begin{bmatrix} Y_c\big(Dc_{in} + K_La(c_s - c)\big) \\ -Y_cv(1+r)Dc \\ Ds_{in}^* - Y_c/Y_s^*\big(Dc_{in} + K_La(c_s - c)\big) \end{bmatrix} \quad ; \quad K^* = \begin{bmatrix} Y_c \\ 0 \\ -Y_c/Y_s^* \end{bmatrix}$$

The symbol * denotes + and −, respectively.

The initial bounds X_0^- and X_0^+ are chosen large enough to make sure that they encompass X_0. We notice that the initial bounds verify the following inequalities:

$$\hat{X}^-(0) \leq X_0^- \leq X_0 \leq X_0^+ \leq \hat{X}^+(0) \tag{14}$$

As in section 4.1, through the same reasoning on positive cooperative systems [25], we can prove that the condition (14) implies that $\hat{X}^-(t) \leq X(t) \leq \hat{X}^+(t) \quad \forall\, t \geq 0$.

If we suppose that there exist positive constants $\bar{c}_{in}$ and M such that: $c_{in}(t) \leq \bar{c}_{in}$ and $\big(s_{in}^+(t) - s_{in}^-(t)\big) \leq M\ \forall\, t \geq 0$ then as in previous sections, we deduce that the asymptotic value of the interval width $E(t) = X^+(t) - \hat{X}^-(t)$ is upper bounded by $\tilde{E}^*$, where:

$$\tilde{E}^* = \begin{bmatrix} 0 \\ 0 \\ (M+R)/(1+r) \end{bmatrix}$$

$R = Y_c(Y_s^+ - Y_s^-)\big((1+r)c_s + \bar{c}_{in} + c_sK_La/\underline{d}\big)/(Y_s^-)^2$

We can state the following proposition:

Proposition 3 *For $s_{in}(t)$ and Y_s uncertain but belonging to $[s_{in}^-(t), s_{in}^+(t)]$ and $[Y_s^-, Y_s^+]$ respectively, and for initial bounds $[X_0^-, X_0^+]$ of X_0, the observer gives an interval $[\hat{X}^-(t), \hat{X}^+(t)]$ containing $X(t)$ for all $t \geq 0$. The observation bounds (x^+, x^-) and (x_r^+, x_r^-) will converge asymptotically to the actual values x and x_r respectively, independently of the uncertainty on $s_{in}(t)$ and Y_s. The interval observation of s is bounded, and converges towards an interval depending on the uncertainty on $s_{in}(t)$ and Y_s.*

5 Simulation results

Simulation results are obtained by integrating the model equations (1) with the Monod growth function and typical values of model parameters and initial conditions inspired from literature data

[27] and given in the following:

Growth function :	$\mu(s) = \mu_{max} \dfrac{s}{K_s + s} \dfrac{c}{K_c + c}$
Kinetic parameters :	$\mu_{max} = 0.2\ h^{-1}$; $K_s = 75\ mg/l$; $K_c = 2\ mg/l$
Stoichiometric parameters :	$Y_c = 1.8$; $Y_s = 0.8$
Process parameters :	$v = 2$; $r = 1$; $w = 0.05$; $c_s = 10\ mg/l$
Initial conditions :	$x_0 = 1225$; $x_{r0} = 2333$; $s_0 = 20$; $c_0 = 6\ (mg/l)$

The dilution rate is considered constant $D(t) = 0.3\ h^{-1}$
The evolution of the substrate concentration in the influent is represented by an uncertain function subject to disturbances and fluctuating between two sinusoidal functions where $s_{in}^{+} - s_{in}^{-} = 30\ mg/l = 20\%$ of the mean value of s_{in} (FIG. 2.a).
For simulation of interval observer (11), we assume that the oxygen mass transfer coefficient is bounded between two constants: $K_La^{-} = 1.5 \leq K_La = 1.8\ h^{-1} \leq K_La^{+} = 2.0$.
For simulation of interval observer (13), we choose $Y_s^{-} = 0.7 \leq Y_s = 0.8 \leq Y_s^{+} = 0.9$.

The variations of the measured variable c are represented in FIG. 2.b.

• s_{in} unknown (section 4.1): The trajectories of the bounds estimates of x, x_r and s under the constraint that s_{in} is uncertain, are given in diagrams (FIG. 3.a and FIG. 3.b). FIG. 3.a shows that, even with loose initial estimations on each bound of the biomass concentrations in the aerator and in the settler, the upper and lower estimated bounds converge to the unmeasured variables independently of the uncertainty of the substrate concentration in the feed stream.

• s_{in} and K_La unknown (section 4.2): When we suppose in addition that the oxygen transfer coefficient K_La is uncertain, the estimation results are degradated (FIG. 4.a and FIG. 4.b). In this case, we notice that the observed bounds for biomass concentrations do not shrink to the actual values, but rather tend toward an interval containing these unmeasured concentrations (FIG. 4.a).

• s_{in} and Y_s unknown (section 4.3): As illustrated in FIG. 5.a, in the case where the substrate concentration in the influent and the yield coefficient are uncertain, the observer performances for the biomass estimation are identical to the results obtained in FIG. 3.a. The interval estimator is able to reconstruct the values of x and x_r independently of the uncertainties acting only on the dynamic of s. However, the uncertainty of the yield coefficient degradates the estimation results of the substrate concentration (FIG. 5.b).

6 Conclusion

Interval observers were derived to estimate the upper and lower bounds of the substrate, the biomass and the recycled biomass concentrations from the measurements of dissolved oxygen concentration and under the following constraints: the specific growth rate was unknown and some variables were uncertain. We have focussed on three basic cases: s_{in} uncertain, s_{in} and K_La uncertain and s_{in} and Y_s uncertain.

Through the study of each case, we have demonstrated that even if the initial bounds of the unmeasured variables are loose and given a guaranteed interval on the uncertainty of inputs or parameters, this estimator converges towards bounded intervals which contain with certainty the varying unmeasured states.
This information can be useful for specific applications. For example, some operational tasks of treatment plants as fault detection do not require the need to reconstruct the exact values of the supervised variables, since the estimation of their maximum or minimum values is sufficient.
We have proved that generally, the width of the observed intervals directly depends on the un-

certainty range of the unknown variables. The main result of this work shows that, although the substrate concentration in the influent (s_{in}) and the yield coefficient (Y_s) are uncertain, the software sensor is able to asymptotically reconstruct the actual values of biomass concentrations in the aeration tank and the settler. However, the results developed in [6] and [7] with only substrate measurements have shown that interval estimation of biomass concentrations depends on the uncertainty on s_{in}. This means that information obtained from dissolved oxygen concentration allows to estimate the biomass concentrations independently of the uncertainty on s_{in}.
This result opens interesting perspectives for a deep exploitation of this type of observers. Further studies are undergoing examination and concern the following points: the improvement of the interval estimation by an adequate choice of measurements, the increase in the convergence rate by calibration of gain parameters, the application on more complex models including other biological mechanisms such as nitrification, the study of the influence of the noise measurement on the estimation results, the extension of interval observers theory to more general structure of models.

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Appendix: cooperative systems

In this appendix we present a definition and a property concerned with the theory of cooperative systems (cf. [25]).

Definition 1 *A vector field in n-space determines a cooperative system of differential equations provided that all the off-diagonal terms of its Jacobian matrix are nonnegative on a convex domain.*

First, we present a property of dynamical cooperative systems in the non linear case. Let us consider two vectors x and y described by two non-autonomous systems:

$$\begin{aligned} \dot{x} &= f(x,t) \quad ; \quad x(0) = x_0 \\ \dot{y} &= g(y,t) \quad ; \quad y(0) = y_0 \end{aligned}$$

where $f, g : U \times \mathbb{R}^+ \rightarrow \mathbb{R}^n$ are continuously differentiable on a convex subset $U \subset \mathbb{R}^n$.

Property 1 *if*

- $\forall z \in U,\ \forall t \geq 0,\ f(z,t) \leq g(z,t)$
- *g is cooperative*
- $x_0 \leq y_0$

then $x(t) \leq y(t)$ for $t > 0$ (cf. [29]).

From the property 1, we can deduce another result for particular structures of the vector fields f and g.

If we choose:

- $f(x,t) = A\ x$ (A is a cooperative matrix)
- $g(y,t) = A\ y + B(t)$ ($B(t) \geq 0\ \ \forall\ t \geq 0$)
- $z = y - x$

then the linear non-autonomous system:

$$\dot{z} = A\ z + B(t) \quad ; \quad z(0) = z_0$$

has the following property:

Property 2 *if $z_0 \geq 0$ then $z(t) \geq 0$ for $t > 0$.*

The property 2 can be used to prove the lemma 1.

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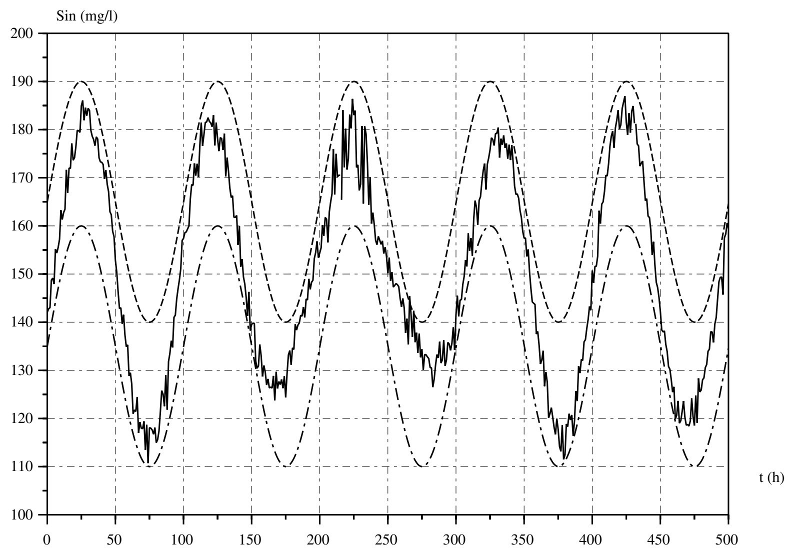
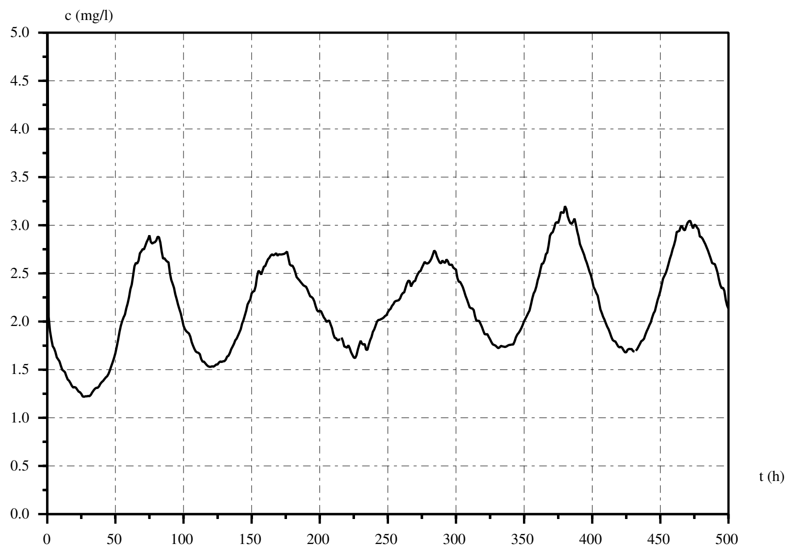
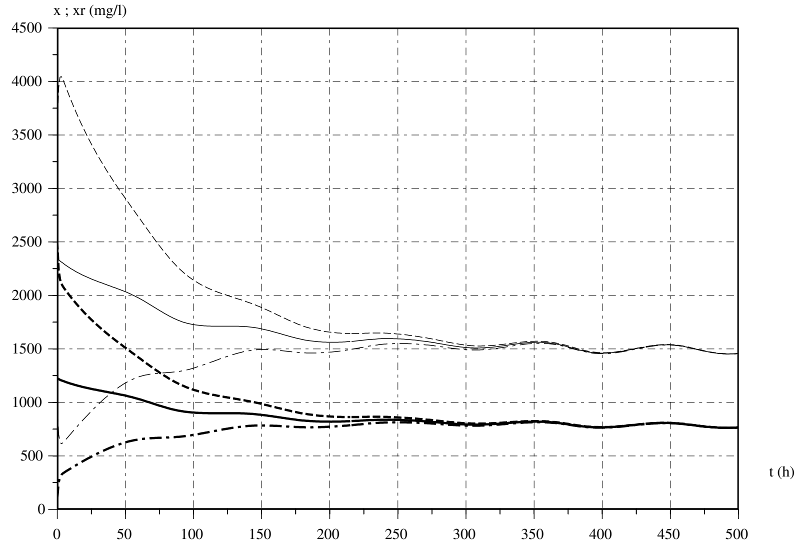


(a) Behavior of s_{in} (solid line), s_{in}^{+} (dashed) and s_{in}^{-} (dot-dashed)

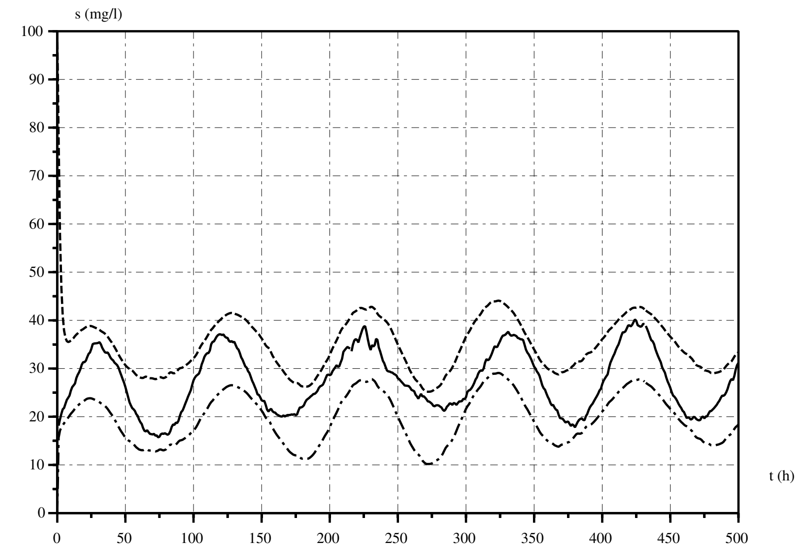


(b) Dissolved oxygen measurements c

Figure 2: Variations of the uncertain input and the measured variable

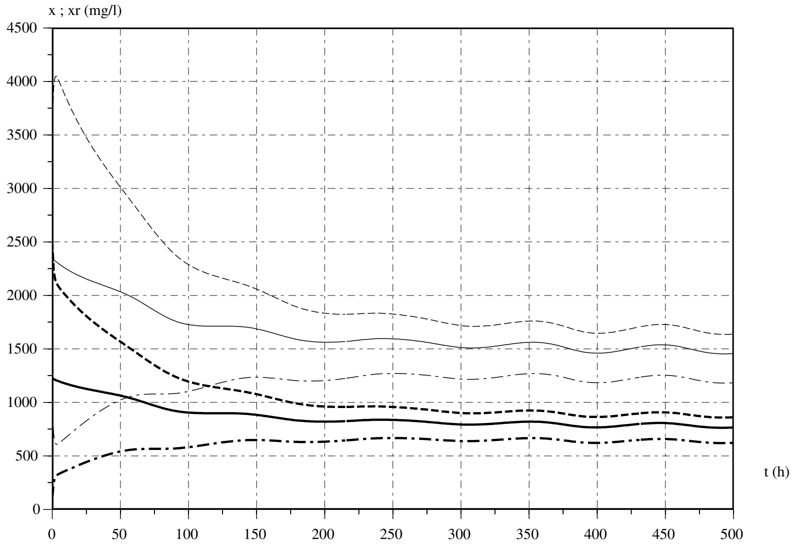


(a) Biomass concentrations: x (dark solid), x^+ (dark dashed), x^- (dark dot-dashed), x_r (light solid), x_r^+ (light dashed) and x_r^- (light dot-dashed)

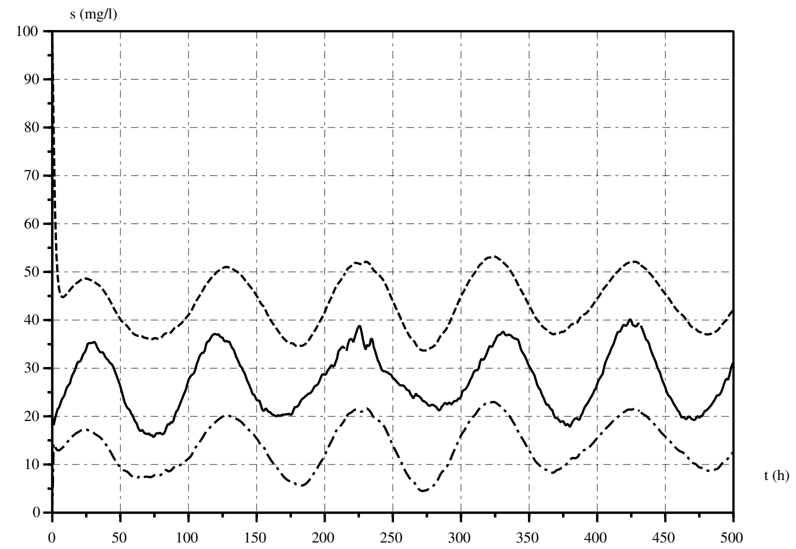


(b) Substrate s (solid), s^+ (dashed) and s^- (dot-dashed)

Figure 3: Bounds estimation with s_{in} uncertain

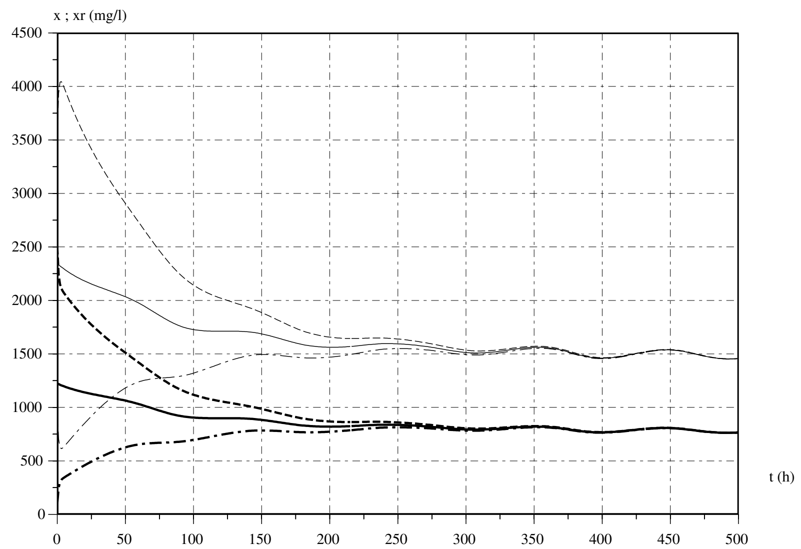


(a) Biomass concentrations: x (dark solid line), x^+ (dark dashed), x^- (dark dot-dashed), x_r (light solid), x_r^+ (light dashed) and x_r^- (light dot-dashed)

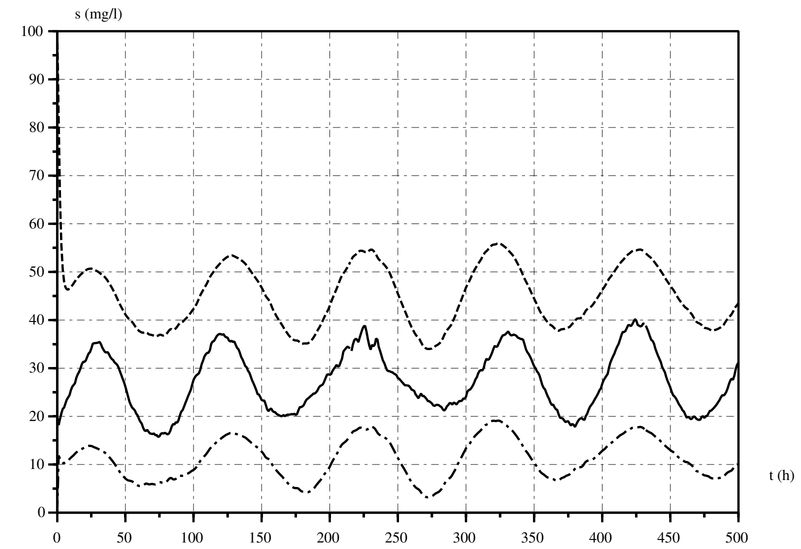


(b) Substrate s (solid), s^+ (dashed) and s^- (dot-dashed)

Figure 4: Bounds estimation with s_{in} and K_La uncertain



(a) Biomass concentrations: x (dark solid), x^+ (dark dashed), x^- (dark dot-dashed), x_r (light solid), x_r^+ (light dashed) and x_r^- (light dot-dashed)



(b) Substrate s (solid line), s^+ (dashed line) and s^- (dot-dashed line)

Figure 5: Bounds estimation with s_{in} and Y_s uncertain