Performance Evaluation – Master UBINET
Assignment 2

Solutions have to be sent by January 19th 2014 to `giovanni.neglia@inria.fr`.

Ex. 1 — Birds

A specie of birds may nidify in two different places. The current population counts $N = 1000$ birds. Let us denote N_i $(i = 1, 2)$ the number of birds at location i. The movement of birds from one location to the other can be modeled as follows. Every month one bird may decide to mate or to move to the other location because of food competition. It mates with probability $p_M = 0.1$: in this case it selects a random partner from the whole population and it moves to the nest of the partner, that may or may not be in the same location. It moves from location i because of food competition with probability $p_F N_i/C_i$, where $p_F = 0.2$, $C_1 = 10000$, $C_2 = 5000$.

1. Show that the system can be modeled as a Discrete Markov Chain.
2. Show that a Mean-Field limit can be correctly derived when N diverges. Write the corresponding system of Ordinary Differential Equations (ODE).
3. Assuming to know the solution of the ODE, how can you approximate the average number of birds at location 1 after 10 months.
4. Determine the equilibrium points of the ODE system. Do these equilibrium points correspond to stationary distributions for the Markov Chain?
5. If you can only simulate the system up to a size $N = 100$, describe which experiment you could carry on to evaluate if the Mean-Field approximation is satisfactory for $N = 1000$.

Student	Graph number
Bernards	1
Carunchio	2
Kaddouri	3
Soni	4
Soroush Haddadi	5
Tetley	6
Toth	7
Varava	8
Zhang	9
Zholtkevych	10

Table 1: Student-trace matching.