Game Theory: introduction and applications to computer networks

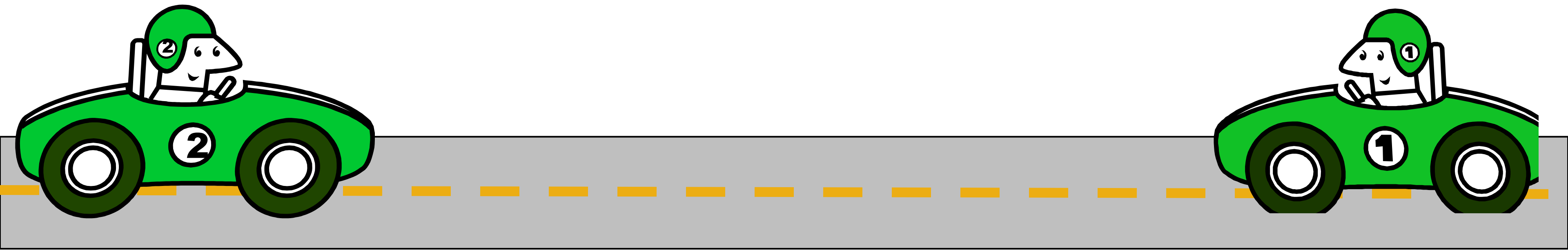
Introduction

Giovanni Neglia
INRIA – EPI Maestro
9 January 2013

Part of the slides are based on a previous course
with D. Figueiredo (UFRJ) and H. Zhang (Suffolk University)

What is Game Theory About?

- Mathematical/Logical analysis of situations of conflict and cooperation



- Game of Chicken
  - driver who steers away looses
  - what should drivers do?
- Goal: to prescribe how rational players should act

What is a Game?

- A Game consists of
  - at least two players
  - a set of strategies for each player
  - a preference relation over possible outcomes
- Player is general entity
  - individual, company, nation, protocol, animal, etc
- Strategies
  - actions which a player chooses to follow
- Outcome
  - determined by mutual choice of strategies
- Preference relation
  - modeled as utility (payoff) over set of outcomes

Short history of GT

- Forerunners:
  - Waldegrave's first minimax mixed strategy solution to a 2-person game (1713), Cournot's duopoly (1838), Zermelo's theorem on chess (1913), Borel's minimax solution for 2-person games with 3 or 5 strategies (20s)
- 1928: von Neumann's theorem on two-person zero-sum games
- 1944: von Neumann and Morgenstern, Theory of Games and Economic Behaviour
- 1950-53: Nash's contributions (Nash equilibrium, bargaining theory)
- 1952-53: Shapley and Gillies' core (basic concept in cooperative GT)
- 60s: Aumann's extends cooperative GT to non-transferable utility games
- 1967-68: Harsanyi's theory of games of incomplete information
- 1972: Maynard Smith's concept of an Evolutionarily Stable Strategy
- Nobel prizes in economics
  - 1994 to Nash, Harsanyi and Selten for "their pioneering analysis of equilibria in the theory of non-cooperative games"
  - 2005 to Aumann and Schelling "for having enhanced our understanding of conflict and cooperation through game-theory analysis"
  - 2012 to Roth and Shapley "for the theory of stable allocations and the practice of market design"
- Movies:
  - 2001 "A beautiful mind" on John Nash's life
- See also:
  - www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm

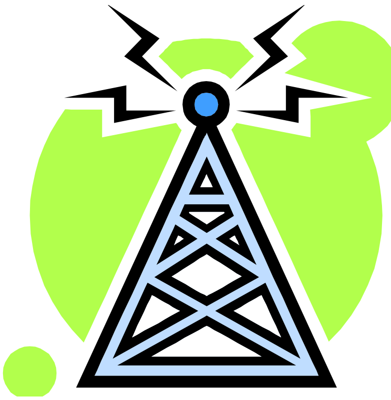
Applications of Game Theory

- Economy
- Politics (vote, coalitions)
- Biology (Darwin's principle, evolutionary GT)
- Anthropology
- War
- Management-labor arbitration
- Philosophy (morality and free will)
- National Football league draft

Applications of Game Theory

- "Recently" applied to computer networks
  - Nagle, RFC 970, 1985
    - "datagram networks as a multi-player game"
  - wider interest starting around 2000
- Which are the strategies available?
  - Network elements follow protocol!!!

Power games



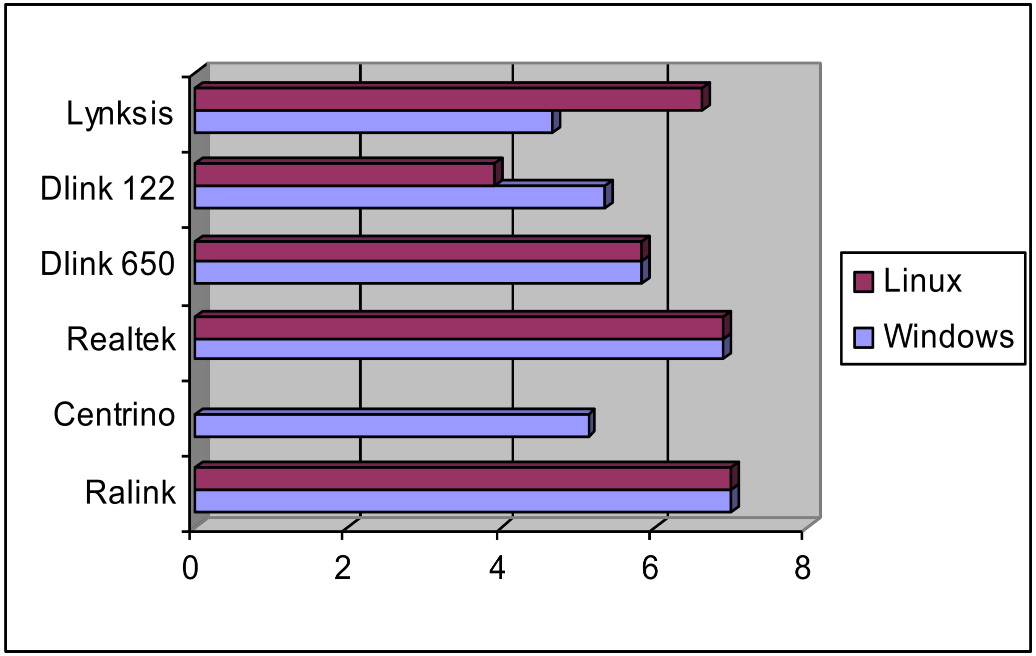
$$SNIR_1 = \frac{H_{1,BS} P_1}{N + H_{2,1} P_2}$$

Medium Access Control Games

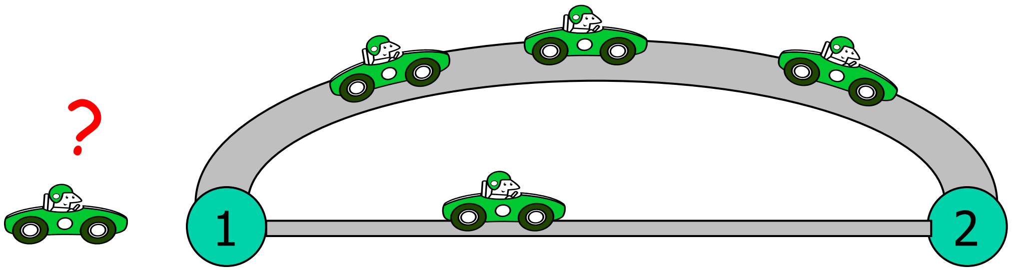
$$Thr_1 = \frac{p_1(1-p_2)P}{(1-p_1)(1-p_2)\sigma + [1-(1-p_1)(1-p_2)]T}$$

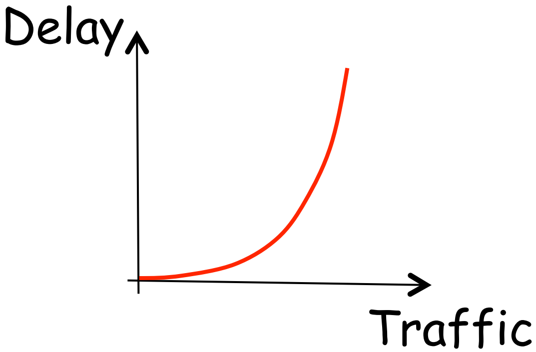
Medium Access Control Games

- Despite of the Wi-Fi certification, several cards exhibit very heterogeneous performance, due to arbitrary protocol implementations
  - "Experimental Assessment of the Backoff Behavior of Commercial IEEE 802.11b Network Cards," G Bianchi et al, INFOCOM 2007



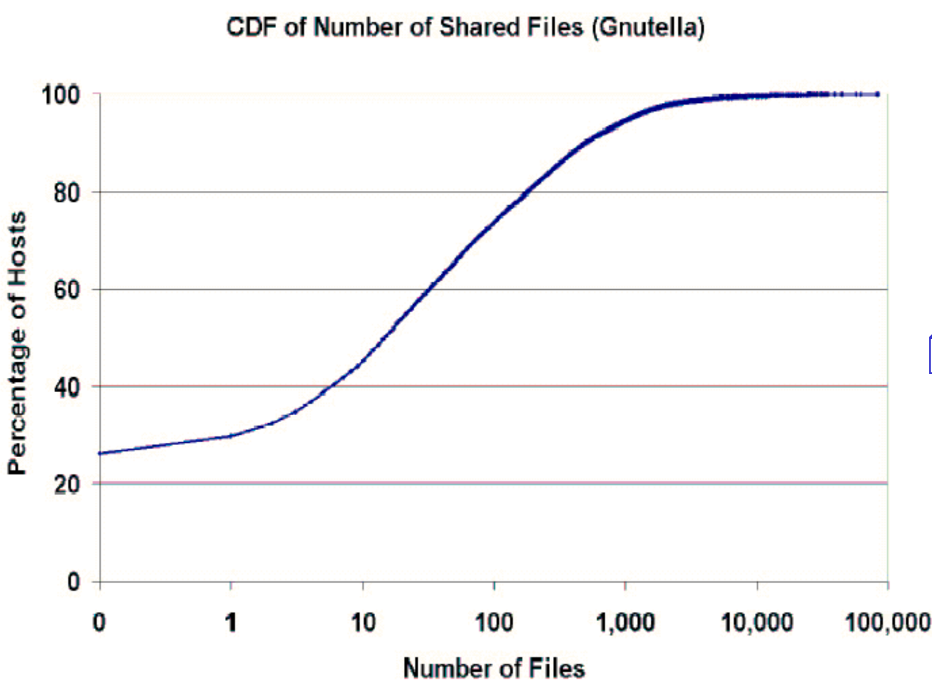
Routing games





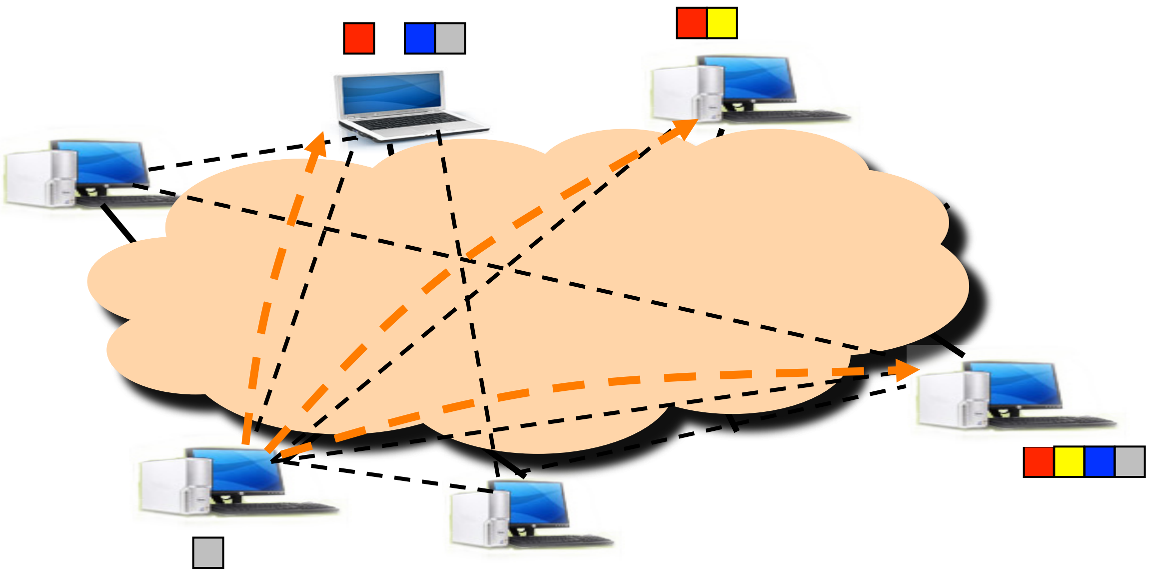
- Possible in the Internet (see later)

Free riders in P2P networks



- Individuals not willing to pay the cost of a public good, they hope that someone else will bear the cost instead
- Few servers become the hot spots: Anonymous?, Copyright?, Privacy? Scalability?, Is it P2P?

Connection games in P2P



- Each peer may open multiple TCP connections to increase its downloading rate

Diffusion of BitTorrent variants

- Try to exploit BitTorrent clients weaknesses

BitThief

- Are they really dangerous?
  - Evolutionary game theory says that

    Yes they can be

Space for GT in Networks

- User behaviors (to share or not to share)
  - Client variants
- Protocols do not specify everything…
  - power level to use
  - number of connections to open
- …and/or are not easy to enforce
  - how control a P2P network
  - not-compliant WiFi implementation
- …and software easy to modify

Limitations of Game Theory

- Real-world conflicts are complex
  - models can at best capture important aspects
- Players are considered rational
  - determine what is best for them given that others are doing the same
  - Men are not, but computers are more
- No unique prescription
  - not clear what players should do

- But it can provide intuitions, suggestions and partial prescriptions
  - the best mathematical tool we have

Syllabus

- References
  - [S] Straffin, Game Theory and Strategy (main one, chapters indicated)
  - [EK] Easley and Kleinberg, Network Crowds and Markets
  - [OR] Osborne and Rubinstein, A course in game theory, MIT Press
- Two-person zero-sum games
  - Matrix games
    - Pure strategy equilibria (dominance and saddle points), [S2]
    - Mixed strategy equilibria, [S3]
  - Game trees (?), [S7]
- Two-person non-zero-sum games
  - Nash equilibria…
    - …And its limits (equivalence, interchangeability, Prisoner's dilemma), [S11-12]
  - Subgame Perfect Nash Equilibria (?)
  - Routing games [EK8]
- Auction theory [EK9]

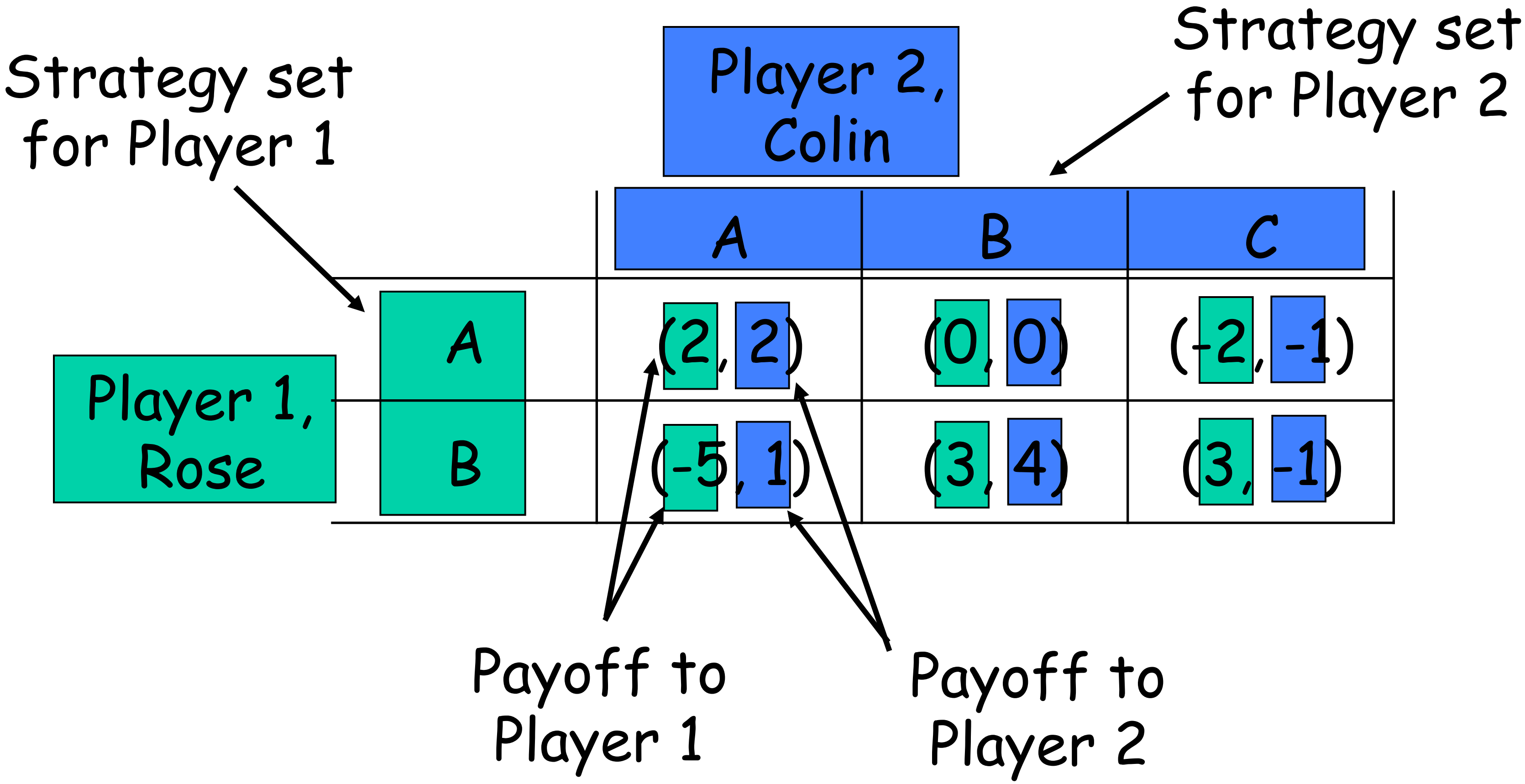
Game Theory: introduction and applications to computer networks

Two-person zero-sum games

Giovanni Neglia
INRIA – EPI Maestro

Slides are based on a previous course
with D. Figueiredo (UFRJ) and H. Zhang (Suffolk University)

Matrix Game (Normal form)



- Simultaneous play
  - players analyze the game and then write their strategy on a piece of paper

More Formal Game Definition

- Normal form (strategic) game
  - a finite set N of players
  - a set strategies S_i for each player $i \in N$
  - payoff function $u_i(s)$ for each player $i \in N$
    - where $s \in S = \times_{j \in N} S_j$ is an outcome
    - sometimes also $u_i(A, B, \ldots)$ $A \in S_1, B \in S_2, \ldots$
    - $u_i : S \rightarrow \Re$

Two-person Zero-sum Games

- One of the first games studied
  - most well understood type of game
- Players interest are strictly opposed
  - what one player gains the other loses
  - game matrix has single entry (gain to player 1)
- A “strong” solution concept

Let's play!

	Colin			
Rose	**A**	**B**	**C**	**D**
A	12	-1	1	0
B	5	1	7	-20
C	3	2	4	3
D	-16	0	0	16

- Divide in pairs, assign roles (Rose/Colin) and play 20 times
- Log how many times you have played each strategy and how much you have won

Analyzing the Game

Colin

Rose		A	B	C	D
	A	12	-1	1	0
	B	5	1	7	-20
	C	3	2	4	3
	D	-16	0	0	16

dominated strategy (dominated by B)

Dominance

- Strategy S *(weakly) dominates* a strategy T if every possible outcome when S is chosen is at least as good as corresponding outcome in T, and one is strictly better
  - S *strictly dominates* T if every possible outcome when S is chosen is strictly better than corresponding outcome in T
- Dominance Principle
  - rational players never choose dominated strategies
- Higher Order Dominance Principle
  - iteratively remove dominated strategies

Higher order dominance may be enough

Colin

Rose

	A	B	C	D
A	12	-1	1	0
B	3	1	4	-18
C	5	2	4	3
D	-16	0	5	-1

Higher order dominance may be enough

GT prescribes:

Rose C - Colin B

Colin

Rose	A	B	C	D
A	12	-1	1	0
B	3	1	4	-18
C	5	2	4	3
D	-16	0	5	-1

(Weakly) Dominated by C

A priori D is **not** dominated by C

Strictly dominated by B

… but not in the first game

		Colin			
		A	B	C	D
Rose	A	12	-1	1	0
	B	5	1	7	-20
	C	3	2	4	3
	D	-16	0	0	16

dominated strategy (dominated by B)

Analyzing the Reduced Game: Movement Diagram

Colin

Rose	A	B	D
A	12	-1	0
B	5	1	20
C	3	$\boxed{2}$	3
D	-16	0	16

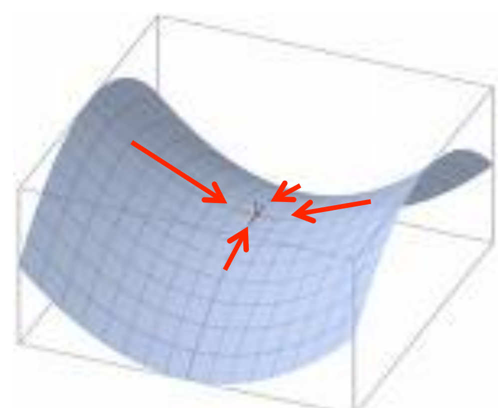
Outcome (C, B) is "stable"

- ***saddle point of game***
- mutual ***best responses***

Saddle Points

- An outcome (x,y) is a *saddle point* if the corresponding entry u(x,y) is both less than or equal to any value in its row and greater than or equal to any value in its column
  - u(x,y) <= u(x,w) for all w in $S_2 = S_{Colin}$
  - u(x,y) >= u(v,y) for all v in $S_1 = S_{Rose}$

	A	B	D
A	12	-1	0
B	5	1	-20
C	3	2	3
D	-16	0	16



Saddle Points Principle

- Players should choose outcomes that are saddle points of the game
  - Because it is an equilibrium…
  - … but not only