Performance Evaluation

Sponsored Search Markets

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4 February 2013

Google

Google

digital photo camera 🔍

Giovanni Neglia | 0 | + Shar

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Best 5 digital cameras - 100 - $200 Digital cameras ... - Digital camera - Than 12X

Digital camera - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Digital_camera**
Jump to **Displaying photos**: Many **digital cameras** include a video output port. Usually sVideo, it sends a standard-definition video signal to a television, ...

Amazon.com: Digital Cameras: Camera & Photo: Point & Sho...
www.amazon.com/**Digital-Cameras-Photo**/b?ie=UTF8

How it works

- Companies bid for keywords
- On the basis of the bids Google puts their link on a given position (first ads get more clicks)
- Companies are charged a given cost for each click (the cost depends on all the bids)

Some numbers

- ≈ 95% of Google revenues (46 billions$) from ads
  - investor.google.com/financial/tables.html
  - 87% of Google-Motorola revenues (50 billions$)
- Costs
  - "calligraphy pens" $1.70
  - "Loan consolidation" $50
  - "mesothelioma" $50 per click
- Click fraud problem

Outline

☐ Preliminaries

  ○ Auctions

  ○ Matching markets

☐ Possible approaches to ads pricing

☐ Google mechanism

☐ References

  ○ Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,10,15

Types of auctions

- 1st price & descending bids
- 2nd price & ascending bids

Game Theoretic Model

□ N players (the bidders)

□ Strategies/actions: b_i is player i's bid

□ For player i the good has value v_i

□ p_i is player i's payment if he gets the good

□ Utility:
  ○ v_i-p_i if player i gets the good
  ○ 0 otherwise

□ Assumption here: values v_i are *independent* and *private*
 ○ i.e. very particular goods for which there is not a reference price

Game Theoretic Model

□ N players (the bidders)
□ Strategies: b_i is player i's bid
□ Utility:
  ○ $v_i - b_i$ if player i gets the good
  ○ 0 otherwise
□ Difficulties:
  ○ Utilities of other players are unknown!
  ○ Better to model the strategy space as continuous
  ○ Most of the approaches we studied do not work!

2nd price auction

☐ Player with the highest bid gets the good and pays a price equal to the 2nd highest bid

☐ There is a dominant strategies

   ○ I.e. a strategy that is more convenient independently from what the other players do

   ○ **Be truthful**, i.e. bid how much you evaluate the good ($b_i = v_i$)

   ○ Social optimality: the bidder who value the good the most gets it!

$b_i=v_i$ is the highest bid

bids

bids

b_i

$b_i'>b_i$

b_k

b_k

$U_i=v_i-b_k>v_i-b_i=0$

$U_i'=v_i-b_k$

b_h

b_h

b_n

b_n
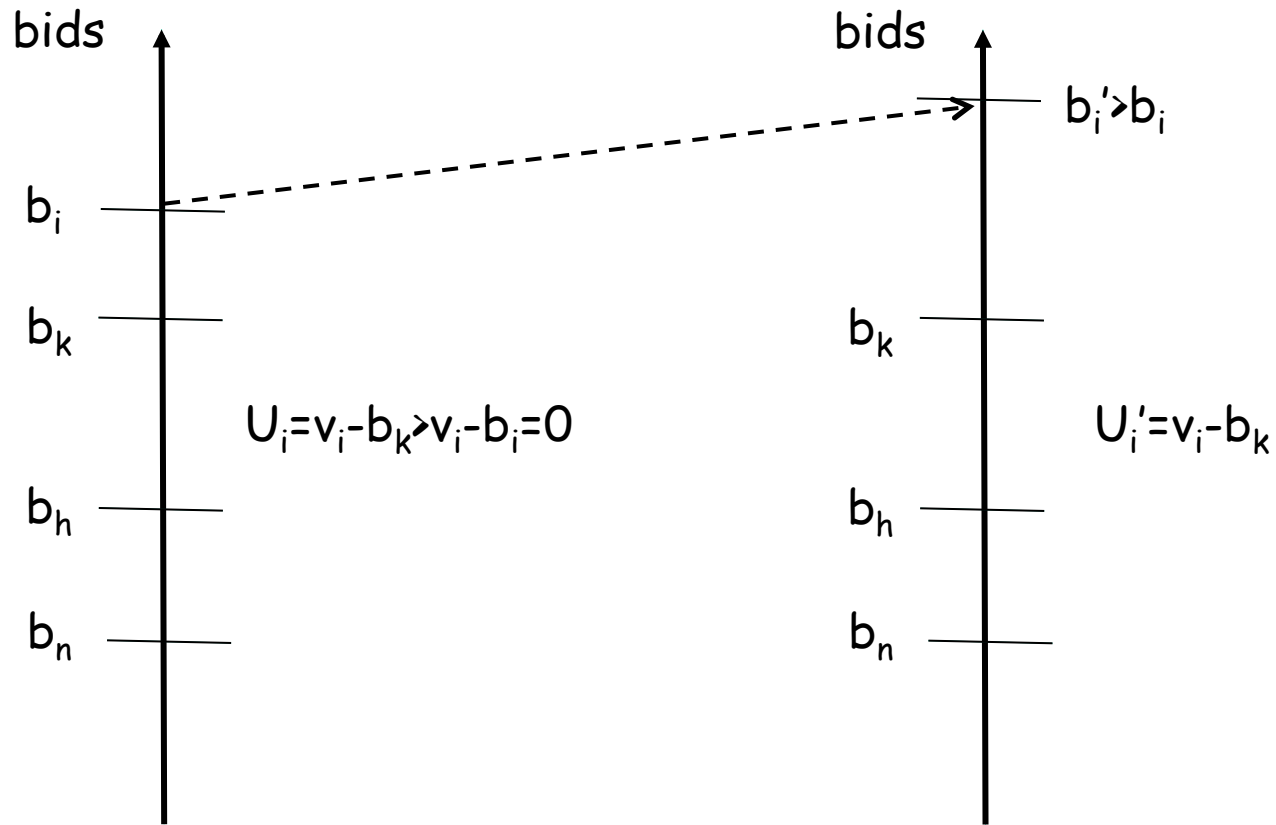
Bidding more than v_i is not convenient

$b_i = v_i$ is the highest bid

bids

bids

b_i

b_k

b_k

$U_i = v_i - b_k > v_i - b_i = 0$

$b_i' < b_i$

$U_i' = 0$

b_h

b_h

b_n

b_n
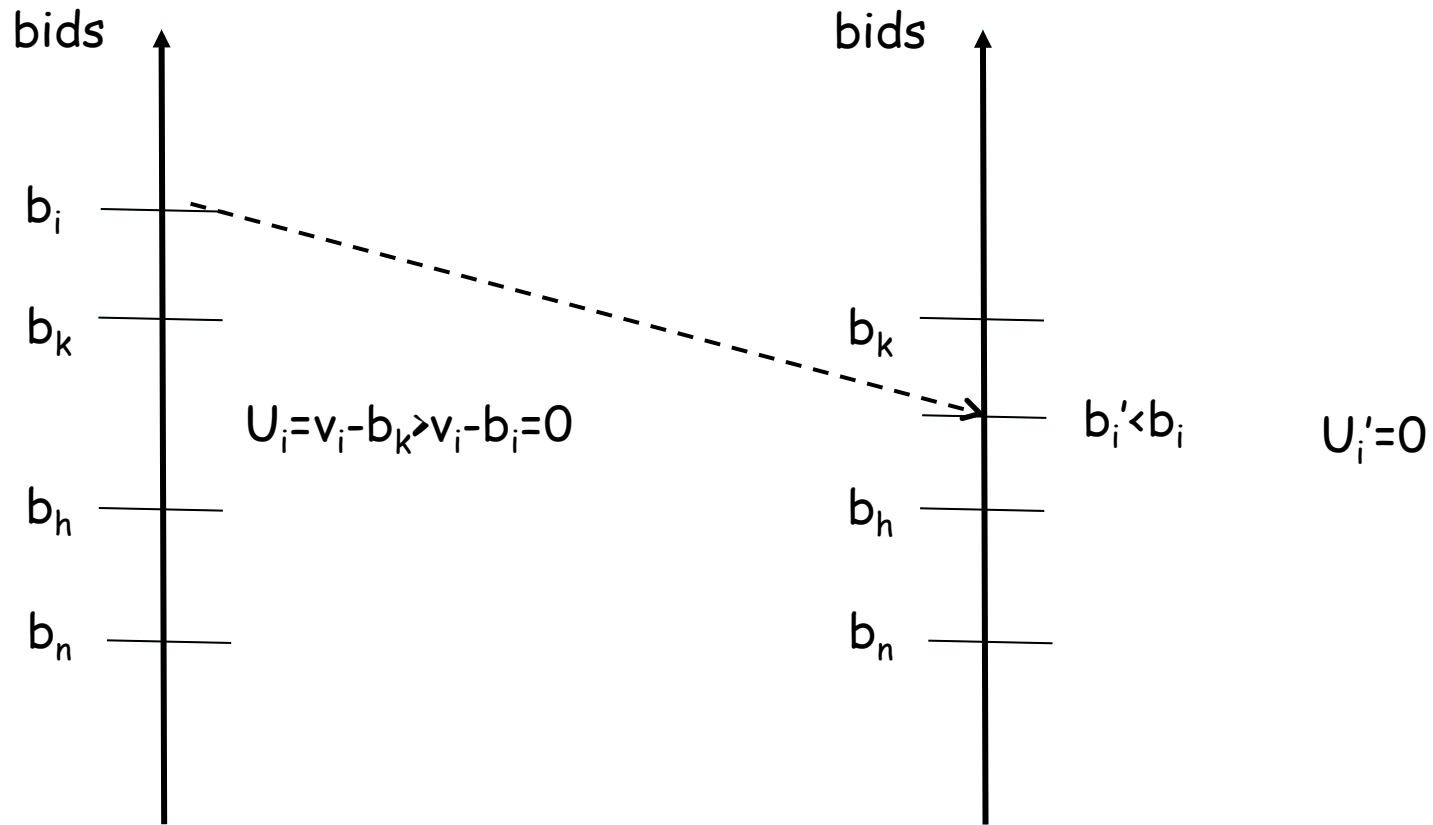
Bidding less than v_i is not convenient (may be unconvenient)
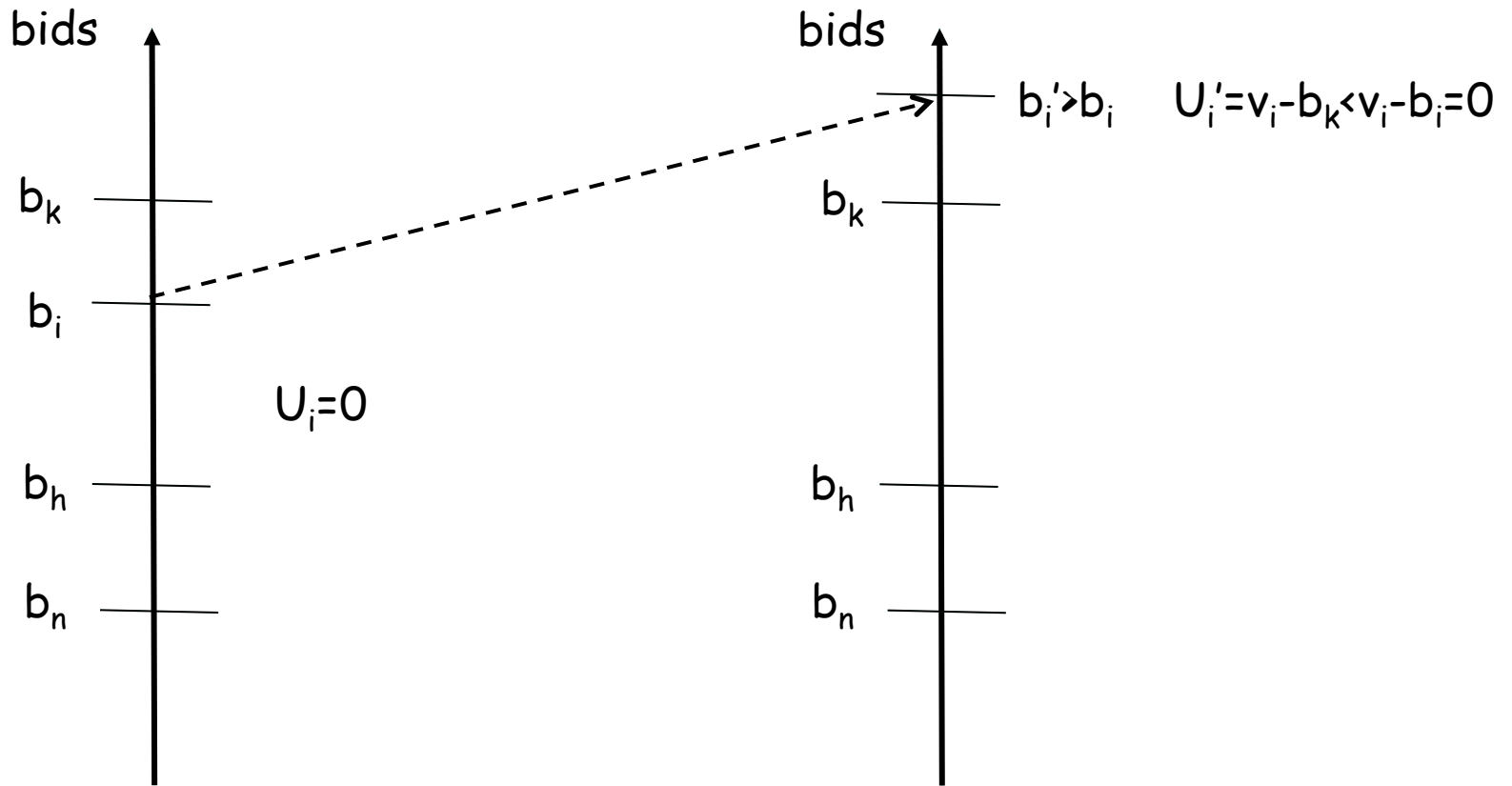
$b_i = v_i$ is not the highest bid

bids

b_k

b_i

$U_i = 0$

b_h

b_n

bids

$b_i' > b_i$ $U_i' = v_i - b_k < v_i - b_i = 0$

b_k

b_h

b_n

Bidding more than v_i is not convenient (may be unconvenient)

$b_i=v_i$ is not the highest bid

bids   bids

b_k b_k

b_i

              $U_i=0$ $b_i'<b_i$ $U_i'=0$

b_h b_h

b_n b_n
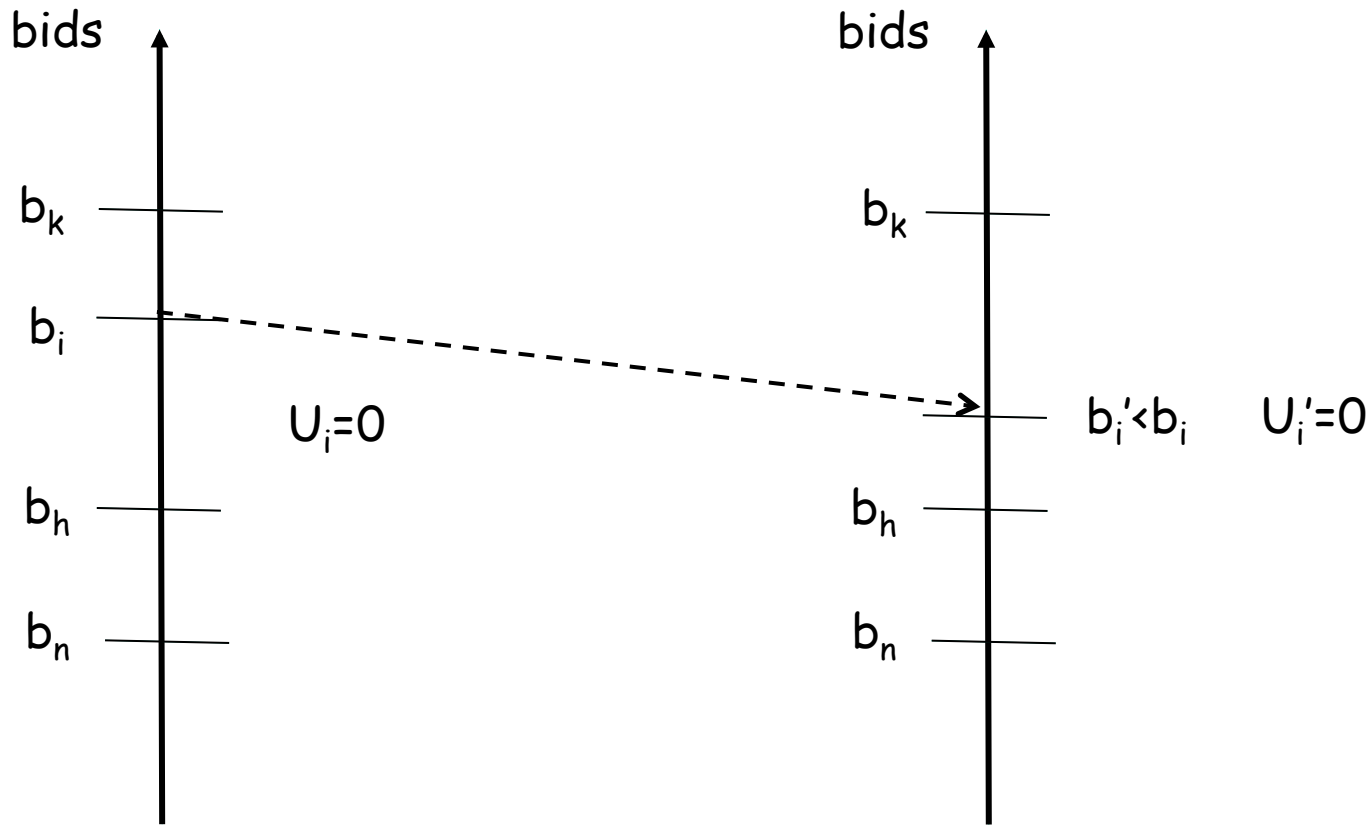
Bidding more than v_i is not convenient

Seller revenue

- N bidders
- Values are independent random values between 0 and 1
- Expected i^{th} largest utility is (N+1-i)/(N+1)
- Expected seller revenue is (N-1)/(N+1)

1st price auction

- Player with the highest bid gets the good and pays a price equal to her/his bid
- Being truthful is not a dominant strategy anymore!
- How to study it?

1st price auction

- Assumption: for each player the other values are i.i.d. random variables between 0 and 1
  - to overcome the fact that utilities are unknown
- Player i's strategy is a function s() mapping value v_i to a bid b_i
  - s() strictly increasing, differentiable function
  - $0 \leq s(v) \leq v$ ➔ $s(0)=0$
- We investigate if there is a strategy s() common to all the players that leads to a Nash equilibrium

1st price auction

- Assumption: for each player the other values are i.i.d. random variables between 0 and 1

- Player i's strategy is a function s() mapping value v_i to a bid b_i

- Expected payoff of player i if all the players plays s():
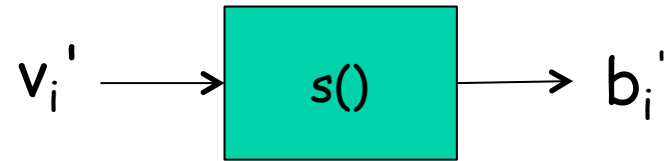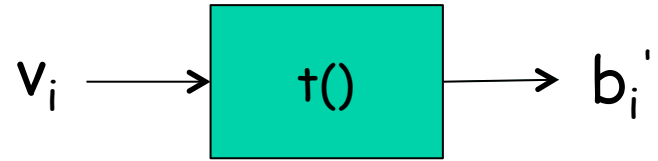   - $U_i(s,\ldots s,\ldots s) = v_i^{N-1}(v_i - s(v_i))$

prob. i wins   i's payoff if he/she wins

1st price auction

□ Expected payoff of player i if all the players play s():

   ○ $U_i(s,...s,...s) = v_i^{N-1} (v_i-s(v_i))$

□ What if i plays a different strategy t()?

   ○ If all players playing s() is a NE, then :

   ○ $U_i(s,...s,...s) = v_i^{N-1} (v_i-s(v_i)) \geq v_i^{N-1} (v_i-t(v_i)) =$
                $= U_i(s,...t,...s)$

□ Difficult to check for all the possible functions t() different from s()

□ Help from the **revelation principle**

The Revelation Principle

$v_i \longrightarrow$ [s()] $\longrightarrow b_i$

$v_i \longrightarrow$ [t()] $\longrightarrow b_i'$

$v_i' \longrightarrow$ [s()] $\longrightarrow b_i'$
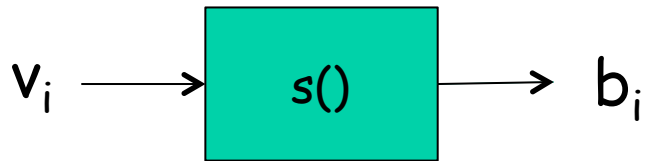
- □ All the strategies are equivalent to bidder i supplying to s() a different value of v_i

1ˢᵗ price auction

- ❑ Expected payoff of player i if all the players plays s():
  - ○ $U_i(v_1,...v_i,...v_N) = U_i(s,...s,...s) = v_i^{N-1} (v_i-s(v_i))$
- ❑ What if i plays a different strategy t()?
- ❑ By the revelation principle:
  - ○ $U_i(s,...t,...s) = U_i(v_1,...v,...v_N) = v^{N-1} (v_i-s(v))$
- ❑ If $v_i^{N-1} (v_i-s(v_i)) \geq v^{N-1} (v_i-s(v))$ for each v (and for each v_i)
  - ○ Then all players playing s() is a NE

1st price auction

- [ ] If $v_i^{N-1} (v_i - s(v_i)) \geq v^{N-1} (v_i - s(v))$ for each v (and for each v_i)
  - ○ Then all players playing s() is a NE
- [ ] $f(v) = v_i^{N-1} (v_i - s(v_i)) - v^{N-1} (v_i - s(v))$ is minimized for $v = v_i$
- [ ] $f'(v) = 0$ for $v = v_i$,
  - ○ i.e. $(N-1) v_i^{N-2} (v_i - s(v)) + v_i^{N-1} s'(v_i) = 0$ for each v_i
  - ○ $s'(v_i) = (N-1)(1 - s(v_i)/v_i)$, $s(0) = 0$
  - ○ Solution: $s(v_i) = (N-1)/N \, v_i$

1st price auction

- All players bidding according to
  $s(v) = (N-1)/N \, v$ is a NE
- Remarks
  - They are not truthful
  - The more they are, the higher they should bid
- Expected seller revenue
  - $(N-1)/N \, E[v_{max}] = (N-1)/N \, N/(N+1) = (N-1)/(N+1)$
  - Identical to 2nd price auction!
  - A general revenue equivalence principle

Outline

□ **Preliminaries**

  ○ Auctions

  ○ **Matching markets**

□ Possible approaches to ads pricing

□ Google mechanism

□ References

  ○ Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,**10**,15

Matching Markets

goods

buyers

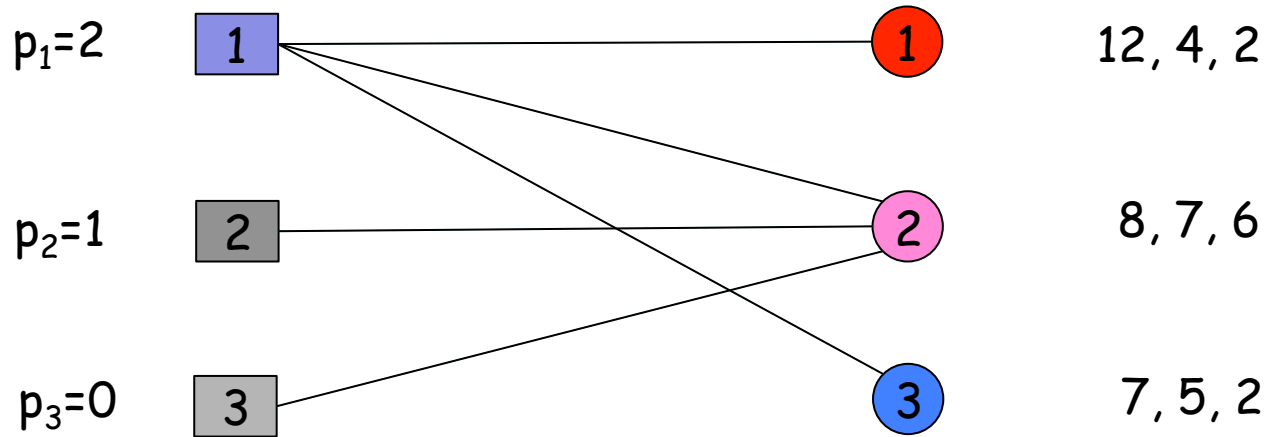|	
1	1 v_{11}, v_{21}, v_{31}
2	2 v_{12}, v_{22}, v_{32}
3	3 v_{12}, v_{22}, v_{32}

v_{ij}: value that buyer j gives to good i

How to match a set of different goods to a set of buyers with different evaluations

Matching Markets

$p_1=2$ [1] ——— (1) 12, 4, 2

$p_2=1$ [2] ——— (2) 8, 7, 6

$p_3=0$ [3] ——— (3) 7, 5, 2

Which goods buyers like most? Preferred seller graph

How to match a set of different goods to a set of buyers with different evaluations

Matching Markets

$p_1 = 2$ 1   1   12, 4, 2

$p_2 = 1$ 2   2   8, 7, 6

$p_3 = 0$ 3   3   7, 5, 2
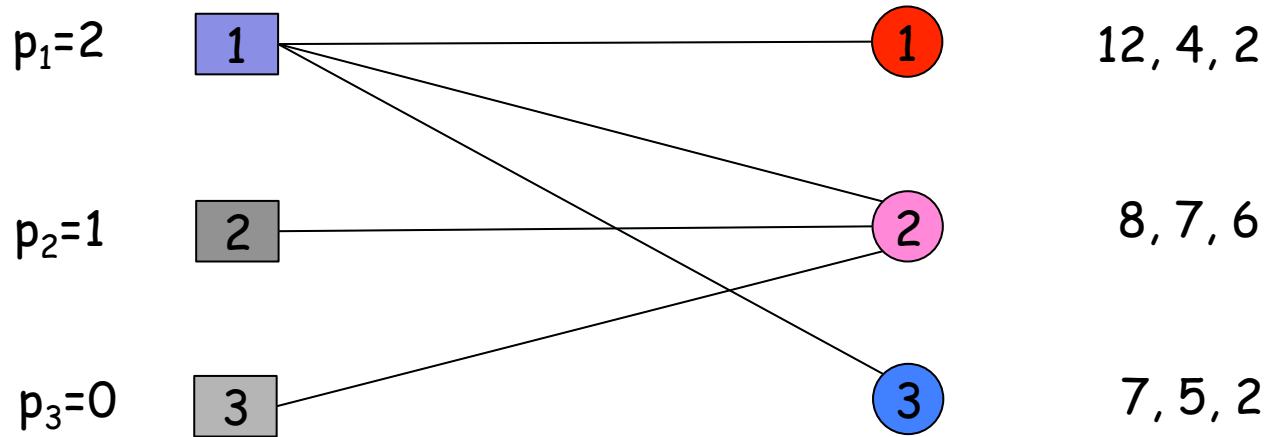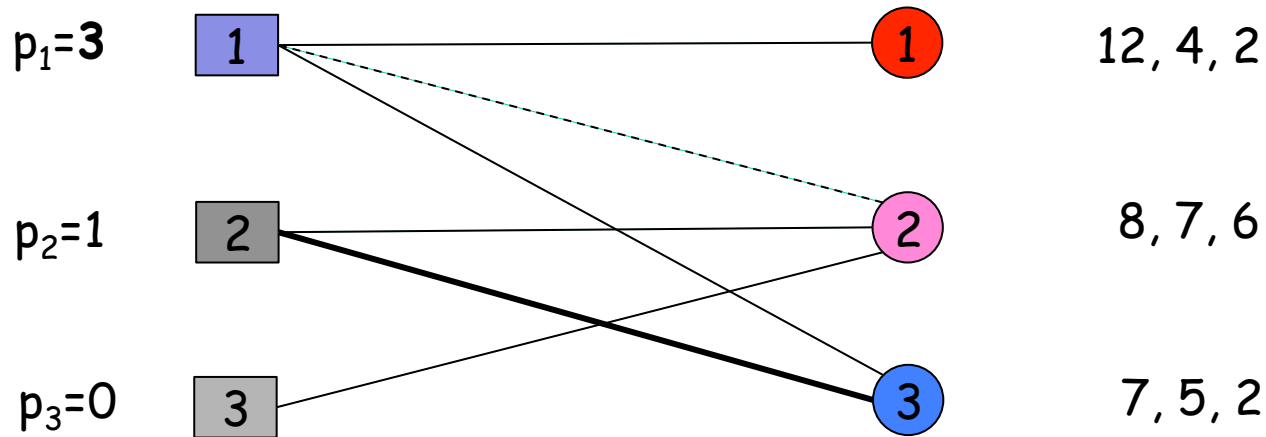
Which goods buyers like most? Preferred seller graph

- Given the prices, look for a perfect matching on the preferred seller graph
- There is no such matching for this graph

Matching Markets

$p_1=\mathbf{3}$ 1 ——— 1 12, 4, 2

$p_2=1$ 2 2 8, 7, 6

$p_3=0$ 3 3 7, 5, 2

Which goods buyers like most? Preferred seller graph

❏ But with different prices, there is

Matching Markets

$p_1 = 3$	1		1	12, 4, 2
$p_2 = 1$	2		2	8, 7, 6
$p_3 = 0$	3		3	7, 5, 2
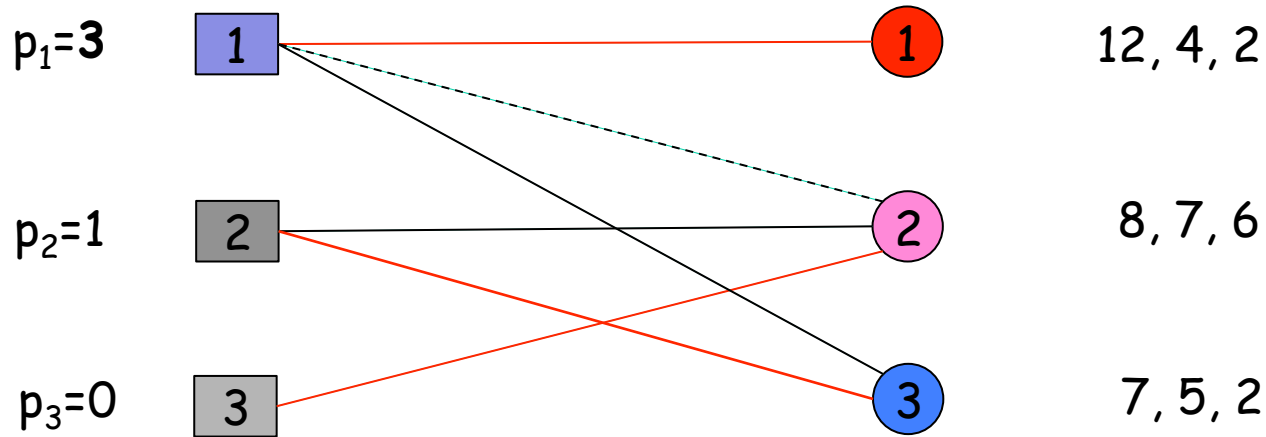
Which goods buyers like most? Preferred seller graph

- ❑ But with different prices, there is
- ❑ Such prices are **market clearing prices**

Market Clearing Prices

- They always exist
    - And can be easily calculated if valuations are known
- They are socially optimal in the sense that they maximize the sum of all the payoffs in the network (both sellers are buyers)

Outline

☐ Preliminaries
  ○ Auctions
  ○ Matching markets

☐ **Possible approaches to ads pricing**

☐ Google mechanism

☐ **References**
  ○ Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,10,**15**

Ads pricing

Ads positions
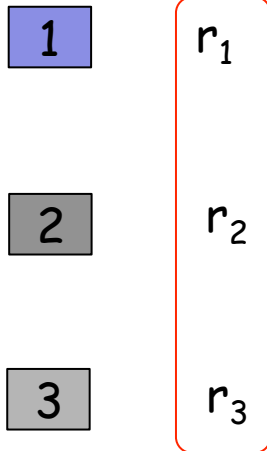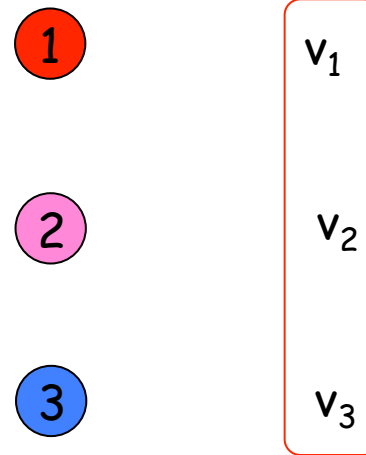
1	r_1
2	r_2
3	r_3

companies

1	v_1
2	v_2
3	v_3

r_i: click rate for an ad in position i
(assumed to be independent
from the ad and known a priori)

v_i: value that company i
gives to a click

How to rank ads from different companies

Ads pricing as a matching market

Ads positions   companies

1   r_1 1 $v_1 r_1, v_1 r_2, v_1 r_3$

2   r_2 2 $v_2 r_1, v_2 r_2, v_2 r_3$

3   r_3 3 $v_3 r_1, v_3 r_2, v_3 r_3$

r_i: click rate for an ad in position i v_i: value that company i
(assumed to be independent   gives to a click
from the ad and known a priori)

☐ Problem: Valuations are not known!

☐ ... but we could look for something as 2nd price auctions

The VCG mechanism

- The correct way to generalize 2nd price auctions to multiple goods
- Vickrey-Clarke-Groves
- Every buyers should pay a price equal to the social value loss for the others buyers
  - Example: consider a 2nd price auction with $v_1 > v_2 > ... v_N$
    - With 1 present the others buyers get 0
    - Without 1, 2 would have got the good with a value v_2
    - then the social value loss for the others is v_2

The VCG mechanism

- The correct way to generalize 2nd price auctions to multiple goods
- Vickrey-Clarke-Groves
- Every buyers should pay a price equal to the social value loss for the others buyers
  - If V_B^S is the maximum total valuation over all the possible perfect matchings of the set of sellers S and the set of buyers B,
  - If buyer j gets good i, he/she should be charged $V_{B-j}^S - V_{B-j}^{S-i}$

VCG example

Ads positions   companies

1      $r_1 = 10$ 1 $v_1 = 3$

2      $r_2 = 5$ 2 $v_2 = 2$

3      $r_3 = 2$ 3 $v_3 = 1$

r_i: click rate for an ad in position i
(assumed to be independent
from the ad and known a priori)

v_i: value that company i
gives to a click

VCG example

Ads positions

companies

| 1 | | 1 | 30, 15, 6 |

| 2 | | 2 | 20, 10, 4 |

| 3 | | 3 | 10, 5, 2 |

VCG example

Ads positions

companies

1 —————————— 1   30, 15, 6

2 —————————— 2   20, 10, 4

3 —————————— 3   10, 5, 2

❒ This is the maximum weight matching
❒ 1 gets 30, 2 gets 10 and 3 gets 2

VCG example

Ads positions

companies

1 — 2

1	30, 15, 6
2	20, 10, 4
3	10, 5, 2

- ❑ If 1 weren't there, 2 and 3 would get 25 instead of 12,
- ❑ Then 1 should pay 13

VCG example

Ads positions   companies

[1] ————————————— (1)   30, 15, 6

[2] ——   ⊗   20, 10, 4
        ——
           ——
              ——
                 —— (3)   10, 5, 2

[3]

□ If 2 weren't there, 1 and 3 would get 35 instead of 32,

□ Then 2 should pay 3

VCG example

Ads positions

companies

| 1 | ——— | 1 | 30, 15, 6 |

| 2 | ——— | 2 | 20, 10, 4 |

| 3 | | ✕ | 10, 5, 2 |
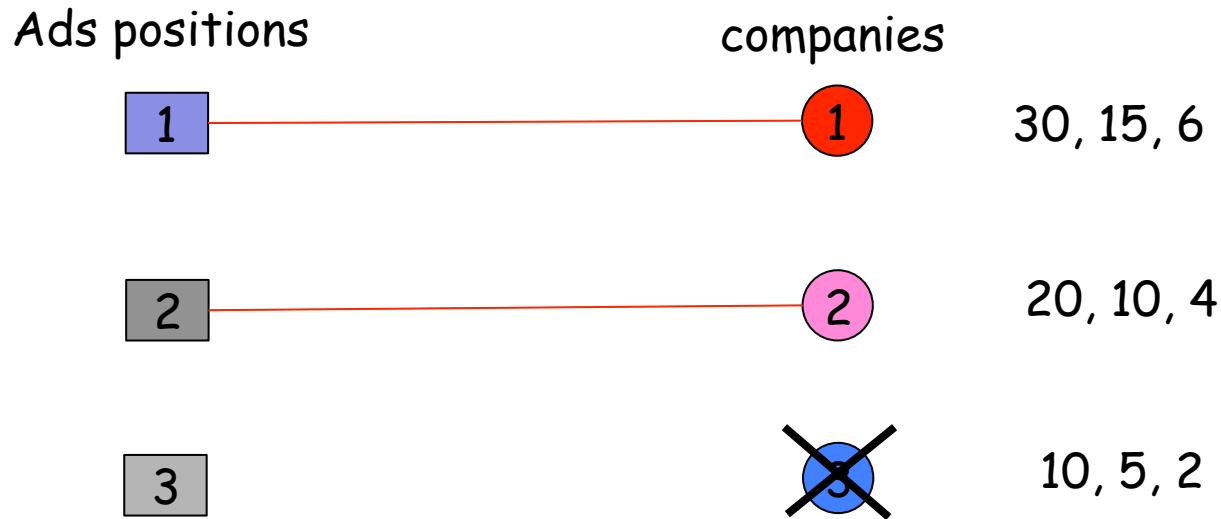
□ If 3 weren't there, nothing would change for 1 and 2,

□ Then 3 should pay 0

The VCG mechanism

☐ Every buyers should pay a price equal to the social value loss for the others buyers

  ○ If V_B^S is the maximum total valuation over all the possible perfect matchings of the set of sellers S and the set of buyers B,

  ○ If buyer j gets good i, he/she should be charged $V_{B-j}^S - V_{B-j}^{S-i}$

☐ Under this price mechanism, truth-telling is a dominant strategy

Outline

□ Preliminaries
  ○ Auctions
  ○ Matching markets
□ Possible approaches to ads pricing
□ **Google mechanism**

□ **References**
  ○ Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,10,**15**

Google's GSP auction

- Generalized Second Price
- Once all the bids are collected $b_1 > b_2 > ... b_N$
- Company i pays b_{i+1}
- In the case of a single good (position), GSP is equivalent to a 2nd price auction, and also to VCG
- But why Google wanted to implement something different???

GSP properties

- Truth-telling may not be an equilibrium

GSP example

Ads positions

1    $r_1=10$

2    $r_2=4$

3    $r_3=0$

companies

1    $v_1=7$

2    $v_2=6$

3    $v_3=1$

r_i: click rate for an ad in position i
(assumed to be independent
from the ad and known a priori)

v_i: value that company i
gives to a click

☐ If each player bids its true evaluation, 1 gets a payoff equal to 10

☐ If 1 bids 5, 1 gets a payoff equal to 24

GSP properties

- ☐ Truth-telling may not be an equilibrium
- ☐ There is always at least 1 NE maximizing total advertiser valuation

GSP example

Ads positions

| 1 | $r_1=10$ |

| 2 | $r_2=4$ |

| 3 | $r_3=0$ |

companies

(1) $v_1=7$

(2) $v_2=6$

(3) $v_3=1$

r_i: click rate for an ad in position i
(assumed to be independent
from the ad and known a priori)

v_i: value that company i
gives to a click

❑ Multiple NE
  ○ 1 bids 5, 2 bids 4 and 3 bids 2
  ○ 1 bids 3, 2 bids 5 and 3 bids 1

GSP properties

☐ Truth-telling may not be an equilibrium

☐ There is always at least 1 NE maximizing total advertiser valuation

☐ Revenues can be higher or lower than VCG

  ○ Attention: the revenue equivalence principle does not hold for auctions with multiple goods!

  ○ Google was targeting higher revenues...

  ○ ... not clear if they did the right choice.

GSP example

Ads positions

$\boxed{1}$ $r_1=10$

$\boxed{2}$ $r_2=4$

$\boxed{3}$ $r_3=0$

companies

1 $v_1=7$

2 $v_2=6$

3 $v_3=1$

☐ Multiple NE
  - 1 bids 5, 2 bids 4, 3 bids 2 ➔ google's revenue=48
  - 1 bids 3, 2 bids 5, 3 bids 1 ➔ google's revenue=34

☐ With VCG, google's revenue=44

Other issues

□ Click rates are unknown and depend on the ad!

  ○ Concrete risk: low-quality advertiser bidding high may reduce the search engine's revenue

  ○ Google's solution: introduce and ad-quality factor taking into account actual click rate, relevance of the page and its ranking

    • Google is very secretive about how to calculate it => the market is more opaque

□ Complex queries, nobody paid for

  ○ Usually engines extrapolate from simpler bids