The points marked for each exercise give an indication of the relative importance.

In some of the following exercises we consider graphs. Unless otherwise specified, each graph is connected and undirected, $V = \{0, 1, 2, \ldots n\}$ denotes the set of nodes, E the set of edges, R the set routes. Moreover we define x_r to be the amount of traffic through route $r \in R$, and y_l to be the amount of traffic on link l, i.e. $y_l = \sum_{r|l \in r} x_r$. If links have finite capacity, then C_l denotes the capacity of link l.

Ex. 1 — (1 point)
Briefly define the teleological, macroscopic and microscopic level descriptions for a distributed system and provide some examples from the course.

Ex. 2 — (1 point) Describe the network utility maximization problem, commenting the equations below:

$$
\begin{aligned}
\underset{\mathbf{x} \in \mathbb{R}^{|R|}}{\text{maximize}} \quad & \sum_{r \in R} U_r(x_r) \\
\text{subject to} \quad & \sum_{r|l \in r} x_r \leq C_l \ \ \forall l \in E \\
& x_r \geq 0 \ \ \forall r \in R
\end{aligned}
$$

(Note: it is not required to discuss how to solve it)

Ex. 3 — (5 point) Consider a graph, where each link $l \in E$ is affected by a delay $D_l(y_l)$ depending on the amount of traffic on that link, y_l. The delay function is assumed to be convex, increasing and its first derivative to exist. Consider the following routing optimization problem:

$$\begin{aligned}
\underset{\mathbf{x}\in\mathbb{R}^{|R|},\mathbf{y}\in\mathbb{R}^{|E|}}{\text{minimize}} \quad & \sum_{l\in E} y_l D_l(y_l) \\
\text{subject to} \quad & f_s = \sum_{r|s(r)=s} x_r \ \ \forall s \in S \\
& y_l = \sum_{r|l\in r} x_r \ \ \forall l \in E \\
& x_r \geq 0 \ \ \forall r \in R
\end{aligned}$$

1. What is the system minimizing?
2. Show that the function attains a minimum over the set defined by the constraints.
3. Write the Lagrangian function relative to the first two sets of constraints (i.e. ignoring $x_r \geq 0$).
4. Characterize the optimal routing.
5. Explain how the problem can be solved in a distributed way. Explain the role of each network element in the optimization process, what the element needs to know and what it does.
6. Consider now the particular case of a network made by two nodes u and v connected by $|E| = 3$ parallel edges. f_{uv} traffic has to be routed between u and v. The delays on the links are respectively:
   - $D_1(y_1) = 1 + y_1$
   - $D_2(y_2) = 2 + \frac{1}{2}y_2$
   - $D_3(y_3) = 4 + \frac{1}{100}y_3$

   Determine the optimal routing if $f_{uv} = 1$ and if $f_{uv} = 3$.

Ex. 4 — (1 point)
Solve the following game:

	A	B	C	D
A	5	-2	4	1
B	-5	-3	1	5

Ex. 5 — (1 point) Consider an auction.

1. What does it mean "to be truthful"?
2. In which of the following auctions being truthful is a dominant strategy?
   - First price single-item auction.
   - Second price single-item auction.
   - Descending bid single-item auction.
   - Multi-item GSP auction
   - Multi-item VCG auction.

Ex. 6 — (3 points) GSP and VCG auctions
In an ads auction for a given keyword, there are three possible positions with expected click rates per-day 12, 5 and 1. Three companies bid for these positions. They value one click respectively 5$, 4$ and 2$.

1. In the case of a Generalized Second Price (GSP) auction, do the following bids produce a Nash equilibrium $b_1 = 3$, $b_2 = 4$, $b_3 = 2$?
2. In the case of a VCG auction, how are the ads priced? How does the seller's revenue changes in comparison to the GSP auction studied above?