Ex. 1 — [2 point] Given the example of Braess's paradox considered in class (or the one in Fig. 4.4 of Kelly&Yudovina's book), calculate the toll $T_j(y) = yD_j'(y)$ to be imposed on each link $j \in E$, so that users collectively minimize the global delay. Show that, if this toll is applied just after the new road is built (i.e. while nobody is using it), there is no incentive for any driver to use the new road.

Ex. 2 — [3 point] In the definition of a Wardrop equilibrium, f_s is the aggregate flow for source–sink pair s, and is assumed fixed. Suppose now that the aggregate flow between source–sink pair s is not fixed, but is a continuous, strictly decreasing function $B_s(\lambda_s)$, where λ_s is the minimal delay over all routes serving the source–sink pair s, for each $s \in \mathcal{S}$. Consider that $B_s(0) = \bar{f}_s < \infty$.

1. For the extended model, show that an equilibrium exists and solves the optimization problem:

$$\begin{aligned}
&\text{minimize} && \sum_{j\in\mathcal{J}} \int_0^{y_j} D_j(u)\mathrm{d}u - \sum_{s\in\mathcal{S}} \int_0^{f_s} B_s^{-1}(u)\mathrm{d}u \\
&\text{subject to} && Hx = f, Ax = y, \\
&\text{over} && x \geq 0, y, f.
\end{aligned}$$

2. Interpret your result in terms of a fictitious additional one-link "stay-at-home" route for each source-destination pair s, with appropriate delay characteristics, where the traffic $\bar{f}_s - f_s$ is routed. What are the delay functions of these links?