UBINET, Master 2 IFI Algorithms for telecommunications

First Exam, October 2018

1 hour 1/2

No documents are allowed. No computers, cellphones.

Instruction and comments: the points awarded for your answer will be based on the correctness of your answer as well as the clarity of the main steps in your reasoning. All proposed solutions must be proved. **All the exercises are independent.** *The points are indicated so you may adapt your effort.*

1 Linear Programming

Exercise 1 (Modeling as a linear program (3 points, 20 minutes)) *A company builds two products and wants to increase the level of production to maximize the benefit.*

Each unit of product 1 brings in a profit of 120 euros, when one unit of product 2 brings in 500 euros. Due to limitations in the production line, one cannot produce more than 200 products 1 and 300 products 2. Furthermore, one cannot produce more that 400 products in total because of the limited workforce.

1) *Write a linear program which maximize the profit of the company.*

2) *Solve graphically the problem.*

Exercise 2 (Duality and Optimality certificates. (4 points, 20 minutes)) *We consider the following linear program.*

$$
\begin{array}{lrcrcrcrcr}
\textit{Minimize} & 8x_1 & + & 6x_2 & + & 10x_3 & + & 2x_4 & & \\
\textit{Subject to:} & & & & & & & & & \\
& 2x_1 & + & 1x_2 & + & 3x_3 & + & 1x_4 & \geq & 6 \\
& 6x_1 & + & 4x_2 & + & 8x_3 & + & 2x_4 & \geq & 3 \\
& 10x_1 & - & 4x_2 & + & 8x_3 & + & 4x_4 & \geq & 2 \\
& -4x_1 & + & 2x_2 & - & 4x_3 & - & 2x_4 & \geq & -3 \\
& & & & & & & x_1, x_2, x_3, x_4, x_5 & \geq & 0.
\end{array}
$$

1) *Write the dual of this linear program.*

2) *Is the solution $y_1^* = 4, y_2^* = 0, y_3^* = 0, y_4^* = 1$ optimal for the dual? Use the method presented during the course and explain every step of the resolution.*

Exercise 3 (Simplex. (4 points, 15 minutes)) *Solve the following linear program using the simplex method.*

$$
\begin{array}{lrcrcrcrcr}
\textit{Maximiser} & 5x_1 & + & 6x_2 & + & 9x_3 & + & 8x_4 & & \\
\textit{Sous les contraintes :} & & & & & & & & & \\
& x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & \leq & 5 \\
& x_1 & + & x_2 & + & 2x_3 & + & 3x_4 & \leq & 3 \\
& & & & & & & x_1, x_2, x_3, x_4 & \geq & 0
\end{array}
$$

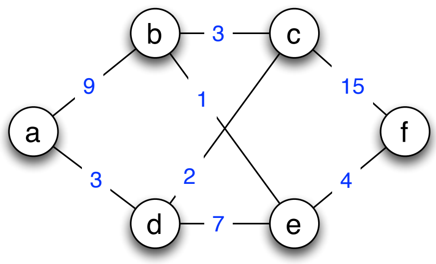

Figure 1: The weighted graph H. Integers on edges represent their length.

Exercise 4 (Dijkstra. 4 points, 15 minutes) Consider the graph H depicted in Figure 1.

- Give the definition of a shortest-path tree rooted in a.
- Apply the Dijkstra algorithm on H to compute a shortest-path tree rooted in a and the distance between any vertex and the vertex a.

  You must explain the execution of the algorithm (you may write the table as seen during the lecture). In particular, indicate the order in which vertices are considered during the execution of the algorithm.
- Give the obtained shortest-path tree rooted in a.

Exercise 5 (Flow. 5 points, 20 minutes) Consider the elementary network flow N depicted in Figure 2 (left) and the initial flow f from s to t in Figure 2 (right).

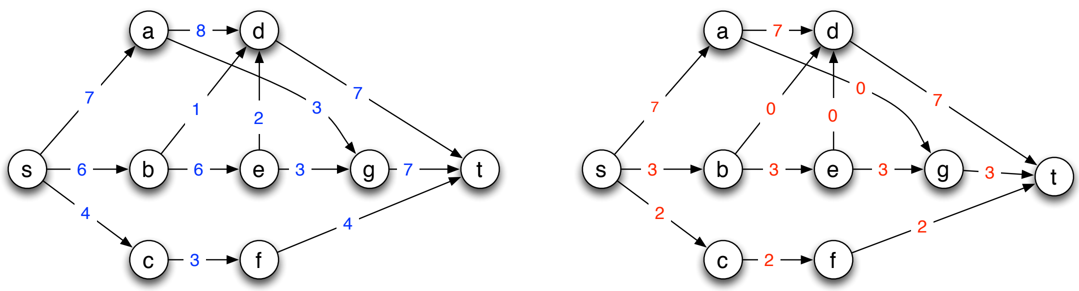

Figure 2: (left) Elementary network flow with arcs' capacity. (right) An initial flow f from s to t: a number close to an arc indicates the amount of flow along it.

- What must be checked to show that f is a flow? What is the value of the initial flow f?
- Apply the Ford-Fulkerson Algorithm to N starting from the flow f. All steps of the execution of the algorithm must be detailed.

  For each iteration, draw the auxiliary graph, give the chosen path and the amount of flow that you will push, and draw the network with the new flow.
- What is the final value of the flow?