Elpi: rule-based meta-languge for Rocq

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This talk

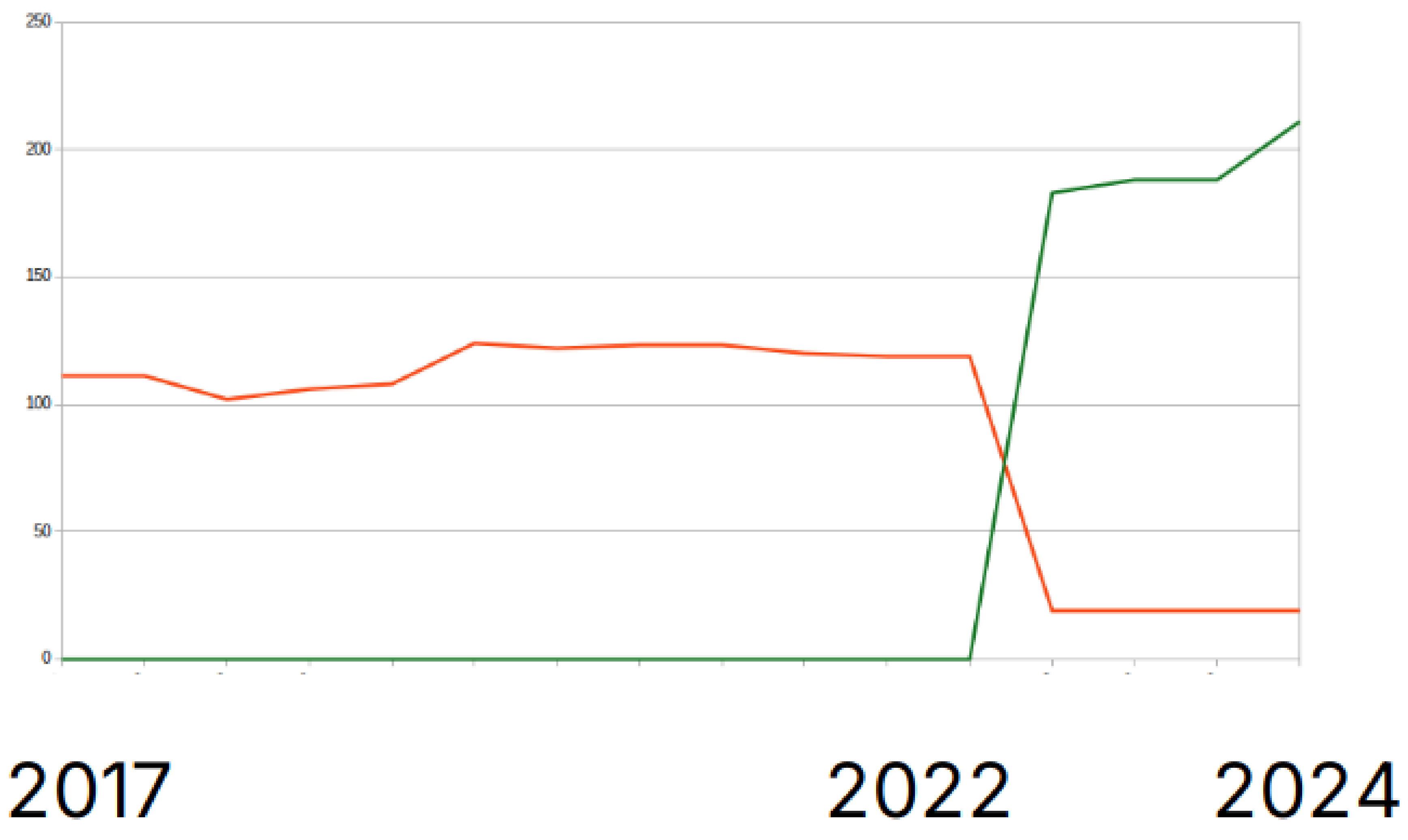
Users of Elpi

- https://github.com/math-comp/hierarchy-builder/
- https://github.com/coq-community/trocq
- https://github.com/LPCIC/coq-elpi/tree/master/apps/derive
- https://github.com/math-comp/algebra-tactics/

Hierarchy Builder

DSL to declare a hierarchy of interfaces

- generates boilerplate via Elpi's API: modules, implicit arguments, canonical structures, ...
- used by the Mathematical Components library and other ~20 libraries
- makes "packed classes" easy




```
From HB Require Import structures.

HB.mixin Record IsAddComoid A := {
  zero : A;
  add : A -> A -> A;
  addrA : forall x y z, add x (add y z) = add (add x y) z;
  addrC : forall x y, add x y = add y x;
  add0r : forall x, add zero x = x;
}.

HB.structure Definition AddComoid := { A of IsAddComoid A }.

Notation "0" := zero.
Infix "+" := add.

Check forall (M : AddComoid.type) (x : M), x + x = 0.
```

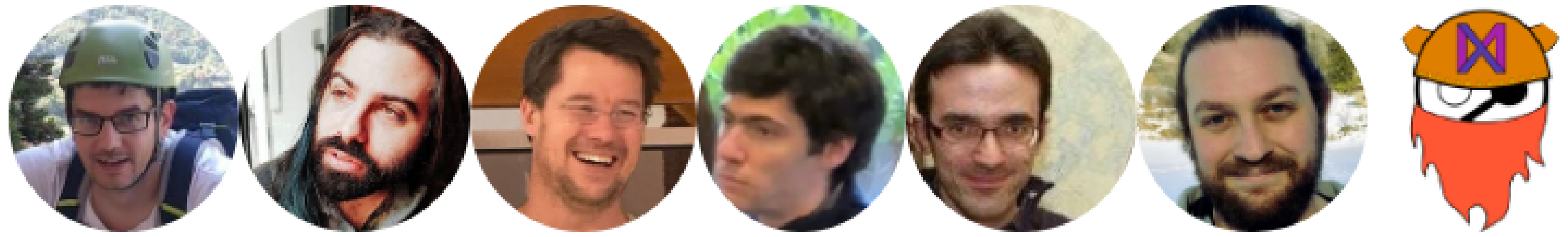
Trocq

Proof transfer via parametricity (with or without univalence).

- Registers in Elpi Databases translation rules
- Synthesizes transfer proofs minimizing the axioms required

```
From Trocq Require Import Trocq.

Definition RN : (N <=> nat)%P := ...
Trocq Use RN.

Lemma RN0 : RN 0%N 0%nat. ...
Lemma RNS m n : RN m n -> RN (N.succ m) (S n). ...
Trocq Use RN0 RNS.

Lemma N_Srec : ∀P : N -> Type, P 0%N ->
    (∀n, P n -> P (N.succ n)) -> ∀n, P n.
Proof.
trocq. (* replaces N by nat in the goal *)
exact nat_rect.
Qed.
```

Derive



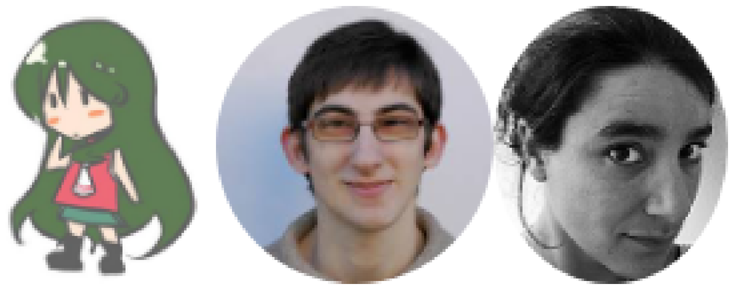
Framework for type driven code synthesis

Derivations:

- parametricity
- deep induction
- equality tests and proofs
- lenses (record update syntax)
- a few more...

```
From elpi.apps Require Import derive.std lens.

#[only(lens_laws, eqb), module] derive
Record Box A := { contents : A; tag : nat }.

About Box. (* Notation Box := Box.t *)

Check Box.eqb :
  ∀A, (A -> A -> bool) -> Box A -> Box A -> bool.

(* the Lens for the second field *)
Check @Box._tag : ∀A, Lens (Box A) (Box A) nat nat.

(* a Lens law *)
Check Box._tag_set_set : ∀A (r : Box A) y x,
  set Box._tag x (set Box._tag y r) = set Box._tag x r.
```

Algebra Tactics



`ring`, `field`, `lra`, `nra`, and `psatz` tactics for the Mathematical Components library.

- works with any instance of the structure: concrete, abstract and mixed like `int * R` where `R` is a variable
- automatically push down ring morphisms and additive functions to leaves of the expression
- reification up to instance unification in Elpi

```
From mathcomp Require Import all_ssreflect.
From mathcomp Require Import all_algebra.
From mathcomp Require Import ring lra.

Lemma test (F : realFieldType) (x y : F) :
  x + 2 * y <= 3 ->
  2 * x + y <= 3 ->
    x + y <= 2.
Proof. lra. Qed.

Variables (R : unitRingType) (x1 x2 x3 y1 y2 y3 : R).
Definition f1 : R := ...
Definition f2 : R := ...
Definition f3 : R := ...

(* 500 lines of polynomials later... *)

Lemma example_from_Sander : f1 * f2 = f3.
Proof. rewrite /f1 /f2 /f3. ring. Qed.
```

Elpi in a nutshell

https://github.com/LPCIC/elpi/

Rules, rules, rules!

Roots

- Elpi is a constraint logic programming language
- Elpi is a dialect of λProlog and CHR
- backtracking is not the point

⚠ Vintage syntax ahead

- variables are capitals `X`
- λx.t is written `x\ t`
- rules are written `goal :- subgoal1, subgoal2...`

What really matters

- Code is organized in rules
- Rule application is guided by a pattern
- Rules can be added statically and dynamically

Elpi: Hello World!

Simply typed λ-calculus in HOAS

```
kind tm type.
type app tm -> tm -> tm.
type lam (tm -> tm) -> tm.

kind ty type.
type arr ty -> ty -> ty.
```

Typing

```
pred of i:tm, o:ty.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

```
goal> of (lam x\ lam y\ x) TyFst.
```

Elpi: Hello World!

Simply typed λ-calculus in HOAS

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type lam (tm -> tm) -> tm.

kind ty type.
type arr ty -> ty -> ty.
```

Typing

```
pred of i:tm, o:ty.
of c S0.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

```
goal> of (lam x\ lam y\ x) (arr S0 T0).
goal> of        (lam y\ c) T0.
```

Elpi: Hello World!

Simply typed λ-calculus in HOAS

```
kind tm type.
type app tm -> tm -> tm.
type lam (tm -> tm) -> tm.

kind ty type.
type arr ty -> ty -> ty.
```

Typing

```
pred of i:tm, o:ty.
of d S1.
of c S0.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

```
goal> of (lam x\ lam y\ x) (arr S0 (arr S1 T1)).
goal> of        (lam y\ c) (arr S1 T1).
goal> of                c  T1.
```

Elpi: Hello World!

Simply typed λ-calculus in HOAS

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```

Typing

```
pred of i:tm, o:ty.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

```
goal> of (lam x\ lam y\ x) TyFst.

Success:
  TyFst = arr S0 (arr S1 S0)
```

Elpi: Hello World!

Simply typed λ-calculus in HOAS

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kind tm type.
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type lam (tm -> tm) -> tm.

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of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

```
goal> of (app H A) T.

Failure.
```

Elpi = λProlog + CHR

Typing (as before)

```
pred of i:tm, o:ty.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

Holes & constraints

```
of (uvar as E) T :-
  declare_constraint (of E T) [E].
```

```
goal> of (app H A) T.
```

Elpi = λProlog + CHR

Typing (as before)

```
pred of i:tm, o:ty.
of (app H A) T :-
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```

Holes & constraints

```
of (uvar as E) T :-
  declare_constraint (of E T) [E].
```

```
goal> of (app H A) T.

Success:

Constraints:
  of A S  /* suspended on A */
  of H (arr S T)  /* suspended on H */
```

Elpi = λProlog + CHR

Typing (as before)

```
pred of i:tm, o:ty.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

Holes & constraints

```
of (uvar as E) T :-
  declare_constraint (of E T) [E].
```

```
goal> of (app H A) T, H = (lam x\ x).

Success:

Constraints:
  of A T  /* suspended on A */
```

Elpi = λProlog + CHR

Typing (as before)

```
pred of i:tm, o:ty.
of (app H A) T :-
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Holes & constraints

```
of (uvar as E) T :-
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```

```
goal> of (app (lam x\ x) A) T.

Success:

Constraints:
  of A T  /* suspended on A */
```

Elpi = λProlog + CHR

Typing (as before)

```
pred of i:tm, o:ty.
of (app H A) T :-
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```

Holes & constraints

```
of (uvar as E) T :-
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```
goal> of (app D D) T.
```

Elpi = λProlog + CHR

Typing (as before)

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pred of i:tm, o:ty.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
  pi x\ of x S => of (F x) T.
```

Holes & constraints

```
of (uvar as E) T :-
  declare_constraint (of E T) [E].
```

```
goal> of (app D D) T.

Success:

Constraints:
  of D S  /* suspended on D */
  of D (arr S T)  /* suspended on D */
```

Elpi = λProlog + CHR

Typing (as before)

```
pred of i:tm, o:ty.
of (app H A) T :-
  of H (arr S T), of A S.
of (lam F) (arr S T) :-
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```

Holes & constraints

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of (uvar as E) T :-
  declare_constraint (of E T) [E].
```

Constraint Handling Rules

```
constraint of {
    rule (of X T1) \ (of X T2) <=> (T1 = T2).
}
```

```
goal> of (app D D) T.

Failure
```

Integration in Rocq

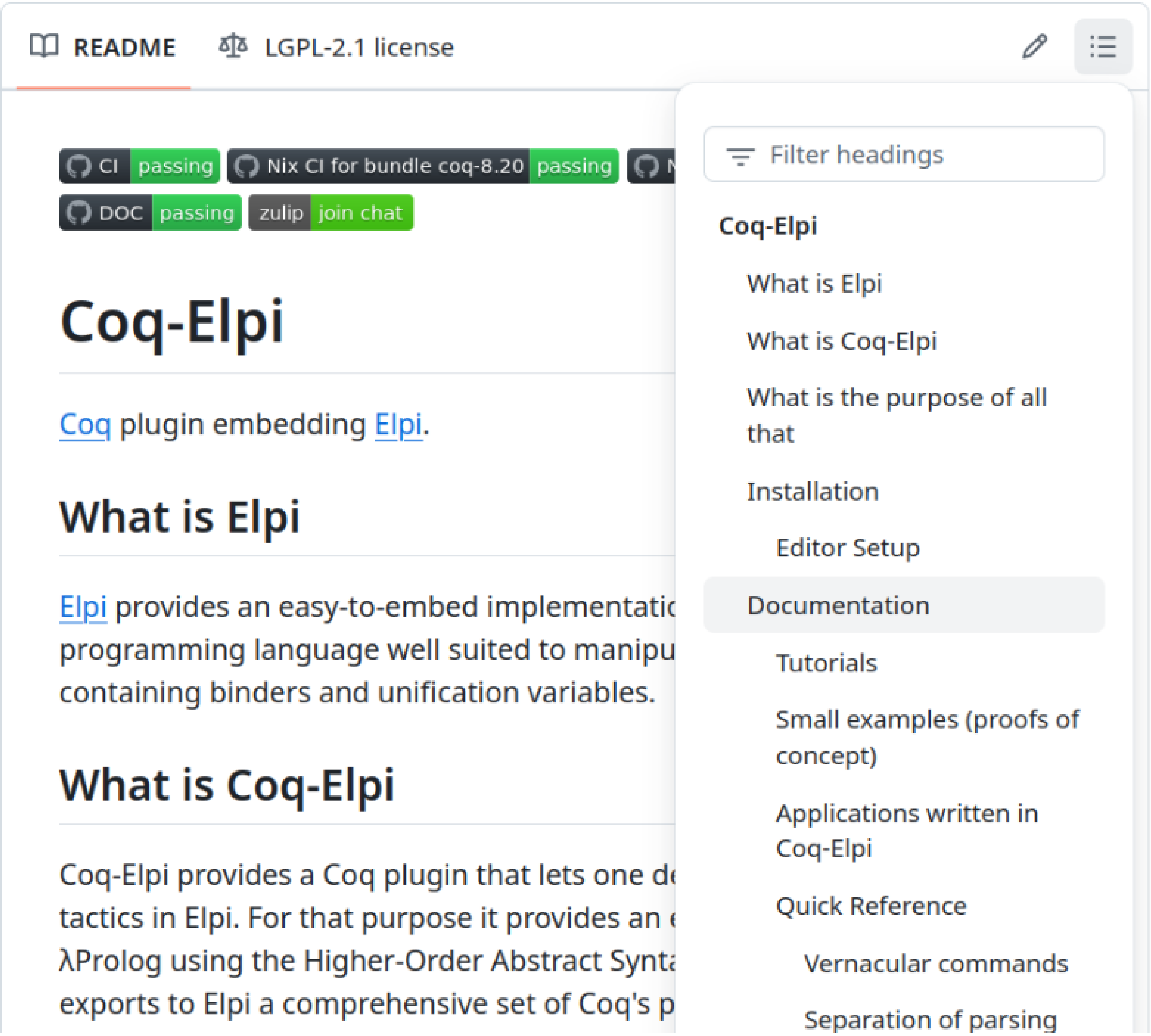
https://github.com/LPCIC/coq-elpi/

Notable features

- HOAS for Gallina
- quotations and anti-quotations

```
  coq.say {{ 1 + lp:{{ app[global S, {{ 0 }} ]  }}   }}
% elpi....  coq..     elpi..........  coq  elpi  coq
```

- Databases of rules
- Extensive API



Demo: from Prop to bool

```
Axiom is_even : nat -> Prop.

Fixpoint even n : bool := match n with
  | O => true
  | S (S n) => even n
  | _ => false
  end.

Lemma evenP n : reflect (is_even n) (even n).
(* Elpi add_tb evenP. *)

Lemma andP  {P Q : Prop} {p q : bool} :
  reflect P p -> reflect Q q ->
    reflect (P /\ Q) (p && q).
(* Elpi add_tb andP. *)

Lemma elimT {P b} : reflect P b -> b = true -> P.
```

```
Lemma test : is_even 6 /\ is_even 4.
Proof.
  refine (elimT (andP (evenP 6) (evenP 4)) _).
  (* elpi to_bool. *)
  simpl. trivial.
Qed.
```

```
(* [tb P R] finds R : reflect P _ *)
Elpi Tactic to_bool.
Elpi Accumulate lp:{{
  pred tb i:term, o:term.
  tb {{ is_even lp:N }} {{ evenP lp:N }}.
  tb {{ lp:P /\ lp:Q }} {{ andP lp:PP lp:QQ }} :- tb P PP, tb Q QQ.

  solve (goal _ _ Ty _ _ as G) GL :-
    tb Ty P, refine {{ elimT lp:P _ }} G GL.              }}.
```

Demo: from Prop to bool

```
Axiom is_even : nat -> Prop.

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  | O => true
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Lemma evenP n : reflect (is_even n) (even n).
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Lemma elimT {P b} : reflect P b -> b = true -> P.
```

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Lemma test : is_even 6 /\ is_even 4.
Proof.
  refine (elimT (andP (evenP 6) (evenP 4)) _).
  (* elpi to_bool. *)
  simpl. trivial.
Qed.
```

```
(* [tb P R] finds R : reflect P _ *)
Elpi Db tb.db lp:{{ pred tb i:term, o:term. }}.

Elpi Tactic to_bool.
Elpi Accumulate Db tb.db.
Elpi Accumulate lp:{{
  solve (goal _ _ Ty _ _ as G) GL :-
    tb Ty P, refine {{ elimT lp:P _ }} G GL.           }}.

Elpi Command add_tb.
Elpi Accumulate Db tb.db.
Elpi Accumulate lp:{{
  pred compile i:term, i:term, i:list prop, o:prop.
  compile {{ reflect lp:P _ }} R Todo (tb P R :- Todo).
  compile {{ reflect lp:S _ -> lp:Ty }} R Todo (pi h\C h) :-
    pi h\ compile Ty {coq.mk-app R [h]} [tb S h|Todo] (C h).
  compile {{ forall x, lp:(Ty x) }} R Todo (pi x\ C x) :-
    pi x\ compile (Ty x) {coq.mk-app R [x]} Todo (C x).

  main [str S] :-
    coq.locate S GR,
    coq.env.typeof GR Ty,
    compile Ty (global GR) [] C,
    coq.elpi.accumulate _ "tb.db" (clause _ _ C).       }}.
```

The good company

https://github.com/coq-community/metaprogramming-rosetta-stone

Comparison

	Elpi	Ltac2	MetaCoq
Gallina	◕ no mutual fix/ind	●	●
Bound Variables	●	◔ quotations	◔ toplevel quotation
Holes	●	◕ tactic monad	◔ only AST
Proof API	◑ weak ltac1 bridge	● (sufficiently close)	◔ only TC search, obligations
Vernacular API	◕ no notations, obligations	○	◔ only env, obligations
Reasoning logic	◔ Abella	○	◕ no holes, unif

To the best of my knowledge, on 1/1/2025

Conclusion

Elpi for Rocq: take home

Extension language

- Use a language (ony) when it is a good fit
- Good FFI → many APIs!

Rule-based is a good fit for

- HOAS (binders and local context)
- prover logical environment (global context)
- (meta) meta programming (homoiconicity)

Ongoing and future work on Rocq-Elpi

- Type Class solver (D.Fissore PhD)
- Obligations (commands that start a proof)
- Mutual fixpoints and inductives (needed by 2 power users)

Ongoing and future work on Elpi

- Mode and determinacy analysis
- Memoization (tabling)

Thanks!

https://github.com/LPCIC/coq-elpi/

For having invited me, for listening, and for **contributing** code:

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Questions?