Dynamics, Consensus, and Distributed Community Detection

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joint work with

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HIGHLIGHTS OF ALGORITHMS

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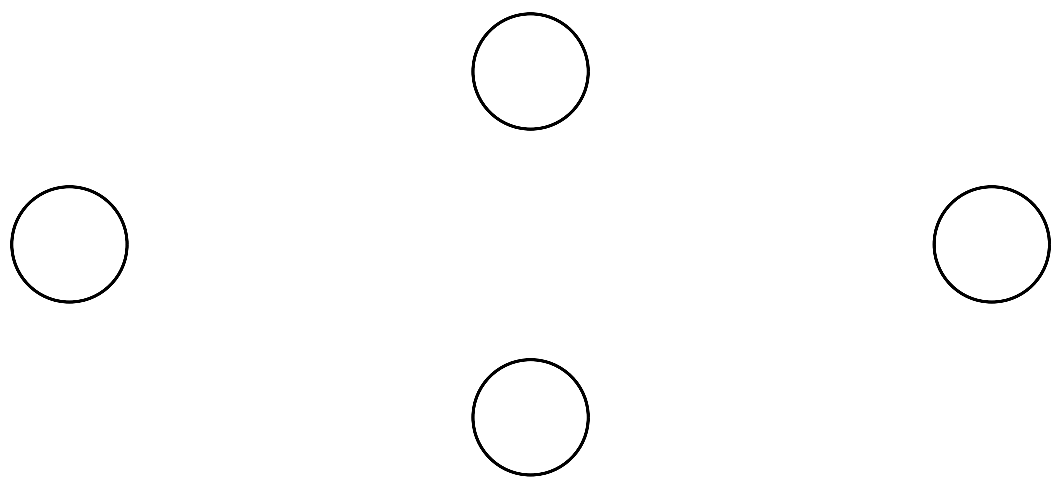
The Morale

Dynamics are cool!

The PULL Model

At each round, each agent observes the states of a fixed-size sample of neighbors, and updates her state accordingly.



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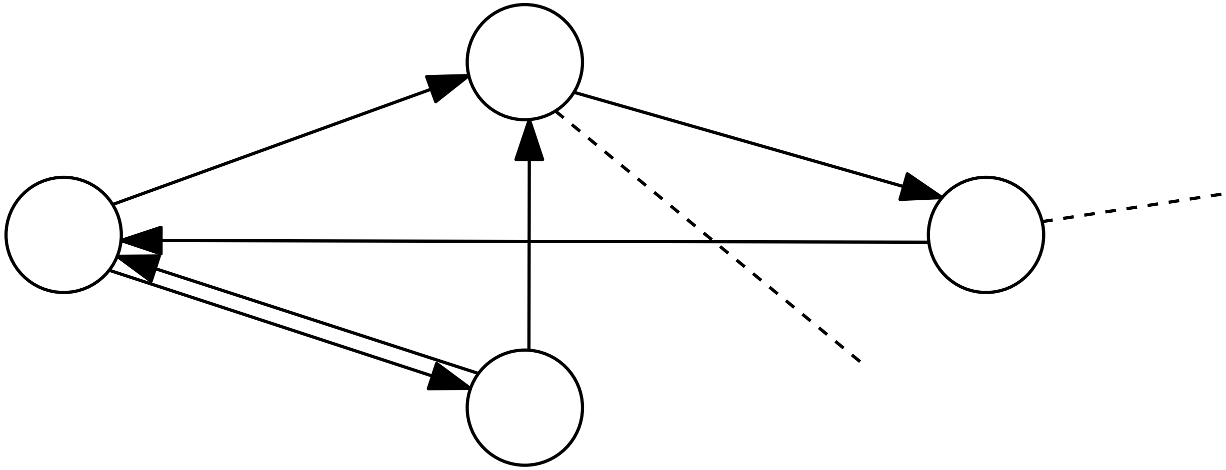

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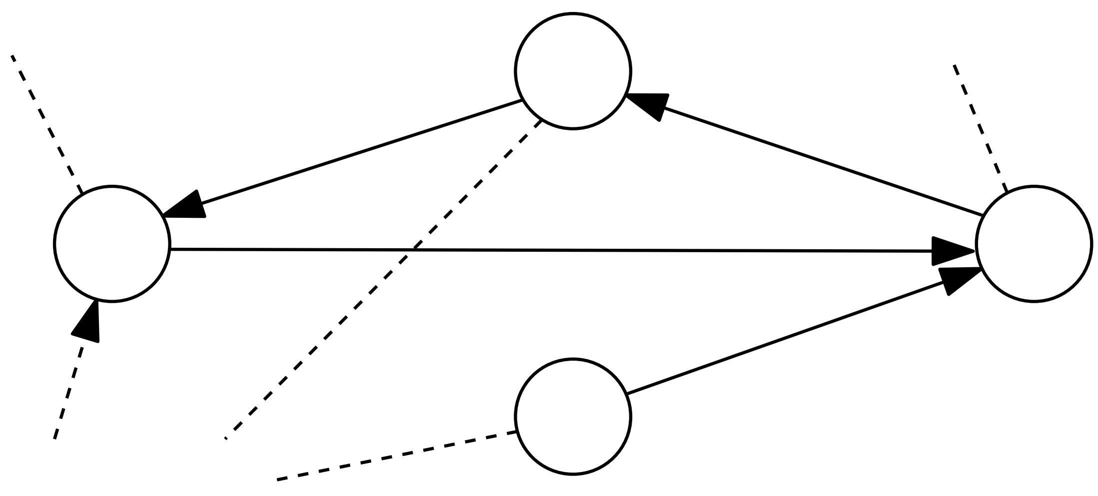

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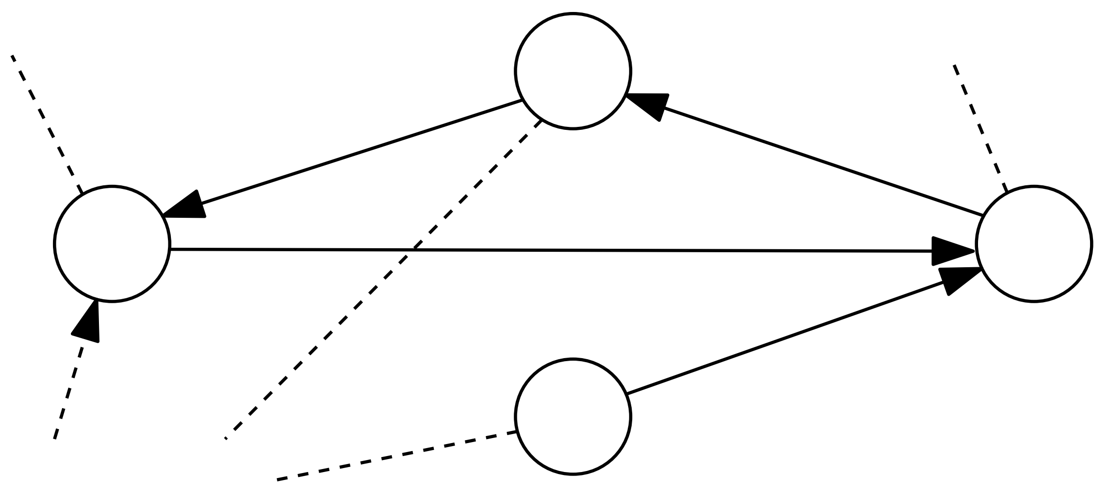

Uniform PULL model $\longrightarrow$ Complete graph

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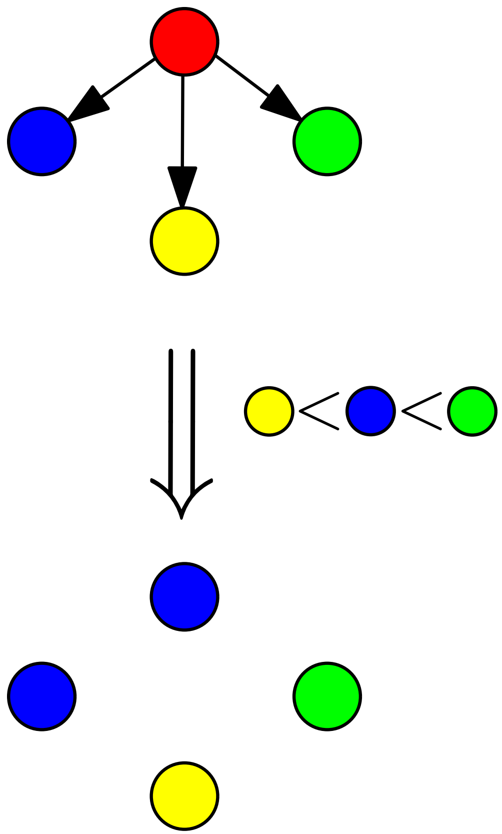
Examples of Dynamics

- 3-Median dynamics



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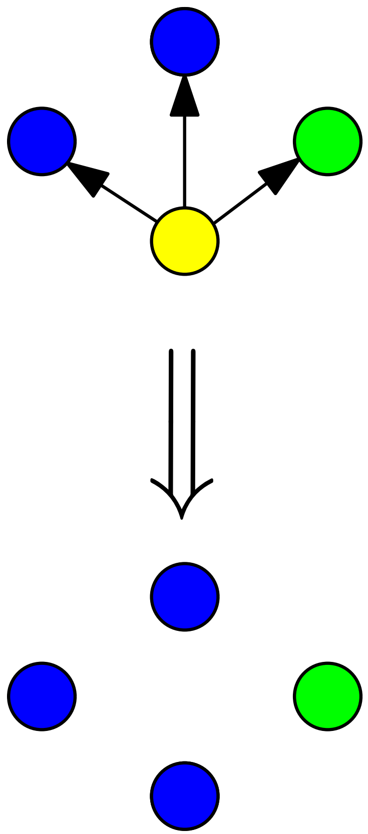

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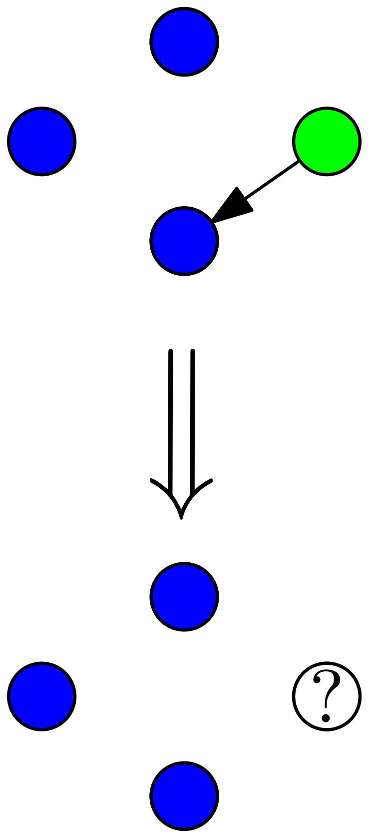

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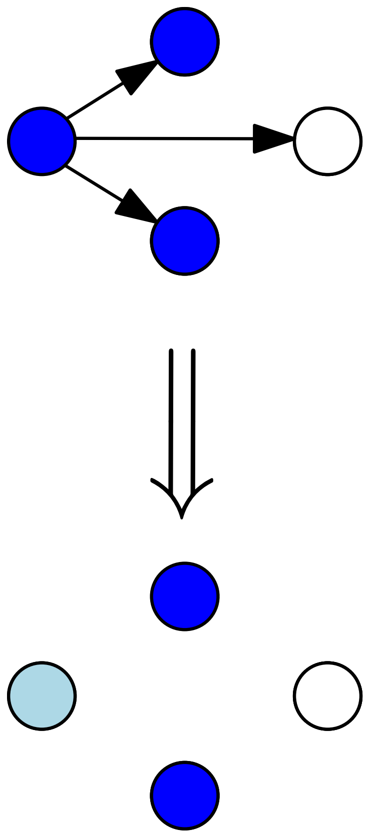

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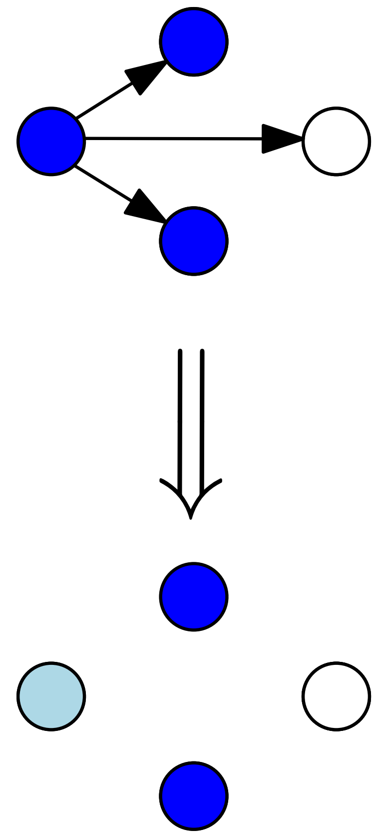
(non-uniform PULL)



The Power of Dynamics

3-Median dynamics [Doerr et al. '11]. Converge to $\mathcal{O}(\sqrt{n \log n})$ approximation of median of system in $\mathcal{O}(\log n)$ rounds w.h.p., even if $\mathcal{O}(\sqrt{n})$ states are arbitrarily changed at each round ($\mathcal{O}(\sqrt{n})$-bounded adversary).

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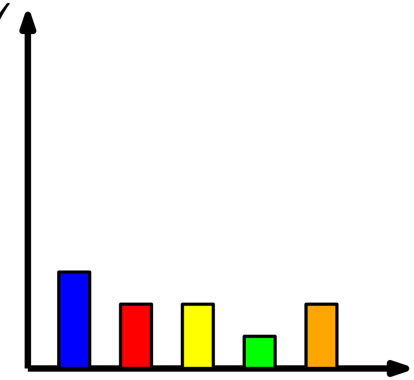
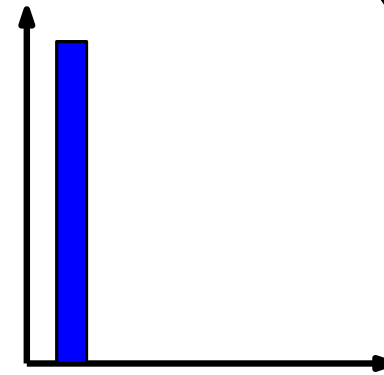
Undecided-State dynamics [Becchetti et al. '15]. If majority/second-majority ($c_{maj}/c_{2^{nd}maj}$) is at least $1+\epsilon$, system converges to plurality within $\tilde{\Theta}(\sum_i c_i^2/c_{maj}^2)$ rounds w.h.p.

A Global Measure of Bias

$$\mathrm{md}(\mathbf{c^{(0)}}) := \sum_{i=1}^{k} \left(\frac{c_i^{(0)}}{c_{maj}^{(0)}} \right)^2 = 1 + \mathcal{D}\Bigg($$

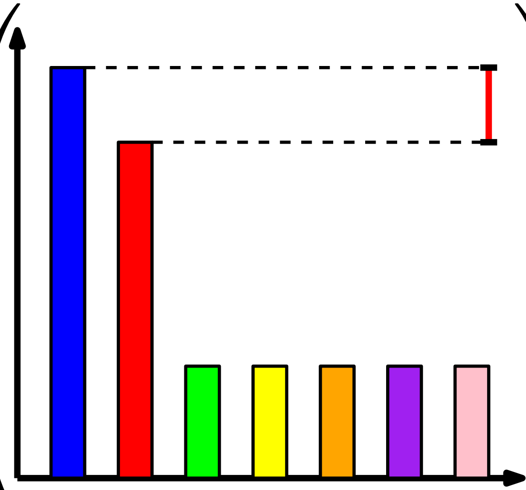


$,$



$\Bigg)$

$1 \leq \mathrm{md}\Bigg($



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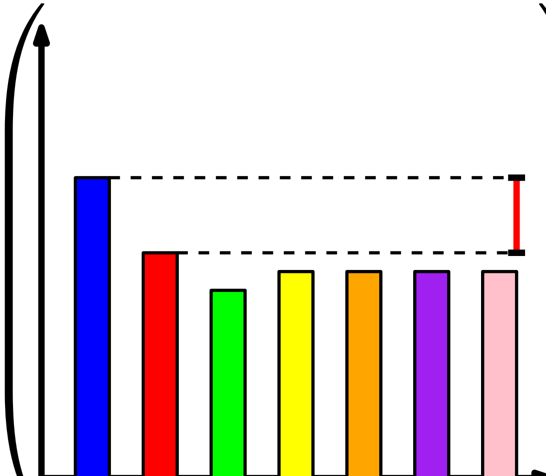

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Averaging dynamics. P matrix of simple random walk on a graph, $x^{(t)}$ config. at time t. Update rule $x^{(t+1)} = P \cdot x^{(t)}$.

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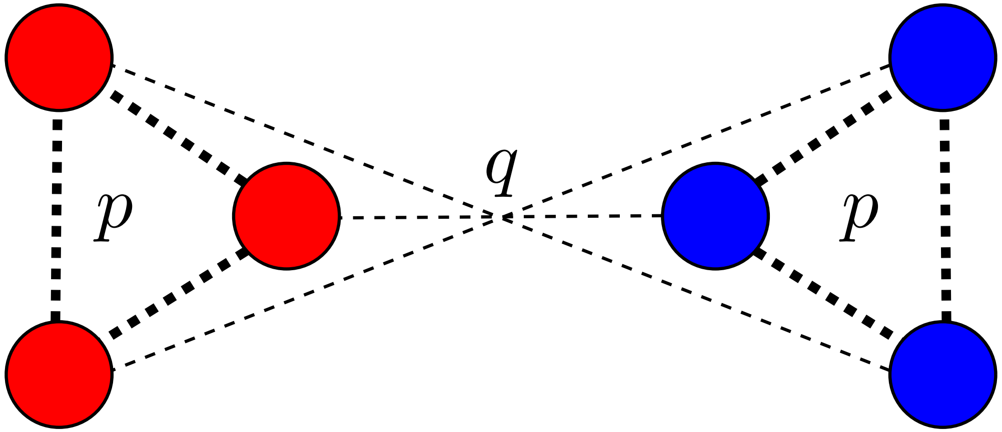
x is eigenvector of $P = D^{-1}A$ iff $D^{1/2}x$ is eigenvector of $N = D^{-1/2}AD^{-1/2}$.

From Consensus to Community Detection

Stochastic Block Model (SBM). Two "communities" of equal size V_1 and V_2, each edge inside a community included with probability p, each edge across communities included with probability $q < p$.



Reconstruction problem. Given graph generated by SBM, find original partition.

Clustering via a Dynamics

Expected matrix

$$\mathbb{E}[P] = \begin{pmatrix} p & \dots & p & q & \dots & q \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p & \dots & p & q & \dots & q \\ q & \dots & q & p & \dots & p \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ q & \dots & q & p & \dots & p \end{pmatrix}$$

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[Becchetti et al. '16] If $x^{(t+1)} - x^{(t)} > 0$ you are blue, if $x^{(t+1)} - x^{(t)} < 0$ you are red: distributed clustering on SBM and other *clusterized* models in $\mathcal{O}(\log n)$ (no dependency on mixing time).

Thank you!