

On Separation Logic, Computational Independence, and Pseudorandomness

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Outline

- ▶ Introduction on separation logic.
- ▶ Barthe et al.'s separation logic for *probabilistic programs*.
- ▶ **Our contribution: a separation logic for *computational cryptography*.**

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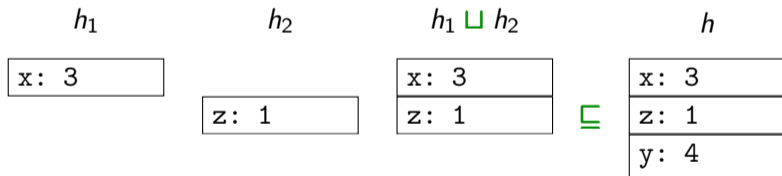
$$h \models \phi * \psi \quad :\Leftrightarrow \quad \exists h_1, h_2 \text{ s.t. } h_1 \models \phi, h_2 \models \psi, \text{ and } h_1 \sqcup h_2 \sqsubseteq h,$$

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$$h_1 \models x = 3$$

$$h_2 \models z = 1$$

$$h \models x = 3 * z = 1$$

Probabilistic Separation Logic (**PSL**) (1/2)

The interpretation of \sqcup and \sqsubseteq determines the semantics of $*$.

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\sqcup	Store union	
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$*$	Locality	

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Example

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PSL can prove perfect secrecy.

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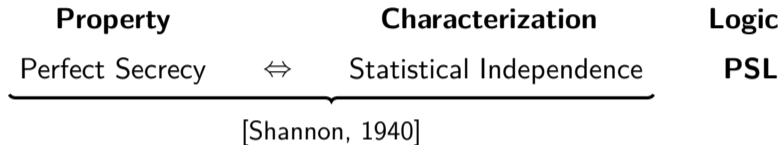
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Theorem (Main Result)

The semantics of the separating conjunction ($*$) in **CSL** is equivalent to Fay's computational independence.

Polytime Programs

Syntax:

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Semantics:

$$\llbracket \Delta \vdash P \rrbracket : \llbracket \Delta \rrbracket \rightarrow \llbracket \Delta \rrbracket$$

Our semantics is polytime in the security parameter *by definition*.

CSL's Syntax and Semantics

$A ::= \mathbf{EQ}(e, g)$
| $\mathbf{CI}(e, g)$

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- ▶ For $((\phi)^\Gamma * (\psi)^\Theta)^\Delta$, Γ and Θ must also be disjoint.

Typed Separating Conjunctions

The standard interpretation of $*$ is ambiguous:

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- ▶ In **PSL**, we do not know which variables are independent.
- ▶ In **CSL**, independent variables are *explicit in formulas*.

$$d \models ((\phi)^\Gamma * (\psi)^\Theta)^\Delta \Rightarrow \text{variables of } \Gamma \text{ and } \Theta \text{ are independent in } d.$$

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Judgments:

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Difficulty with loops

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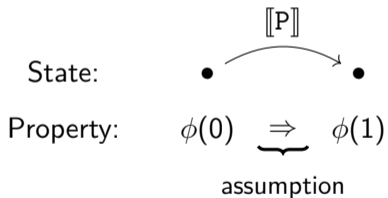
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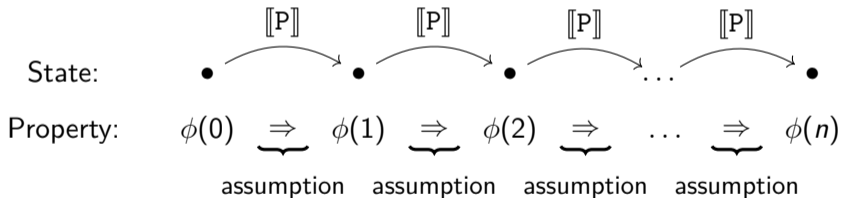
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POTP := key ← unif( $n$ );  
        $r$  ←  $g(\text{key})$ ;  
       cyph ←  $\text{msg} \oplus r$ .
```

g is a *pseudorandom* generator.

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The Pseudo One Time Pad is the *computationally secret* variant of the One Time Pad.

```
POTP := key ← unif(n);  
       r ← g(key);  
       cyph ← msg ⊕ r.
```

g is a *pseudorandom* generator.

$$\vdash_{\text{CSL}} \{(\top)^\Delta\} \Delta \vdash \text{POTP} \underbrace{\{((\top)^{\{msg:\dots\}} * (\mathbf{CU}(cyph))^{\{cyph:\dots\}})^\Delta\}}_{\text{computational secrecy}}$$

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Future work

- ▶ Extend the language supported by **CSL** with for-loops.