

# On Separation Logic, Computational Independence, and Pseudorandomness

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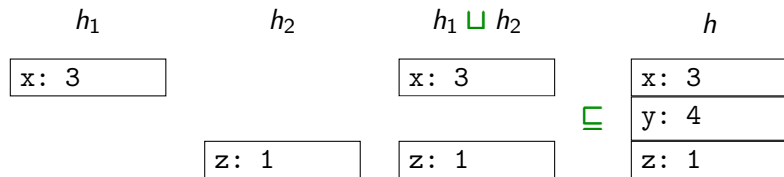


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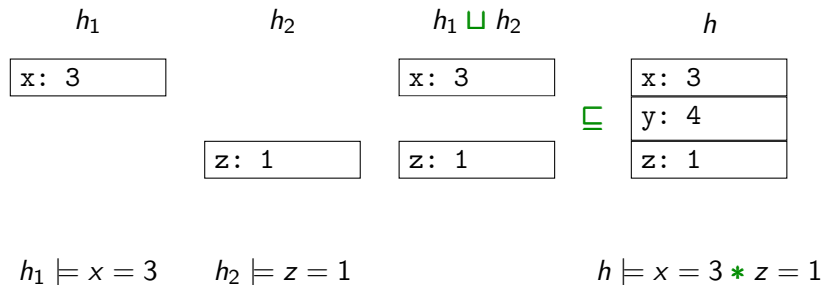
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# Probabilistic Separation Logic (PSL) (1/2)

The interpretation of  $\sqcup$  and  $\sqsubseteq$  determines the semantics of  $*$ .

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PSL can prove perfect secrecy.

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## Theorem (Main Result)

*The semantics of the separating conjunction ( $*$ ) in CSL is equivalent to Fay's computational independence.*

# Polytime Programs

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Programs are polytime in the security parameter *by construction*.

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- ▶ They must also be disjoint for  $*$ .

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- ▶ CSL *formulas* tell us which variables are independent.

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The standard Hoare rule for for-loops is *unsound* in CSL:

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After  $n$  iterations, the bound on the advantage is:  $n^n/2^n$ .



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$$\vdash_{\text{PSL}} \underbrace{\{\mathbf{D}(msg)\}}_{msg \text{ is defined}} \text{OTP} \underbrace{\{\mathbf{D}(msg) * \mathbf{U}(chip)\}}_{\text{perfect secrecy}}$$

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## Future work

- ▶ Extend the language supported by CSL with for-loops.