Kinematic analysis of a spatial four-wire driven parallel crane without constraining mechanism

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Abstract We are interested in wire-driven parallel robot with four wires and at least two distinct attachment points on the end-effector. Such type of robot is non redundant, it exhibits 4 d.o.f. and can be used as a crane. This paper addresses the inverse and forward kinematics problem, taking into account the mechanical equilibrium equations. We show that surprisingly the forward kinematics can be solved, either for determining all solutions or in a real-time context, but that the inverse kinematics is still an open issue.

1 Introduction

We are interested in a parallel robot whose end-effector is driven by 4 wires, without any additional constraining mechanism [10]. We denote by B_i the location of the attachment points on the end-effector, by A_i the location of the attachment points on the base and by C the center of the platform. Such robot exhibits various interesting configurations (figure 1):

- planar motion with only a single attachment point on the end-effector (a): the robot exhibits 2 translational d.o.f. and has a degree of redundancy of 2
- planar motion with 2 to 4 distinct attachment points on the end-effector (b): the robot exhibits 3 d.o.f. and has a degree of redundancy of 1
- non planar motion with only a single attachment point on the end-effector (c): the robot exhibits 3 translational d.o.f. and has a degree of redundancy of 1
- non planar motion with at least two distinct attachment points on the end-effector (d): the robot exhibits 4 d.o.f. and is not redundant

The two first categories offer interesting applications (such as fast pick-and place, windows washing). We are however more interested in the two categories exhibiting spatial motions, which can be used as a crane e.g. for rehabilitation or patient lifting at home or in hospitals.

The kinematics of the three first categories is mastered for robots having rigid legs and can be managed for wire legs for which we have to ensure that the tension are always positive [1, 2, 3, 5, 6, 7, 8, 12]. As for the fourth category it appears that to the best of our knowledge the kinematic problems have never been addressed.

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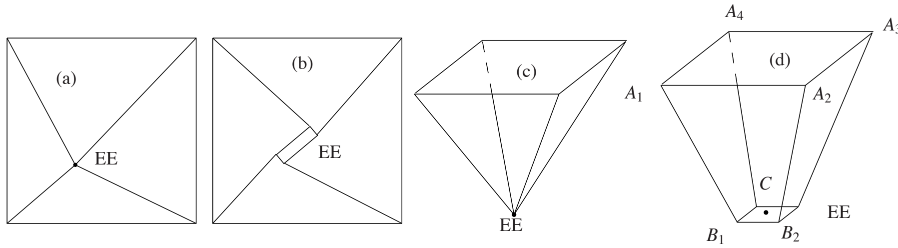

Fig. 1 4 different configurations of a wire-driven parallel robot with 4 wires

2 Forward kinematics

In this problem the lengths ρ_i of the 4 wires are known and we have to determine the pose of the end-effector. We will study this problem for a crane configuration, assuming that the end-effector is only submitted to a load consisting of a mass attached at the center C of the platform, which is supposed to be the center of mass of the platform.

Without loss of generality we will assume that the end-effector has 4 distinct attachment points for the wires and that these points are coplanar, threem of them at least being distinct. Under this assumption a pose of the platform may be parametrized by the coordinates of 3 attachments points in an arbitrary reference frame with origin O. The choice of these 3 points being also arbitrary (provided that they are not collinear) we will use as parameters the 9 coordinates of points B_1, B_2, B_3. For a planar platform we know that there exist 3 constants $\lambda_1, \lambda_2, \lambda_3$ such that:

$$\mathbf{OB_4} = \lambda_1 \mathbf{OB_1} + \lambda_2 \mathbf{OB_2} + \lambda_3 \mathbf{OB_3} \tag{1}$$

Furthermore we have the following 3 constraint equations on the unknowns:

$$||\mathbf{B_1B_2}|| = d_{12} \quad ||\mathbf{B_1B_3}|| = d_{13} \quad ||\mathbf{B_3B_2}|| = d_{32} \tag{2}$$

where d_{ij} denotes the known distance between the attachment points B_i and B_j.

Note also that the coordinates of the A_i points are supposed to be known in the same reference frame and that the square of the wire lengths is obtained as:

$$\rho_i^2 = ||\mathbf{A_iB_i}||^2 \tag{3}$$

Decomposing $\mathbf{A_iB_i}$ as $\mathbf{A_iB_i} = \mathbf{OB_i} - \mathbf{OA_i}$ we get 4 equations:

$$\begin{aligned} &\rho_1^2 = ||\mathbf{OB_1} - \mathbf{OA_1}||^2 \;\; \rho_2^2 = ||\mathbf{OB_2} - \mathbf{OA_2}||^2 \;\; \rho_3^2 = ||\mathbf{OB_3} - \mathbf{OA_3}||^2 \\ &\rho_4^2 = ||\lambda_1 \mathbf{OB_1} + \lambda_2 \mathbf{OB_2} + \lambda_3 \mathbf{OB_3} - \mathbf{OA_4}||^2 \end{aligned} \tag{4}$$

We have then to consider that the system is in mechanical equilibrium. Let τ_i denotes the tension in the i-th wire, τ be the 4 wires tension vector, $\mathscr{F}$ be the wrench exerted on the end-effector (here reduced to $0,0,-mg,0,0,0$), where m is the load mass) and $\mathbf{J^{-T}}$ be the transpose of the inverse jacobian matrix of the robot. Assuming no mass for the wires we have

$$\mathscr{F} = \mathbf{J^{-T}}\tau \tag{5}$$

Note that the inverse jacobian matrix is a 4×6 matrix whose formulation is well known. The above relation defines 6 equations having as unknowns the coordinates x_j, y_j, z_j of B_1, B_2, B_3 and the 4 tensions in the wires. The set of equations (2,4,5) define a square system of 13 equations in the 13 unknowns (the four τ_i and the 9 coordinates of the $\mathbf{OB_i}$). We may also solve the four first mechanical equilibrium equations in the τ_i, thereby obtaining a *reduced system* of 9 equations in 9 unknowns. Note that the initial equations (5) involves the mass m of the load but there is a linear relationship between m and the τ_i's which indicates that we may assign an arbitrary mass to the load, solve the system and then apply a scaling factor to calculate the τ_i corresponding to the real mass of the load.

2.1 Computing all solutions

This system is much more complicated than the forward kinematics of more classical parallel robots because of the involvement of the static equilibrium equations. For determining the possible solutions we may however try to use similar solving methods than for robot with rigid legs. As we are interested in certified solutions we may rely only on the Groebner basis [4] and interval analysis approaches. Drawbacks of the Groebner basis approach is that it requires to have only algebraic equations with rational coefficients and has a complexity that is exponential in terms of the number of unknowns. It is unclear if we will be able to obtain the Groebner basis for the system. On the other hand the interval analysis approach may still be tried as its exponential complexity is valid only in the worst case, while its practical complexity is quite often much lower, as soon as bounds may be determined for the unknowns. Such bounds may easily be obtained in that case: if x^a_j, y^a_j, z^a_j are the coordinates of A_j and if d_{ij} denotes the distance between B_i, B_j , then $x_j \in [\mathrm{Max}(x^a_j - \rho_j, x^a_i - \rho_i - d_{ij}), \mathrm{Min}(x^a_j + \rho_j, x^a_i + \rho_i + d_{ij}), i \neq j]$, $y_j \in [\mathrm{Max}(y^a_j - \rho_j, y^a_i - \rho_i - d_{ij}), \mathrm{Min}(y^a_j + \rho_j, y^a_i + \rho_i + d_{ij}), i \neq j]$, $z_j \in [z^a_j - \rho_j, z^a_j]$ (we are interested only in the poses where the end-effector is under the base). We need also bounds for the τ_i: a lower bound is 0 as we want the wires to be in tension and the upper bound may be reasonably set to twice the value of mg.

2.2 Real-time algorithm

Forward kinematics is a key point for the real-time control of parallel robot. The problem is here somewhat different from the one of computing all solutions, as forward kinematics is computed at each sampling time of the controller. Usually the Newton-Raphson scheme (NR) is used with as initial guess the solution obtained at the previous sampling time. This is a dangerous process as the NR scheme may not converge, or, worse, converge to a solution that is not the one corresponding to the current pose of the robot. However we have already designed a certified NR method that uses both interval analysis and the classical NR method to provide the right solution [9]. This method may be used as well for the wire-driven robot.

2.3 Example

We consider the robot with the A_i, B_i points defined as:

$$A_1 = (185.6, 0.6) \;\; A_2 = (199.4, 105.9) \;\; A_3 = (14.7, 119.4) \;\; A_4 = (0.6, 14.5)$$
$$B_1 = (0.6, -5.25) \;\; B_2 = (0.6, 5.25) \;\; B_3 = (-0.6, 5.25) \;\; B_4 = (-0.6, -5.25)$$

The wire lengths are:

$$\rho_1 = 138.471017 \quad \rho_2 = 149.42176 \quad \rho_3 = 145.908576 \quad \rho_4 = 143.793263$$

The forward kinematics has 4 solutions, shown on figure 2, that are computed in about 13 minutes on a DELL D620 laptop. Note that the reduced system that has only 9 equations requires more computation time as the two last of the equations of the system are quite complex. It must be noted that solutions 2 and 4 exhibit

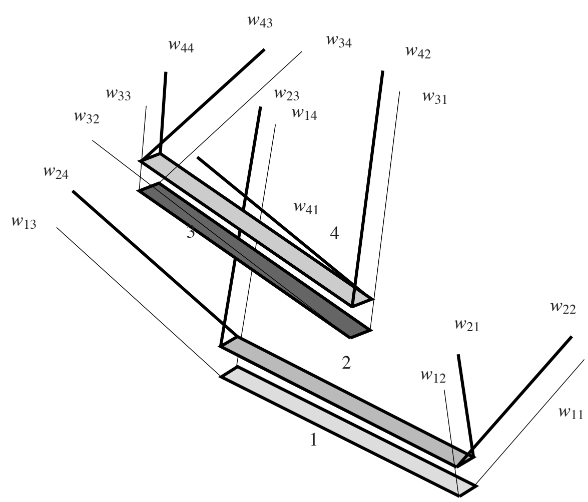

Fig. 2 The four solutions of the forward kinematics. The wire w_{ij} is the wire j for solution i.

a crossing of the wires and a normal to the platform that is pointed downward: such solution should not be retained. As for the certified NR scheme the average computation time is less than 0.1 ms.

3 Inverse kinematics

For the spatial configuration we have a 4 d.o.f. robot and hence we have motion constraints between the 6 d.o.f. of the end-effector. We will first investigate these motion constraints.

3.1 *Motion constraints*

Equations (5, 2) define a set $\mathscr{S}$ of 9 equations in 13 unknowns (the 9 coordinates of B_1, B_2, B_3 and the 4 joint forces τ_i). Hence as expected we have four degrees of freedom. We may also parametrize the pose of the end-effector by the 3 coordinates x_c, y_c, z_c of C and three orientation angles (e.g. the Euler angles ψ, θ, ϕ) in which case we have 10 unknowns and the 6 equations (5). Hence we have a coupling between the pose parameters and we may determine a constraint equations as follows. Consider the first four equations of (5) as a linear system in τ and solve this system; then we substitute the result in the last two equations of (5) which lead to two constraints equations $\mathscr{S}_1, \mathscr{S}_2$ in the unknowns $x_c, y_c, z_c, \psi, \theta, \phi$ which have respectively degree 2, 2, 2 and 2, 3, 1 in x_c, y_c, z_c. Hence we may solve the second equation in z_c and substitute the result in the first constraint equation to get a single constraint equation relating $x_c, y_c, \psi, \theta, \phi$ which is of degree 5 in x_c and 6 in y_c.

In a crane application the first priority may be to be able to reach a desired location for the center C of the platform and we will be interested in determining a single constraint equation involving only the rotation angles. For that purpose we use the Weierstrass substitution to transform the two constraint equations $\mathscr{S}_1, \mathscr{S}_2$ into algebraic constraints involving $T_1 = \tan(\psi/2), T_2 = \tan(\theta/2), T_3 = \tan(\phi/2)$. The degrees of these equations are respectively 2, 6, 6 and 4, 6, 6 in T_1, T_2, T_3. We then compute the resultant in T_1 of these two equations to get a single constraint equation in T_2, T_3. This resultant factors out in two expressions respectively of degree 8, 8 and 24, 20 in T_2, T_3, that are represented in figure 3. Note that we may expect a symmetry in the constraint curve as if a triplet $(\psi_0, \theta_0, \phi_0)$ is a solution of the two constraint equations, then the triplet $(\psi_0 - \pi, -\theta_0, \phi_0 + \pi)$ is also a solution. It must also be mentioned that not all solutions of the constraint equations lead to a feasible end-effector orientation as we must also check that the tensions in the wires are positive.

Note that equation (5) indicates that the wire lines and the vertical lines going trough C belong to a linear complex and hence that the infinitesimal motion at a pose is an helical motion whose axis and pitch can be calculated [11].

3.2 *Finding an end-effector orientation*

For the crane application we have to find at least one orientation that allows to reach a given location in a mechanical equilibrium. We use the following algorithm:

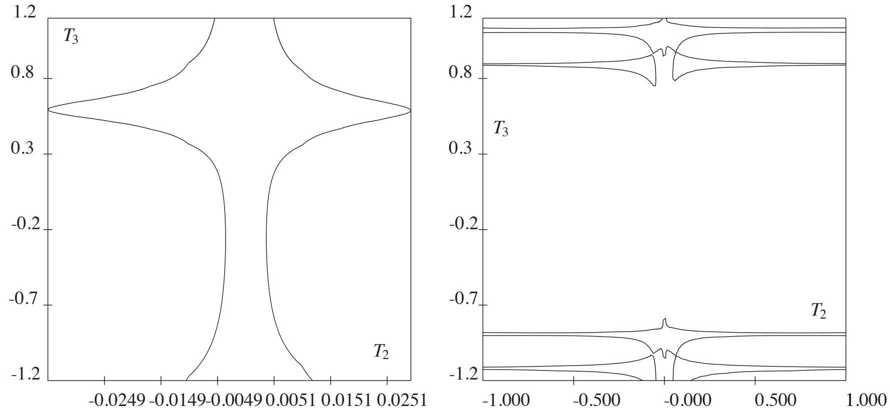

Fig. 3 The allowed region in the $T_2, T3$ space

1. compute the wire lengths for the desired pose assuming rigid legs
2. compute the pose of the end-effector with the forward kinematics
3. if the distance between the current pose and the desired one is lower than a given threshold, then exit
4. let $\Delta\mathbf{X}$ be the difference vector between the desired pose and the current one (no correction is applied for the orientation part). Compute a correction $\Delta\rho$ of the wire lengths as $\mathbf{J^{-1}}\Delta\mathbf{X}$, goto step 2

Although this algorithm is quite efficient we may consider that we may use the fourth d.o.f. to determine the ρ such that the obtained orientation is ”close” in some sense to a desired orientation of the end-effector. However a major problem is to define an appropriate metric to measure the ”closeness” between two orientations.

We propose to use as closeness index $\mathscr{C}$ the sum of the distances between the location of the points B_1^d, B_2^d, B_3^d at the desired orientation and the location of these points at an orientation that is compatible with the constraint equations $\mathscr{S}_1, \mathscr{S}_2$:

$$\mathscr{C} = \sum_{j=1}^{j=3} ||B_j - B_j^d||$$

Determining this minimum is however a difficult task as it amounts to solve a constrained optimization problem. It may be solved by using the Lagrange multipliers method. We define H as

$$H = \mathscr{C} + l_1\mathscr{S}_1 + l_2\mathscr{S}_2$$

where l_1, l_2 are the Lagrange multipliers. The minimum of $\mathscr{C}$ must satisfy

$$\frac{\partial H}{\partial\psi} = \frac{\partial H}{\partial\theta} = \frac{\partial H}{\partial\phi} = \frac{\partial H}{\partial l_1} = \frac{\partial H}{\partial l_2} = 0 \tag{6}$$

The two first equations are linear in l_1, l_2 and the result substituted in the third equations. Consequently we get a system of 3 equations in ψ, θ, ϕ. This system is relatively large but may still be solved with interval analysis in about 8 minutes, which is however incompatible with a real time use. However as soon a good initial estimate of the solution is known the certified NR scheme is working efficiently. Hence if we assume that the task should be performed with an orientation that is close to a given one we may determine the best orientation at the beginning of the task and maintains its value through the NR scheme during the task, which requires to calculate the best orientation only once, at the start of the task. We propose to use a continuation method [13] for getting this initial guess. The solutions of the system $\mathscr{S}_1, \mathscr{S}_2$ is determined for a given $\theta = \theta_0$ in the range $[0, \pi/2]$: only a positive θ is needed as the solutions are symmetrical with respect to that variable and we limit this angle to $\pi/2$ as we want to have the normal of the platform oriented upward. Starting from these solutions we use the NR scheme to determine the solutions for $\theta = \theta_0 + \Delta\theta$, where $\Delta\theta$ is a small increment that is automatically determined by the algorithm. For each solution we calculate the wire tensions and, if they are positive, the closeness index. The solution presenting the lowest closeness is retained as initial guess. As an example we choose the following coordinates for C: 90, 60, -80 and gives as desired orientation $\psi = \theta = \phi = 0$. Figure 4 presents the closeness index curve that are obtained. The initial guess is obtained as $\psi = 1.853rd, \theta = 0.103rd, \phi = 4.427rd$ that leads to a closeness index of 0.0616, which is almost optimal. This scheme is relatively fast: about 1 minute is necessary to compute the initial guess.

Fig. 4 The closeness index as a function of θ and a close view of the minimum.

4 Conclusion

In this paper we have addressed the difficult problem of the kinematic analysis of a four wire-driven spatial parallel crane. This analysis shows that both the geometrical relations and the static equilibrium have to be taken into account. It appears that the forward kinematics, although difficult as for any parallel robot, may still be managed in real-time.

Surprisingly the inverse kinematics still remains an open issue. We have identified the relationship between the pose parameters and have proposed some strategies for computing the joint variables in specific cases but a generic solution still has to be developed.

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