Ontology-Based Query Answering over Datalog-Expressible Rule Sets is Undecidable

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Preliminaries

The OBQA Problem for Existential Rules

Definition: The OBQA Problem

Given a set $\mathcal{R}$ of existential rules, a set $\mathcal{F}$ of facts, and a fact φ; decide if $\langle \mathcal{R}, \mathcal{F} \rangle \models \varphi$ under standard FO semantics.

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Example: Existential Rules

$$\forall x, y, z. \big(\mathit{Connected}(x, y) \land \mathit{Connected}(y, z) \rightarrow \mathit{Connected}(x, z) \big)$$

$$\forall x. \big(\mathit{Human}(x) \rightarrow \exists y. \mathit{HasParent}(x, y) \land \mathit{Human}(y) \big)$$

$$\forall x, y, z. \big(P(x, y, z) \land Q(x, z) \rightarrow \exists w, v. R(x, y, v, w) \land S(w, y) \land P(z, z, w) \big)$$

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Example: Facts

$$\mathit{Connected}(\mathit{paris}, \mathit{montpellier}) \qquad P(c, d, e)$$

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Theorem

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1. Bottom-up/Materialisation: compute a finite representation of a *universal model* $\mathcal{U}$ of $\langle \mathcal{R}, \mathcal{F} \rangle$ and check if $\mathcal{U} \models \varphi$.

2. Top-down/Rewriting: compute a rewriting γ of the *rule query* $\langle \mathcal{R}, \varphi \rangle$ and check if $\mathcal{F} \models \gamma$.

Example: A Chase Terminating Rule Set

$$Colleague(x, y) \wedge MainAff(x, z) \wedge MainAff(y, w) \rightarrow z \approx w \quad (1)$$

$$Colleague(x, y) \rightarrow Colleague(y, x) \quad (2)$$

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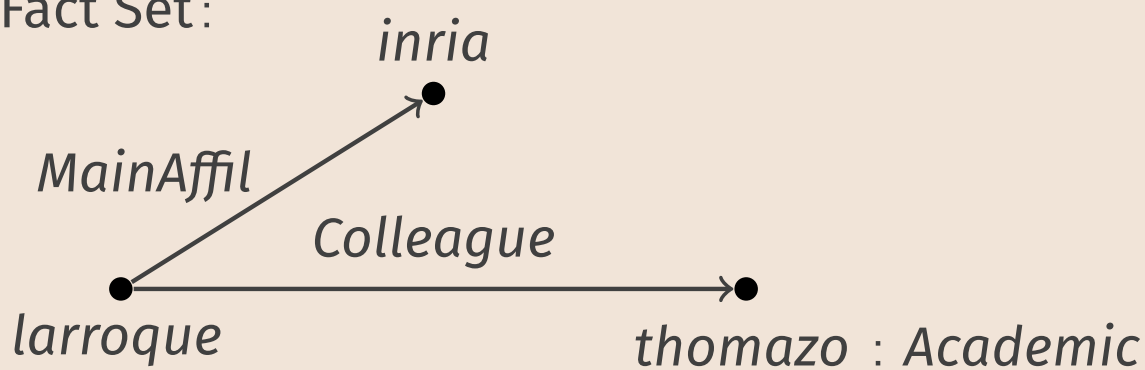
Input Fact Set :



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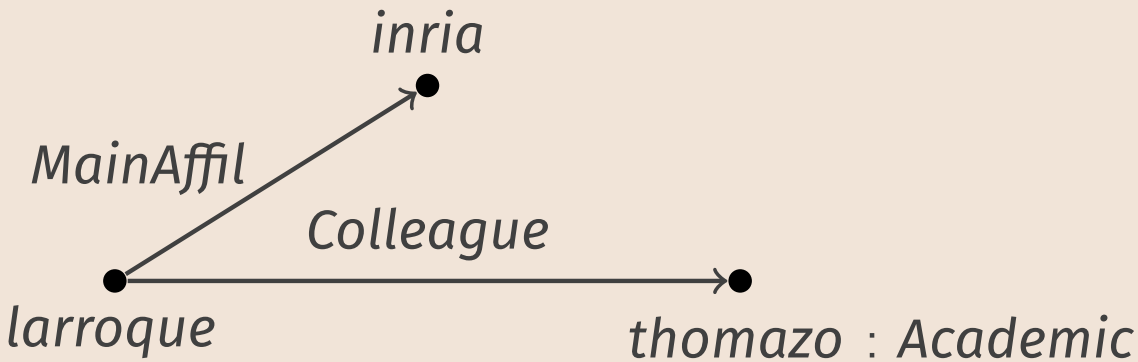

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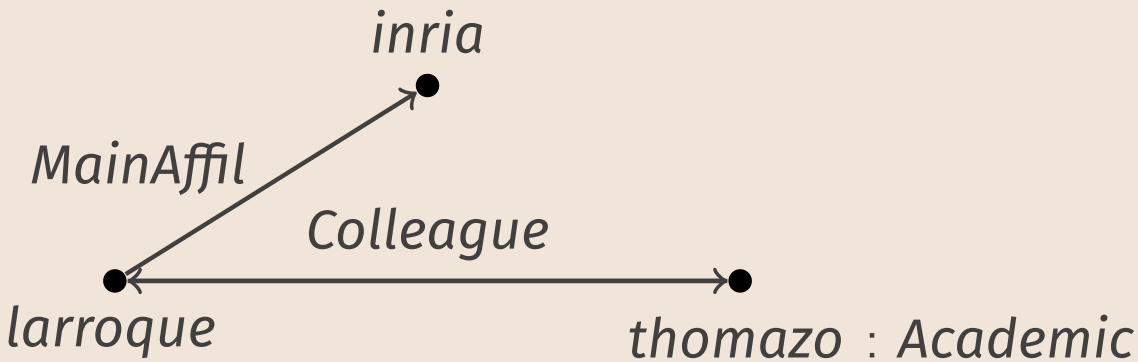

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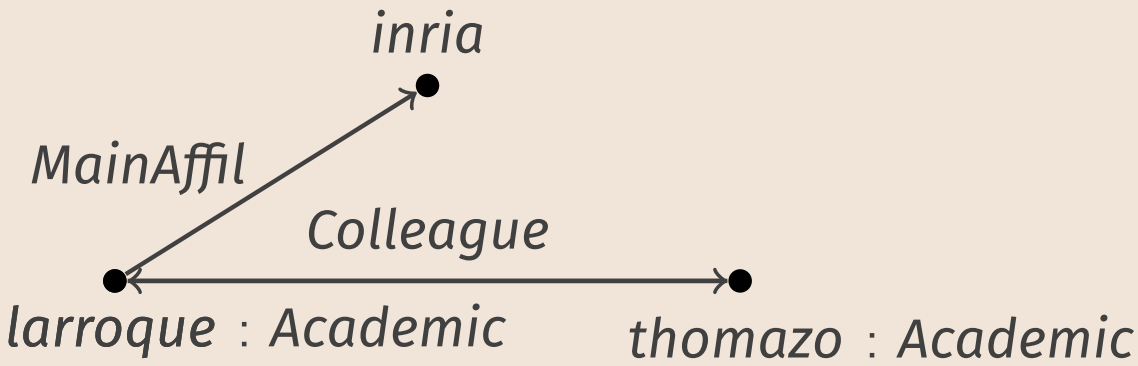

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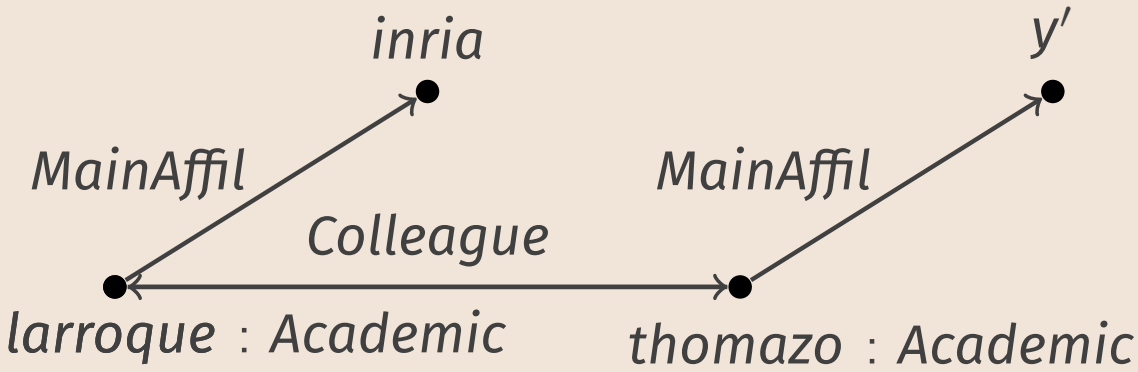

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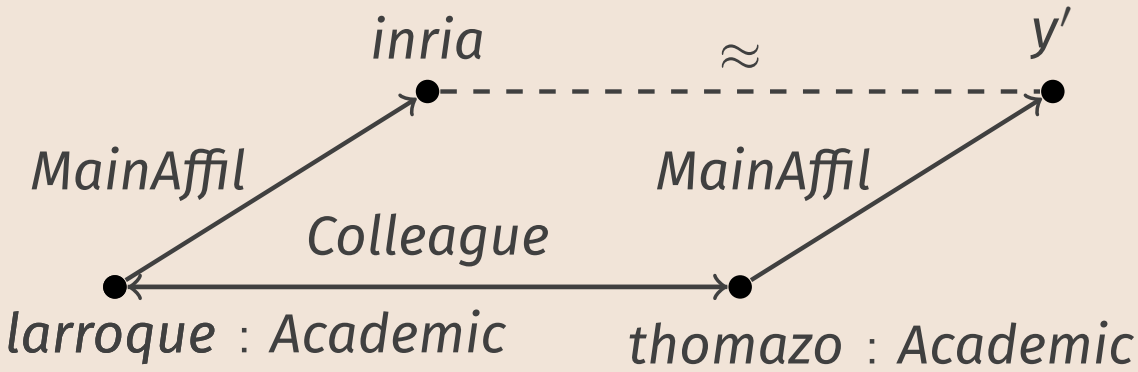

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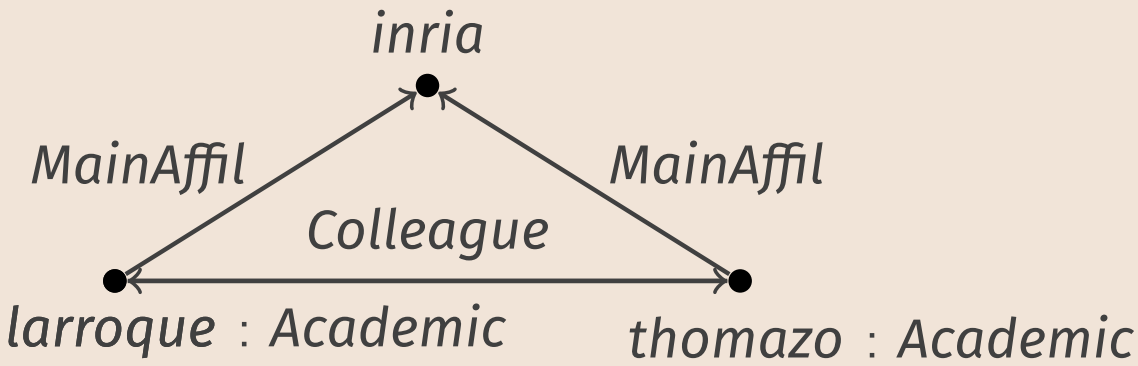

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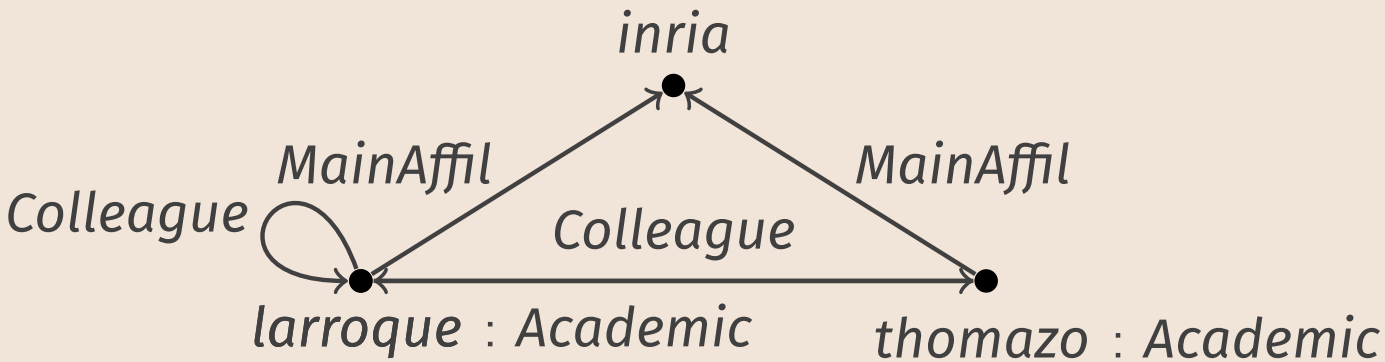

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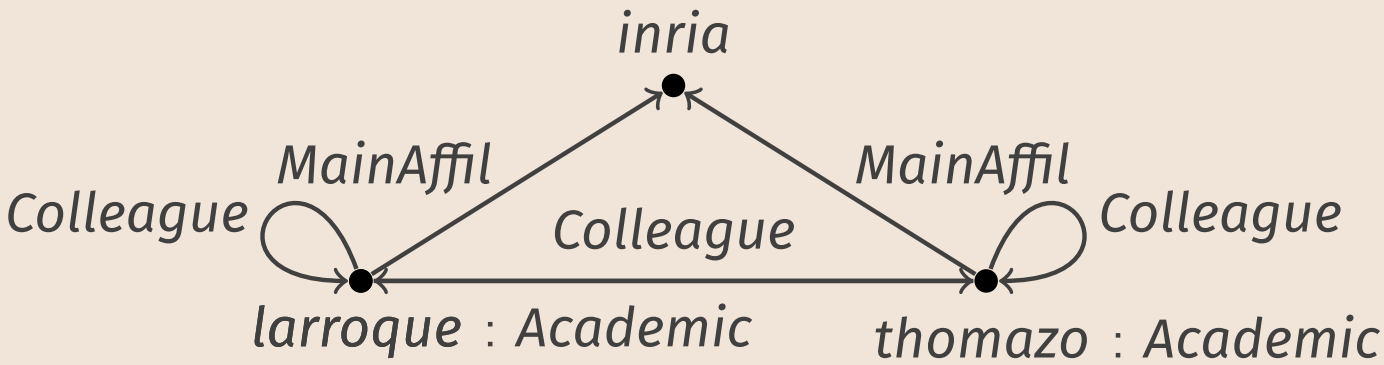

Example: A UBCQ-Expressible Rule Query

Consider the rule set $\mathcal{R}$:

$$MathProfessor(x) \rightarrow Professor(x) \tag{6}$$

$$MathClass(x) \rightarrow Class(x) \tag{7}$$

$$Teaches(x, y) \wedge Class(y) \rightarrow Professor(x) \tag{8}$$

$$Class(x) \rightarrow \exists y.IsTaughtBy(x, y) \wedge Professor(y) \tag{9}$$

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The rule query $\langle \mathcal{R}, \mathit{Professor}(al) \rangle$ admits the UBCQ-rewriting:

$$\gamma = \mathit{Professor}(al) \vee \mathit{MathProfessor}(al) \vee \big(\exists y.\mathit{Teaches}(al, y) \wedge \mathit{Class}(y)\big) \vee \big(\exists y.\mathit{Teaches}(al, y) \wedge \mathit{MathClass}(y)\big)$$

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That is, for every fact set $\mathcal{F}$, we have that:

$$\langle \mathcal{R}, \mathcal{F} \rangle \models \mathit{Professor}(al) \iff \mathcal{F} \models \gamma$$

Motivation

One Procedure to Rewrite Them All

Definition: UBCQ-Expressibility

A rule query $\langle \mathcal{R}, \varphi \rangle$ is *UBCQ-expressible* if it admits a *UBCQ-rewriting*. That is, if there is a UBCQ γ such that, for every fact set $\mathcal{F}$:

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Theorem (see [KLMT15] for more info)

There is a procedure that, on input $\langle \mathcal{R}, \varphi \rangle$ a rule query,

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Remark

We can reuse the very same procedure to rewrite every UBCQ-expressible fragment: OWL QL, linear, sticky...

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Consider the rule set $\mathcal{R}$:

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The rule query $\langle \mathcal{R}, \mathit{MainAff}(\mathit{alice}, \mathit{inria}) \rangle$ is not UBCQ-expressible. However, we obtain a datalog-rewriting by replacing (15) in $\mathcal{R}$ with

$$\mathit{Colleague}(x,y) \wedge \mathit{MainAff}(x,z) \rightarrow \mathit{MainAff}(y,z)$$

Many Procedures to Rewrite into Datalog

Source Language	Target Language	Implemented	Reference
$\mathcal{SHIQ}$	Disj. Datalog	Yes	[HMS07]
$\mathcal{SHIQ}b_S$	Disj. Datalog	No	[RKH12]
Horn-$\mathcal{ALCHOIQ}$	Datalog	Yes	[CDK18]
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Frontier Guarded Rules	Datalog	No	[BBtC13]
Nearly Guarded Rules	Datalog	No	[GRS14]
Guarded Disj. Rules	Disj. Datalog	No	[AOS18]
Guarded Rules	Datalog	Yes	[BBG$^+$22]

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Guarded Rules	Datalog	Yes	[BBG$^+$22]
Warded rules	Datalog	Yes	[BGPS22]
Linear	Non-Rec. Datalog	No	[GS12]
Sticky(-Join)	Non-Rec. Datalog	No	[GS12]

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Is the datalog-expressible fragment datalog-rewritable?

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Corollary

The class of all datalog-expressible rule queries is not datalog-rewritable.

Reasoning with Datalog-Expressible Rule Queries: An Undecidable Problem

Reasoning with Dat-Expressible Rule Queries

Theorem

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- Lemma 1. The machine M halts on ϵ if and only if the KB $\langle \mathcal{R}_M, \emptyset \rangle$ entails the (nullary) fact *Halt*.

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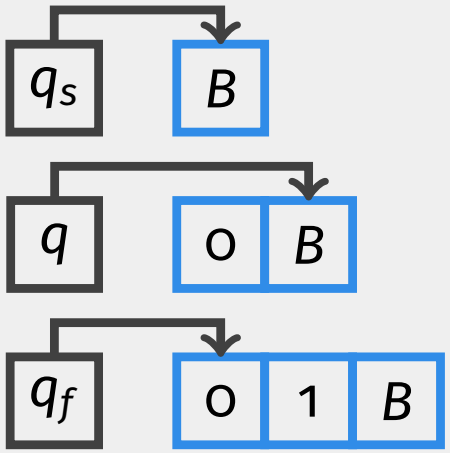
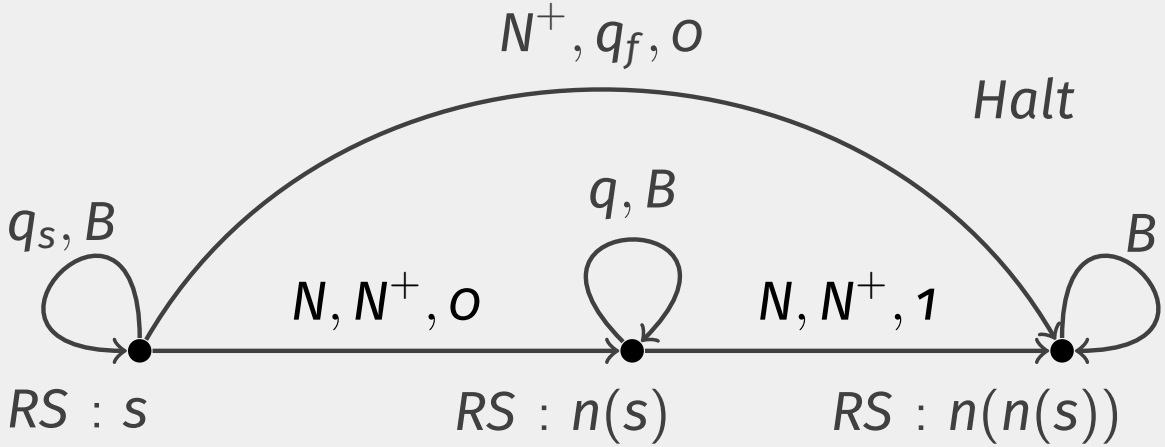
After proving the above, the theorem follows by contradiction.

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



A universal model of $\langle \mathcal{R}_M, \emptyset \rangle$:



Lemma 1. The machine M halts on the empty word if and only if the KB $\langle \mathcal{R}_M, \emptyset \rangle$ entails the fact *Halt*.

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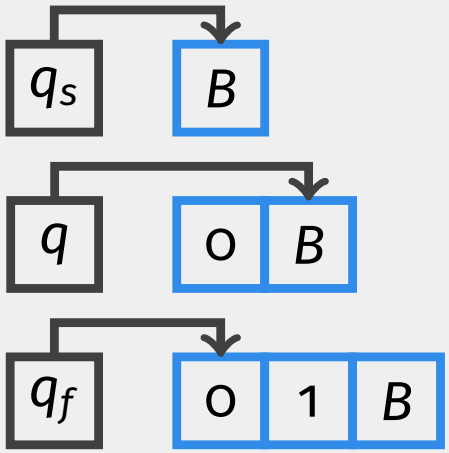

The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:

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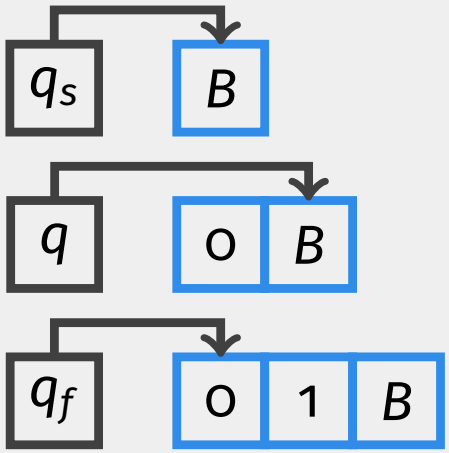

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$$\text{Apply} \rightarrow \exists s.q_s(s, s) \in \mathcal{R}_M$$

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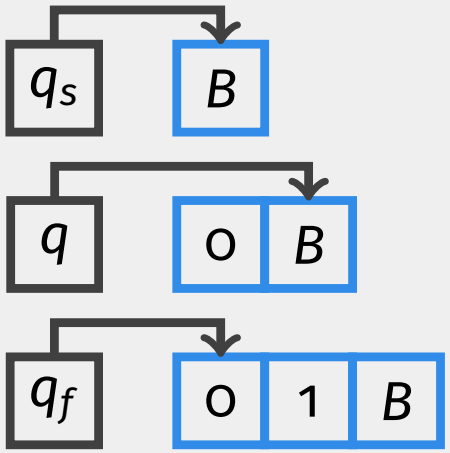

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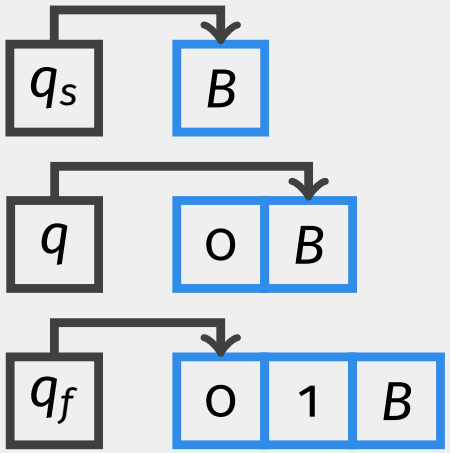

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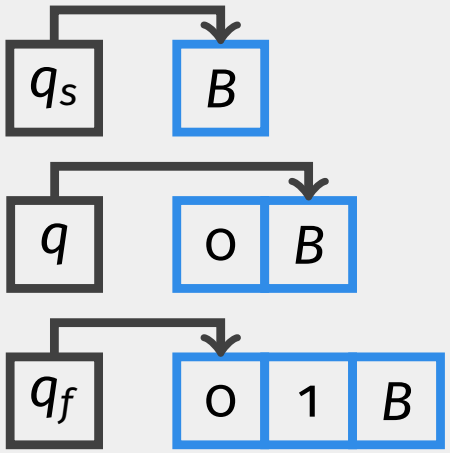

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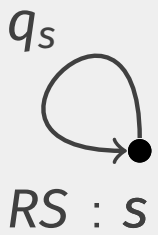

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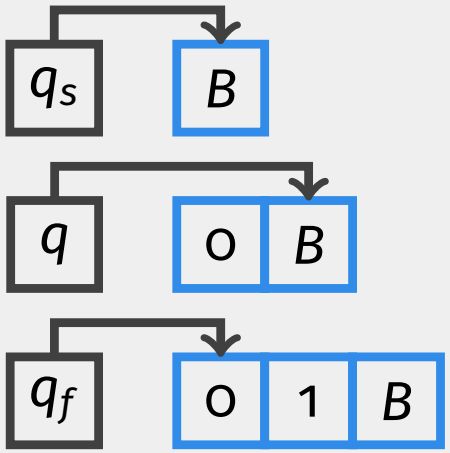

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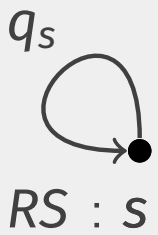

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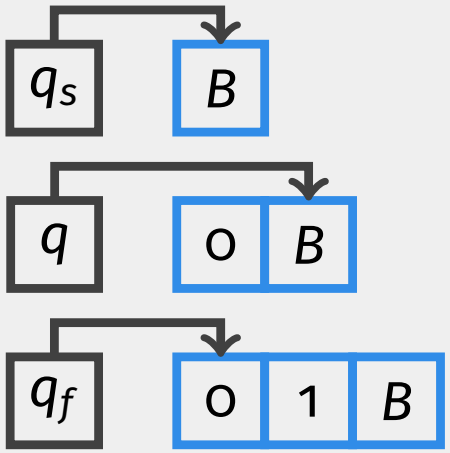

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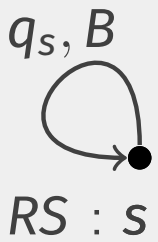

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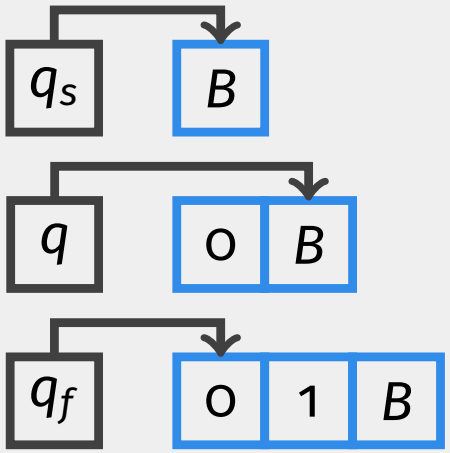

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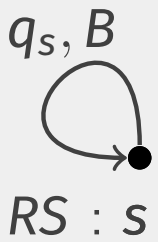

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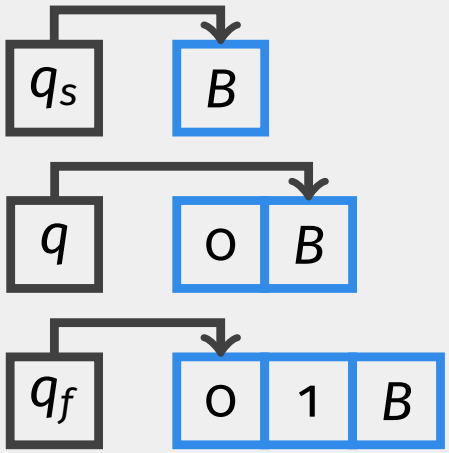

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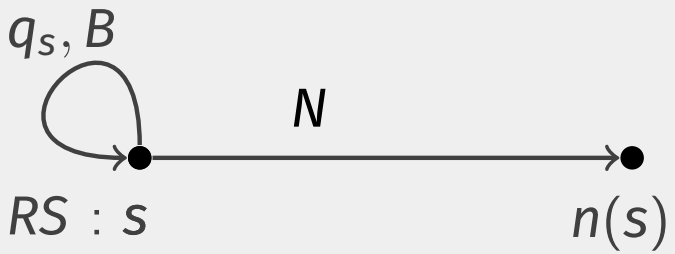

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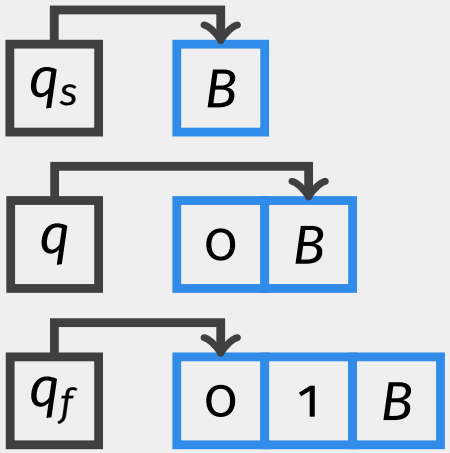

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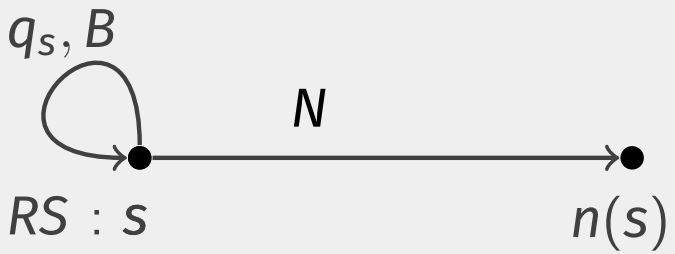

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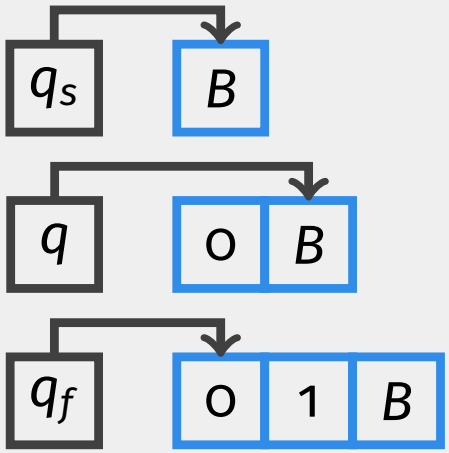

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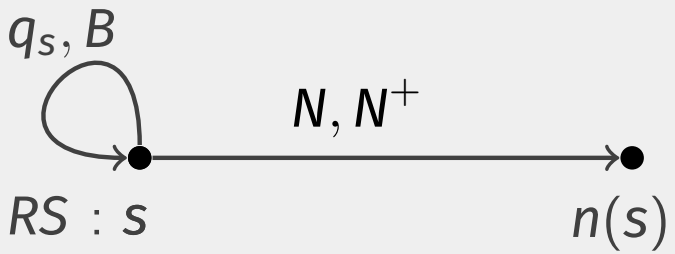

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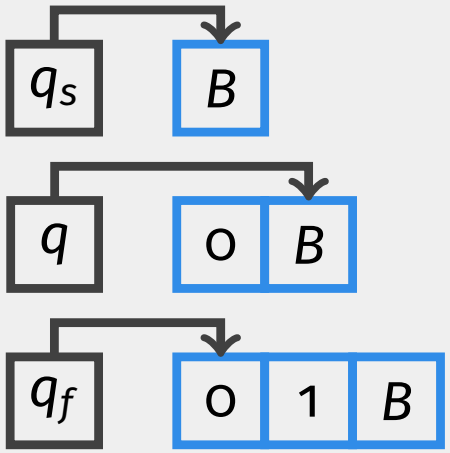

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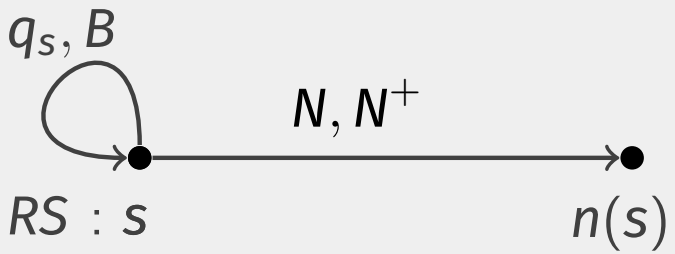

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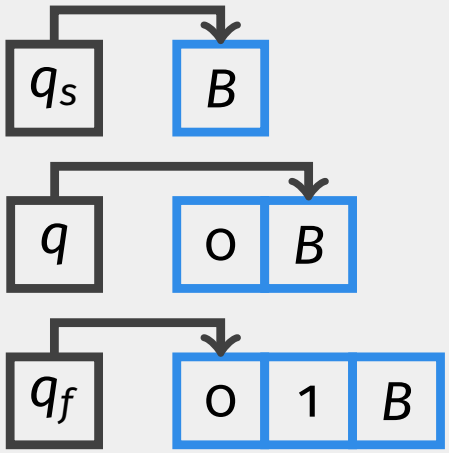

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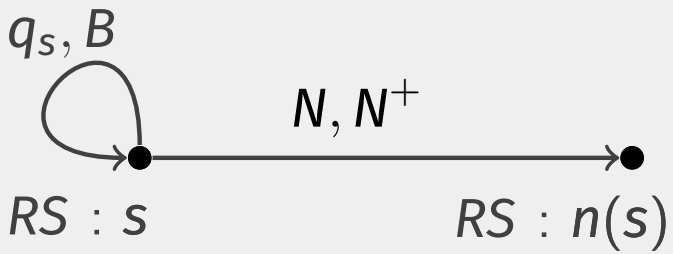

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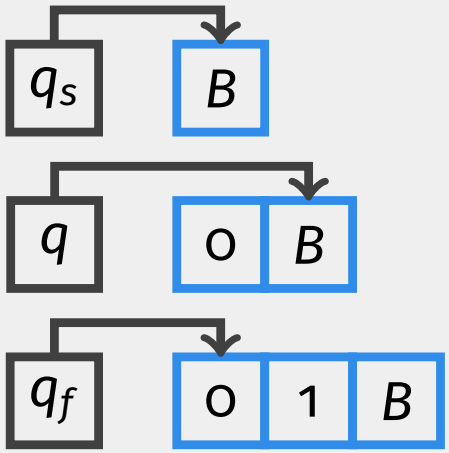

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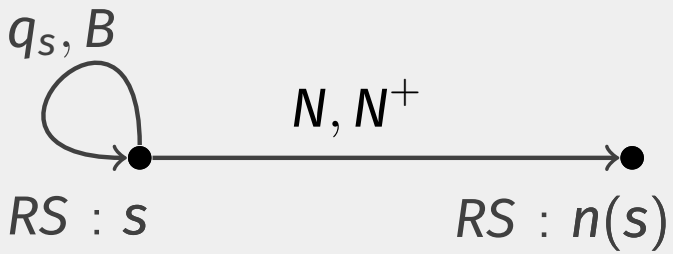

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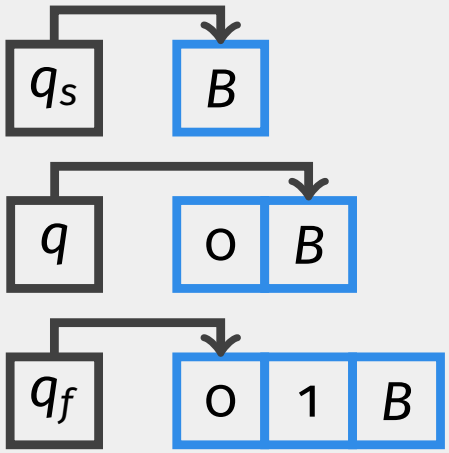

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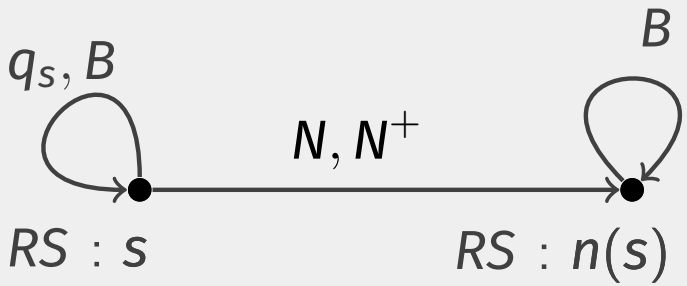

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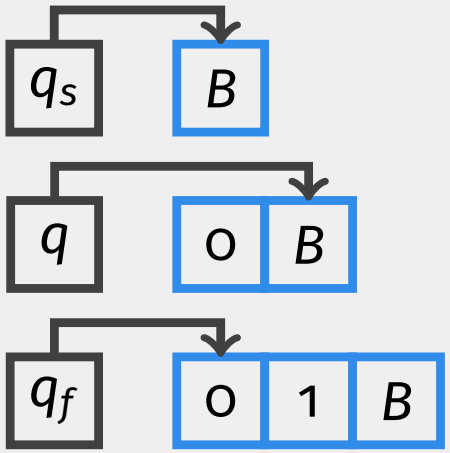

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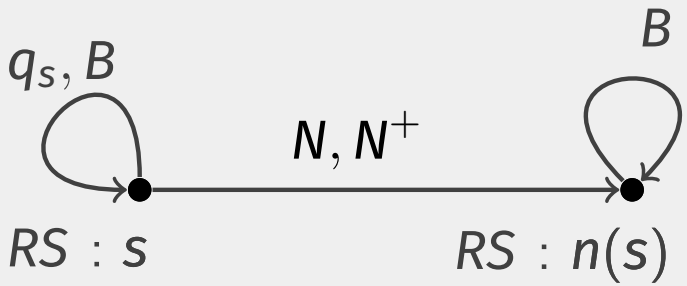

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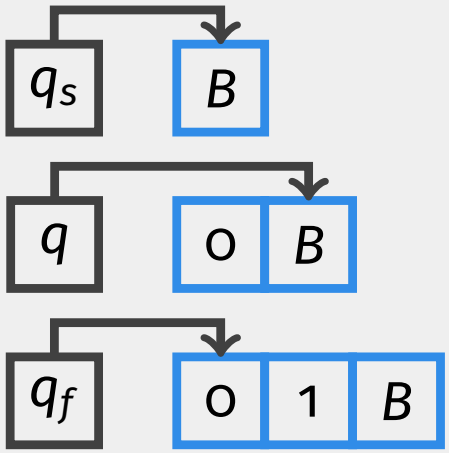

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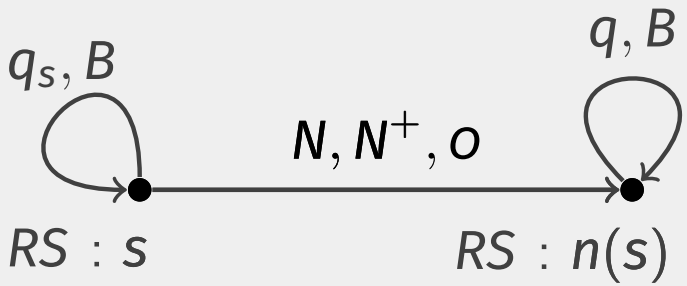

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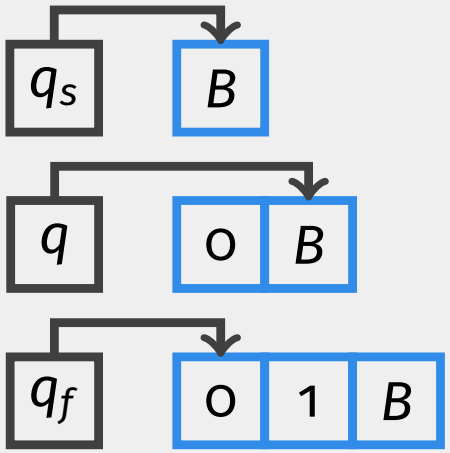

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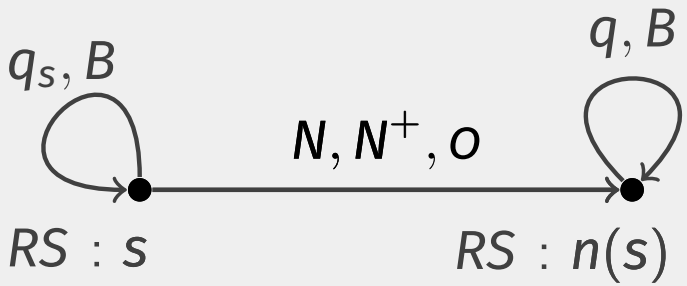

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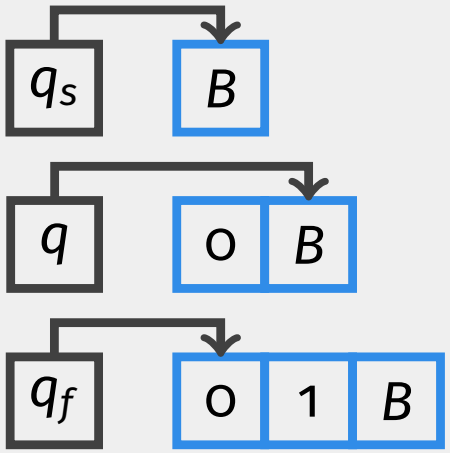

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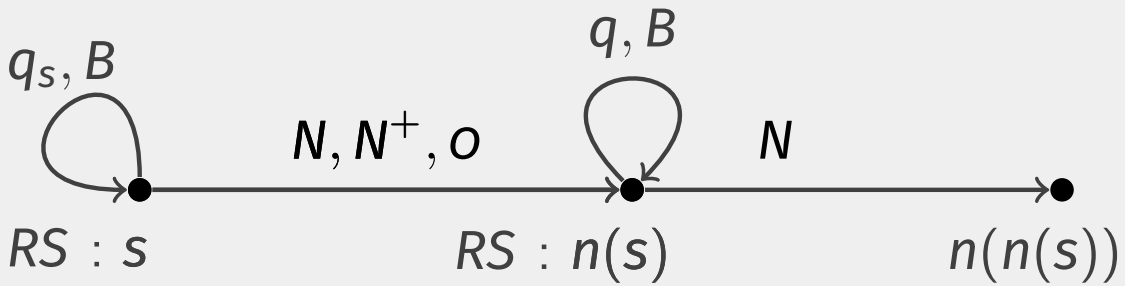

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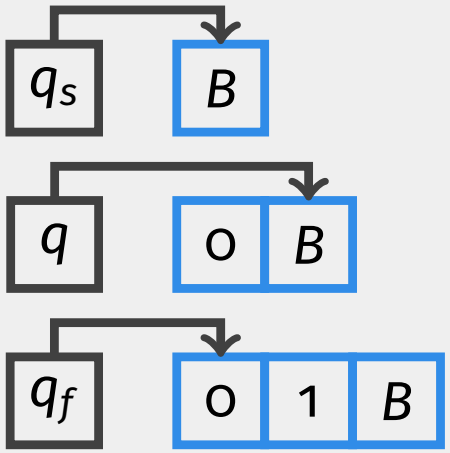

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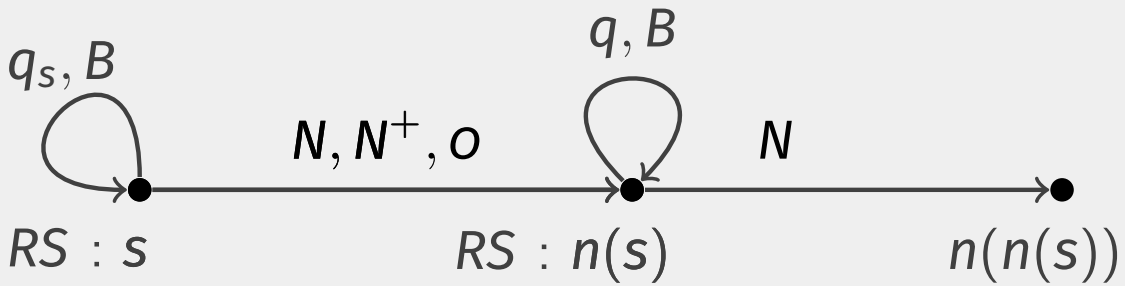

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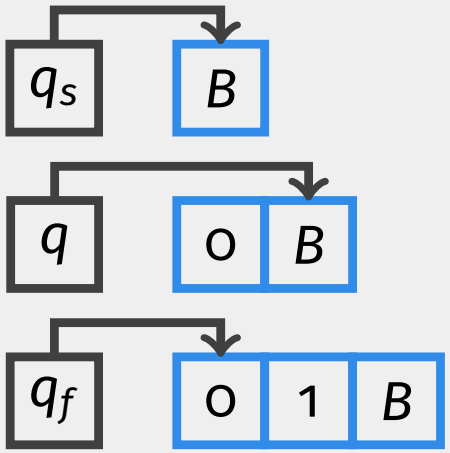

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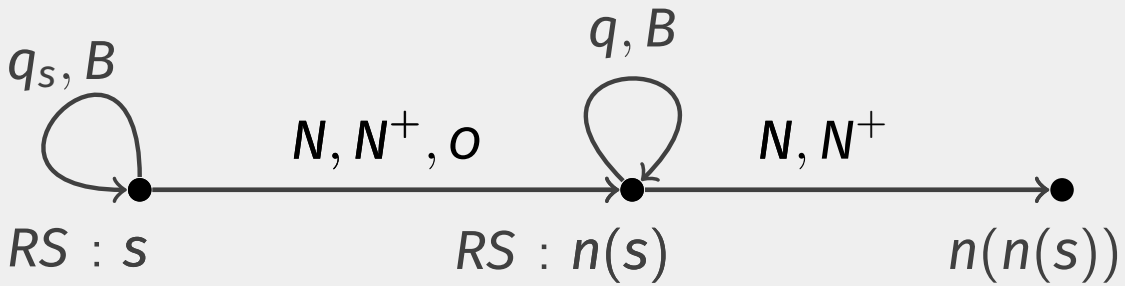

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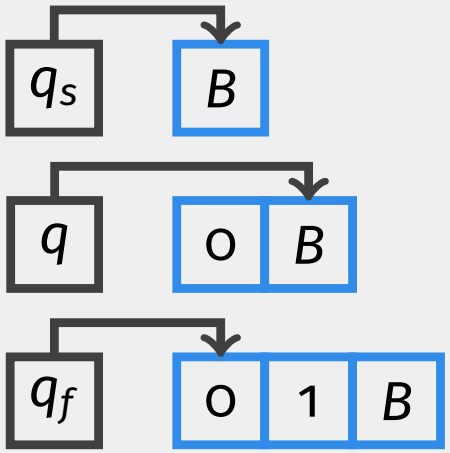

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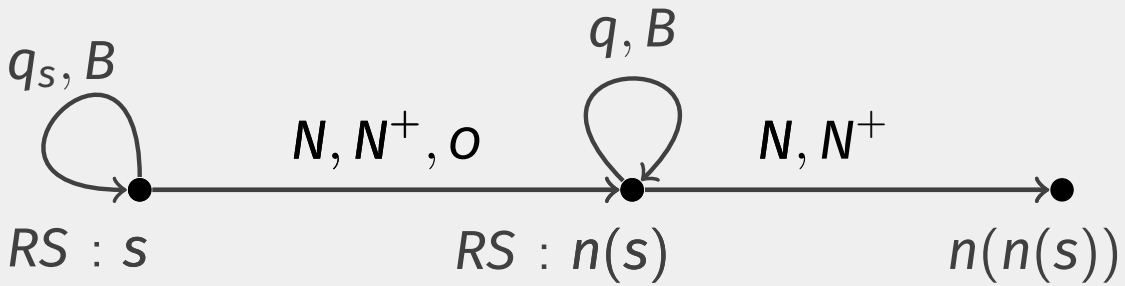
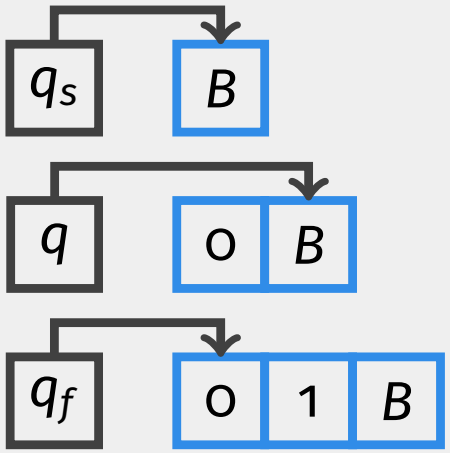


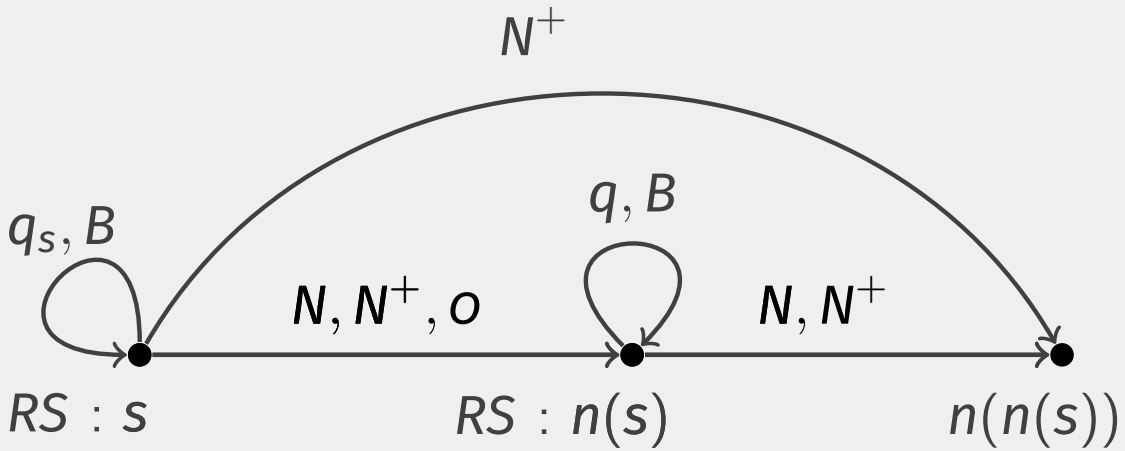
Apply $N(\ell_i, \ell_{i+1}) \wedge N^+(\ell_{i+1}, \ell_j) \rightarrow N^+(\ell_i, \ell_j) \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



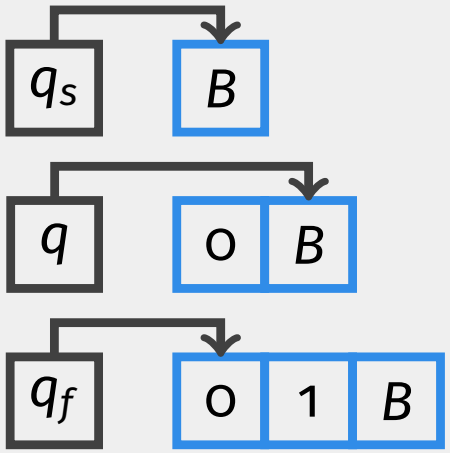
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



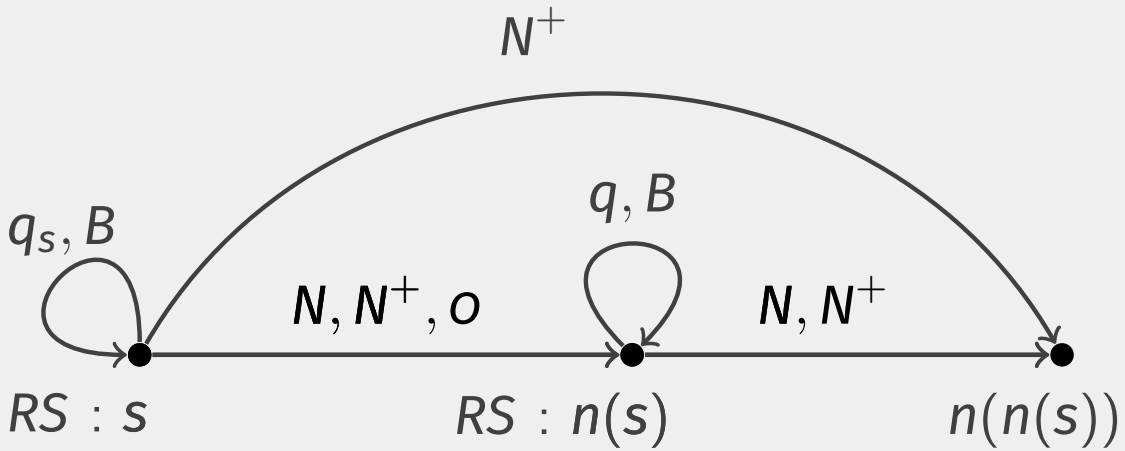
Apply $N(\ell_i, \ell_{i+1}) \wedge N^+(\ell_{i+1}, \ell_j) \rightarrow N^+(\ell_i, \ell_j) \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



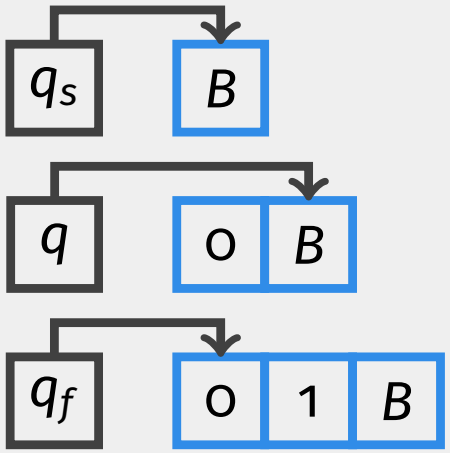
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



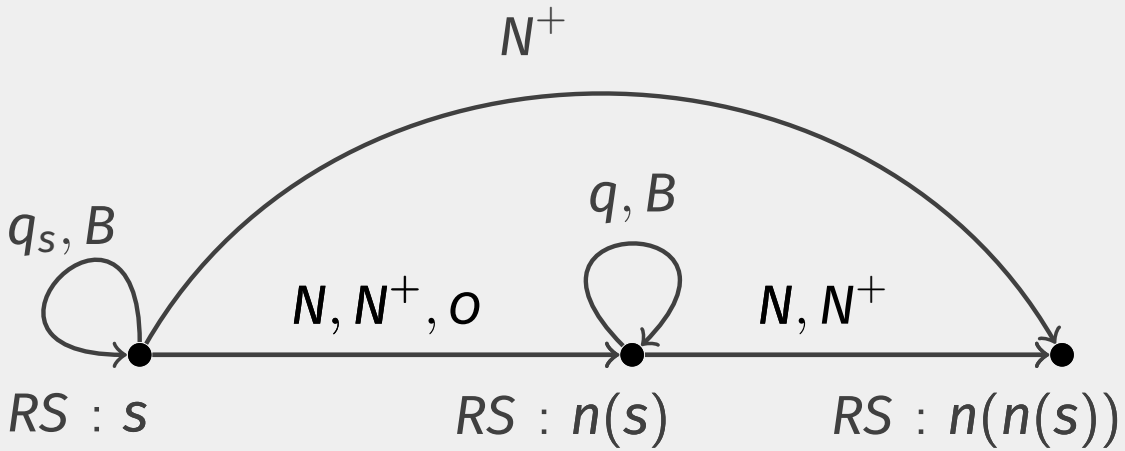
Apply $q_s(\ell_1, \ell_1) \wedge N^+(\ell_1, \ell_i) \rightarrow RS(\ell_i) \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



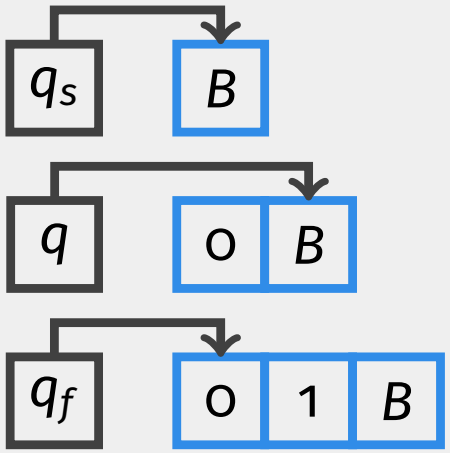
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



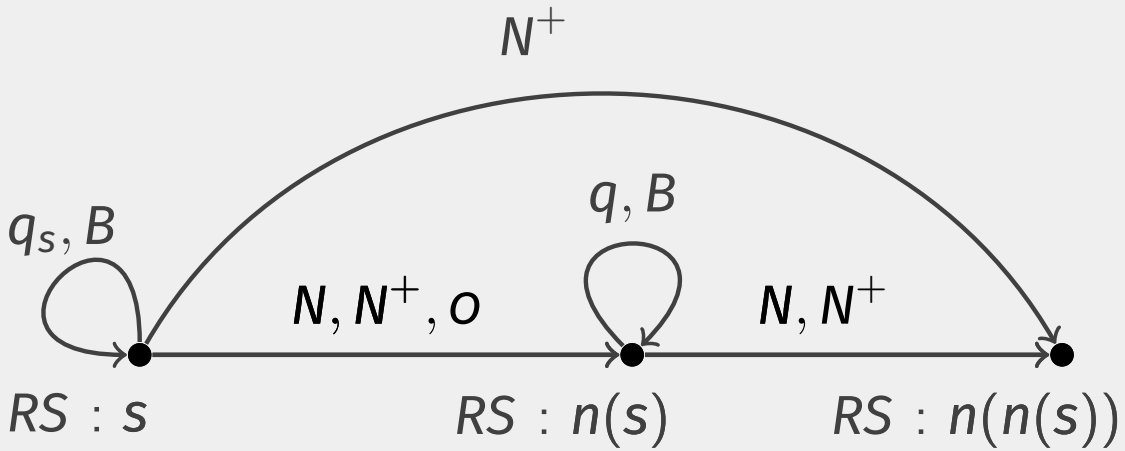
Apply $q_s(\ell_1, \ell_1) \wedge N^+(\ell_1, \ell_i) \rightarrow RS(\ell_i) \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



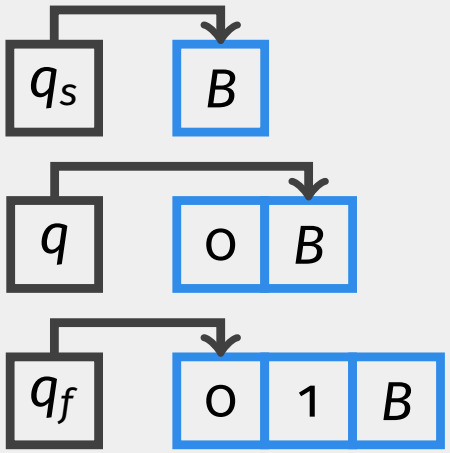
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



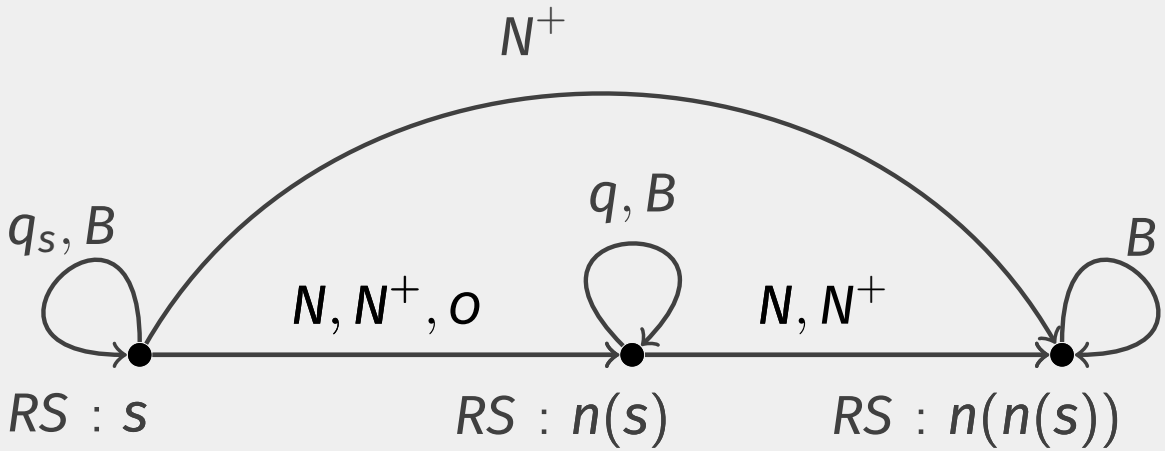
Apply $RS(x) \rightarrow B(x, x) \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



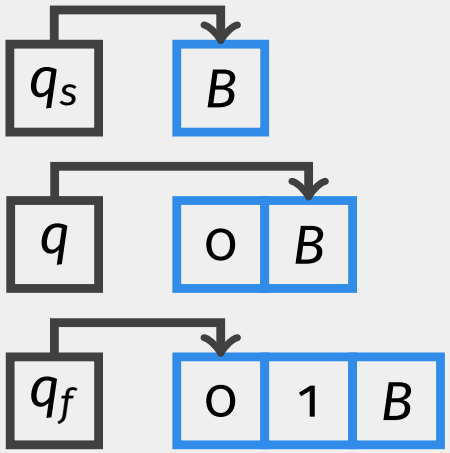
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



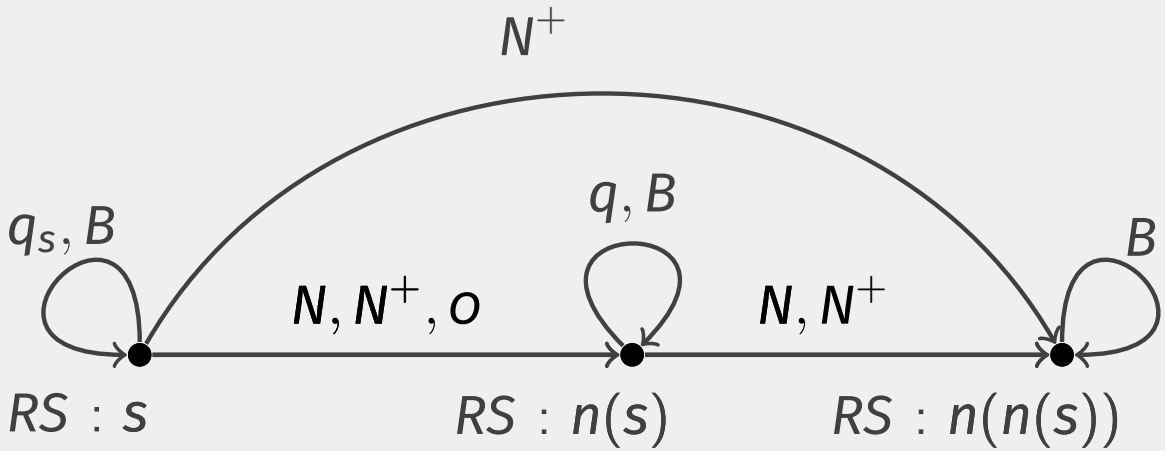
Apply $RS(x) \rightarrow B(x, x) \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:

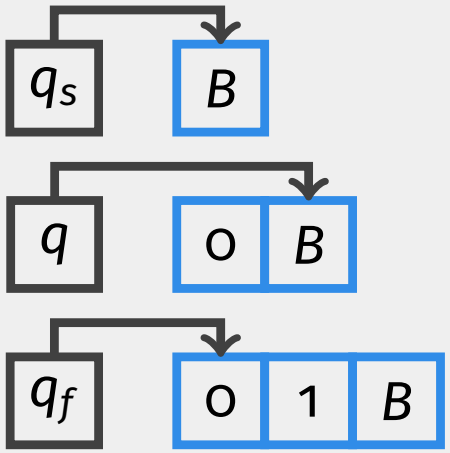


Since $M(q, B) = \langle q_f, 1, L \rangle$, we can apply

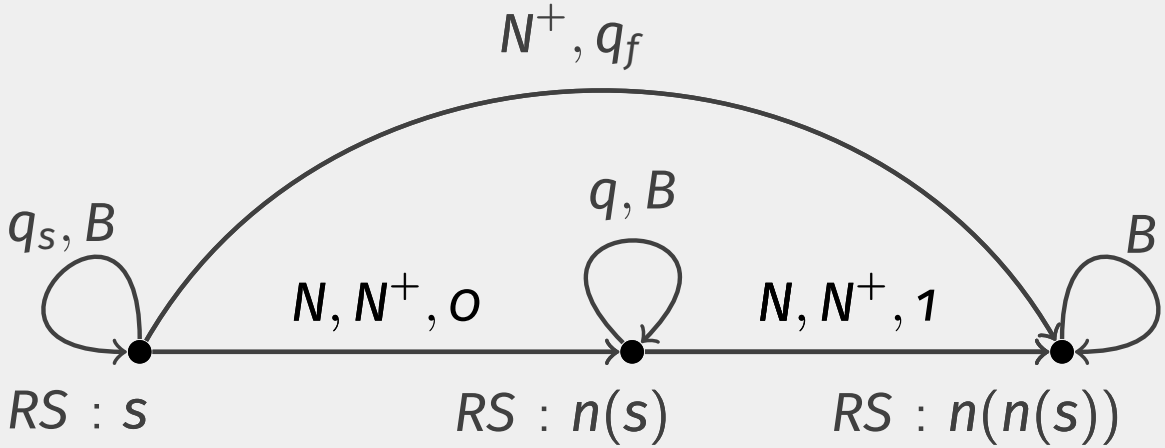
$$q(p_i, c_j) \wedge B(p_i, c_j) \wedge N(p_{i-1}, p_i) \wedge N(c_j, c_{j+1}) \wedge N^+(p_i, c_{j+1}) \wedge RS(p_i) \wedge RS(c_{j+1}) \rightarrow q_f(p_{i-1}, c_{j+1}) \wedge 1(p_i, c_{j+1}) \in \mathcal{R}_M$$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:

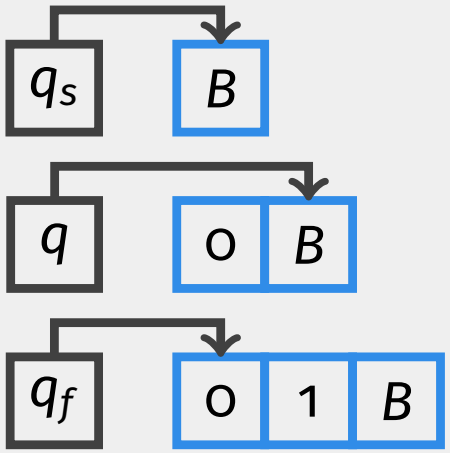


Since $M(q, B) = \langle q_f, 1, L \rangle$, we can apply

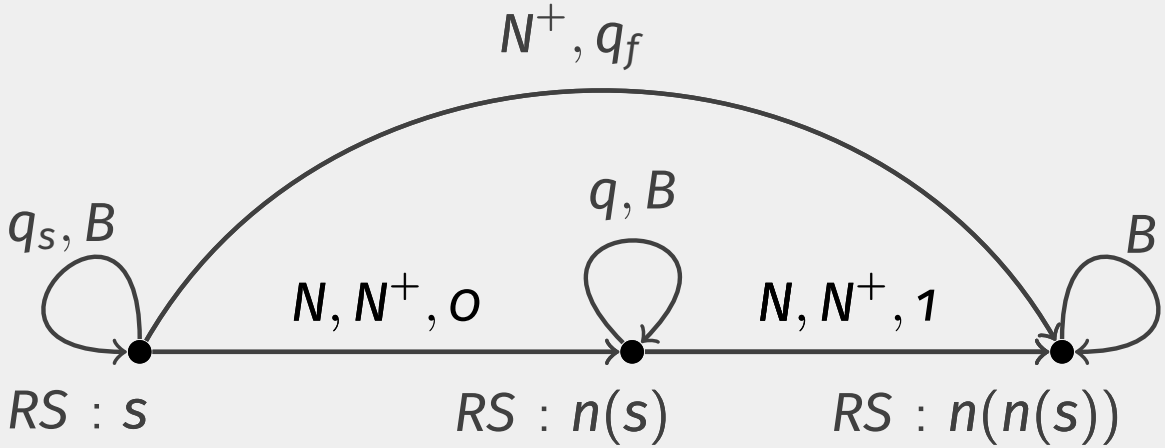
$$q(p_i, c_j) \wedge B(p_i, c_j) \wedge N(p_{i-1}, p_i) \wedge N(c_j, c_{j+1}) \wedge N^+(p_i, c_{j+1}) \wedge RS(p_i) \wedge RS(c_{j+1}) \rightarrow q_f(p_{i-1}, c_{j+1}) \wedge 1(p_i, c_{j+1}) \in \mathcal{R}_M$$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:

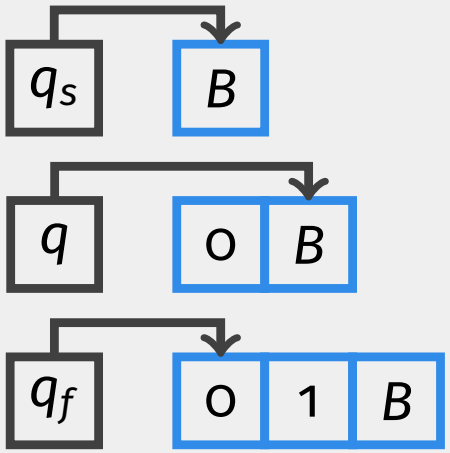


Since q is a non-final state in M, we can apply

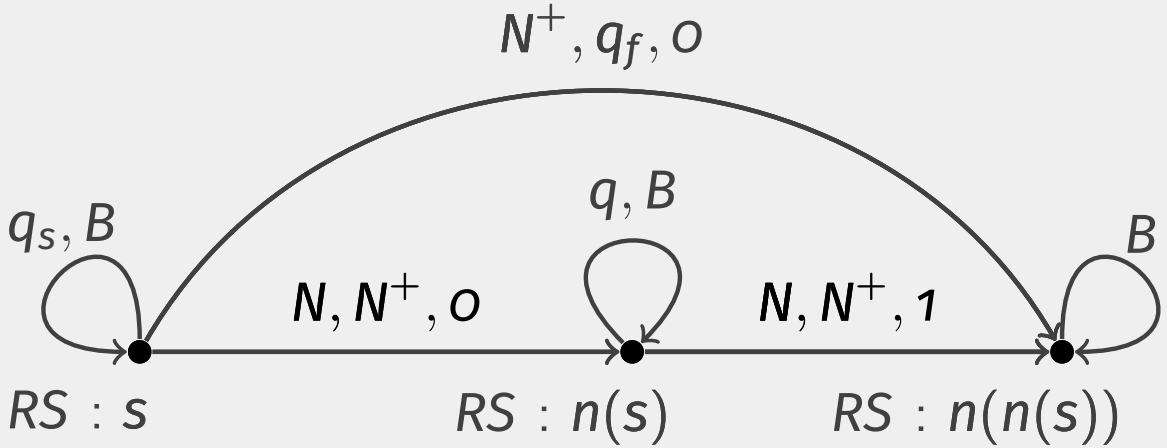
$$0(p_i, c_j) \wedge q(p_k, c_j) \wedge N(c_j, c_{j+1}) \wedge N^+(p_i, p_k) \wedge N^+(p_k, c_{j+1}) \wedge N^+(p_i, c_{j+1}) \wedge RS(p_i) \wedge RS(p_k) \wedge RS(c_{j+1}) \rightarrow 0(p_i, c_{j+1}) \in \mathcal{R}_M$$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:

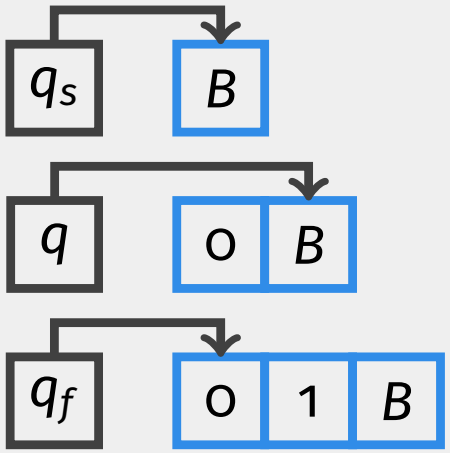


Since q is a non-final state in M, we can apply

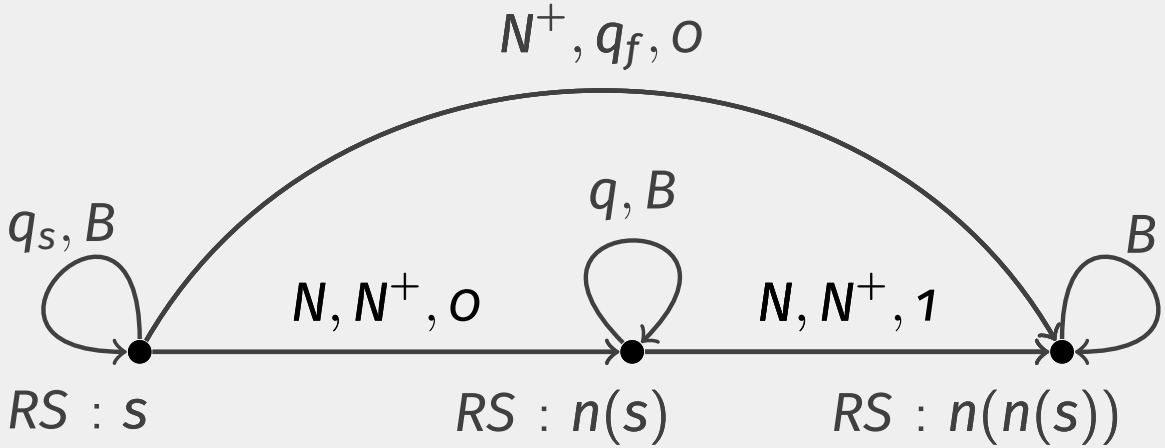
$$o(p_i, c_j) \wedge q(p_k, c_j) \wedge N(c_j, c_{j+1}) \wedge N^+(p_i, p_k) \wedge N^+(p_k, c_{j+1}) \wedge N^+(p_i, c_{j+1}) \wedge RS(p_i) \wedge RS(p_k) \wedge RS(c_{j+1}) \rightarrow o(p_i, c_{j+1}) \in \mathcal{R}_M$$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



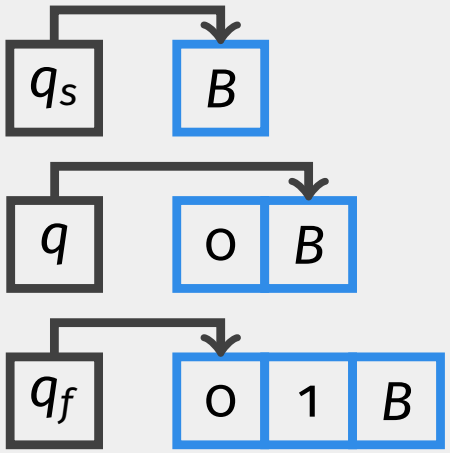
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



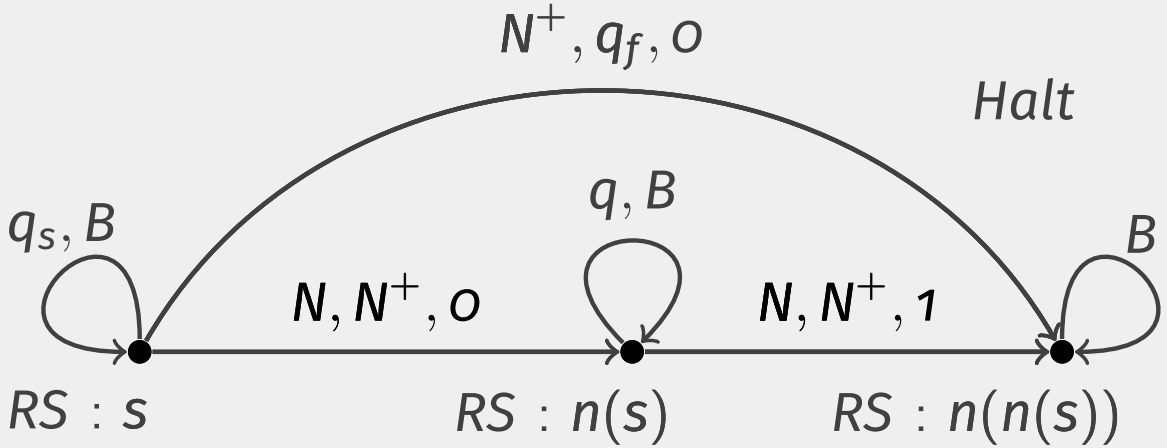
Apply $q_f(p, c) \rightarrow Halt \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



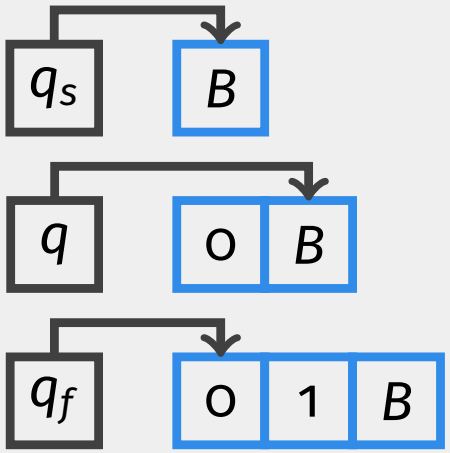
The chase of $\langle \mathcal{R}_M, \emptyset \rangle$:



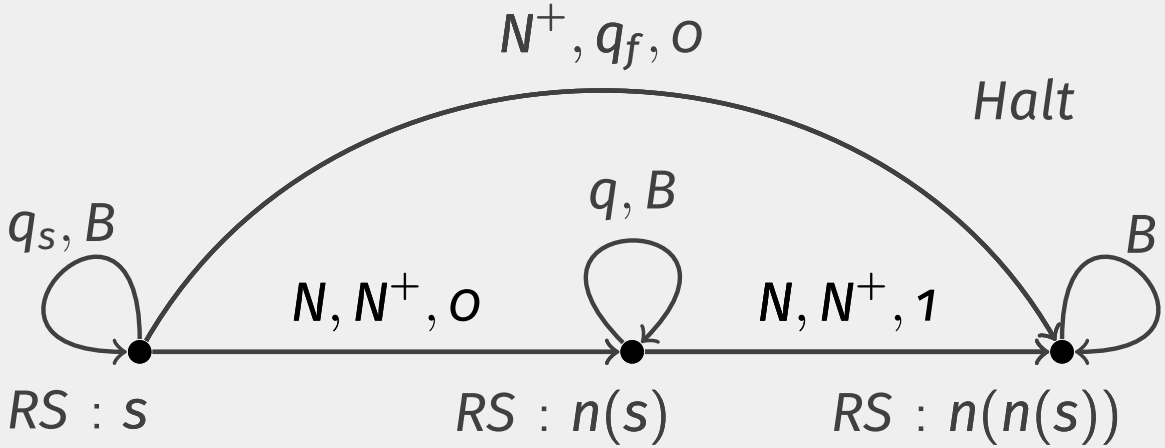
Apply $q_f(p, c) \rightarrow Halt \in \mathcal{R}_M$

Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:

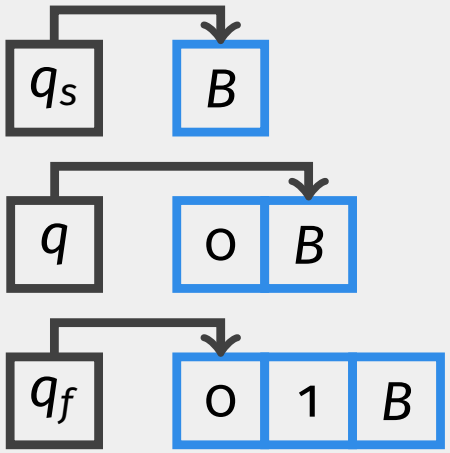


A universal model of $\langle \mathcal{R}_M, \emptyset \rangle$:

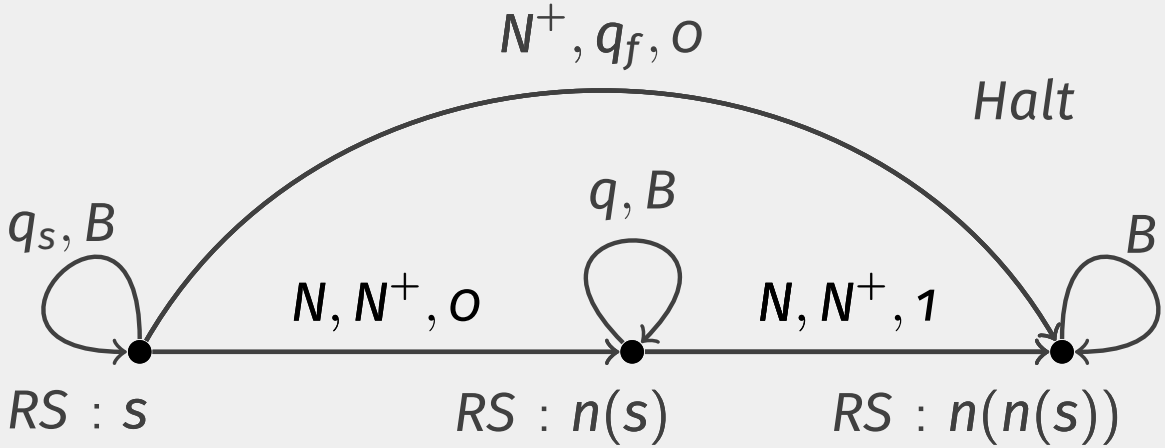


Emulating a Machine M with the KB $\langle \mathcal{R}_M, \emptyset \rangle$

The computation of M on ϵ:



A universal model of $\langle \mathcal{R}_M, \emptyset \rangle$:



Lemma 1. The machine M halts on the empty word if and only if the KB $\langle \mathcal{R}_M, \emptyset \rangle$ entails the fact *Halt*.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

Proof.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

Proof.

- If M halts on ϵ, then $\langle \{\rightarrow \mathit{Halt}\}, \mathit{Halt} \rangle$ is a datalog-rewriting since $\langle \mathcal{R}_M, \emptyset \rangle \models \mathit{Halt}$ by Lemma 1.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

Proof.

- If M halts on ϵ, then $\langle \{\rightarrow \mathit{Halt}\}, \mathit{Halt} \rangle$ is a datalog-rewriting since $\langle \mathcal{R}_M, \emptyset \rangle \models \mathit{Halt}$ by Lemma 1.
- If M does not halt on ϵ; then $\langle \mathcal{R}_M^{\forall}, \mathit{Halt} \rangle$ is a datalog-rewriting of $\langle \mathcal{R}_M, \mathit{Halt} \rangle$, where $\mathcal{R}_M^{\forall}$ is the set of datalog rules in $\mathcal{R}_M$.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

Proof.

- If M halts on ϵ, then $\langle \{\rightarrow \mathit{Halt}\}, \mathit{Halt} \rangle$ is a datalog-rewriting since $\langle \mathcal{R}_M, \emptyset \rangle \models \mathit{Halt}$ by Lemma 1.
- If M does not halt on ϵ; then $\langle \mathcal{R}_M^{\forall}, \mathit{Halt} \rangle$ is a datalog-rewriting of $\langle \mathcal{R}_M, \mathit{Halt} \rangle$, where $\mathcal{R}_M^{\forall}$ is the set of datalog rules in $\mathcal{R}_M$. That is, for every fact set $\mathcal{F}$, we show that $\langle \mathcal{R}_M, \mathcal{F} \rangle \models \mathit{Halt}$ iff $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \models \mathit{Halt}$.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

Proof.

- If M halts on ϵ, then $\langle \{\rightarrow \mathit{Halt}\}, \mathit{Halt} \rangle$ is a datalog-rewriting since $\langle \mathcal{R}_M, \emptyset \rangle \models \mathit{Halt}$ by Lemma 1.
- If M does not halt on ϵ; then $\langle \mathcal{R}_M^\forall, \mathit{Halt} \rangle$ is a datalog-rewriting of $\langle \mathcal{R}_M, \mathit{Halt} \rangle$, where $\mathcal{R}_M^\forall$ is the set of datalog rules in $\mathcal{R}_M$. That is, for every fact set $\mathcal{F}$, we show that $\langle \mathcal{R}_M, \mathcal{F} \rangle \models \mathit{Halt}$ iff $\langle \mathcal{R}_M^\forall, \mathcal{F} \rangle \models \mathit{Halt}$.
  - If $\langle \mathcal{R}_M^\forall, \mathcal{F} \rangle \models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \models \mathit{Halt}$ since $\mathcal{R}_M^\forall \subseteq \mathcal{R}_M$.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 2

The rule query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is datalog-expressible.

Proof.

- If M halts on ϵ, then $\langle \{\rightarrow \mathit{Halt}\}, \mathit{Halt} \rangle$ is a datalog-rewriting since $\langle \mathcal{R}_M, \emptyset \rangle \models \mathit{Halt}$ by Lemma 1.
- If M does not halt on ϵ; then $\langle \mathcal{R}_M^\forall, \mathit{Halt} \rangle$ is a datalog-rewriting of $\langle \mathcal{R}_M, \mathit{Halt} \rangle$, where $\mathcal{R}_M^\forall$ is the set of datalog rules in $\mathcal{R}_M$. That is, for every fact set $\mathcal{F}$, we show that $\langle \mathcal{R}_M, \mathcal{F} \rangle \models \mathit{Halt}$ iff $\langle \mathcal{R}_M^\forall, \mathcal{F} \rangle \models \mathit{Halt}$.
  - If $\langle \mathcal{R}_M^\forall, \mathcal{F} \rangle \models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \models \mathit{Halt}$ since $\mathcal{R}_M^\forall \subseteq \mathcal{R}_M$.
  - To-do: If $\langle \mathcal{R}_M, \mathcal{F} \rangle \models \mathit{Halt}$, then $\langle \mathcal{R}_M^\forall, \mathcal{F} \rangle \models \mathit{Halt}$.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

Proof. Suppose that the fact set $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ does not contain *Halt*.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

Proof. Suppose that the fact set $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ does not contain *Halt*.

- The constants in $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ form a disjoint union of N-chains.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

Proof. Suppose that the fact set $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ does not contain Halt.

- The constants in $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ form a disjoint union of N-chains.
  - Because of $N^+(x,x) \to \mathit{Halt}$, $N(x,y) \to N^+(x,y)$, and $N(x,y) \wedge N^+(y,z) \to N^+(x,z)$; there are no N-cycles.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

Proof. Suppose that the fact set $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ does not contain Halt.

- The constants in $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ form a disjoint union of N-chains.
  - Because of $N^+(x,x) \to \mathit{Halt}$, $N(x,y) \to N^+(x,y)$, and $N(x,y) \wedge N^+(y,z) \to N^+(x,z)$; there are no N-cycles.
  - Because of $N(x,y) \wedge N(x,z) \to y \approx z$ and $N(y,x) \wedge N(z,x) \to y \approx z$, constants have at most one N-successor/predecessor.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

Proof. Suppose that the fact set $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ does not contain *Halt*.

- The constants in $\mathrm{Chase}(\mathcal{R}_M^{\forall}, \mathcal{F})$ form a disjoint union of N-chains.

The Rule Query $\langle \mathcal{R}_M, \mathit{Halt} \rangle$ is Datalog-Expressible

Lemma 3.

If M does not halt on ϵ and $\langle \mathcal{R}_M^{\forall}, \mathcal{F} \rangle \not\models \mathit{Halt}$, then $\langle \mathcal{R}_M, \mathcal{F} \rangle \not\models \mathit{Halt}$.

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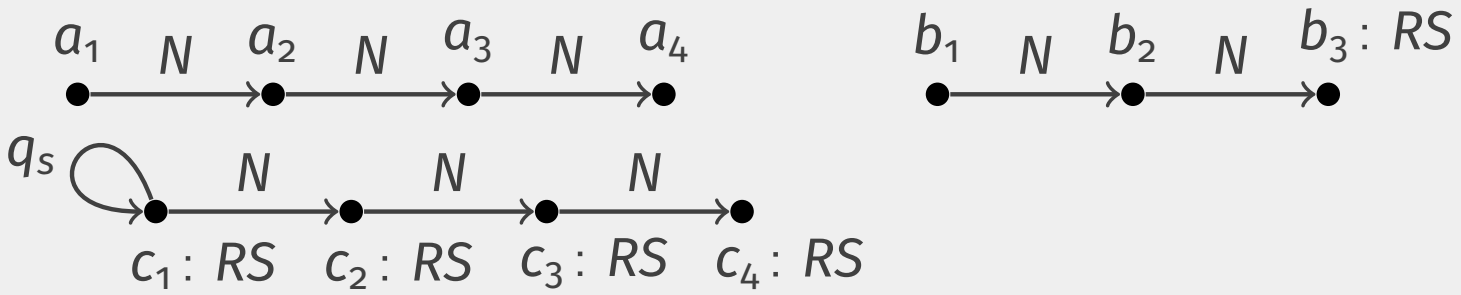

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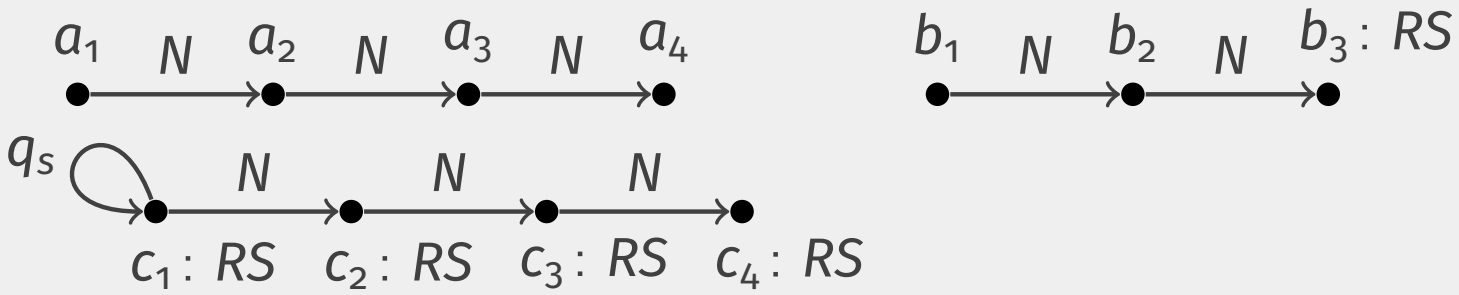

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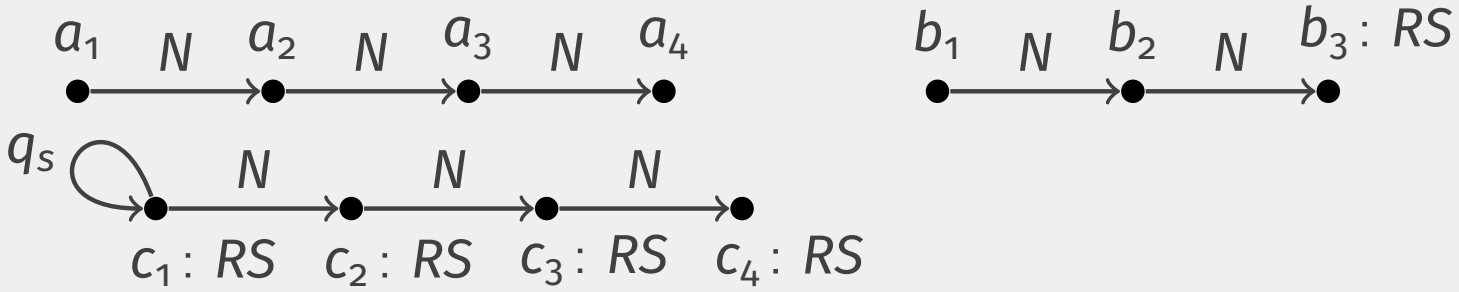

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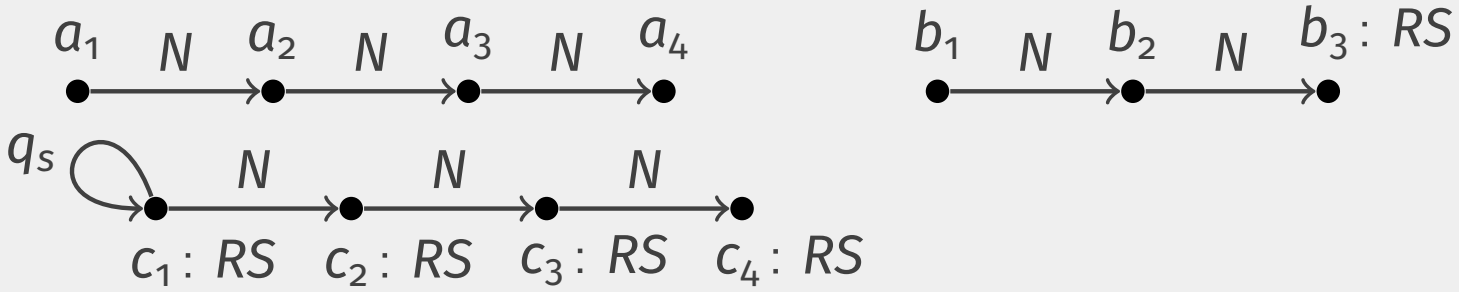

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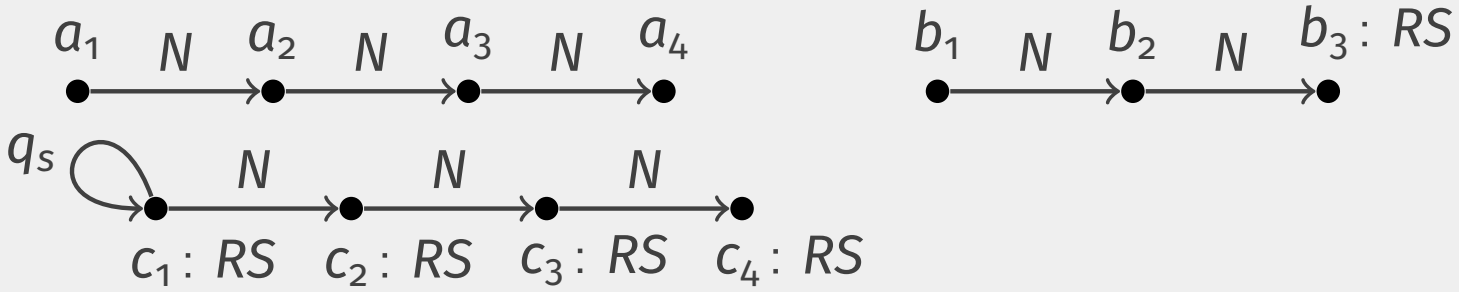

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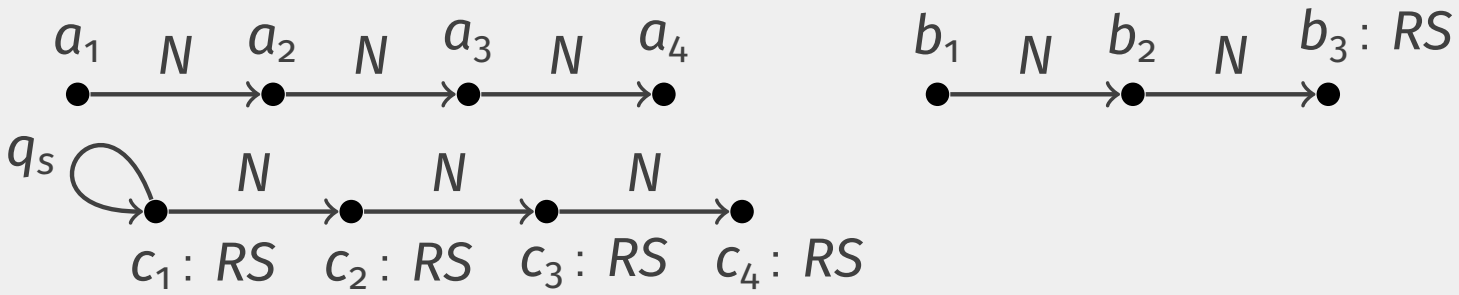

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Future Work and Conclusions

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Thanks for your attention!

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