A Journey to the Frontiers of Query Rewritability

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June 7, 2022

The setting: existential rules

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Usually omitted

The setting: knowledge representation using ∃-rules

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"Everyone has a mother"

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Query entailment problem

Given a ruleset $\mathcal{R}$ and a database $\mathcal{D}$ we ask if some query $\mathcal{Q}$ holds in every model of $\mathcal{R}$ and $\mathcal{D}$?

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To denote that a query holds in every model of $\mathcal{R}$ and $\mathcal{D}$ we write

$$\mathcal{R}, \mathcal{D} \models \mathcal{Q}.$$

The main character: BDD class of ∃-rules

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BDD class definition (By FO-rewritings)

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It is thought that **BDD is well understood**. Soon, however, we will see that **there is a lot more to learn about it.**

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(not a part of this talk)

The chase

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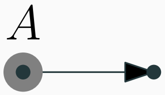

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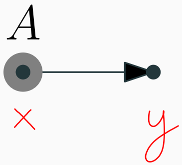

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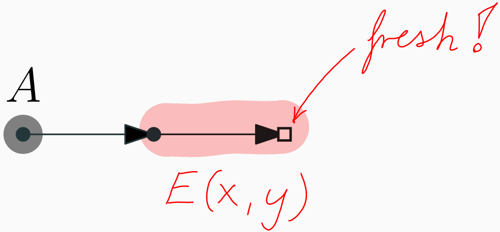

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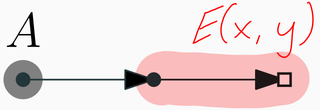

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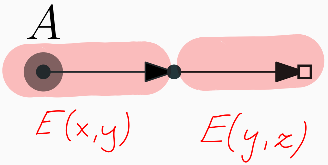

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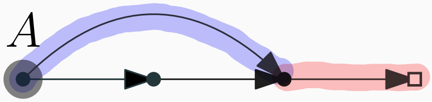

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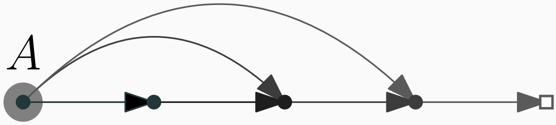

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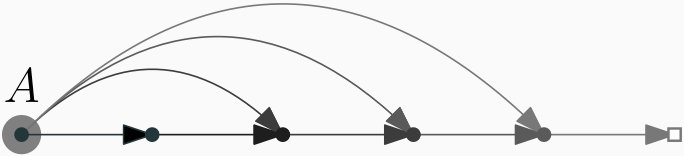

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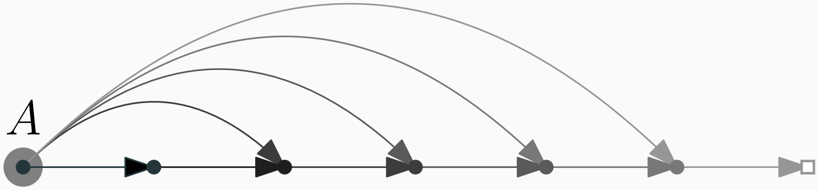

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Query

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Query $\quad$ $\in \mathbb{N}$

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Query ($\mathcal{Q}$) — $k \in \mathbb{N}$ — Database ($\mathcal{D}$)

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Database

k steps

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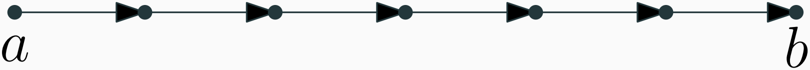

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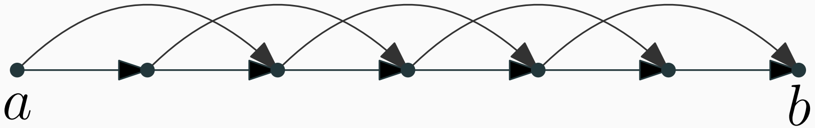

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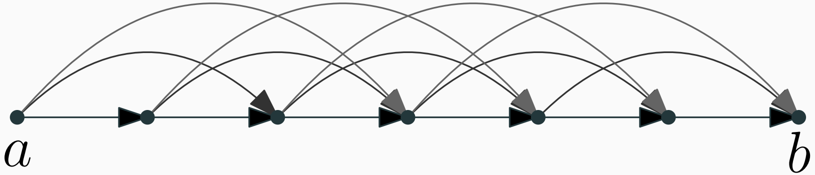

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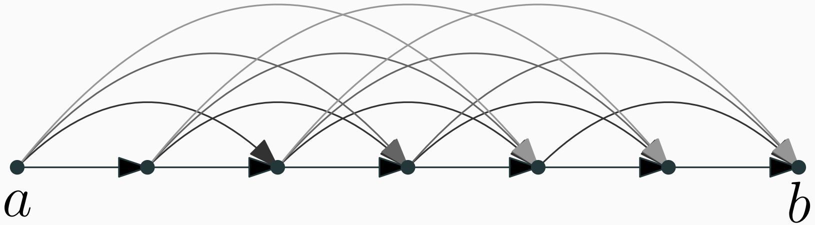

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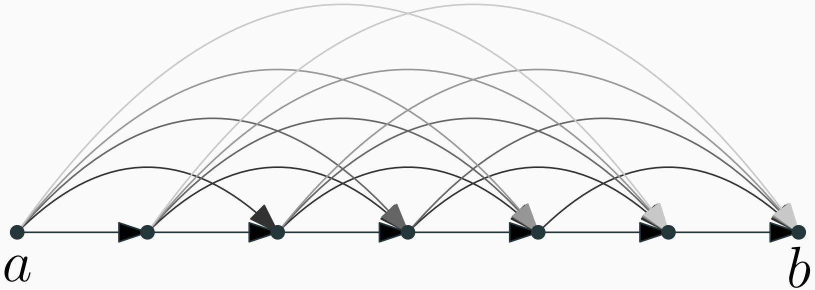

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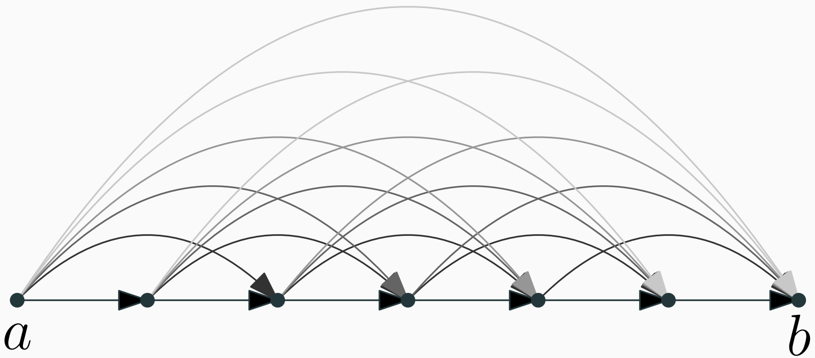

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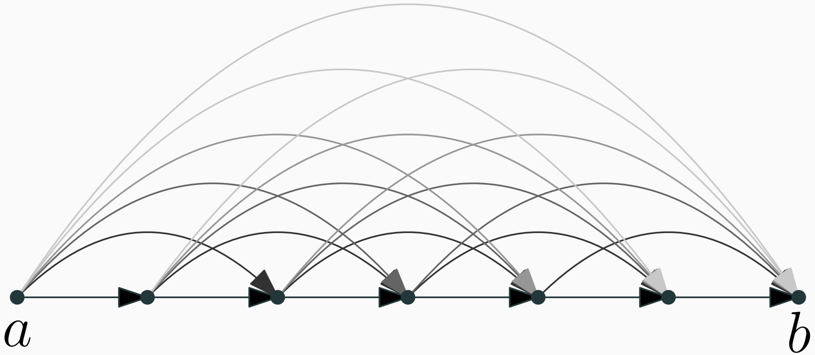

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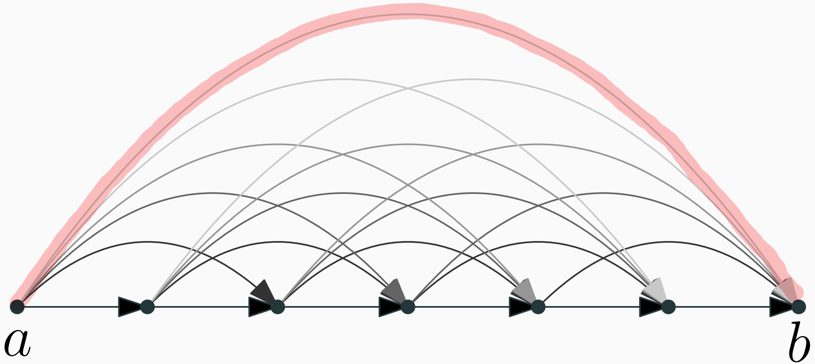

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$Q = E(a, b)$

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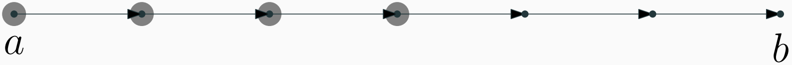

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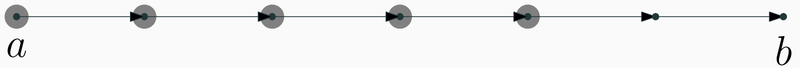

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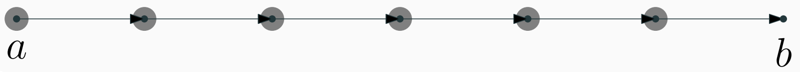

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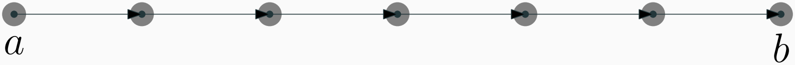

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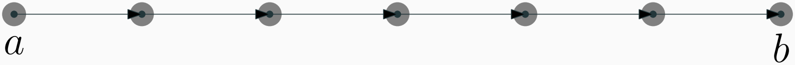

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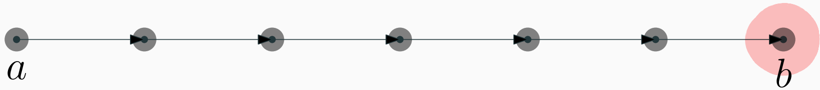

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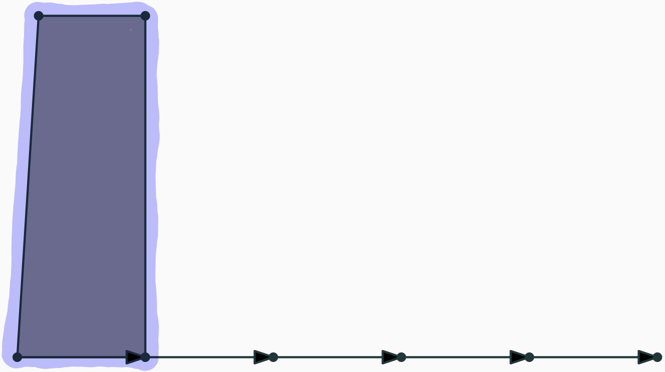

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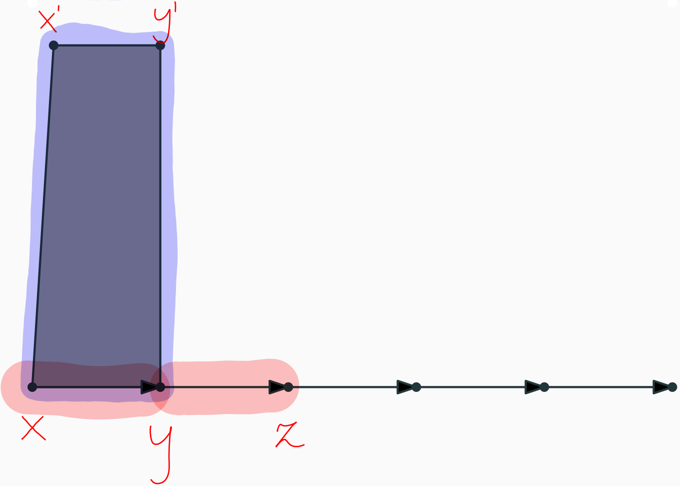

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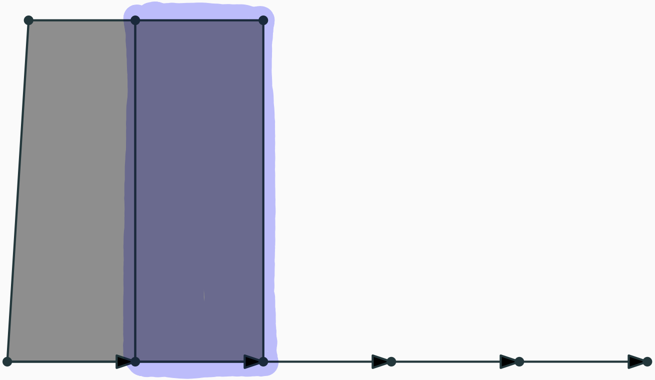

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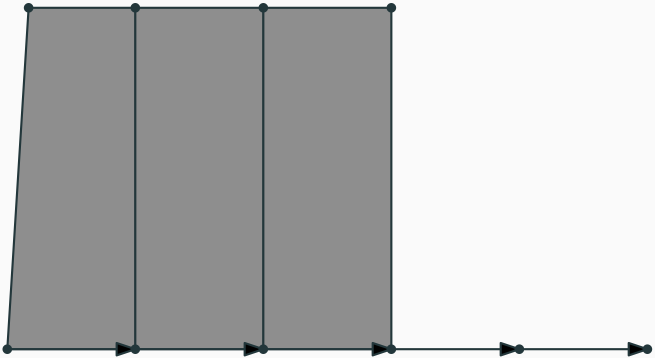

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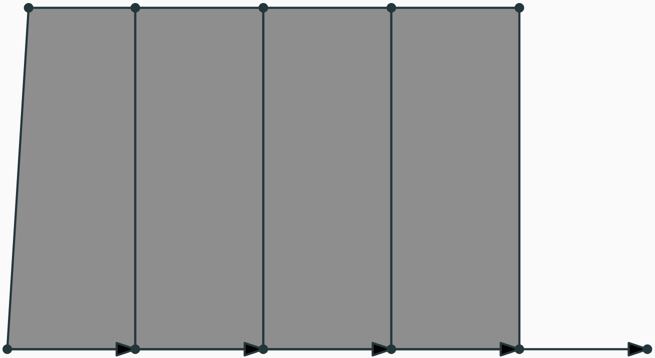

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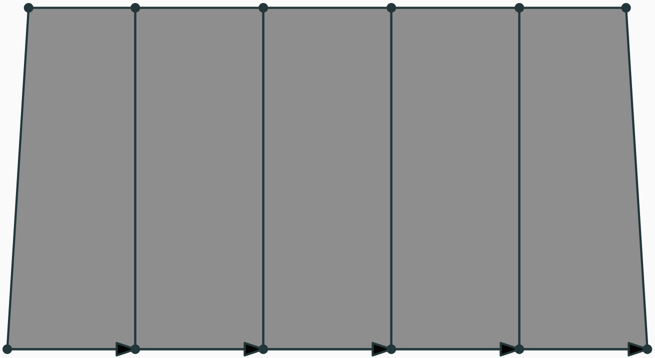

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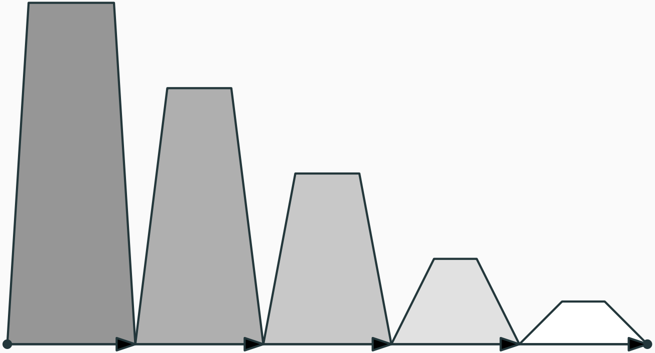

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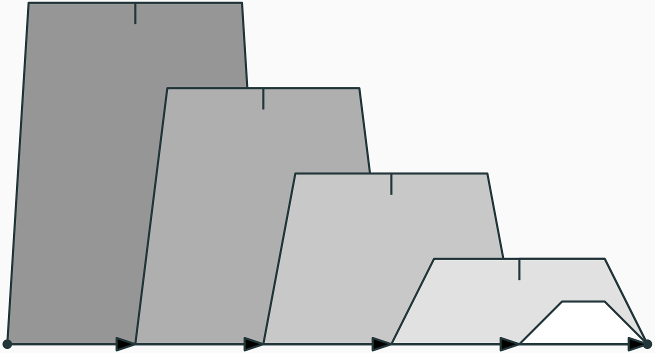

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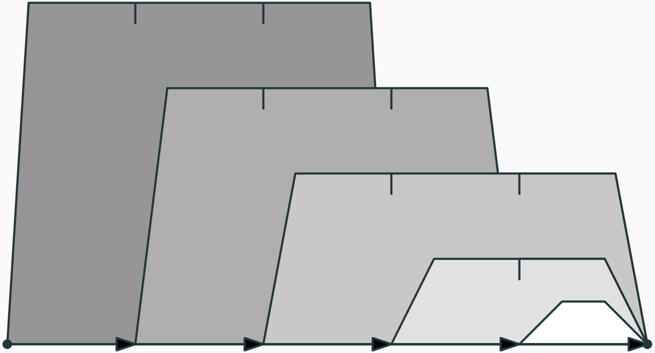

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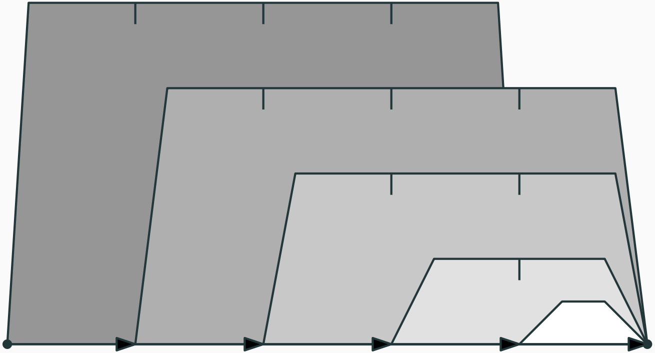

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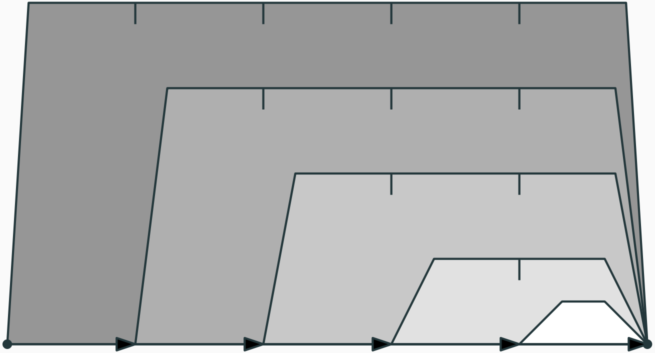

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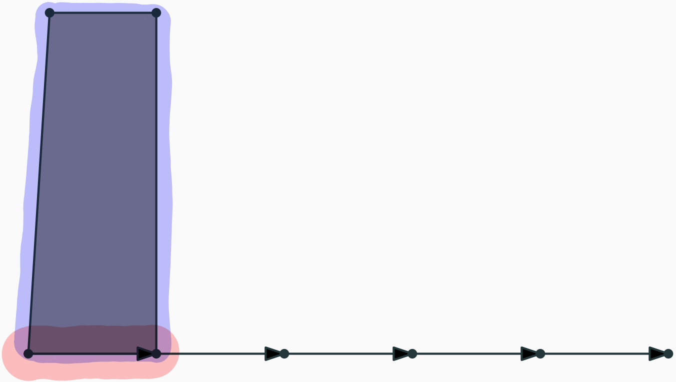

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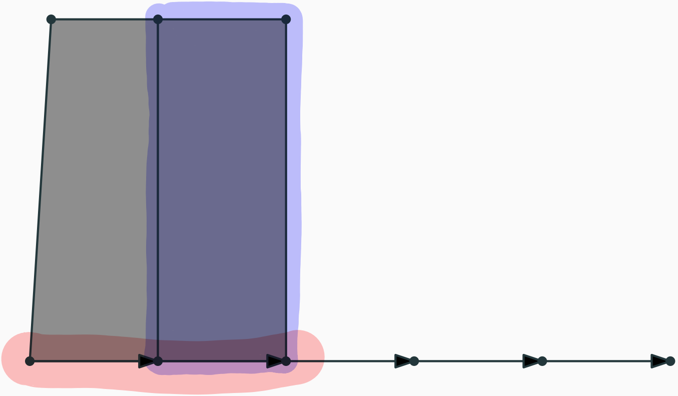

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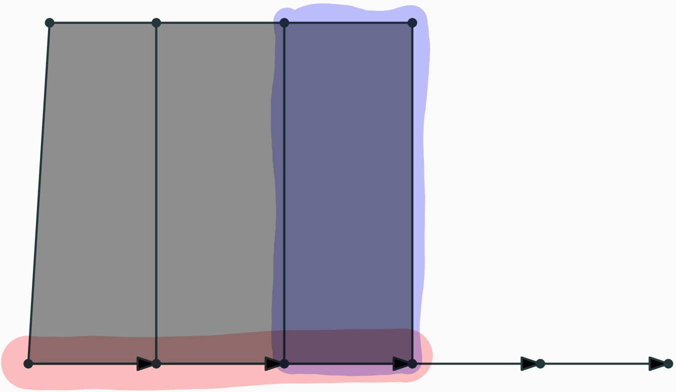

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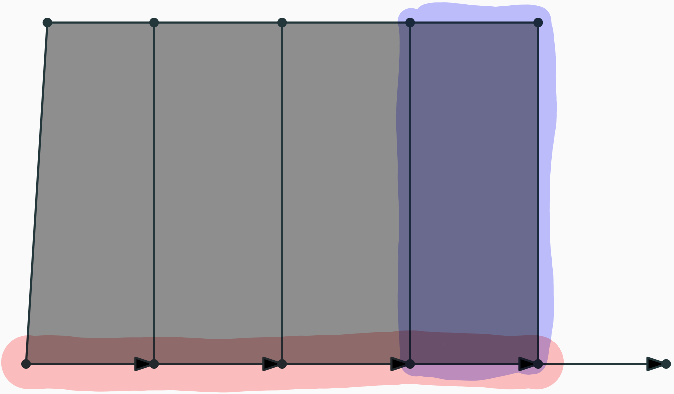

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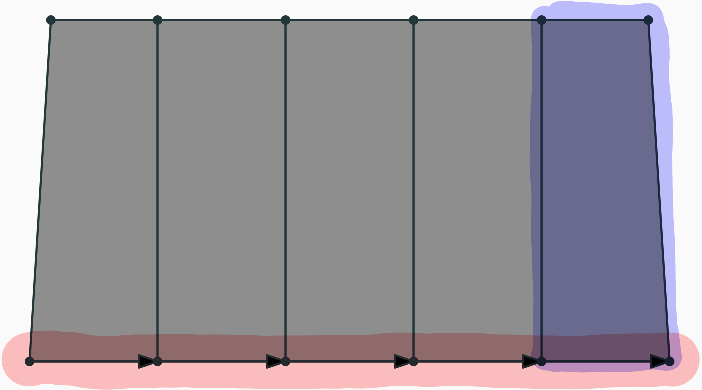

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Linear-width rewritings

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BDD definition (by FO-rewritings)

A ruleset $\mathcal{R}$ is BDD (FO-rewritable) if for every CQ $\mathcal{Q}$ there exists a UCQ $\mathcal{Q}'$ such that for every database $\mathcal{D}$ the following holds

$$\mathcal{D}, \mathcal{R} \models \mathcal{Q} \iff \mathcal{D} \models \mathcal{Q}'$$

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Every known decidable subclass of BDD admits linear-width rewritings!

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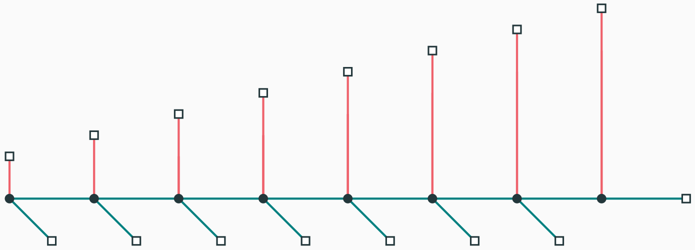

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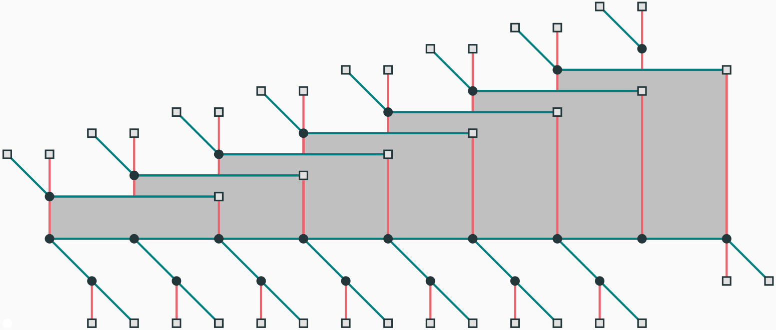

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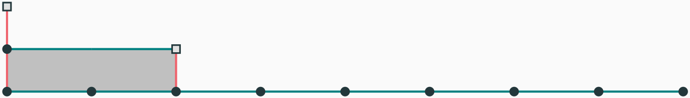

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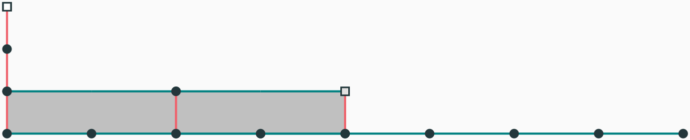

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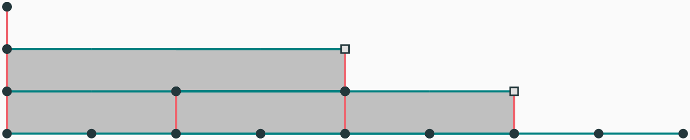

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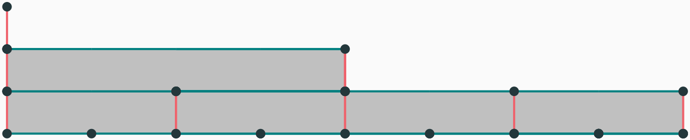

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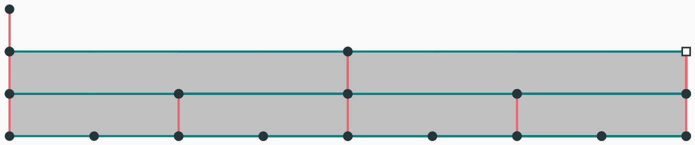

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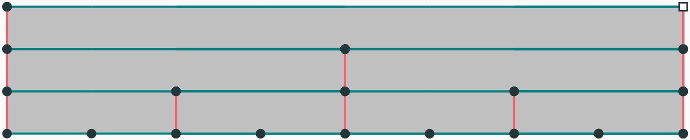

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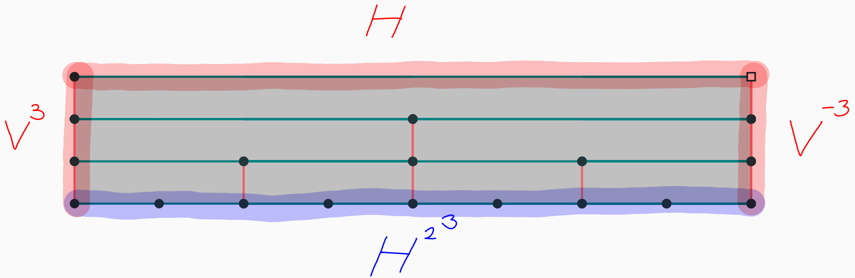

Rewriting of $V^n H V^{-n}$ contains a disjunct H^{2^n}!

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It gives a concrete example of behaviour that can be formalized and used for more expressive knowledge representation.

Thank You