A Journey
to the Frontiers of Query Rewritability

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June 7, 2022
The setting: existential rules

Definition (by example)

An \( \exists \)-rule or a Tuple Generating Dependency is an FO sentence of the following form:

\[
\forall x \ Human(x) \rightarrow \exists y \ Mother(x, y)
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\forall x, y \ Mother(x, y) \rightarrow Human(y)
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The setting: knowledge representation using $\exists$-rules
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“Everyone has a mother”

$\text{Human}(x) \rightarrow \exists y \ \text{Mother}(x, y)$

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"$E$ is transitive"

$$E(x, y), E(y, z) \rightarrow E(x, z)$$
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Given a ruleset $\mathcal{R}$ and a database $\mathcal{D}$ we ask if some query $Q$ holds in every model of $\mathcal{R}$ and $\mathcal{D}$?
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To denote that a query holds in every model of $\mathcal{R}$ and $\mathcal{D}$ we write

$\mathcal{R}, \mathcal{D} \models Q$. 
The main character: BDD class of ∃-rules
The main character: BDD class of $\exists$-rules

BDD class definition (By FO-rewritings)
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A ruleset $\mathcal{R}$ admits *Bounded Derivation Depth property (is FO-rewritable)* if for every CQ $Q$ there exists a UCQ $Q'$ such that for every database $D$ the following holds:

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The BDD class

If a ruleset is BDD then the entailment problem for conjunctive queries is decidable. BDD is an undecidable property. Thus, a lot of decidable subclasses of BDD were invented, such as: Linear, Sticky, Sticky-Join, Backward Shy. It is thought that BDD is well understood. Soon, however, we will see that there is a lot more to learn about it.
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Our contributions

We identify a class of "local theories" subsuming every known decidable subclass of BDD. We show that this class is strictly contained in BDD. And we show that the FUS/FES conjecture (BDD ∩ Core Terminating = Uniform BDD) holds for this class.
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(not a part of this talk)
The chase

\[ (x, y) \rightarrow \exists z \ E(y, z) \]

\[ A(x), E(x, y), E(y, z) \rightarrow E(x, z) \]
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BDD definition (by chase)
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A ruleset $\mathcal{R}$ is BDD if:

\[
\forall Q \exists k \forall D \mathcal{D}, \mathcal{R} \models Q \iff \text{Chase}^k(\mathcal{R}, \mathcal{D}) \models Q
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A ruleset $\mathcal{R}$ is BDD if:

$$\forall Q \exists k \forall D \quad D, \mathcal{R} \models Q \iff \text{Chase}_k(\mathcal{R}, D) \models Q$$
Transitivity is not BDD

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$E(x, y), E(y, z) \rightarrow E(x, z)$

$(\neg \text{BDD}) \exists Q \ \forall k \ \exists D \ \mathcal{D}, \mathcal{R} \models Q \land \text{Chase}_k(\mathcal{R}, \mathcal{D}) \nvdash Q$
Transitivity is not BDD

\[ E(x, y), E(y, z) \rightarrow E(x, z) \]

\[ (\neg \text{BDD}) \quad \exists Q \quad \forall k \quad \exists D \quad D, R \models Q \land \text{Chase}_k(R, D) \not\models Q \]

\[ Q = E(a, b) \]
Reachability is not BDD

\[ a \xrightarrow{E(x, y)} A(y) \]
Reachability is not BDD

$A(x), E(x, y) \rightarrow A(y)$
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Class of local theories

Definition of the class of local theories

We say that a ruleset $R$ is local if every atom of the chase can be derived only from a constant number of atoms of the database.

Every local ruleset is BDD!

Moreover, it contains every known decidable subclass of BDD.

Is converse true as well?

No!
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\[ E(x, y) \rightarrow \exists x', y' \ Box(x, y, x', y') \]
BDD is not local!

\( E(x, y) \rightarrow \exists x', y' \ Box(x, y, x', y') \)

\( \Box(x, y, x', y'), E(x, y), E(y, z) \rightarrow \exists z' \ Box(y, z, y', z') \)
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Linear-width rewritings

BDD definition (by FO-rewritings)

A ruleset $R$ is BDD (FO-rewritable) if for every CQ $Q$ there exists a UCQ $Q'$ such that for every database $D$ the following holds:

$$D, R \models Q \iff D \models Q'$$

The linear-width rewritings class

The width of a UCQ is the maximal size of its disjuncts. We say that a ruleset $R$ admits linear-width rewritings if it is BDD and for every CQ its UCQ rewriting is of linear size width.

Every known decidable subclass of BDD admits linear-width rewritings!
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BDD is not about linear-width rewritings
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\[ \rightarrow \exists x \; H(x, x), \; V(x, x) \]
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\[ \rightarrow \exists x \ H(x, x), V(x, x) \]

\[ \top(x) \rightarrow \exists y, z \ H(x, y), V(x, z) \]
BDD is not about linear-width rewritings

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→ ∃x H(x, x), V(x, x)
T(x) → ∃y, z H(x, y), V(x, z)
H(x, y), H(y, z), V(x, x') → ∃z' V(z, z'), H(x', z')
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\[
\rightarrow \exists x \ H(x, x), \ V(x, x) \\
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H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z')
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BDD is not about linear-width rewritings

\[ \to \exists x \ H(x, x), \ V(x, x) \]

\[ \text{T}(x) \to \exists y, z \ H(x, y), \ V(x, z) \]

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$$\top(x) \rightarrow \exists y, z \ H(x, y), \ V(x, z)$$

$$H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z')$$
BDD is not about linear-width rewritings

\[ \rightarrow \exists x \ H(x, x), \ V(x, x) \]

\[ \top(x) \rightarrow \exists y, z \ H(x, y), \ V(x, z) \]

\[ H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z') \]
BDD is not about linear-width rewritings

\[
\rightarrow \exists x \; H(x, x), \; V(x, x)
\]

\[
\top(x) \rightarrow \exists y, z \; H(x, y), \; V(x, z)
\]

\[
H(x, y), \; H(y, z), \; V(x, x') \rightarrow \exists z' \; V(z, z'), \; H(x', z')
\]
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\[ \rightarrow \exists x \ H(x, x), \ V(x, x) \]
\[ \top(x) \rightarrow \exists y, z \ H(x, y), \ V(x, z) \]
\[ H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z') \]
BDD is not about linear-width rewritings

→ ∃x H(x, x), V(x, x)

T(x) → ∃y, z H(x, y), V(x, z)

H(x, y), H(y, z), V(x, x') → ∃z' V(z, z'), H(x', z')
BDD is not about linear-width rewritings

\[ \rightarrow \exists x \ H(x, x), \ V(x, x) \]

\[ \top(x) \rightarrow \exists y, z \ H(x, y), \ V(x, z) \]

\[ H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z') \]
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\[ H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z') \]
BDD is not about linear-width rewritings

\[ \rightarrow \exists x \ H(x, x), \ V(x, x) \]

\[ T(x) \rightarrow \exists y, z \ H(x, y), \ V(x, z) \]

\[ H(x, y), \ H(y, z), \ V(x, x') \rightarrow \exists z' \ V(z, z'), \ H(x', z') \]
Why is this important?
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It shows deficiency in the understanding of an impactful class of existential rules.
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How can we use it?
Why is this important?
It shows deficiency in the understanding of an impactful class of existential rules.

How can we use it?
It gives a concrete example of behaviour that can be formalized and used for more expressive knowledge representation.
Thank You