Deciding Hyperproperties Combined with Functional Specifications

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[3] *LIRMM, Inria, University of Montpellier, CNRS, France*

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		no LTL spec.	with LTL spec.
Temporal Safety	complete fragment	coRE [Thm. 3.7]	Σ_1^1 [Thm. 3.11]
	$\forall^*\exists^*.\bigcirc^*$	NEXP [Thm. 3.12]	NEXP [Thm. 3.12]
	$\forall^*\exists^*.\square$	NEXP [Lem. 3.13]	Σ_1^1 [Thm. 3.11]
	$\forall^*\exists^*.\square(\bigcirc^*)$	coRE [Lem. 3.10]	Σ_1^1 [Thm. 3.11]
Temporal Liveness	complete fragment	Σ_1^1 [Thm. 4.2]	Σ_1^1 [Thm. 4.2]
	$\forall\exists^*$. det-liveness	trivial [Prop. 4.15]	Σ_1^1 [Cor. 4.16]
	$\forall\exists^*.\Diamond(\bigcirc^*)$	NP [Lem. 4.4]	dec. [Thm. 4.6]
	$\forall^*\exists^*.\Diamond\wedge\cdots\wedge\Diamond$	NP [Lem. 4.4]	Σ_1^1 [Thm. 4.12]

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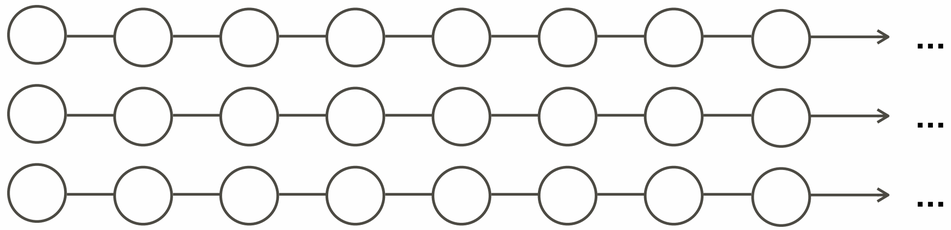
- Sound algorithm for largest models for $\forall\exists^*$ HyperLTL

Hyperproperties

Hyperproperties

Trace property P: set of traces

system



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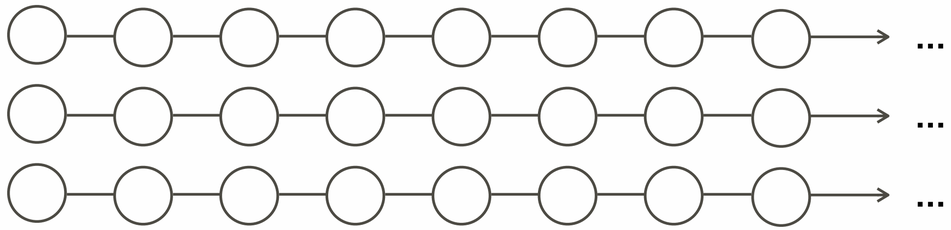

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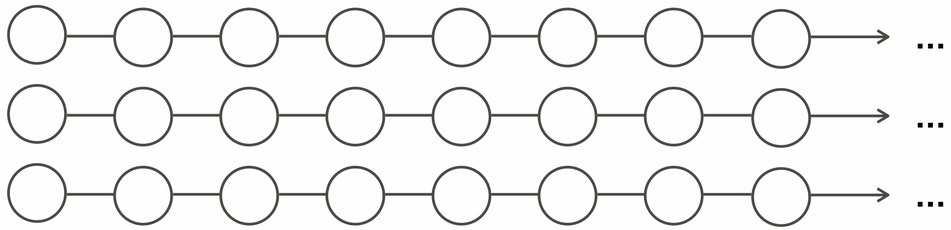

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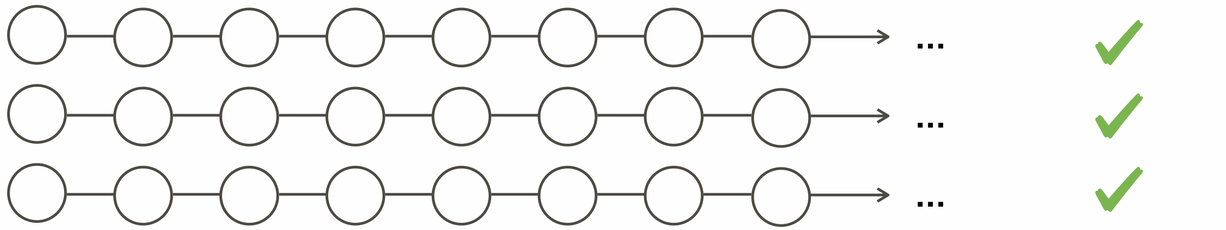

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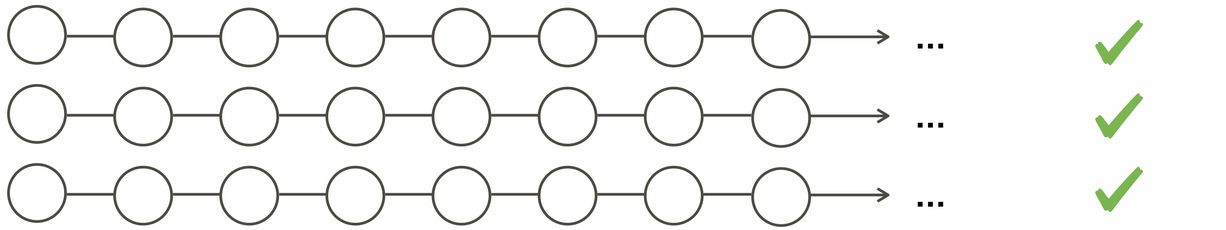

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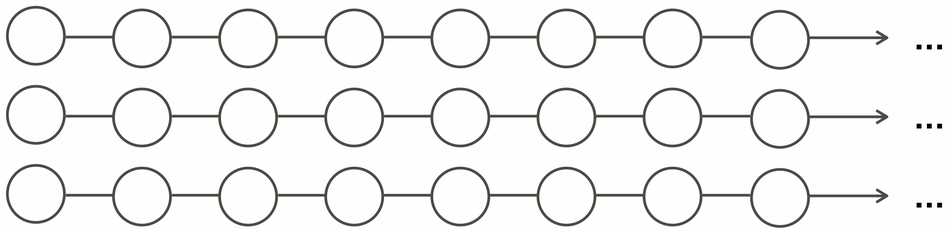

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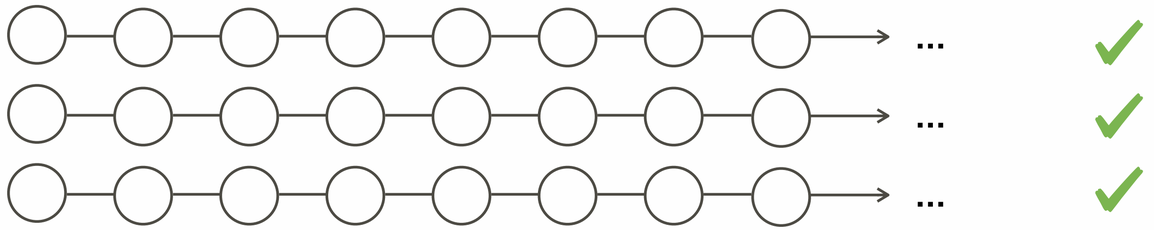

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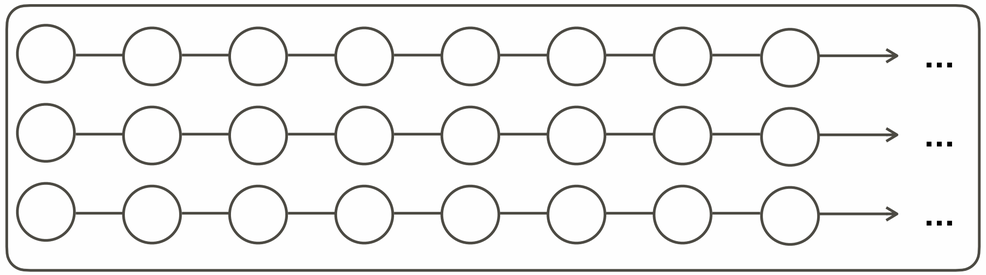

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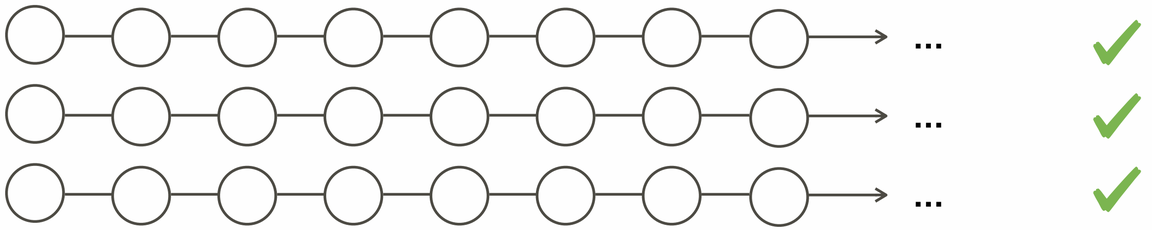

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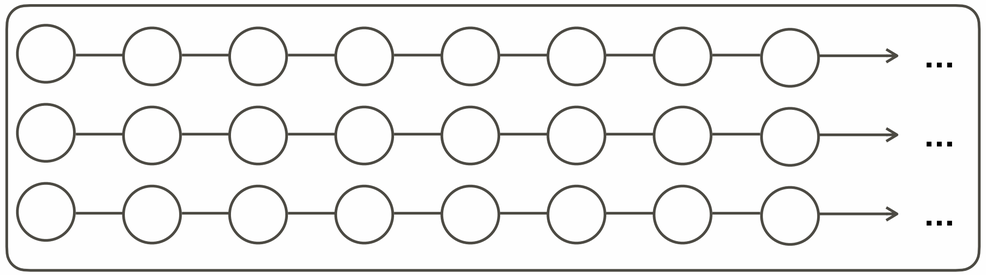

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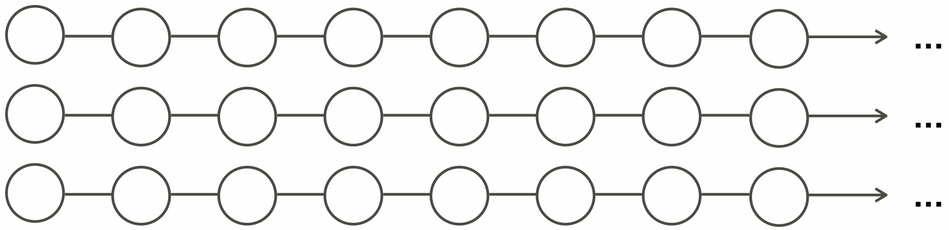

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Functional properties: safety, liveness, …

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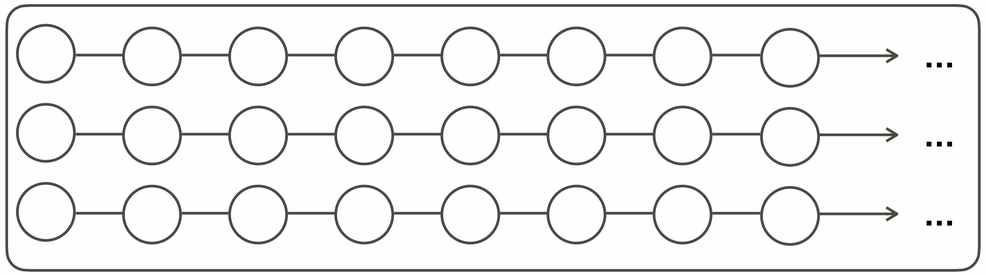

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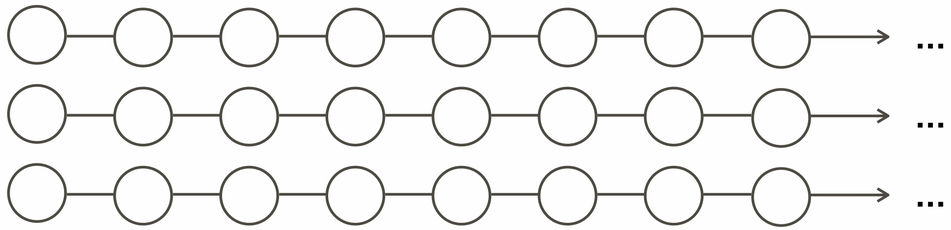

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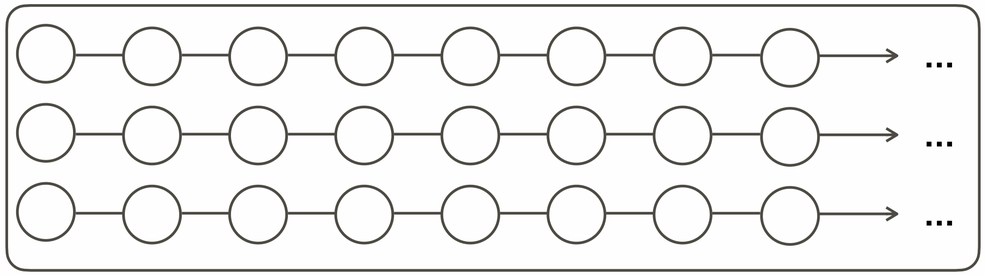

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Relational properties:

"Does a change of inputs lead to a change of outputs?"

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HyperLTL[1]

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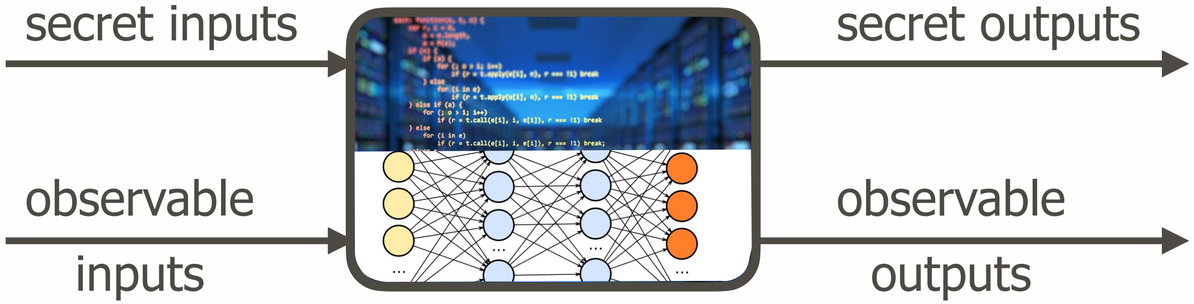

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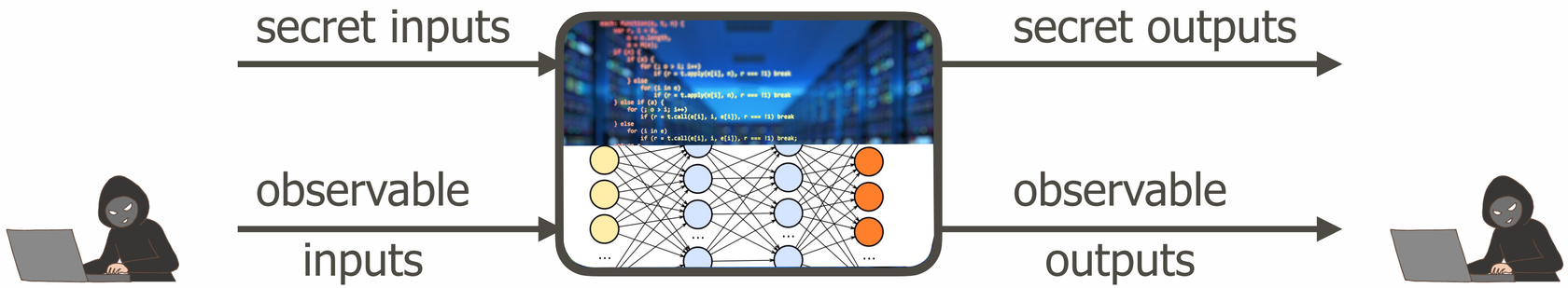

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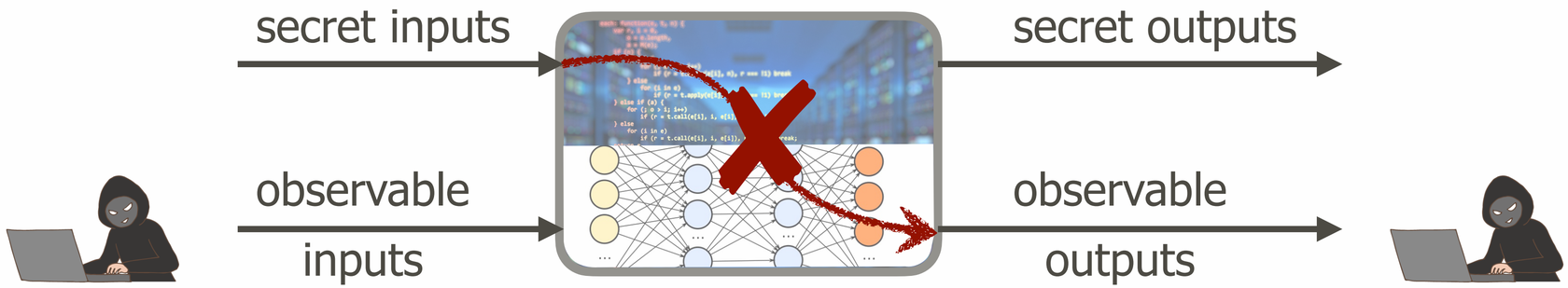

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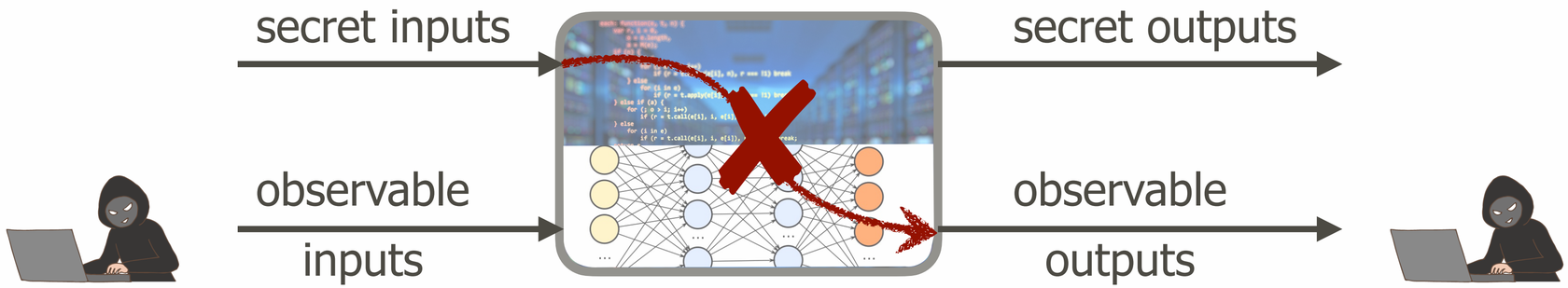

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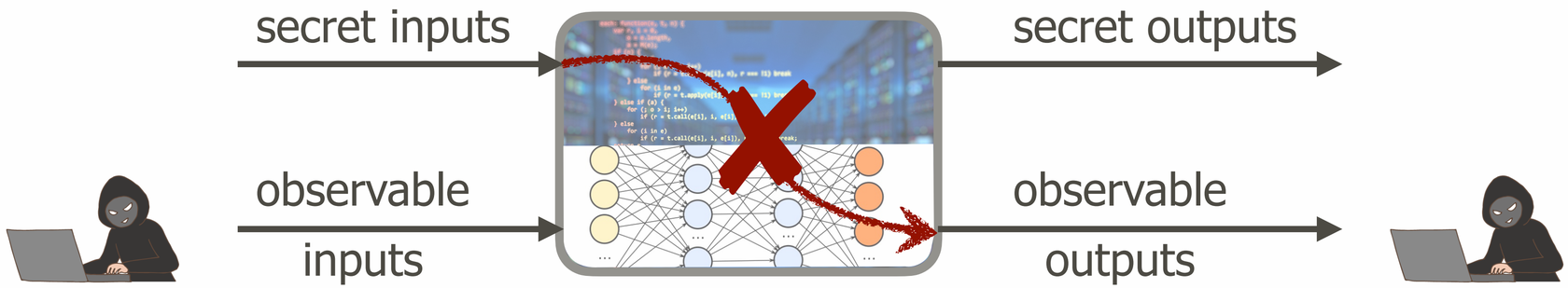

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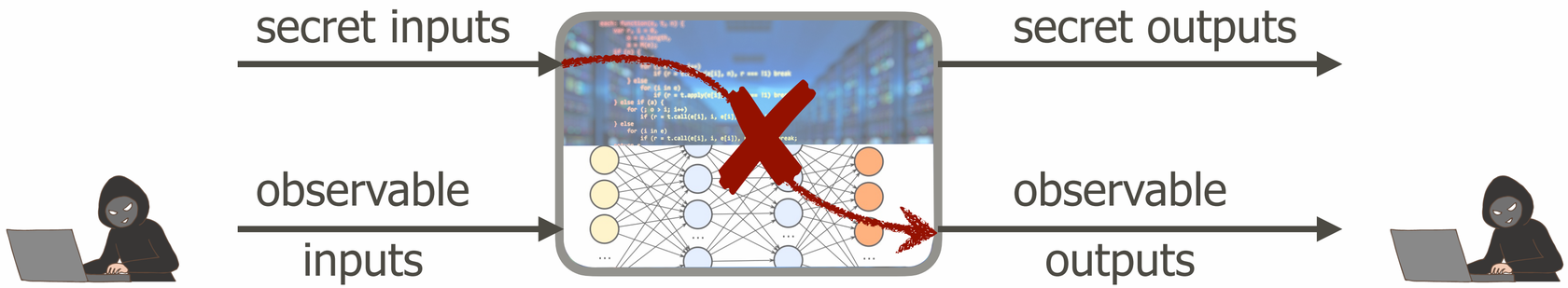

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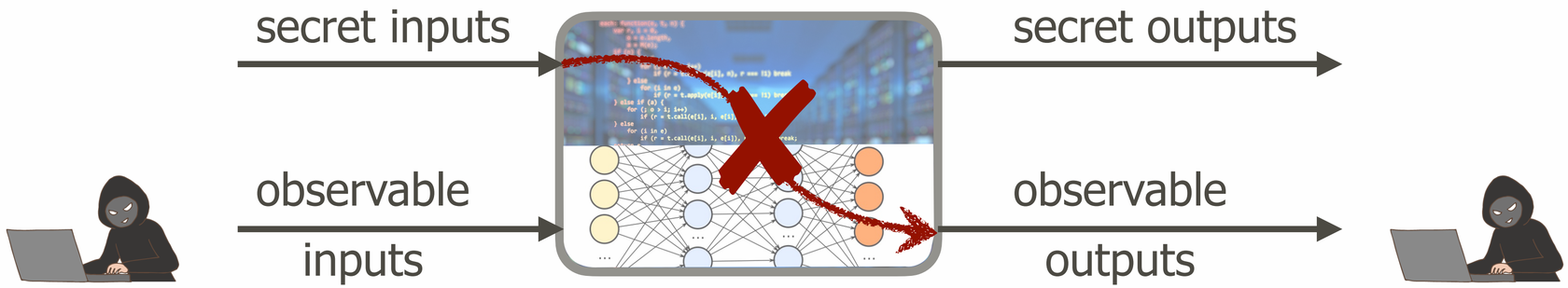

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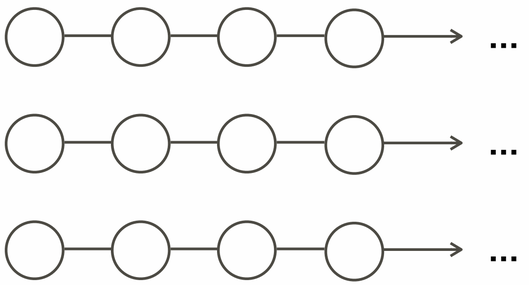

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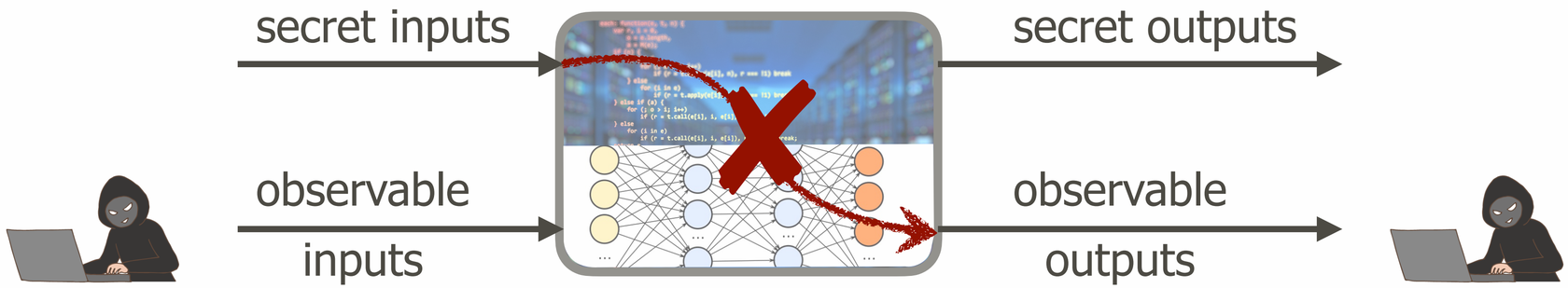

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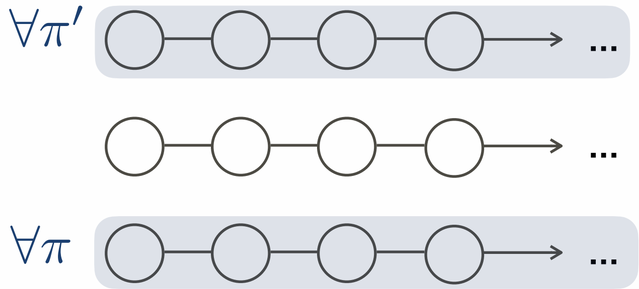

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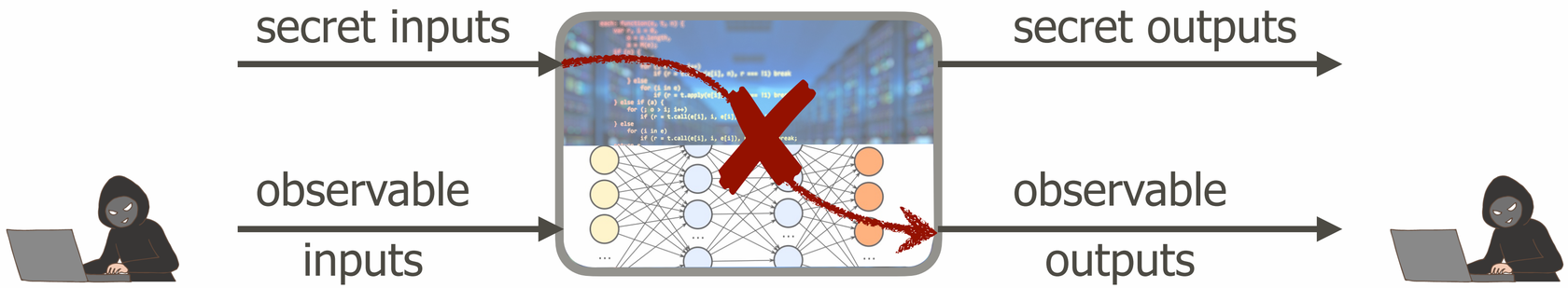

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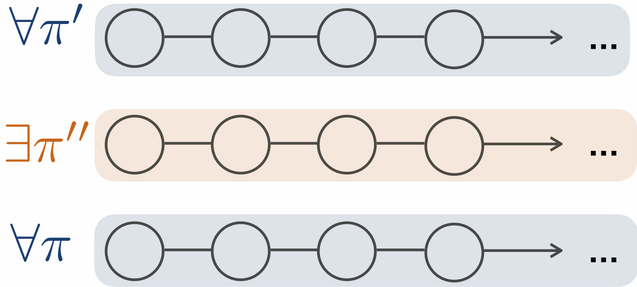

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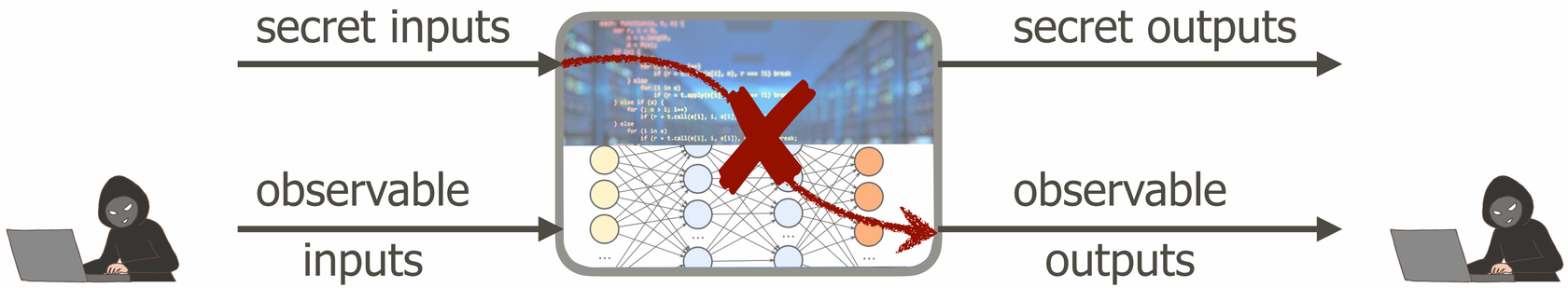

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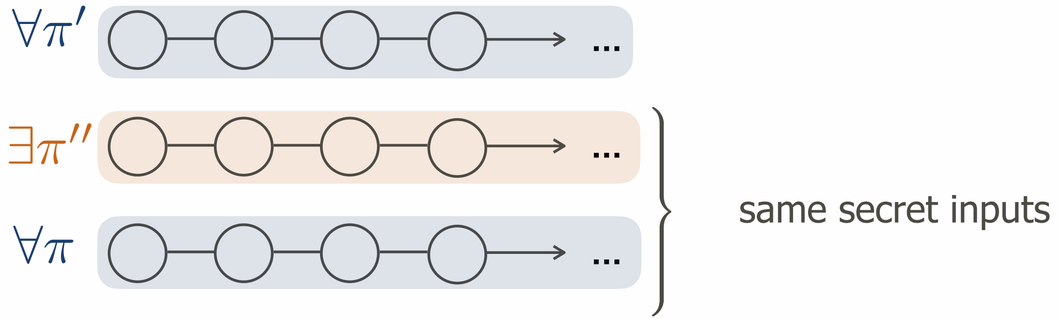

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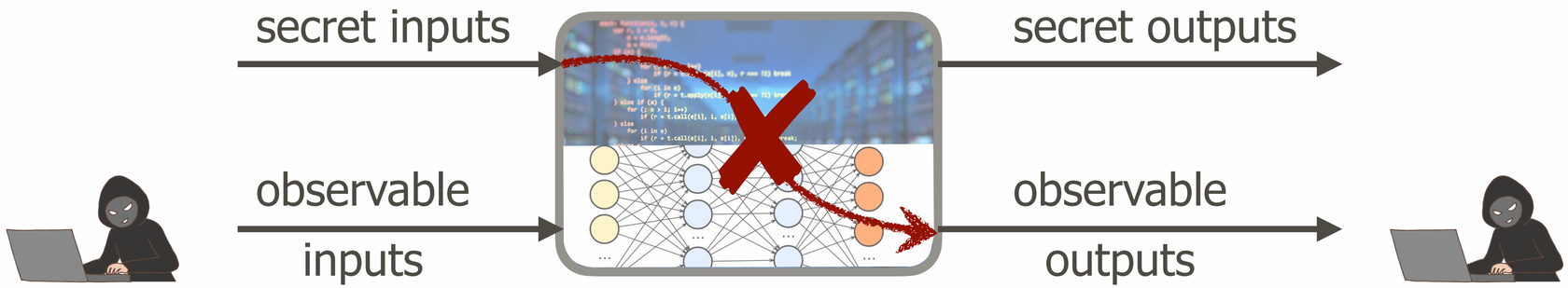

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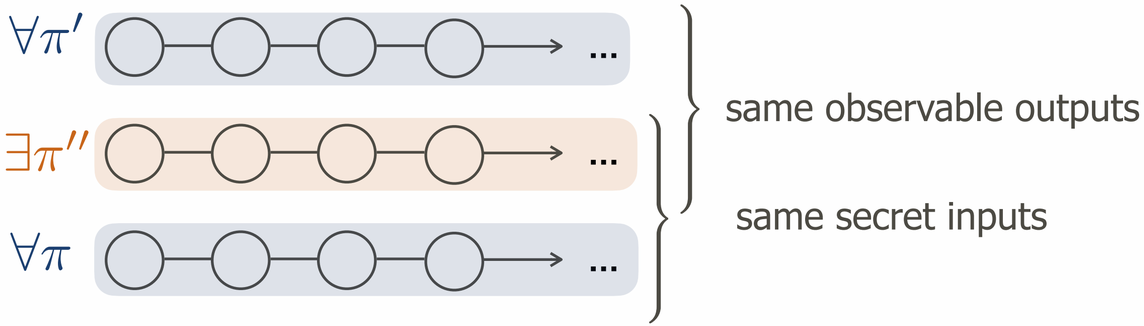

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Satisfiability of ∀*∃* HyperLTL

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⇨ New perspectives on HyperLTL satisfiability?

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New Perspectives on ∀*∃* HyperLTL SAT

2 approaches:

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1) Split hyperproperty into trace property + hyperproperty

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Trace property in LTL: describes functional behavior

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Trace property in LTL: describes functional behavior

Hyperproperty in HyperLTL: simple relational property

New Perspectives on ∀* ∃* HyperLTL SAT

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1) Split hyperproperty into trace property + hyperproperty

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$\forall^* \exists^* . \psi$ — safety / liveness LTL formula

Temporal Safety vs Hypersafety

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Hypersafety[1]: Does every counterexample have a "finite reason" for being a counterexample?

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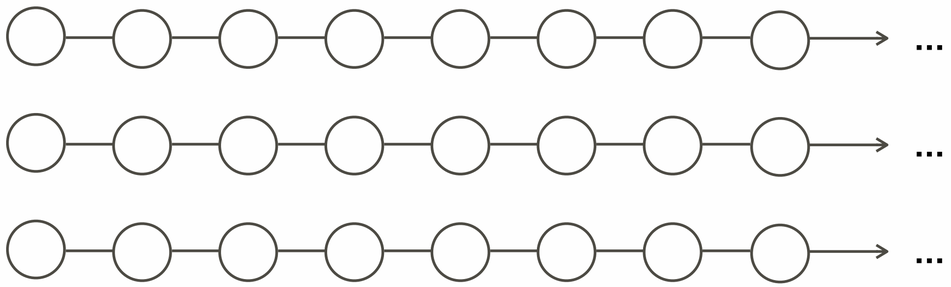

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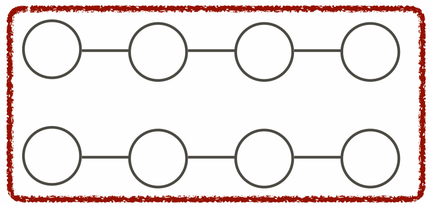

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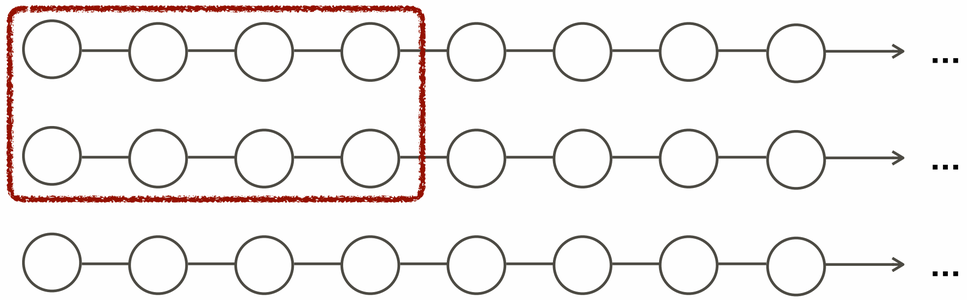

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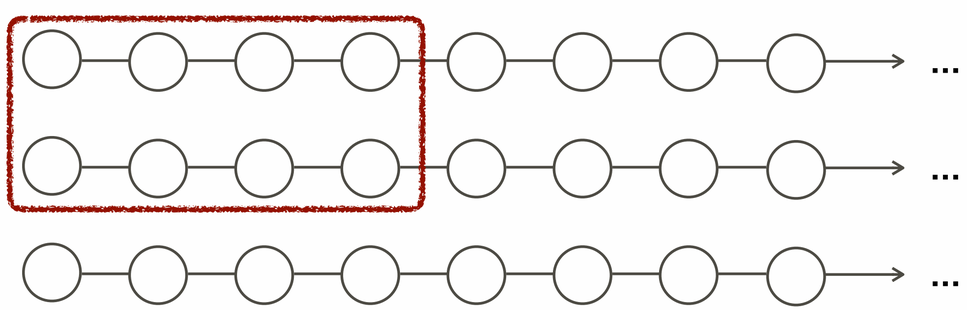

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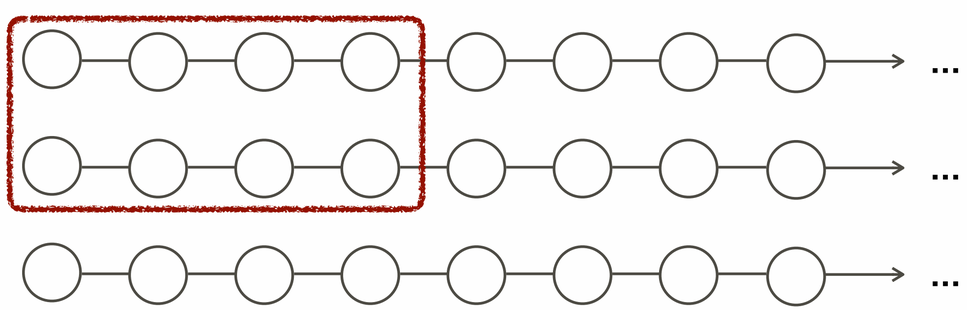

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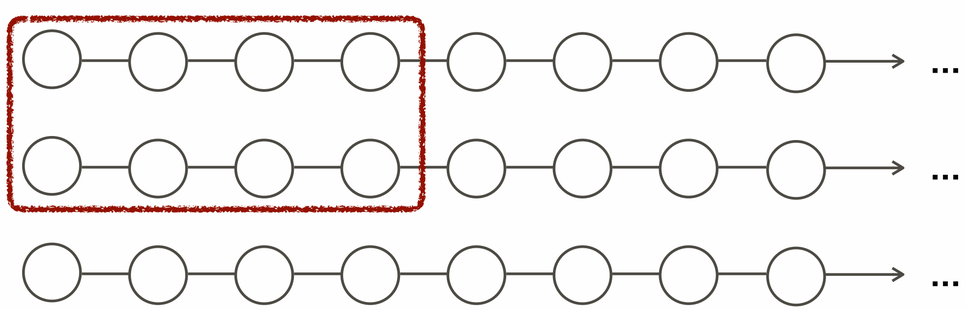

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Hypersafety does not help with satisfiability:

1) Whether HyperLTL formula φ is hypersafety is highly undecidable (in Π^1_1)[2]

2) If we know that φ is hypersafety, deciding SAT is in PSPACE - no harder than LTL

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Results

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Reduction to first-order logic

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First decidability result for formulas that can enforce models with infinitely many traces

Finding Largest Models for ∀∃* HyperLTL

1: **procedure** FINDMODEL($\mathcal{A}$)
2: **if** $\mathcal{L}(\mathcal{A}^{\forall}) = \emptyset$ **then**
3: **return** UNSAT;
4: **if** $\mathcal{L}(\mathcal{A}^{\exists}) \subseteq \mathcal{L}(\mathcal{A}^{\forall})$ **then**
5: **return** SAT, model: $\mathcal{L}(\mathcal{A}^{\forall})$;
6: $\mathcal{A}_{\text{new}} := \mathcal{A} \cap \mathcal{A}^{\forall}_{\pi'}$;
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6:     𝒜_new := 𝒜 ∩ 𝒜^∀_π';
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$\forall\pi.\exists\pi'.\ \square(a_\pi \wedge (b_\pi \leftrightarrow \bigcirc b_{\pi'}))$

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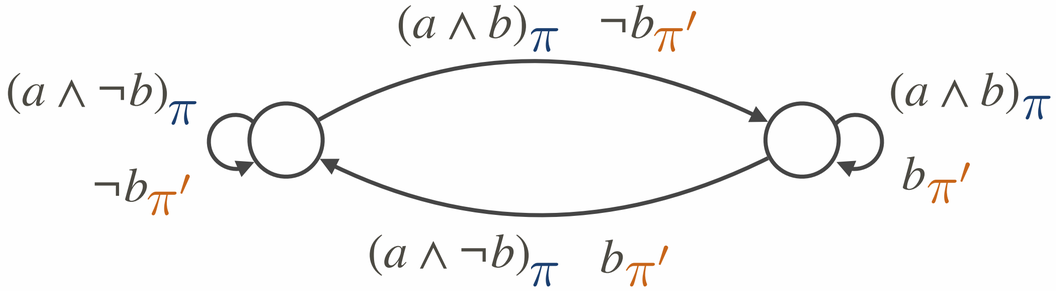

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Remove ∀-traces that produce wrong ∃-traces

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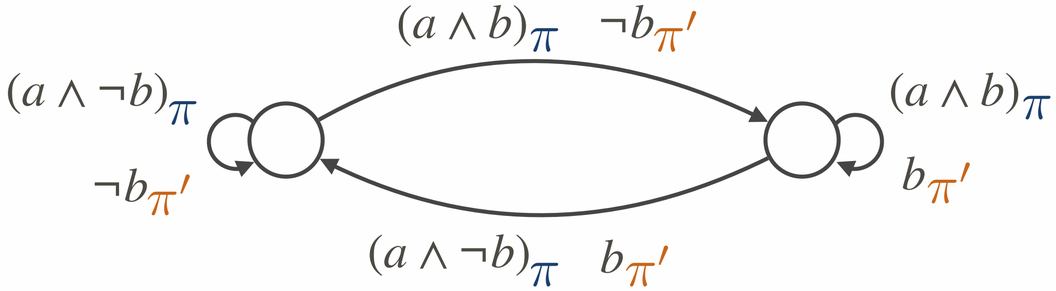

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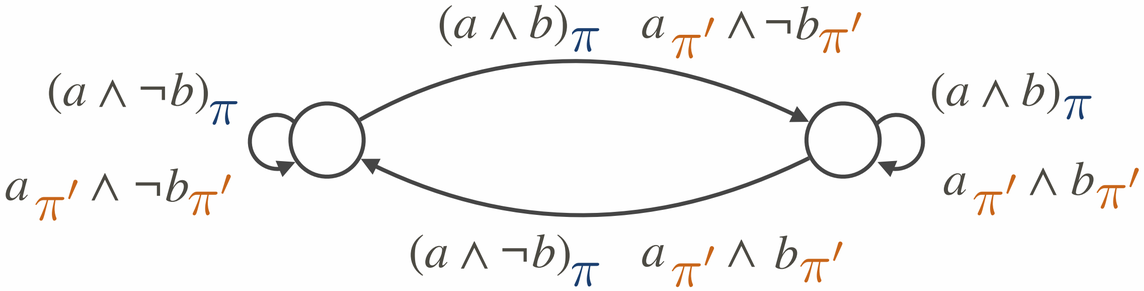

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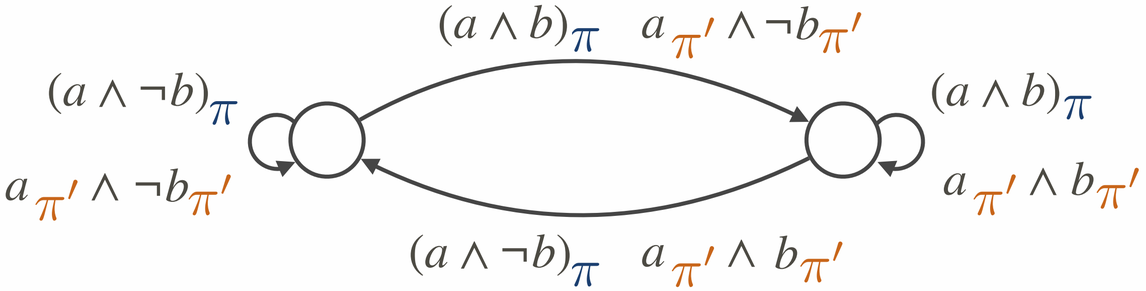

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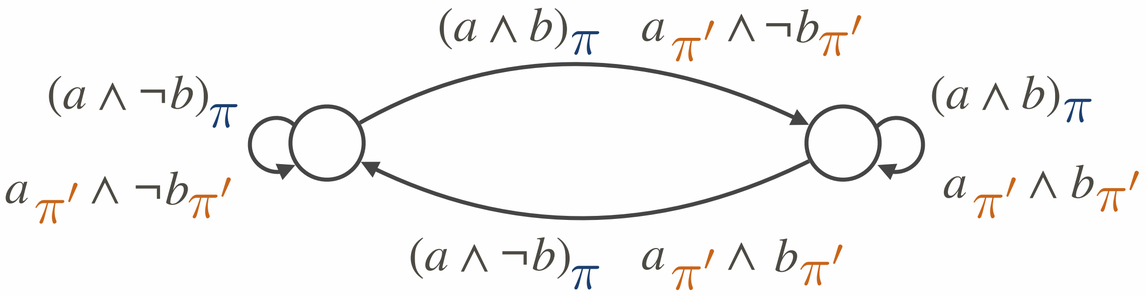

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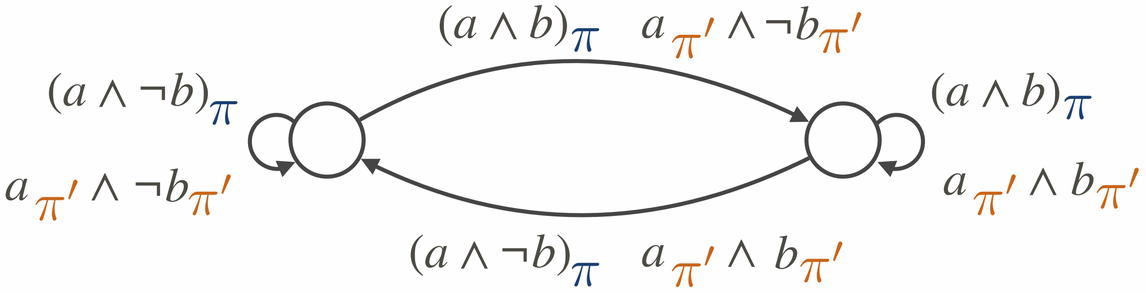

- Finds largest models
- Sound but necessarily not complete
- Evaluation: finds models that MGHyper[1] does not find, can show unsatisfiability

[1] Finkbeiner, Hahn, Hans. MGHyper: *Checking Satisfiability of HyperLTL formulas beyond the ∃*∀* Fragment*. ATVA 2018.

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New $\forall^*\exists^*$ decidability results beyond purely syntactic restrictions

Fixpoint algorithm for SAT and UNSAT

- Sound algorithm for largest models for $\forall\exists^*$ HyperLTL