Modelling Multiple Perspectives with Standpoint Logics

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Outline

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➡ Standpoint Logic at a Glance (Motivation, Syntax and Semantics)

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➡ Decidability and Complexity Landscape

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    - ➡ Sentential Fragments (Decidability, Complexity, Implementations)

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  - ➡ Monodic Fragments (Decidability, Complexity, Implementations)

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  - ➡ Monodic Fragments (Decidability, Complexity, Implementations)
- ➡ Conclusions and Future Work

Standpoint Logic

(Motivation)

Knowledge Integration

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(2,3) Process(c)

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(2,3) Process(c) ← Tumor(c) → ¬Process(c) (1,4)

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- **Multimodal logic** inspired by the **supervaluationist theory** of natural language.

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- **Standpoints can be hierarchically organised, combined and put in relation to one another.**

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OPT-2 Representation of the diversity (Standpoint Logic):

- Knowledge relative to ``points of view'' (standpoints)
- **Multimodal logic** inspired by the **supervaluationist theory** of natural language.
- **Standpoints can be hierarchically organised, combined and put in relation to one another.** Eg: $(P \preceq S) \wedge (T \preceq S)$

Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg \exists x \ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\square_{\mathsf{P}}(\forall x \ \mathsf{Tumor}(x) \rightarrow \mathsf{AbnormalGrowthProcess}(x))$
3. $\square_{\mathsf{P}}(\forall x \ \mathsf{AbnormalGrowthProcess}(x) \rightarrow \mathsf{Process}(x))$
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4. $\square_{\mathsf{T}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
5. $\mathsf{T} \preceq \mathsf{S} \ \wedge\ \mathsf{P} \preceq \mathsf{S}$

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4. $\square_{T}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
5. $\mathrm{T} \preceq \mathrm{S} \ \wedge \ \mathrm{P} \preceq \mathrm{S}$
6. $\forall x\ \lozenge_{S}(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_{T}\ \mathrm{Tumor}(x)$

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2. $\square_P(\forall x \; \text{Tumor}(x) \rightarrow \text{AbnormalGrowthProcess}(x))$
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5. $\text{T} \preceq \text{S} \;\wedge\; \text{P} \preceq \text{S}$
6. $\forall x \; \lozenge_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_T \, \text{Tumor}(x)$
7. $\forall x \; \square_P \, (\exists y \; \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \lozenge_T \text{Tumor}(x)$

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6. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \square_T\, \mathsf{Tumor}(x)$
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4. $\square_T(\forall x \ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
5. $T \preceq S \ \wedge \ P \preceq S$
6. $\forall x \ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_T \text{Tumor}(x)$
7. $\forall x \ \square_P (\exists y \ \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_T \text{Tumor}(x)$
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9. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

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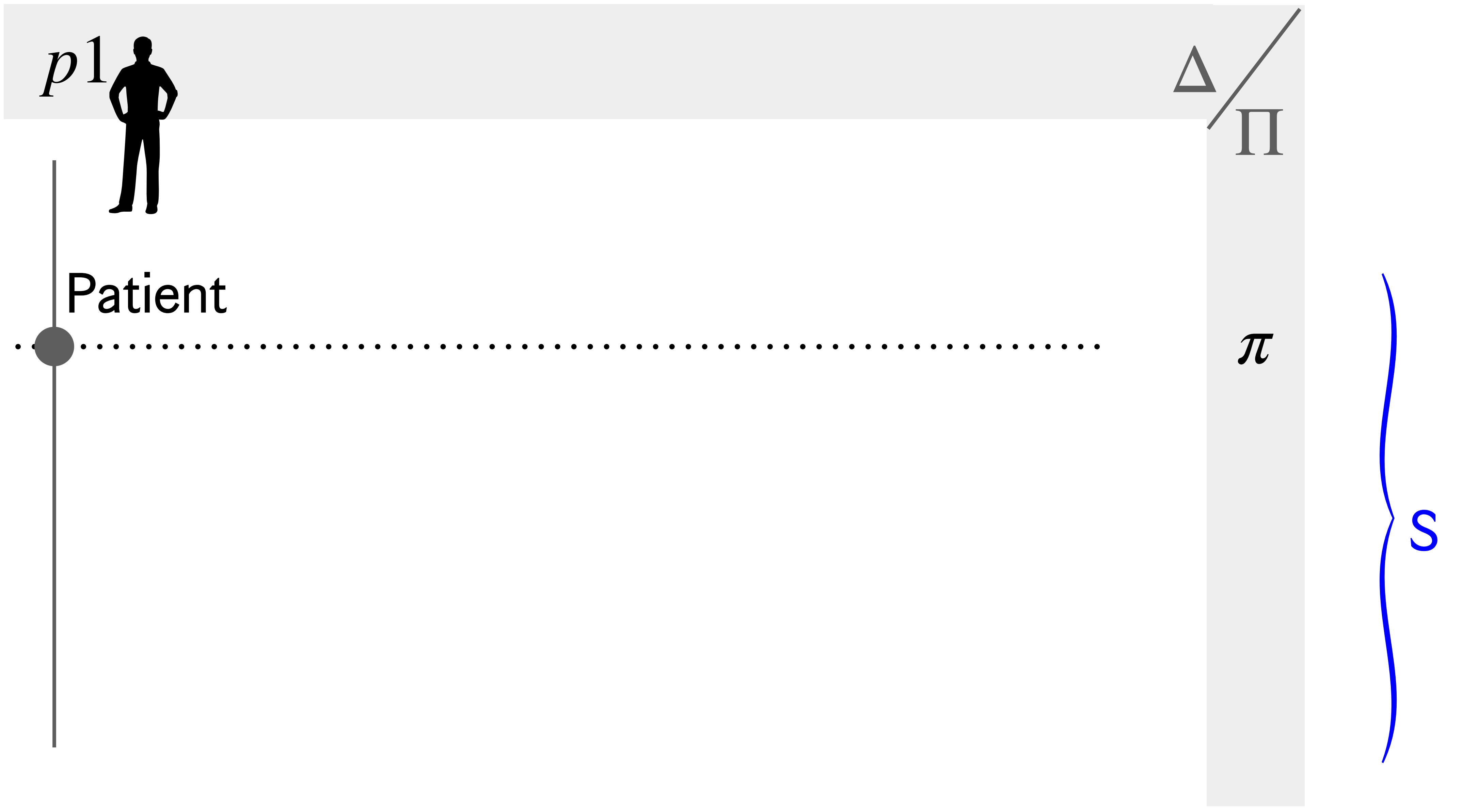

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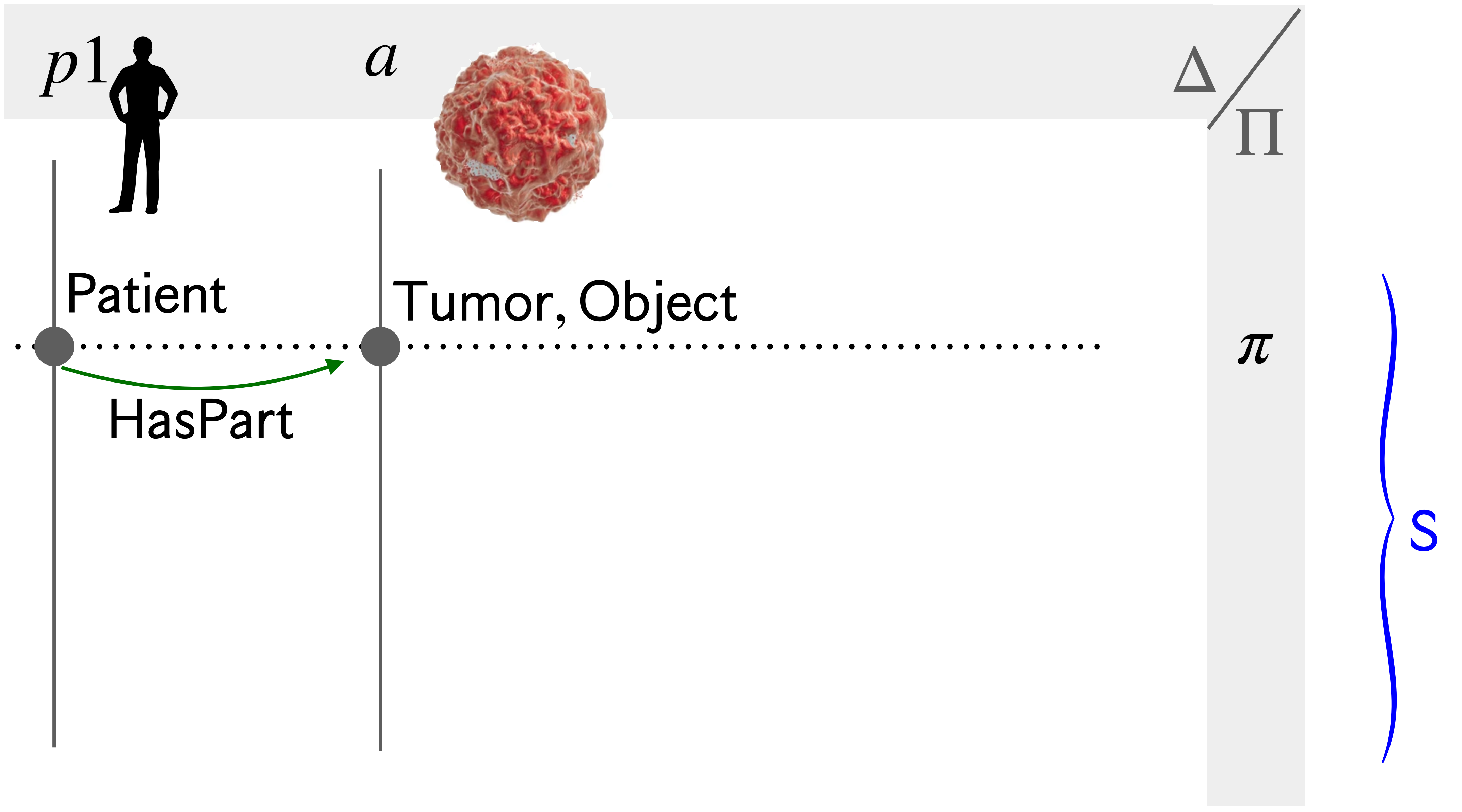

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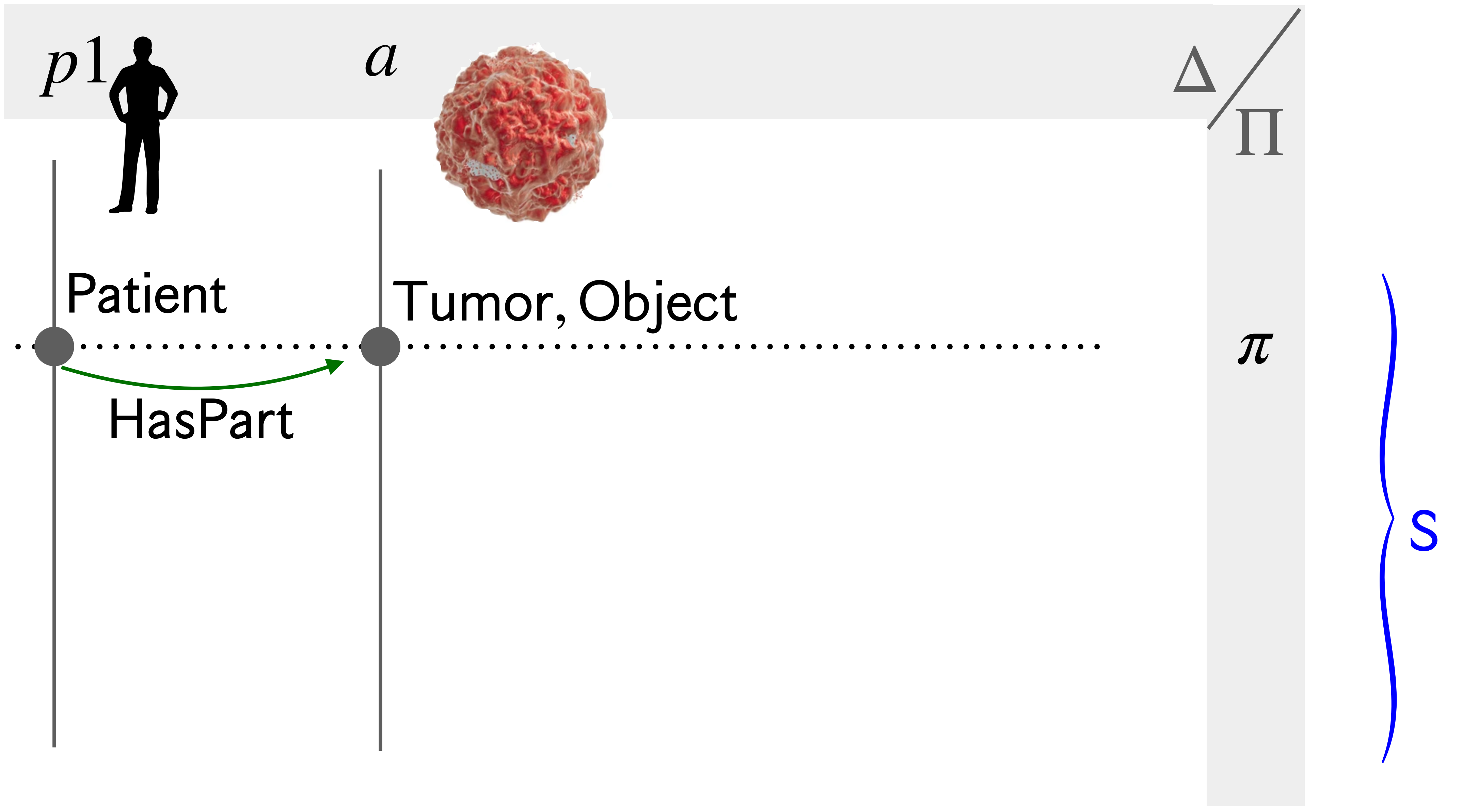


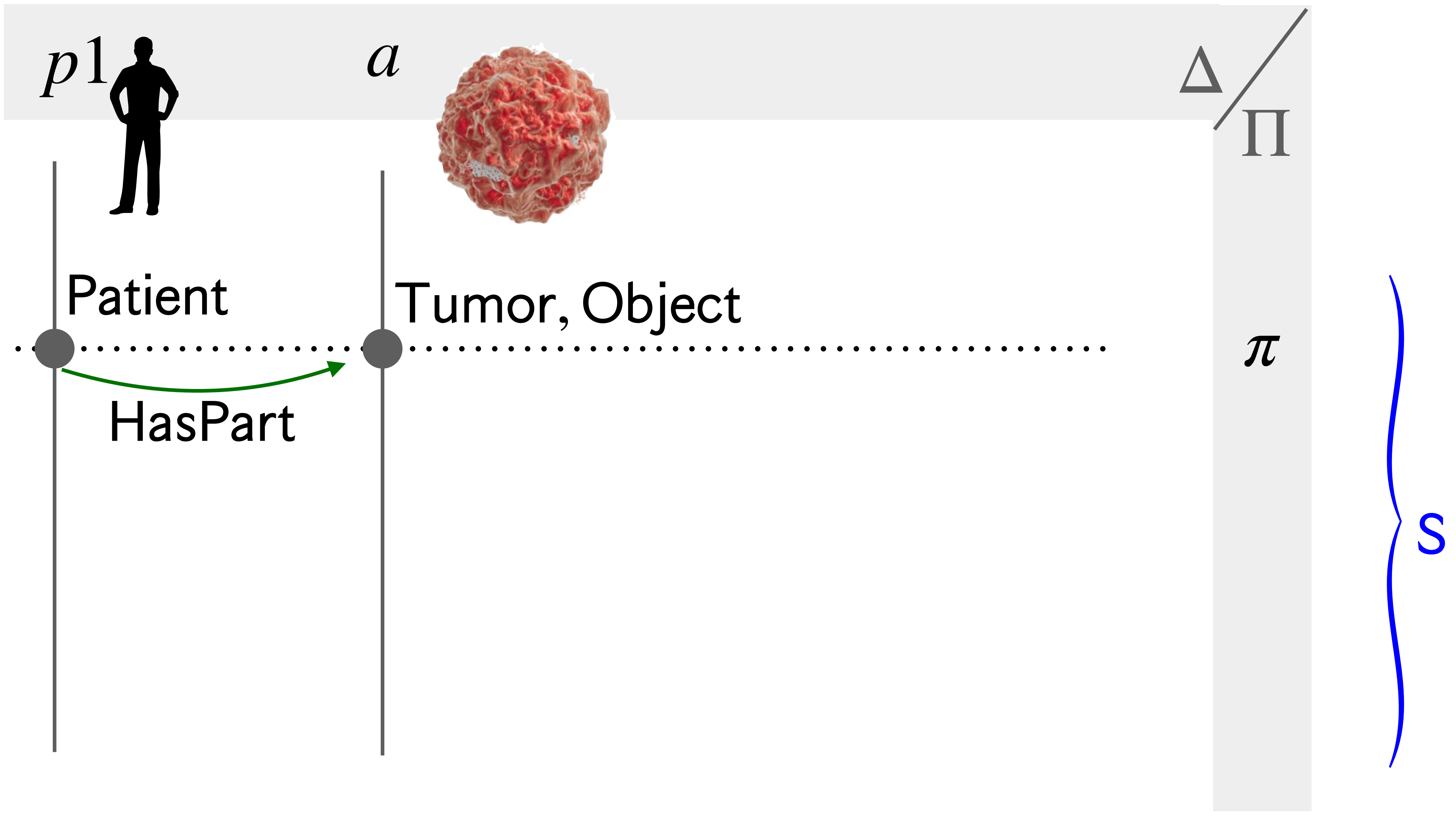
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg \exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\Box_P(\forall x\ \text{Tumor}(x) \rightarrow \text{AbnormalGrowthProcess}(x))$
3. $\Box_P(\forall x\ \text{AbnormalGrowthProcess}(x) \rightarrow \text{Process}(x))$
4. $\Box_T(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
5. $T \preceq S \ \wedge\ P \preceq S$
6. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \Box_T\,\text{Tumor}(x)$
7. $\forall x\ \Box_P(\exists y\ \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_T\text{Tumor}(x)$
8. $\forall x\ \Box_T\,\text{Tissue}(x) \rightarrow \Box_P\,\text{Tissue}(x)$

9. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

Inferences:

10. $\Box_T\text{Tumor}(a)$ (6,9)

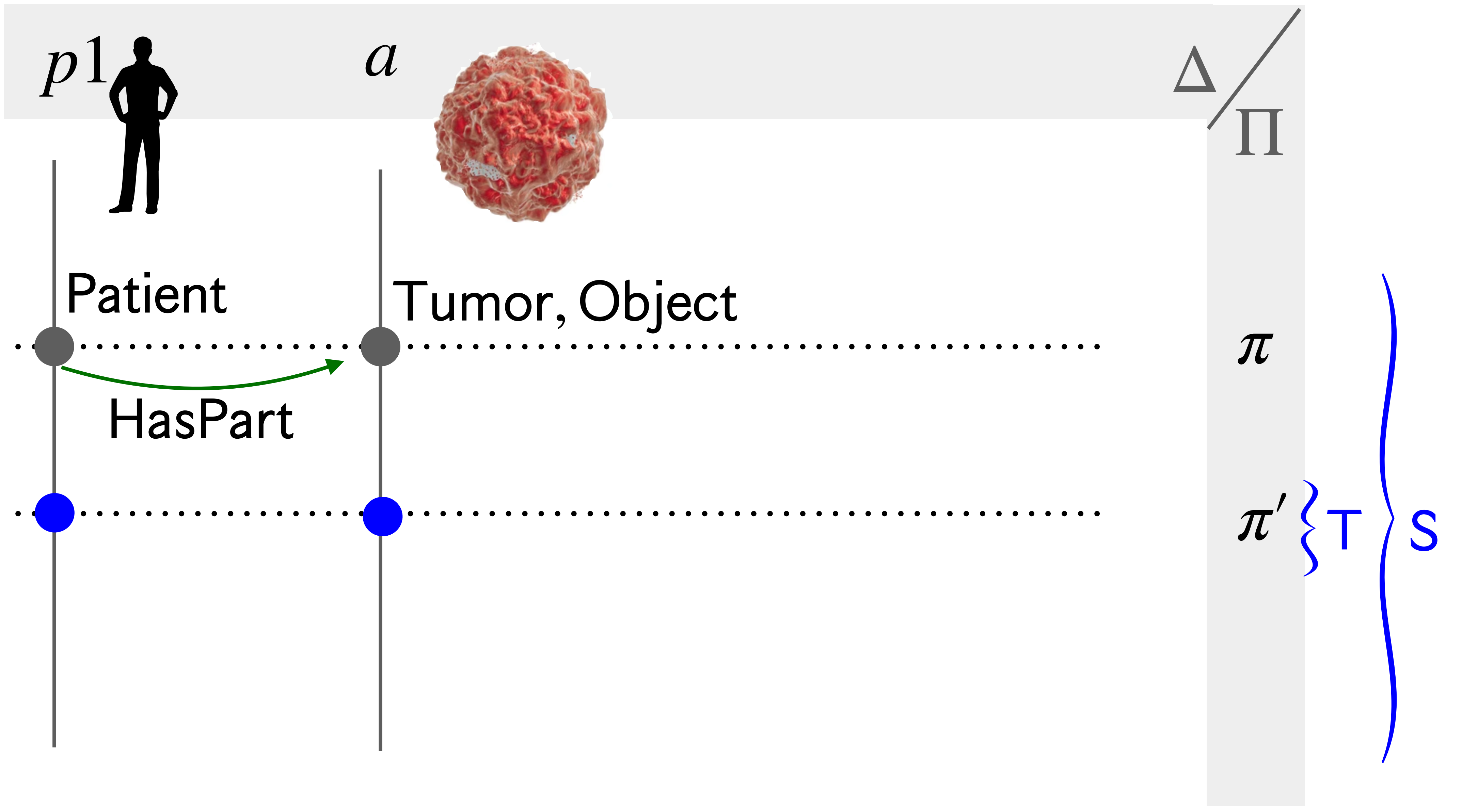


Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\Box_P(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{AbnormalGrowthProcess}(x))$
3. $\Box_P(\forall x\ \mathsf{AbnormalGrowthProcess}(x) \rightarrow \mathsf{Process}(x))$
4. $\Box_T(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
5. $\mathsf{T} \preceq \mathsf{S} \ \land\ \mathsf{P} \preceq \mathsf{S}$
6. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \Box_T\,\mathsf{Tumor}(x)$
7. $\forall x\ \Box_P\,(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \Diamond_T\mathsf{Tumor}(x)$
8. $\forall x\ \Box_T\,\mathsf{Tissue}(x) \rightarrow \Box_P\,\mathsf{Tissue}(x)$
9. $\Diamond_S(\mathsf{Patient}(p1) \land \mathsf{HasPart}(p1,a) \land \mathsf{Tumor}(a) \land \mathsf{Object}(a))$

Inferences:

10. $\Box_T\mathsf{Tumor}(a)$ (6,9)



Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathrm{Process}(x) \land \mathrm{Tissue}(x))$
2. $\Box_P(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{AbnormalGrowthProcess}(x))$
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5. $\mathsf{T} \preceq \mathsf{S} \ \land\ \mathsf{P} \preceq \mathsf{S}$
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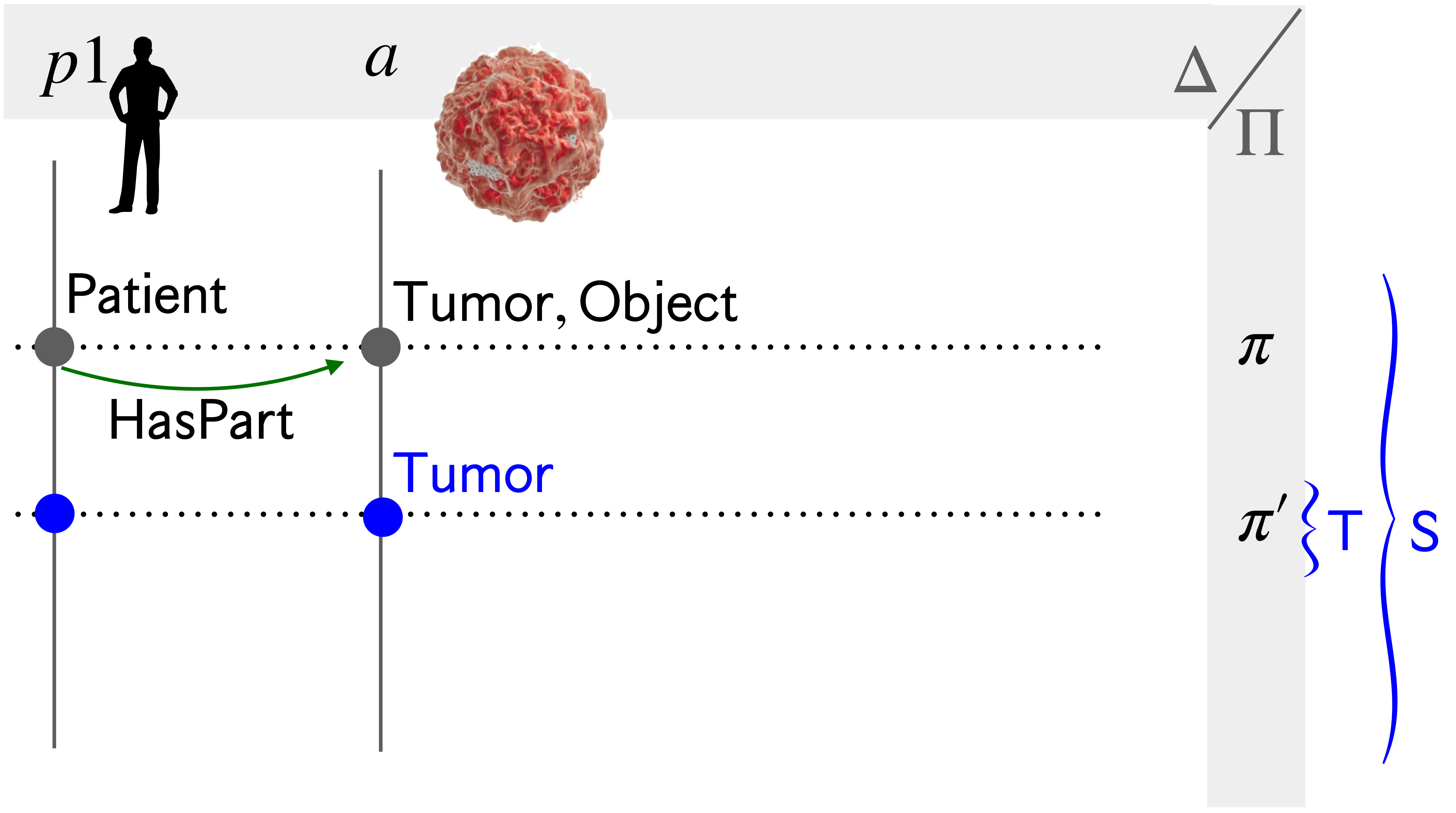
7. $\forall x\ \Box_P(\exists y\ \mathrm{productOf}(x,y) \land \mathrm{Tumor}(y)) \leftrightarrow \Diamond_T\mathrm{Tumor}(x)$
8. $\forall x\ \Box_T\,\mathrm{Tissue}(x) \rightarrow \Box_P\,\mathrm{Tissue}(x)$

9. $\Diamond_S(\mathrm{Patient}(p1) \land \mathrm{HasPart}(p1,a) \land \mathrm{Tumor}(a) \land \mathrm{Object}(a))$

Inferences:

10. $\Box_T\mathrm{Tumor}(a)$ (6,9)

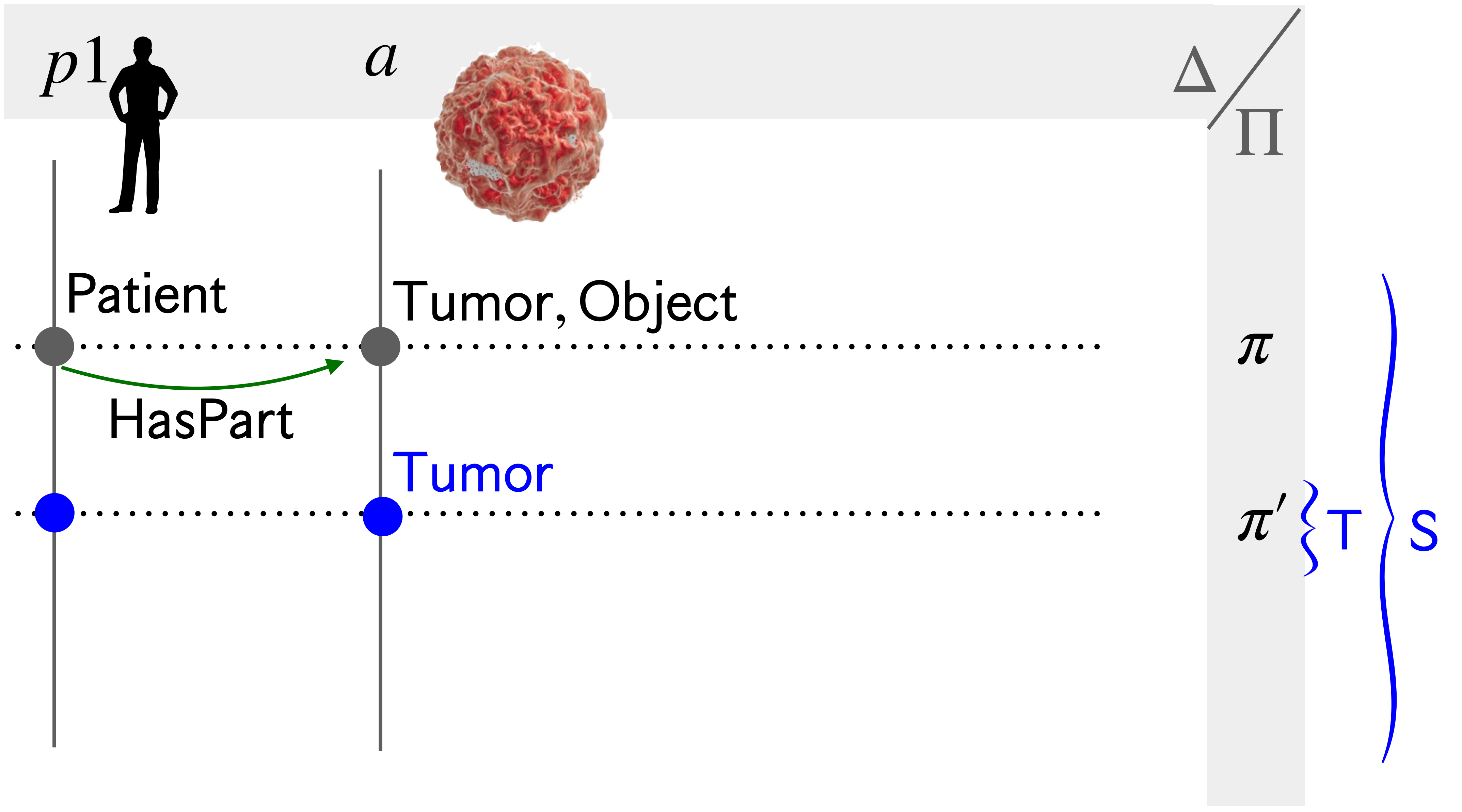


Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x \; \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_P(\forall x \; \mathsf{Tumor}(x) \rightarrow \mathsf{AbnormalGrowthProcess}(x))$
3. $\square_P(\forall x \; \mathsf{AbnormalGrowthProcess}(x) \rightarrow \mathsf{Process}(x))$
4. $\square_T(\forall x \; \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
5. $\mathsf{T} \preceq \mathsf{S} \;\wedge\; \mathsf{P} \preceq \mathsf{S}$
6. $\forall x \; \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_T \mathsf{Tumor}(x)$
7. $\forall x \; \square_P (\exists y \; \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_T \mathsf{Tumor}(x)$
8. $\forall x \; \square_T \mathsf{Tissue}(x) \rightarrow \square_P \mathsf{Tissue}(x)$
9. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

10. $\square_T \mathsf{Tumor}(a)$ (6,9)
11. $\square_T \mathsf{Tissue}(a)$ (10,4)

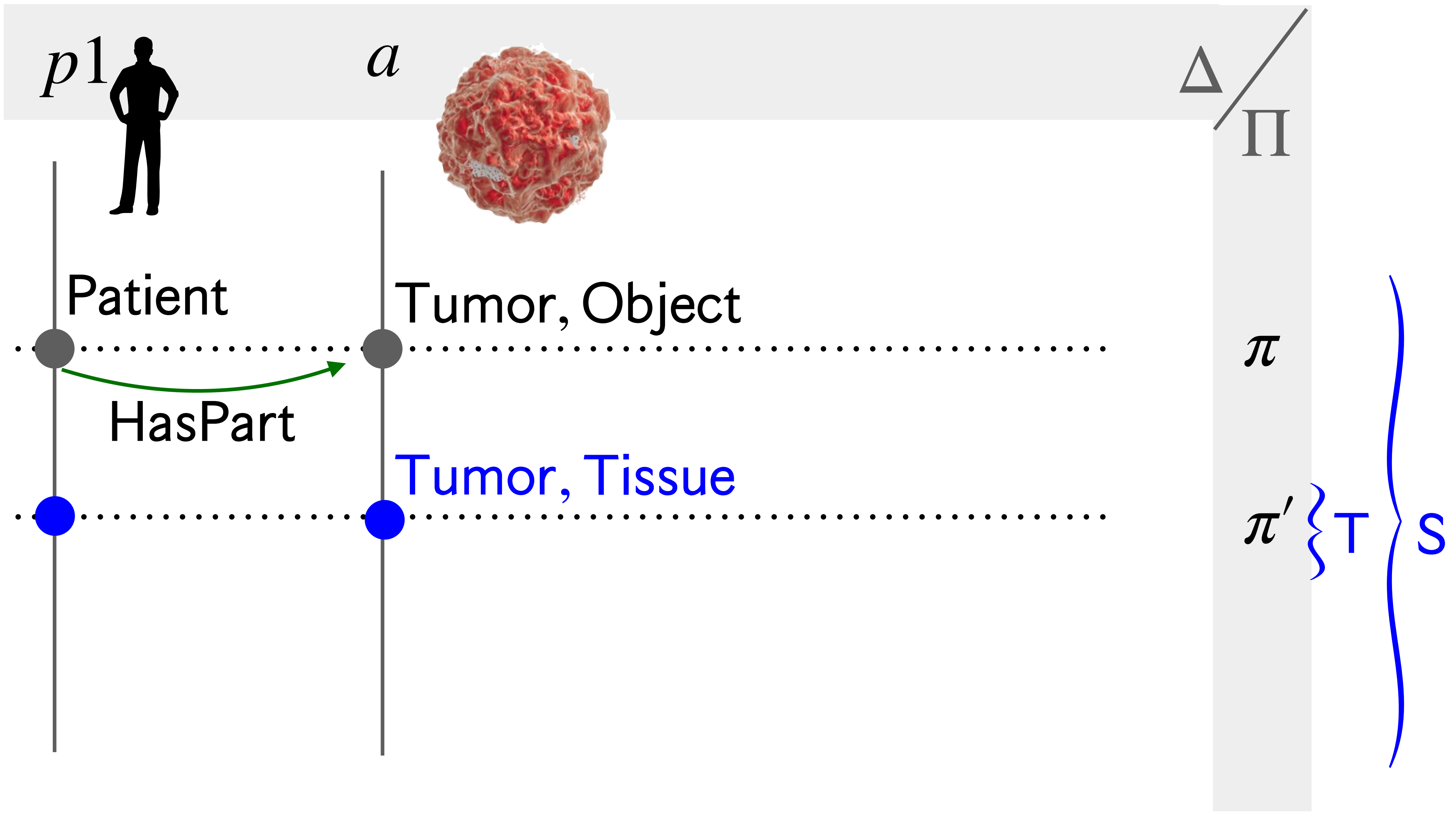


Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\Box_P(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{AbnormalGrowthProcess}(x))$
3. $\Box_P(\forall x\ \mathrm{AbnormalGrowthProcess}(x) \rightarrow \mathrm{Process}(x))$
4. $\Box_T(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
5. $\mathrm{T} \preceq \mathrm{S} \ \wedge\ \mathrm{P} \preceq \mathrm{S}$
6. $\forall x\ \Diamond_S(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \Box_T\,\mathrm{Tumor}(x)$
7. $\forall x\ \Box_P(\exists y\ \mathrm{productOf}(x,y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \Diamond_T\mathrm{Tumor}(x)$
8. $\forall x\ \Box_T\,\mathrm{Tissue}(x) \rightarrow \Box_P\,\mathrm{Tissue}(x)$
9. $\Diamond_S(\mathrm{Patient}(p1) \wedge \mathrm{HasPart}(p1,a) \wedge \mathrm{Tumor}(a) \wedge \mathrm{Object}(a))$

Inferences:

10. $\Box_T\mathrm{Tumor}(a)$ (6,9)
11. $\Box_T\mathrm{Tissue}(a)$ (10,4)

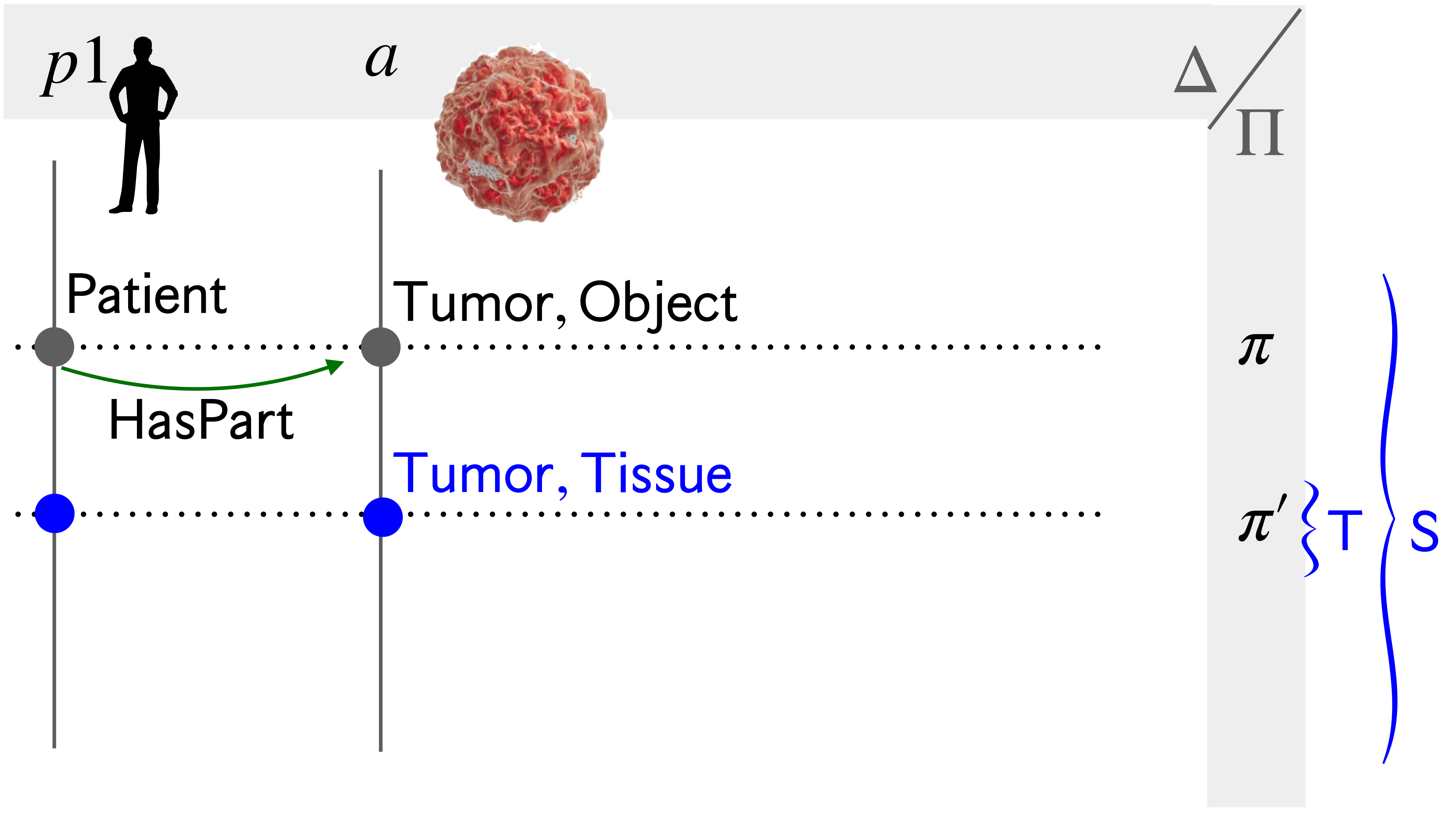


Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg \exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\Box_P(\forall x\ \text{Tumor}(x) \rightarrow \text{AbnormalGrowthProcess}(x))$
3. $\Box_P(\forall x\ \text{AbnormalGrowthProcess}(x) \rightarrow \text{Process}(x))$
4. $\Box_T(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
5. $T \preceq S\ \ \wedge\ \ P \preceq S$
6. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \Box_T\,\text{Tumor}(x)$
7. $\forall x\ \Box_P\,(\exists y\ \text{productOf}(x,y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_T\text{Tumor}(x)$
8. $\forall x\ \Box_T\,\text{Tissue}(x) \rightarrow \Box_P\,\text{Tissue}(x)$
9. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

Inferences:

10. $\Box_T\text{Tumor}(a)$ (6,9)
11. $\Box_T\text{Tissue}(a)$ (10,4)
12. $\Box_P\text{Tissue}(a)$ (11,8)

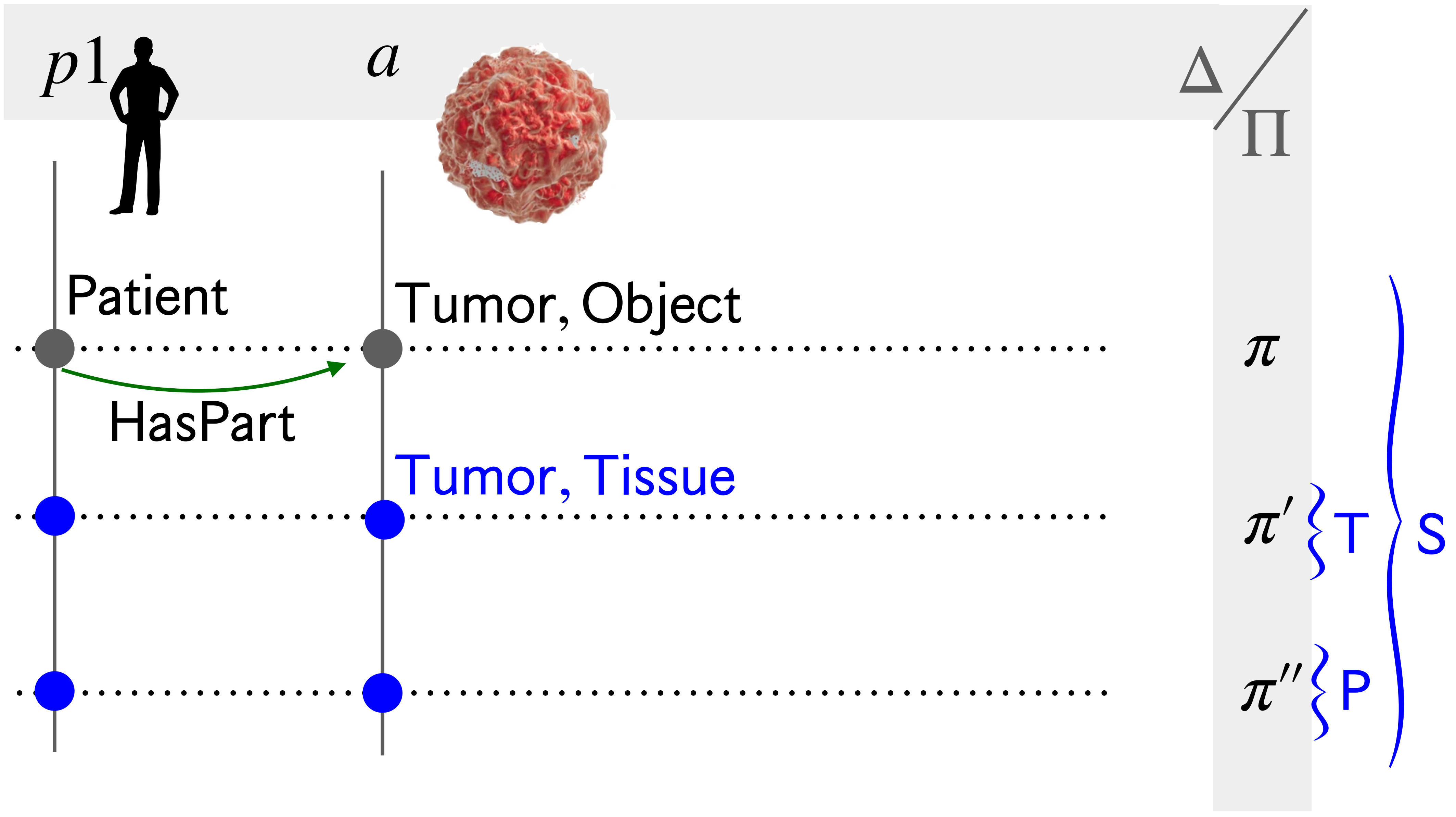


Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\square_P(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{AbnormalGrowthProcess}(x))$
3. $\square_P(\forall x\ \mathrm{AbnormalGrowthProcess}(x) \rightarrow \mathrm{Process}(x))$
4. $\square_T(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
5. $T \preceq S \ \wedge\ P \preceq S$
6. $\forall x\ \lozenge_S(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_T \mathrm{Tumor}(x)$
7. $\forall x\ \square_P(\exists y\ \mathrm{productOf}(x, y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \lozenge_T \mathrm{Tumor}(x)$
8. $\forall x\ \square_T \mathrm{Tissue}(x) \rightarrow \square_P \mathrm{Tissue}(x)$
9. $\lozenge_S(\mathrm{Patient}(p1) \wedge \mathrm{HasPart}(p1,a) \wedge \mathrm{Tumor}(a) \wedge \mathrm{Object}(a))$

Inferences:

10. $\square_T\mathrm{Tumor}(a)$ (6,9)
11. $\square_T\mathrm{Tissue}(a)$ (10,4)
12. $\square_P\mathrm{Tissue}(a)$ (11,8)



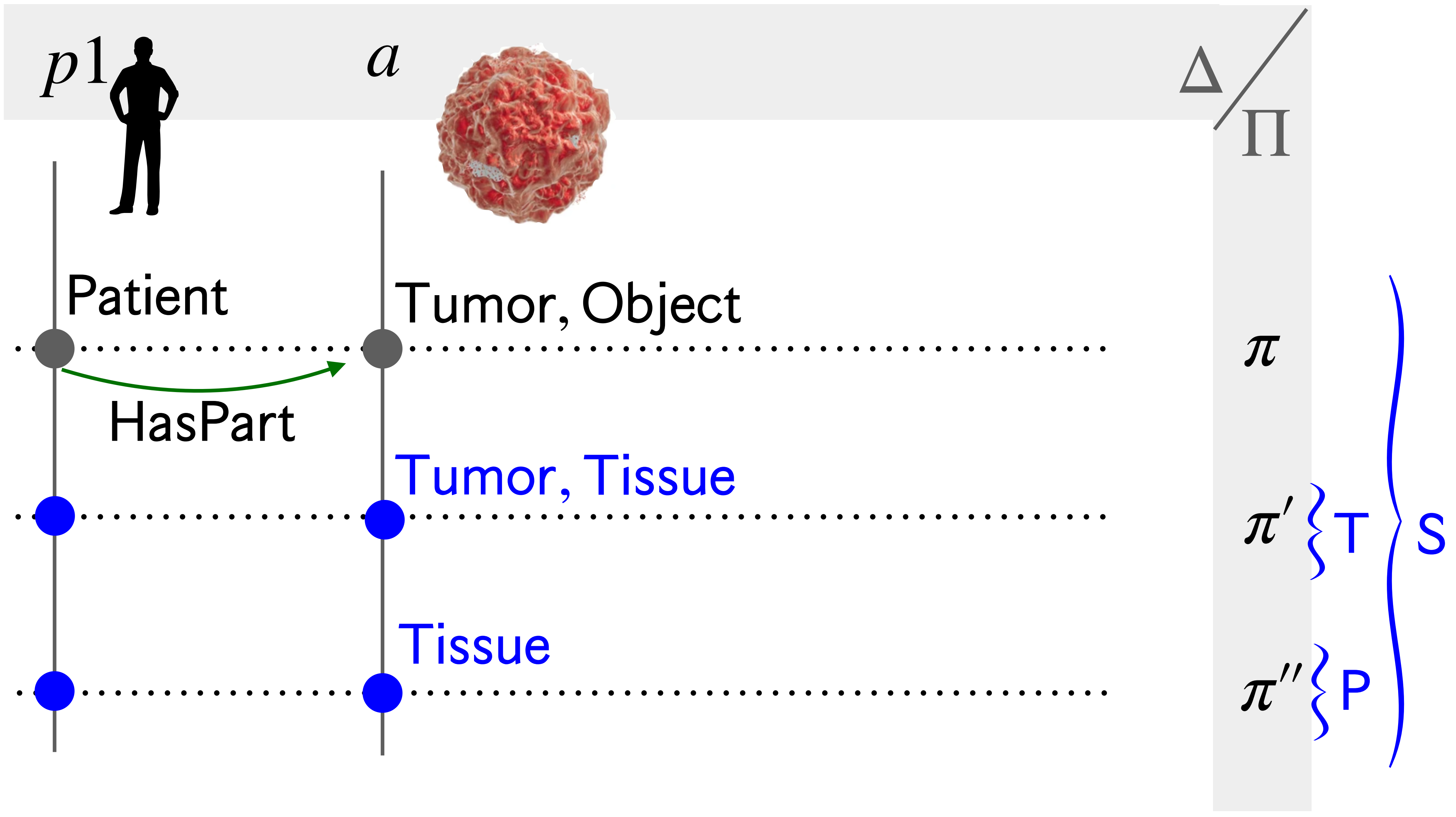
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\square_P(\forall x\ \text{Tumor}(x) \rightarrow \text{AbnormalGrowthProcess}(x))$
3. $\square_P(\forall x\ \text{AbnormalGrowthProcess}(x) \rightarrow \text{Process}(x))$
4. $\square_T(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
5. $T \preceq S \ \wedge\ P \preceq S$
6. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_T \text{Tumor}(x)$
7. $\forall x\ \square_P(\exists y\ \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_T \text{Tumor}(x)$
8. $\forall x\ \square_T \text{Tissue}(x) \rightarrow \square_P \text{Tissue}(x)$

9. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

Inferences:

10. $\square_T\text{Tumor}(a)$ (6,9)
11. $\square_T\text{Tissue}(a)$ (10,4)
12. $\square_P\text{Tissue}(a)$ (11,8)



Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \text{Process}(x) \land \text{Tissue}(x))$
2. $\square_P(\forall x\ \text{Tumor}(x) \rightarrow \text{AbnormalGrowthProcess}(x))$
3. $\square_P(\forall x\ \text{AbnormalGrowthProcess}(x) \rightarrow \text{Process}(x))$
4. $\square_T(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
5. $T \preceq S \ \land\ P \preceq S$
6. $\forall x\ \lozenge_S(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \square_T\,\text{Tumor}(x)$
7. $\forall x\ \square_P\,(\exists y\ \text{productOf}(x,y) \land \text{Tumor}(y)) \leftrightarrow \lozenge_T\text{Tumor}(x)$
8. $\forall x\ \square_T\,\text{Tissue}(x) \rightarrow \square_P\,\text{Tissue}(x)$

9. $\lozenge_S(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

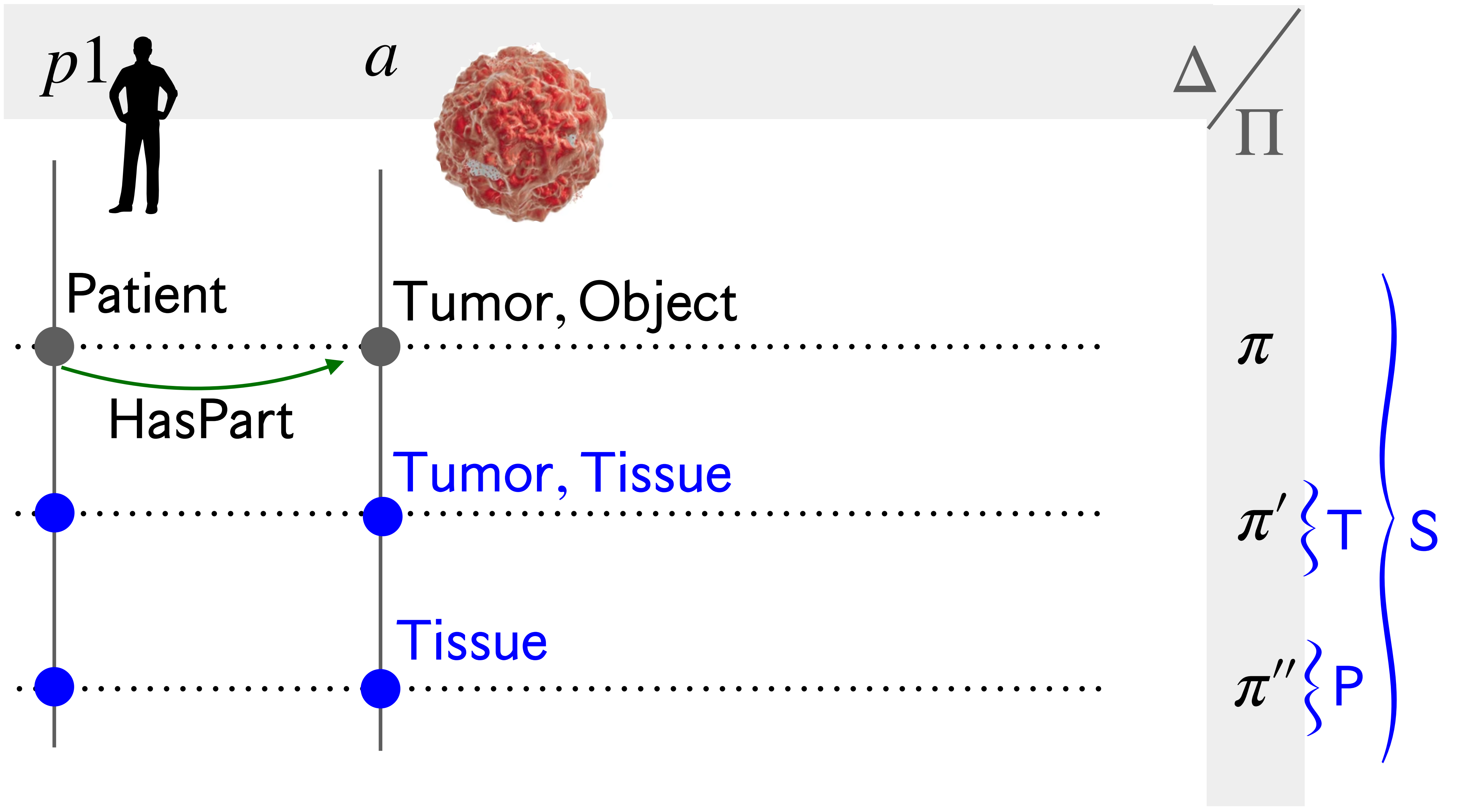
Inferences:

10. $\square_T\text{Tumor}(a)$ (6,9)

11. $\square_T\text{Tissue}(a)$ (10,4)

12. $\square_P\text{Tissue}(a)$ (11,8)

13. $\square_P\,\exists y\ \text{productOf}(a,y) \land \text{Tumor}(y)$ (12,7)



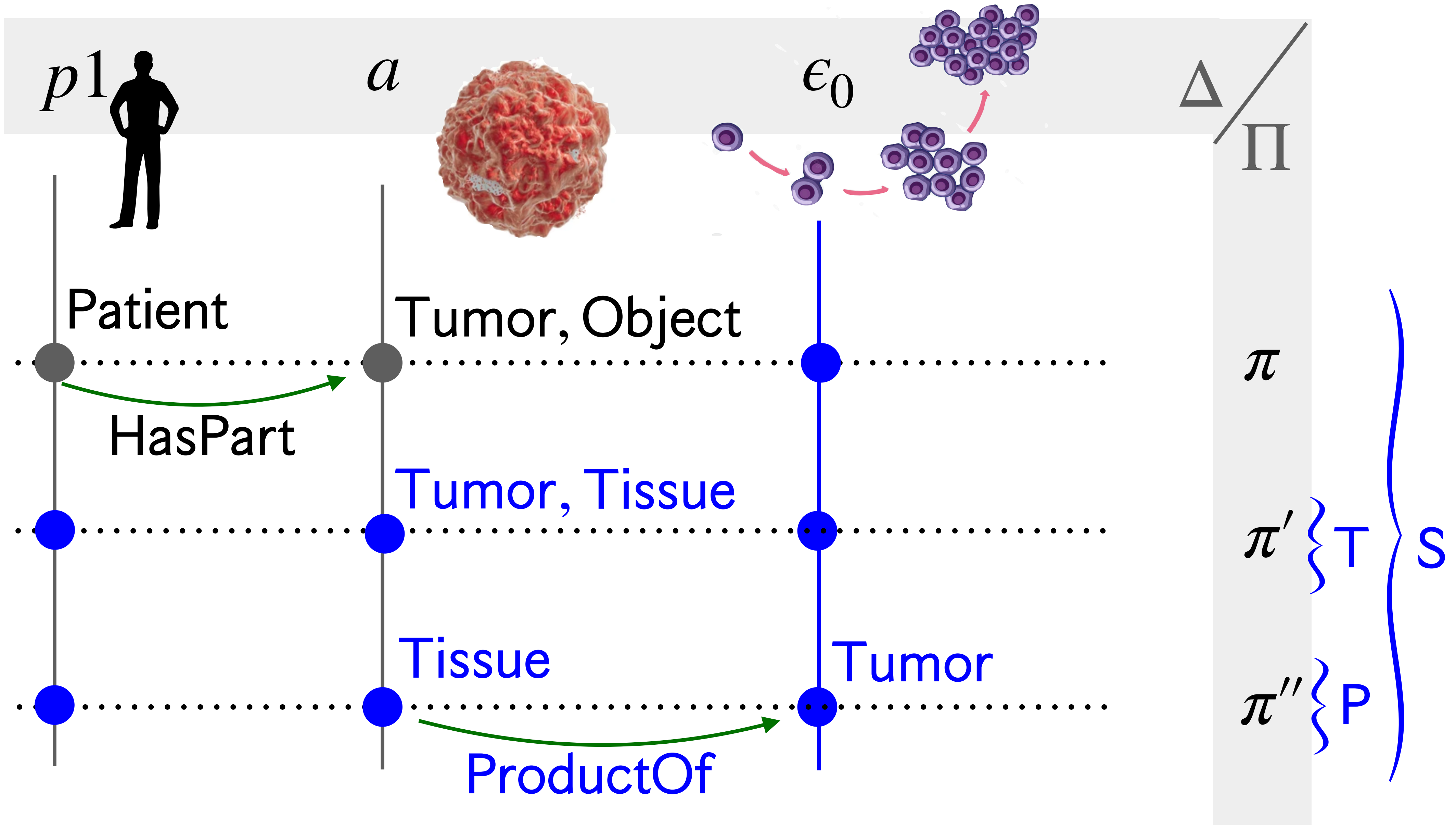
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_P(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{AbnormalGrowthProcess}(x))$
3. $\square_P(\forall x\ \mathsf{AbnormalGrowthProcess}(x) \rightarrow \mathsf{Process}(x))$
4. $\square_T(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
5. $\mathsf{T} \preceq \mathsf{S} \ \wedge\ \mathsf{P} \preceq \mathsf{S}$
6. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_T\, \mathsf{Tumor}(x)$
7. $\forall x\ \square_P\,(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_T \mathsf{Tumor}(x)$
8. $\forall x\ \square_T\, \mathsf{Tissue}(x) \rightarrow \square_P\, \mathsf{Tissue}(x)$

9. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

10. $\square_T \mathsf{Tumor}(a)$ (6,9)
11. $\square_T \mathsf{Tissue}(a)$ (10,4)
12. $\square_P \mathsf{Tissue}(a)$ (11,8)
13. $\square_P\, \exists y\ \mathsf{productOf}(a,y) \wedge \mathsf{Tumor}(y)$ (12,7)

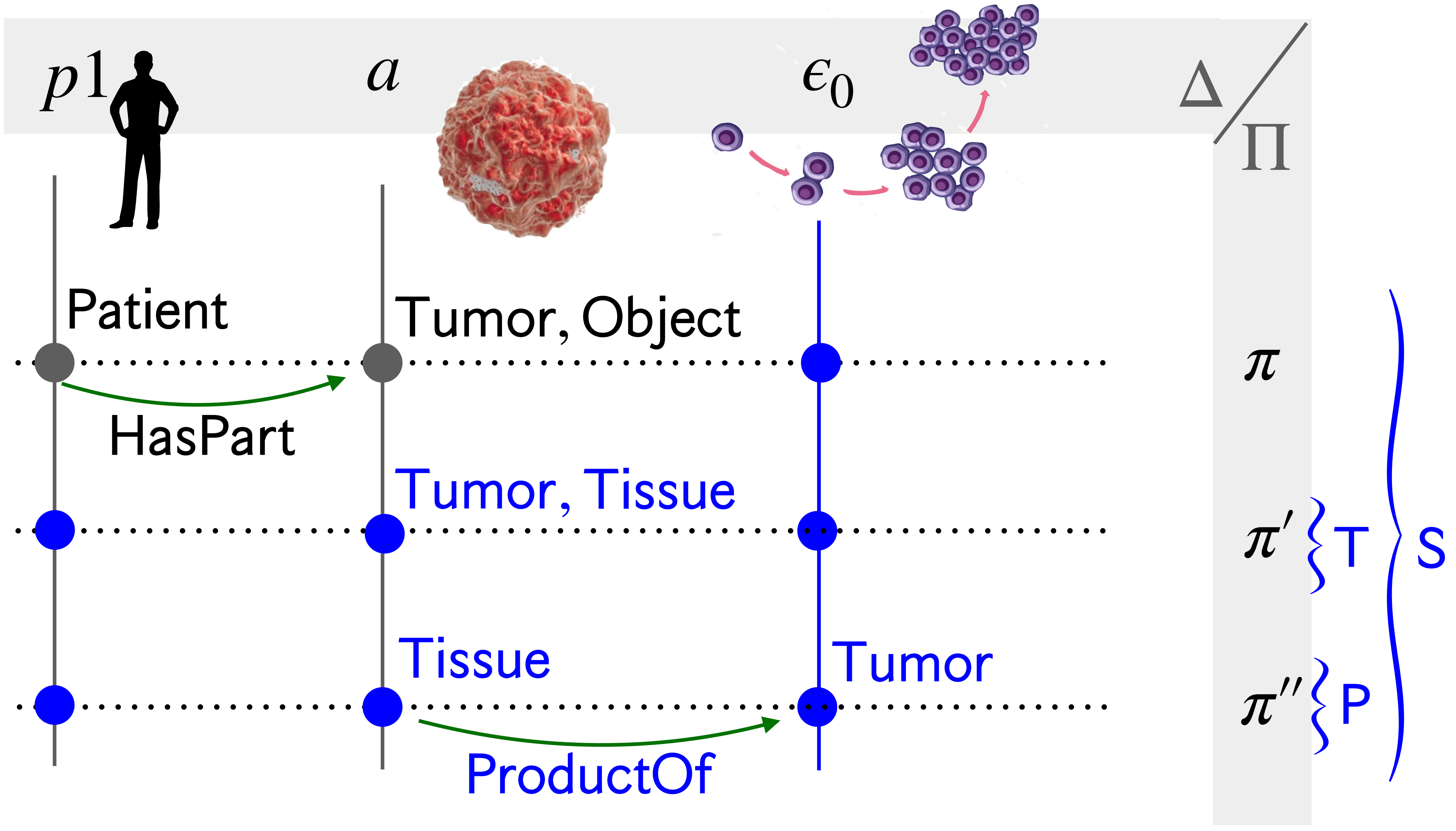


Knowledge Integration - Standpoint Logic

1. $\Box_{\mathsf{S}}(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\Box_{\mathsf{P}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{AbnormalGrowthProcess}(x))$
3. $\Box_{\mathsf{P}}(\forall x\ \mathsf{AbnormalGrowthProcess}(x) \rightarrow \mathsf{Process}(x))$
4. $\Box_{\mathsf{T}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
5. $\mathsf{T} \preceq \mathsf{S} \ \wedge\ \mathsf{P} \preceq \mathsf{S}$
6. $\forall x\ \Diamond_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \Box_{\mathsf{T}}\,\mathsf{Tumor}(x)$
7. $\forall x\ \Box_{\mathsf{P}}\,(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \Diamond_{\mathsf{T}}\mathsf{Tumor}(x)$
8. $\forall x\ \Box_{\mathsf{T}}\,\mathsf{Tissue}(x) \rightarrow \Box_{\mathsf{P}}\,\mathsf{Tissue}(x)$
9. $\Diamond_{\mathsf{S}}(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

10. $\Box_{\mathsf{T}}\mathsf{Tumor}(a)$ (6,9)
11. $\Box_{\mathsf{T}}\mathsf{Tissue}(a)$ (10,4)
12. $\Box_{\mathsf{P}}\mathsf{Tissue}(a)$ (11,8)
13. $\Box_{\mathsf{P}}\,\exists y\ \mathsf{productOf}(a,y) \wedge \mathsf{Tumor}(y)$ (12,7)
14. $\Box_{\mathsf{P}}\,\exists y\ \mathsf{AbnormalGrouthProcess}(y) \wedge \mathsf{Process}(y)$ (13,2,3)



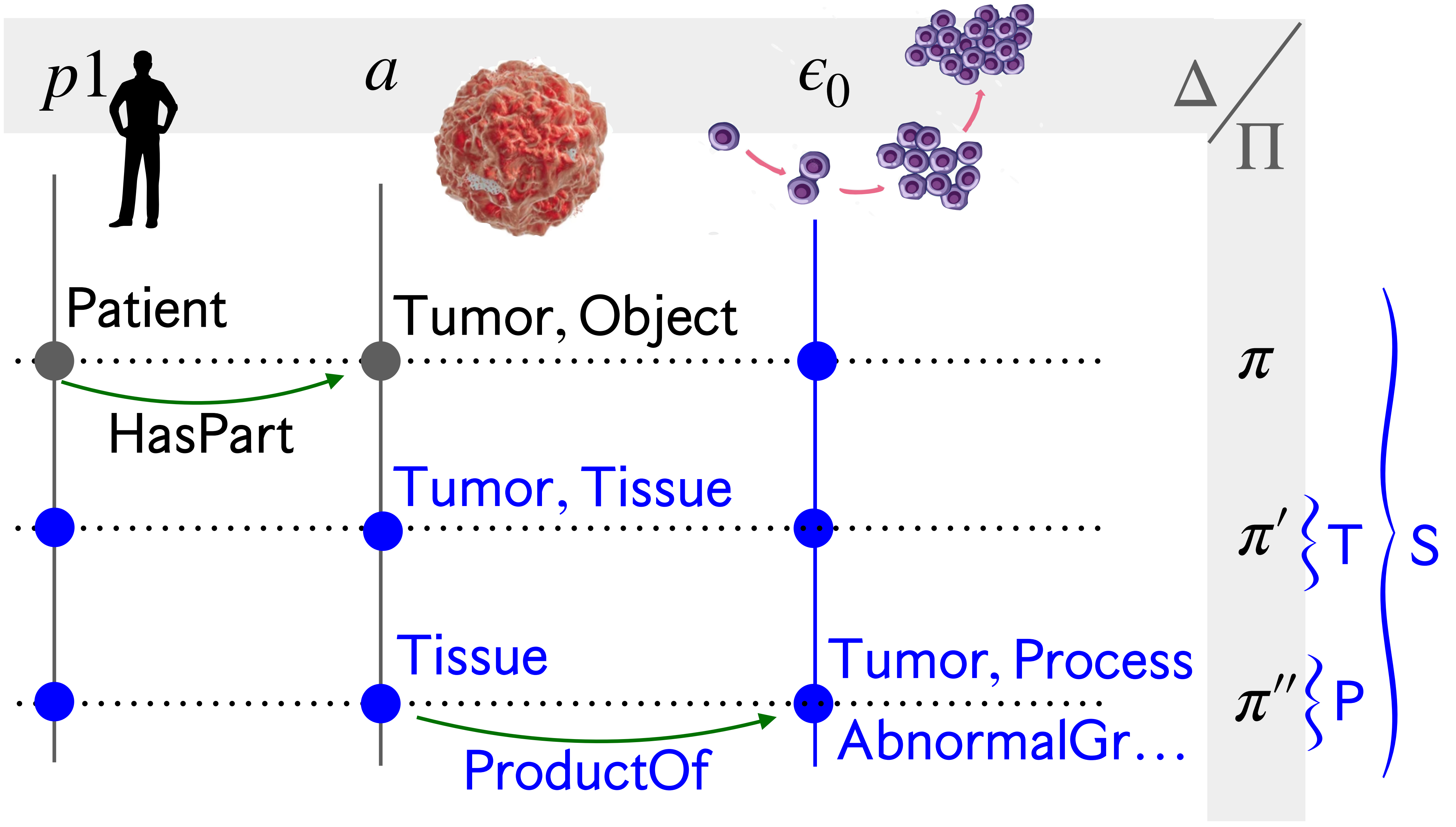
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\Box_P(\forall x\ \text{Tumor}(x) \rightarrow \text{AbnormalGrowthProcess}(x))$
3. $\Box_P(\forall x\ \text{AbnormalGrowthProcess}(x) \rightarrow \text{Process}(x))$
4. $\Box_T(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
5. $T \preceq S \ \wedge\ P \preceq S$
6. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \Box_T\,\text{Tumor}(x)$
7. $\forall x\ \Box_P\,(\exists y\ \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_T\text{Tumor}(x)$
8. $\forall x\ \Box_T\,\text{Tissue}(x) \rightarrow \Box_P\,\text{Tissue}(x)$

9. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

Inferences:

10. $\Box_T\text{Tumor}(a)$ (6,9)
11. $\Box_T\text{Tissue}(a)$ (10,4)
12. $\Box_P\text{Tissue}(a)$ (11,8)
13. $\Box_P\,\exists y\ \text{productOf}(a, y) \wedge \text{Tumor}(y)$ (12,7)
14. $\Box_P\,\exists y\ \text{AbnormalGrouthProcess}(y) \wedge \text{Process}(y)$ (13,2,3)



The general setting:

First-Order Standpoint Logics

Standpoint Logic: Syntax

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

- $\Box_s \phi \longrightarrow$ "it is **unequivocal**, according to s, that ϕ"

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

- $\Box_s \phi \longrightarrow$ "it is **unequivocal**, according to s, that ϕ"
- $\Diamond_s \phi \longrightarrow$ "it is **conceivable**, according to s, that ϕ"

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \square_e \phi$$

$$\square_e \neg\phi \equiv \neg\lozenge_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

- $\square_s \phi \longrightarrow$ “it is **unequivocal**, according to s, that ϕ”
- $\lozenge_s \phi \longrightarrow$ “it is **conceivable**, according to s, that ϕ”

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

- $\Box_s \phi \longrightarrow$ "it is **unequivocal**, according to s, that ϕ"
- $\Diamond_s \phi \longrightarrow$ "it is **conceivable**, according to s, that ϕ"
- $\Box_{s \cup s'} \phi \longrightarrow$ "it is unequivocal, according to both s and s', that ϕ"

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x\phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

- $\Box_s \phi \longrightarrow$ "it is **unequivocal**, according to s, that ϕ"
- $\Diamond_s \phi \longrightarrow$ "it is **conceivable**, according to s, that ϕ"
- $\Box_{s \cup s'} \phi \longrightarrow$ "it is unequivocal, according to both s and s', that ϕ"
- $\Box_{s \cap s'} \phi \longrightarrow$ "it is unequivocal, according to the fusion of s and s', that ϕ"

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

+ (definable) sharpening statements: $e \preceq e'$ denoting $\Box_{e_1 \backslash e_2} \mathbf{f}$

- $\Box_s \phi \longrightarrow$ “it is **unequivocal**, according to s, that ϕ”
- $\Diamond_s \phi \longrightarrow$ “it is **conceivable**, according to s, that ϕ”
- $\Box_{s \cup s'} \phi \longrightarrow$ “it is unequivocal, <u>according to both s and s’</u>, that ϕ”
- $\Box_{s \cap s'} \phi \longrightarrow$ “it is unequivocal, <u>according to the fusion of s and s’</u>, that ϕ”

Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

+ (definable) sharpening statements: $e \preceq e'$ denoting $\Box_{e_1 \backslash e_2} \mathbf{f}$

- $\Box_s \phi \longrightarrow$ "it is **unequivocal**, according to s, that ϕ"
- $\Diamond_s \phi \longrightarrow$ "it is **conceivable**, according to s, that ϕ"
- $\Box_{s \cup s'} \phi \longrightarrow$ "it is unequivocal, <u>according to both s and s'</u>, that ϕ"
- $\Box_{s \cap s'} \phi \longrightarrow$ "it is unequivocal, <u>according to the fusion of s and s'</u>, that ϕ"
- $s \preceq s' \longrightarrow$ "*s* inherits or **extends** *s'*"

Standpoint Logic: Semantics

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Semantics of $\mathbb{S}$

Relational semantics:

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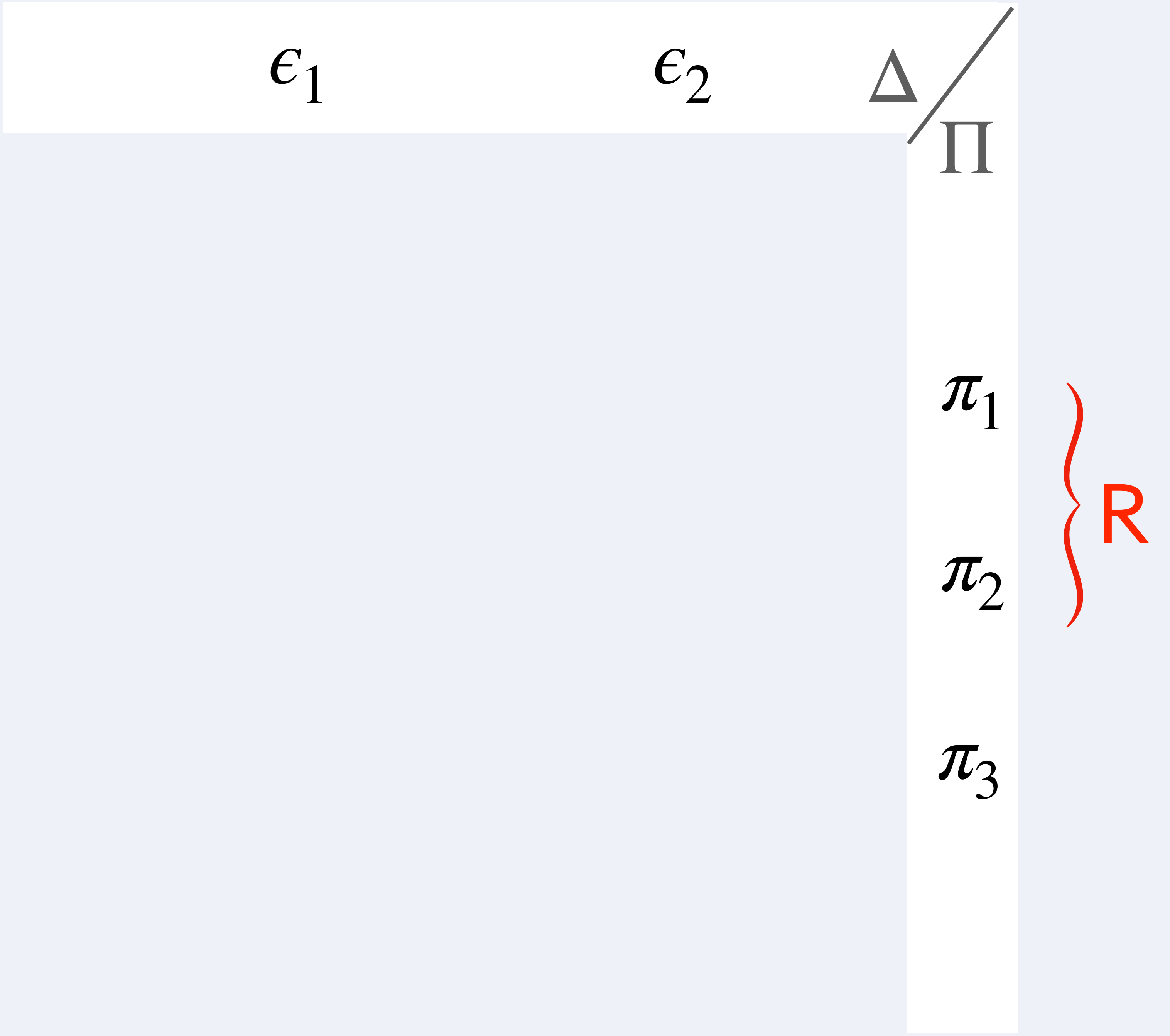

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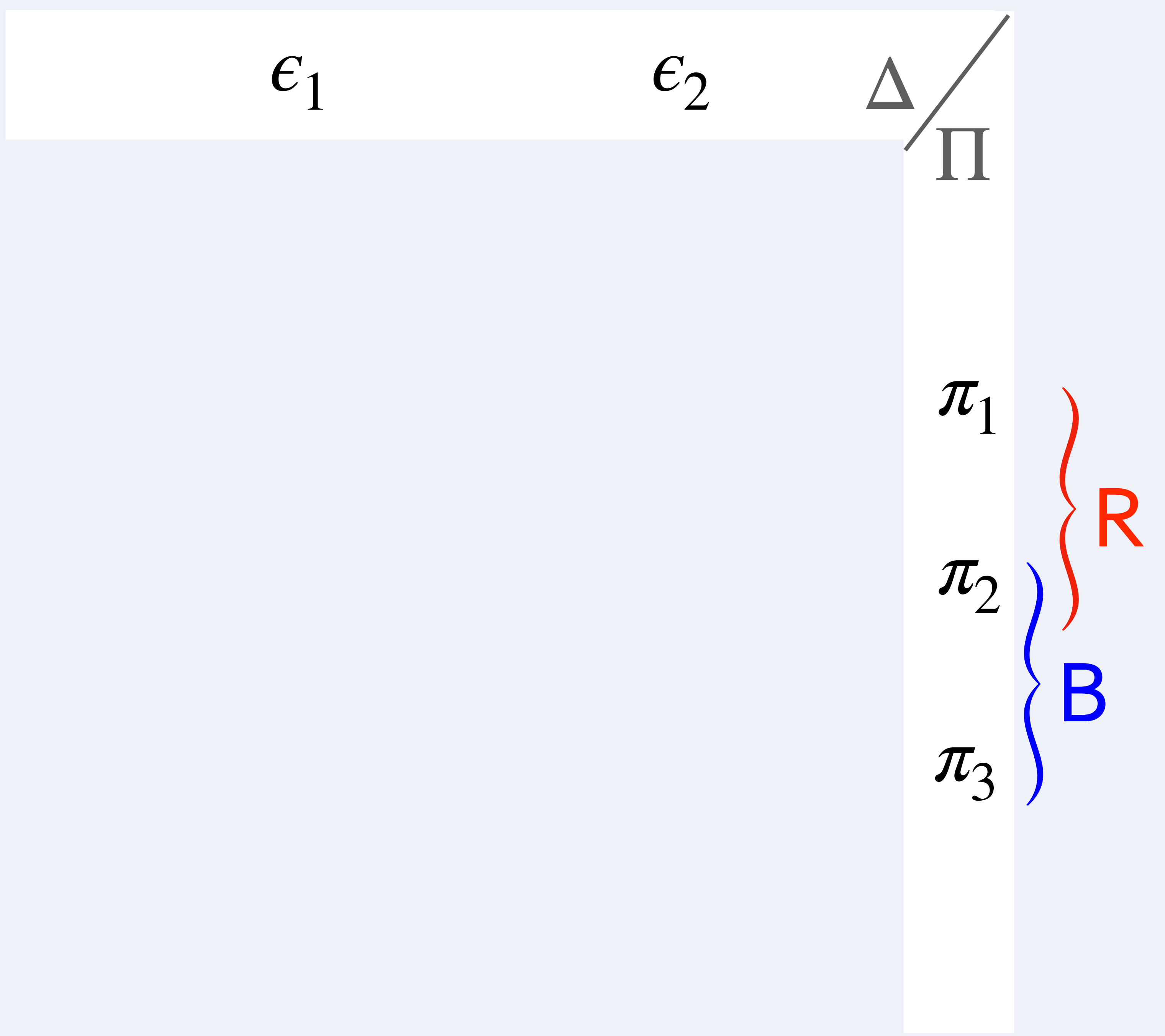

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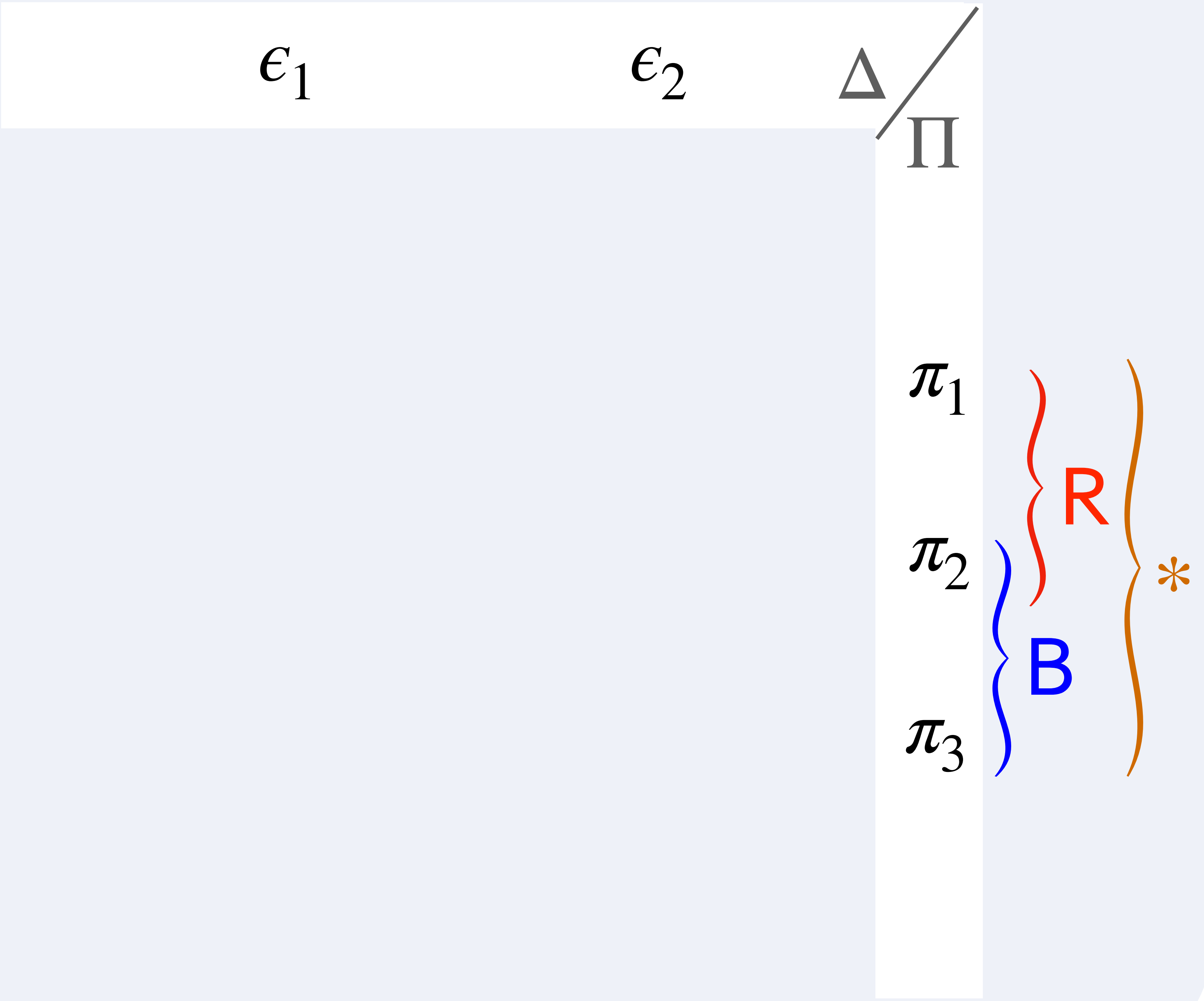

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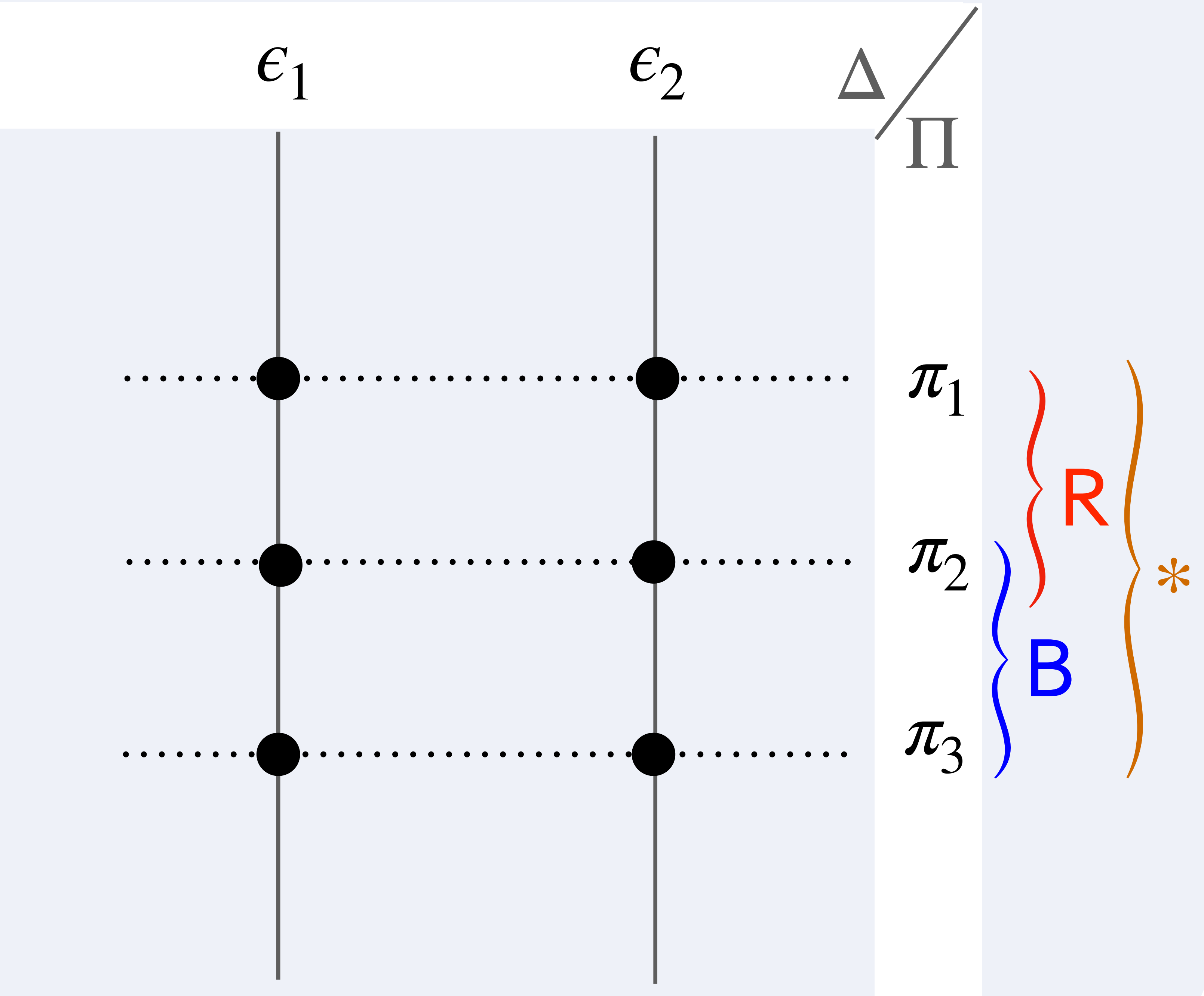

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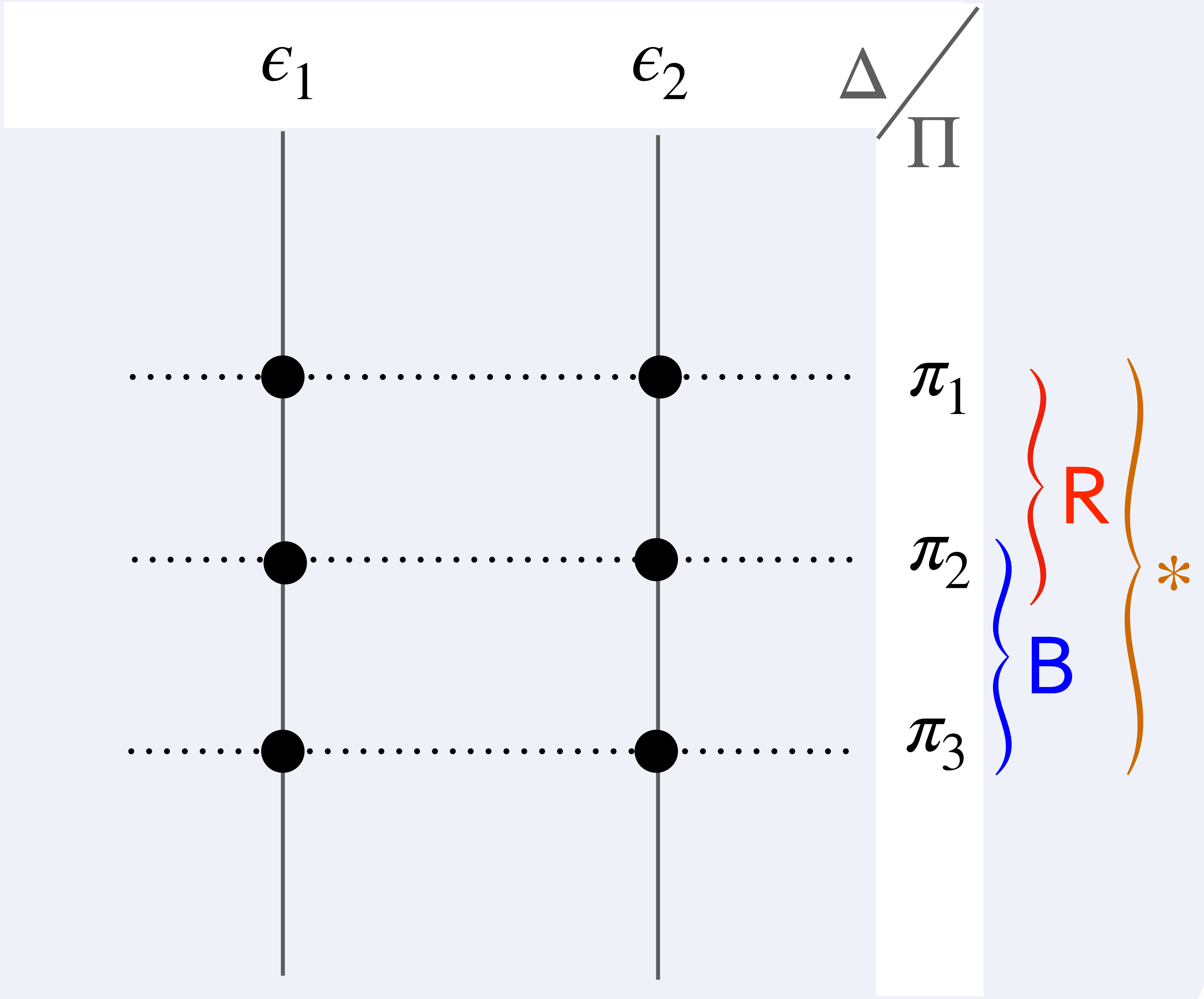

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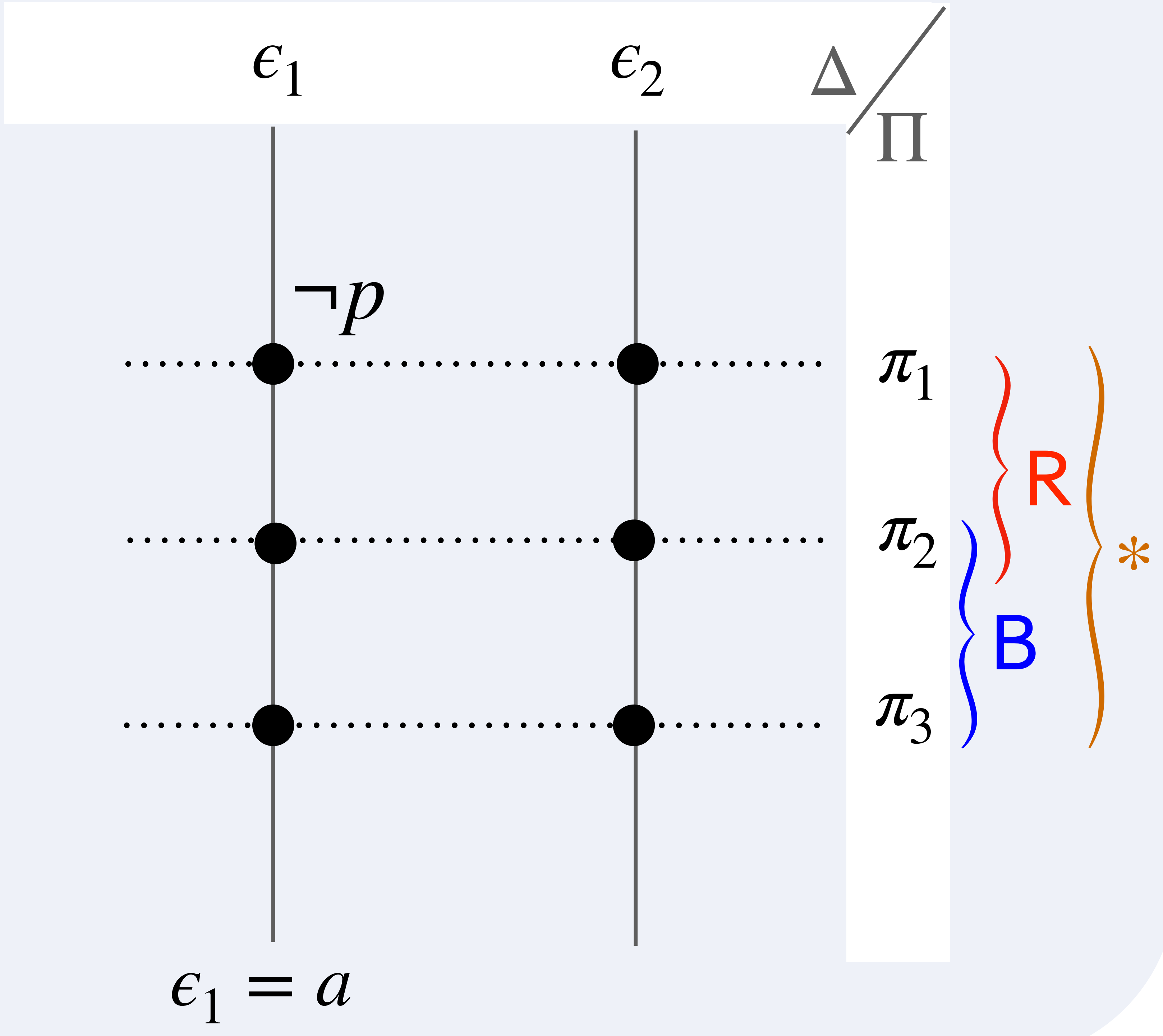

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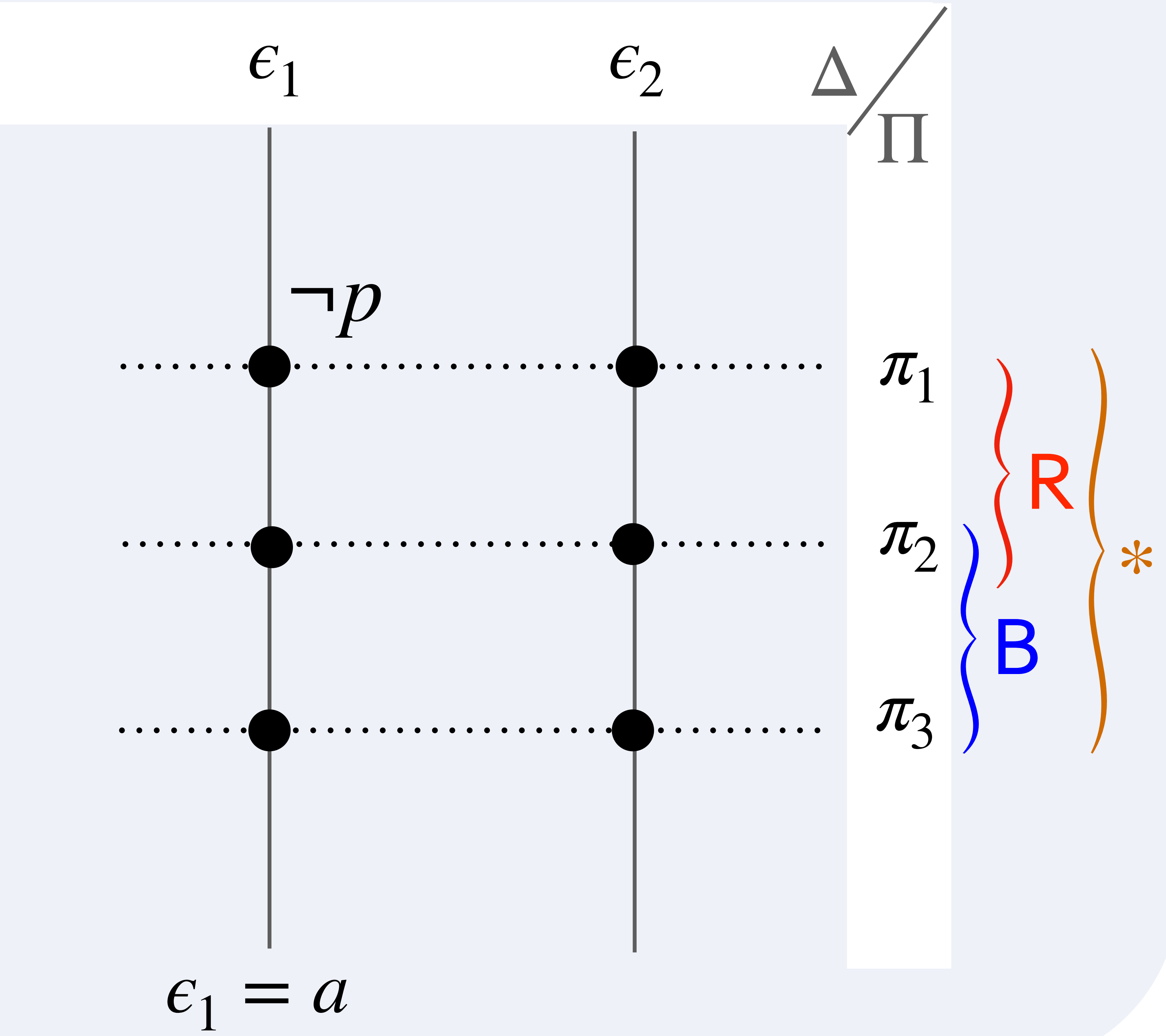

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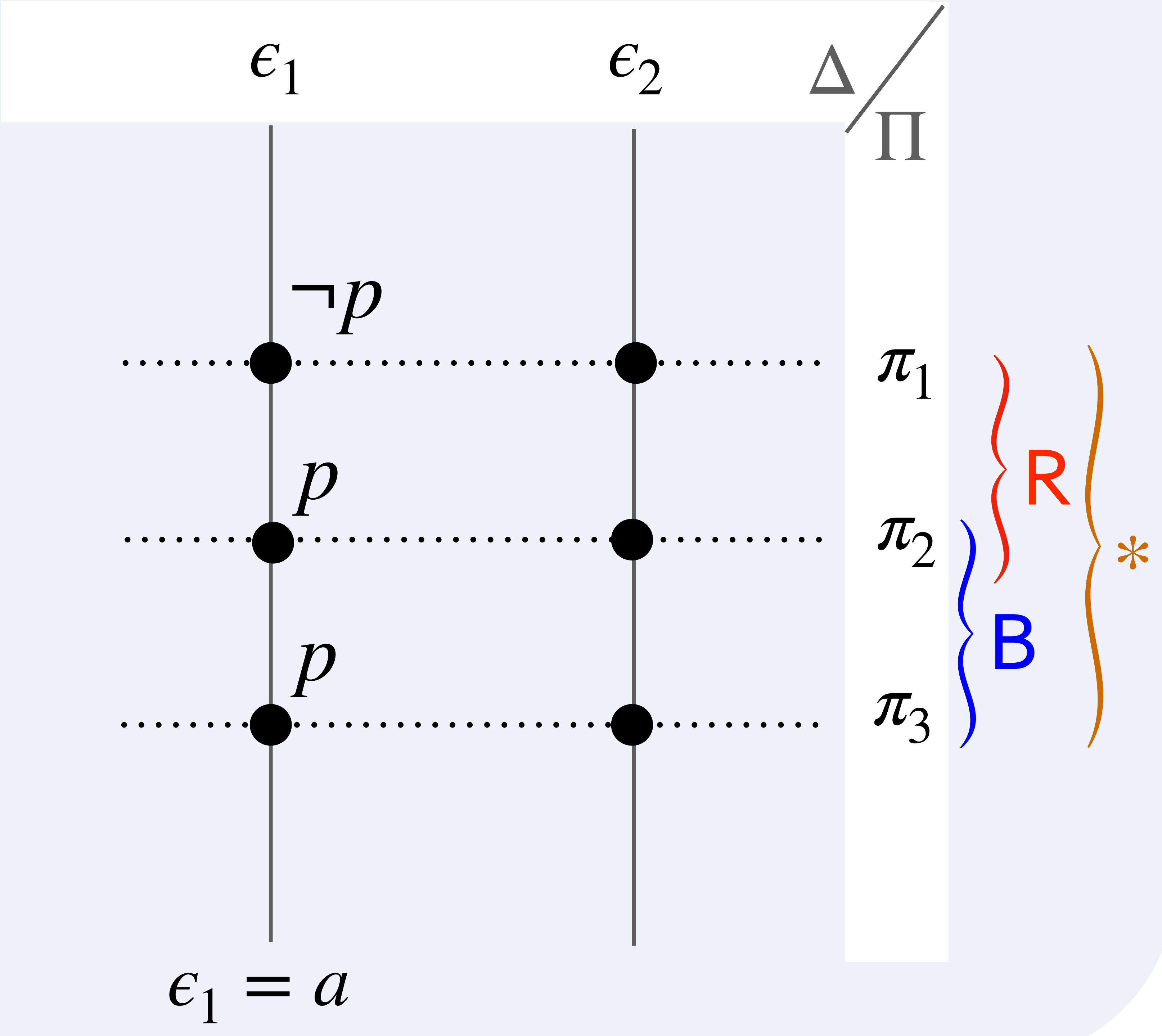

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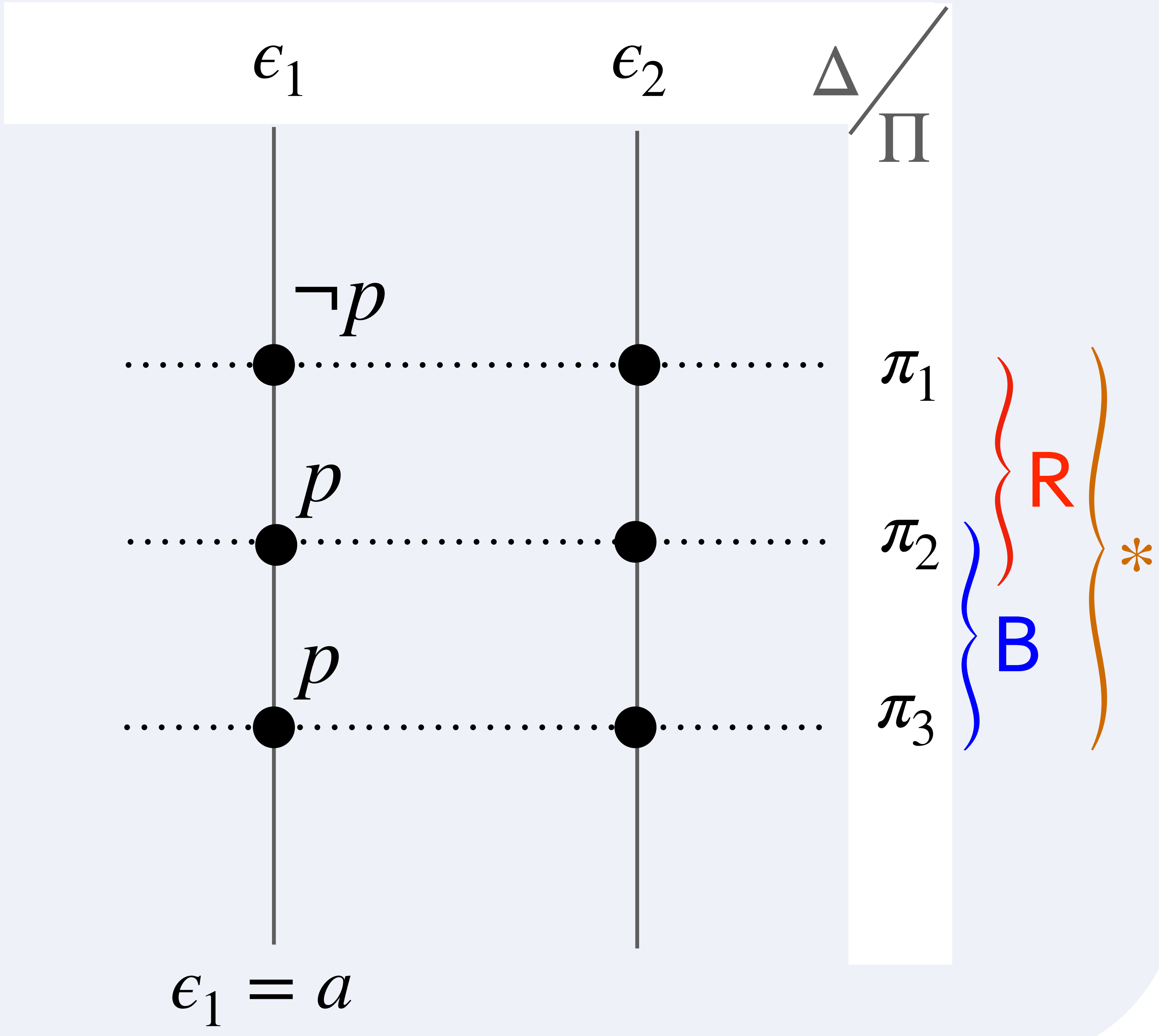

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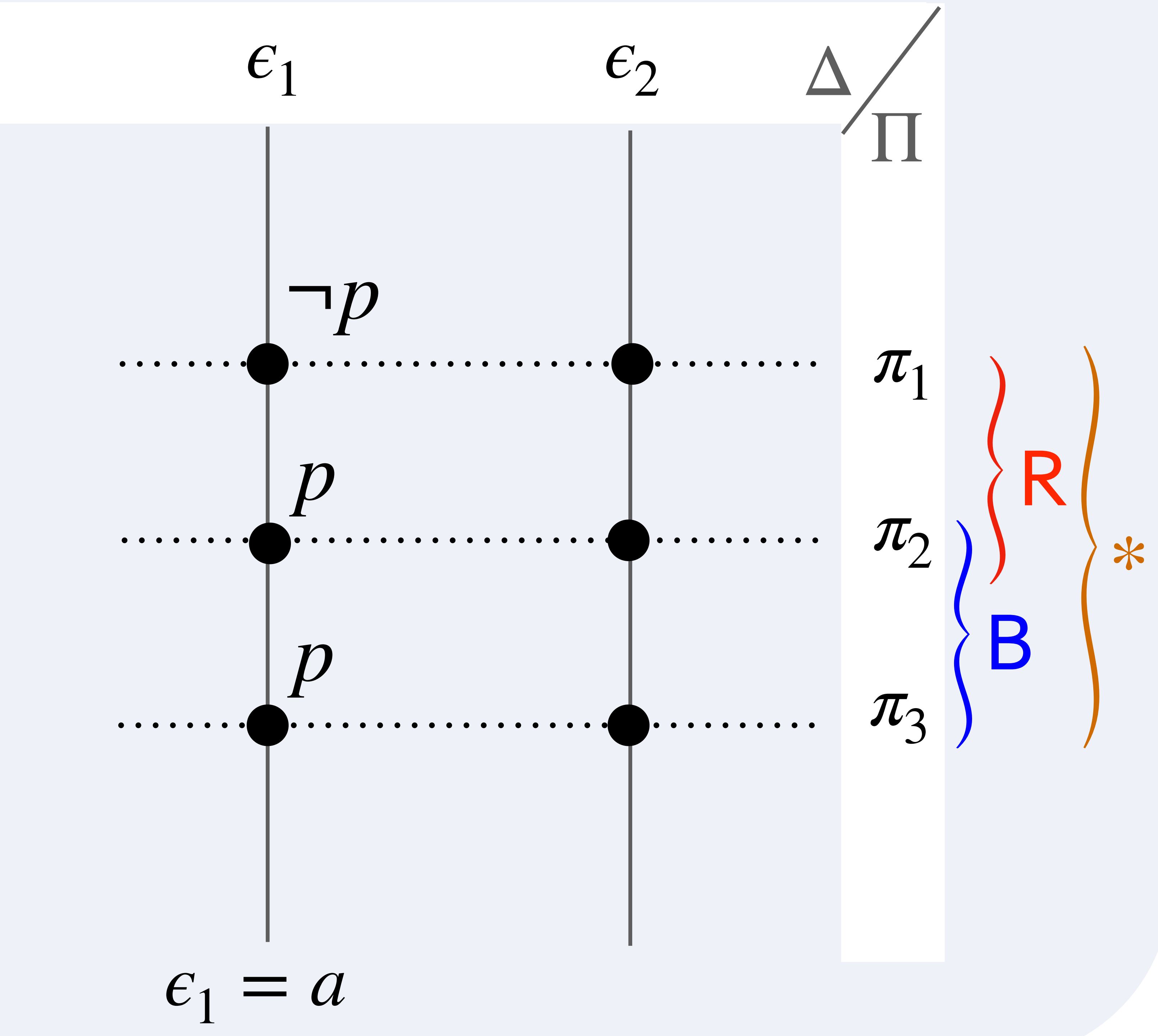

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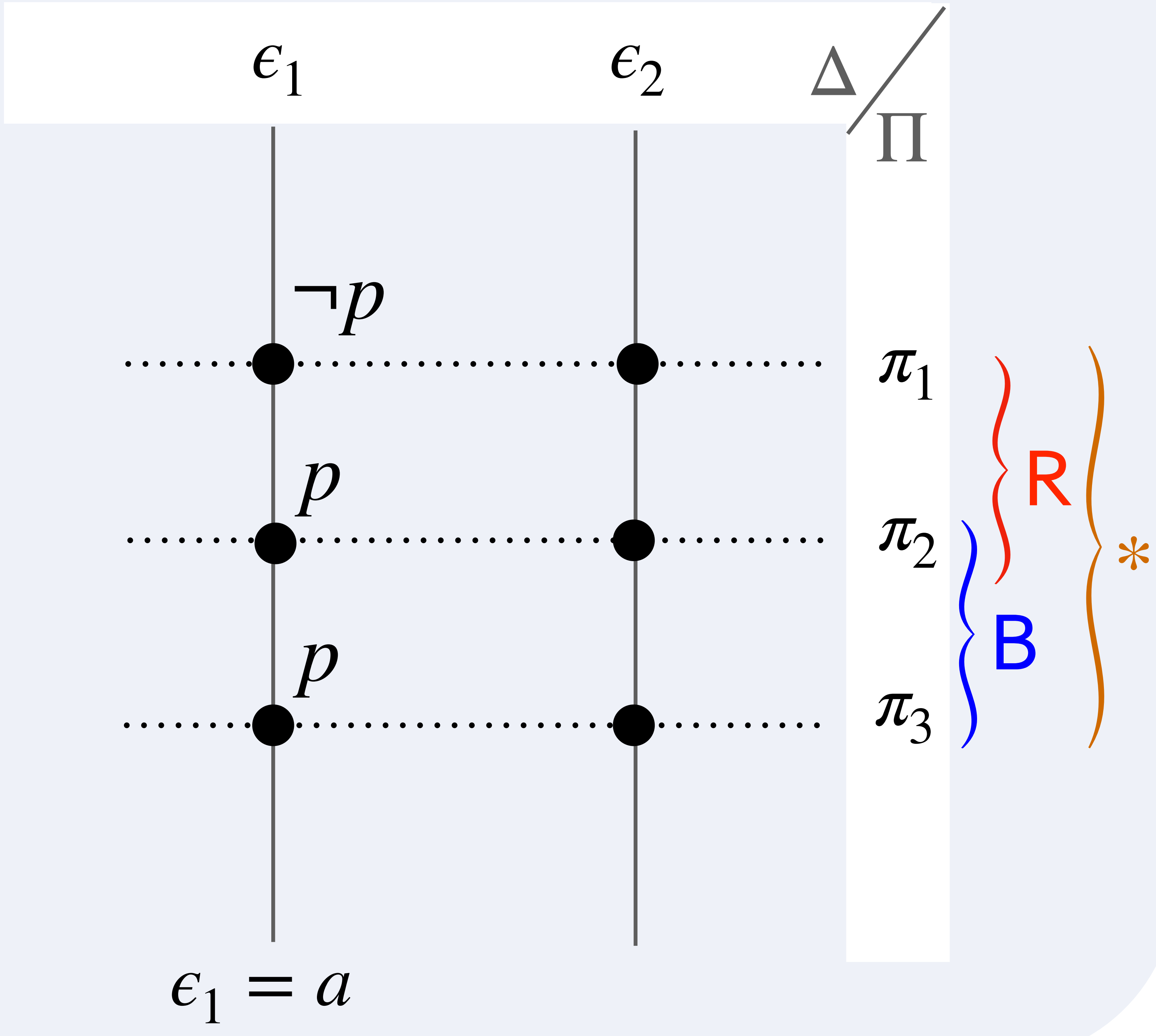

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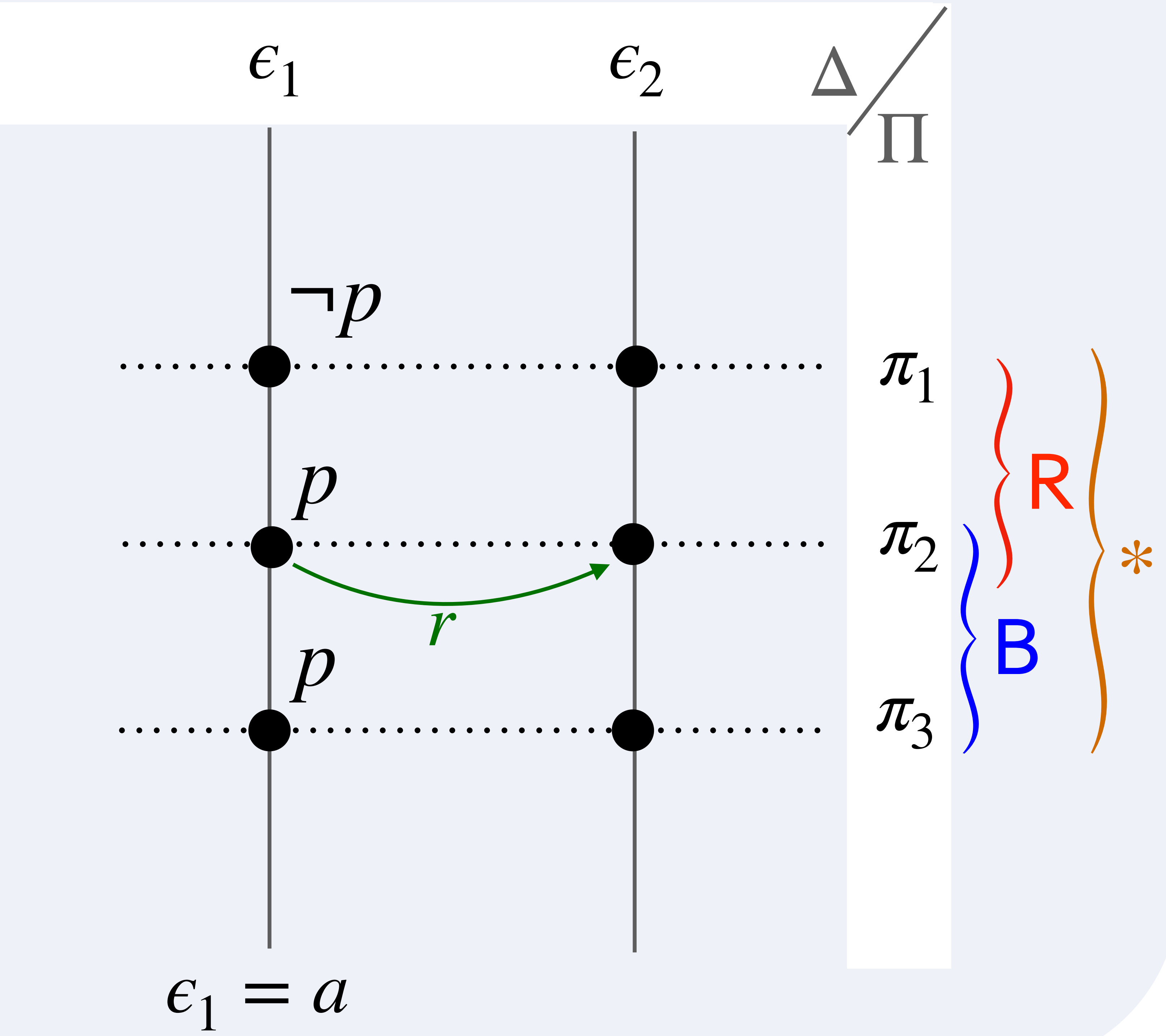

Decidability, Complexity and Implementations

Decidability and Complexity Landscape

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Remark: The semantics of standpoint logic can be expressed in standard Kripke (relational) semantics.

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Definition 1 (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\Box_e \psi$, all variables occurring in ψ are bound by a quantifier.

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2. $\lozenge_T[\top] \land \lozenge_P[\top]$ ✓
3. $\forall x \square_P[\mathrm{Tissue}(x)] \rightarrow \square_T[\mathrm{Tissue}(x)]$ ✗

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2. $\lozenge_T[\top] \wedge \lozenge_P[\top]$
3. $\forall x \square_P[\mathtt{Tissue}(x)] \rightarrow \square_T[\mathtt{Tissue}(x)]$

Theorem 1 (Small Model Property):
A sentential FOSL formula ϕ is satisfiable iff it has a model with at most $|\phi|$ precisifications. That is, for sentential FOSL, satisfiability and $|\phi|$-satisfiability coincide.

Sentential Fragments

(n-)Equisatisfiable Translation to Plain First-Order Logic

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain First-Order Logic

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\mathrm{trans}_n \left\{ \begin{aligned} \mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\ \mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\ \mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\ \mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\ \mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \to \mathrm{trans}_n(\pi, \psi)) \end{aligned} \right.$$

$$\begin{aligned} \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\ \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\ \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\ \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \end{aligned}$$

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$$\mathrm{trans}_{\mathcal{E}} \left\{ \begin{aligned} \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\ \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\ \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\ \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \end{aligned} \right.$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \dots, t_k)) &= \mathtt{P}_\pi(t_1, \dots, t_k) \\
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\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
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Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

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\end{aligned}$$

Example: $\phi = \neg\, \Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

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\end{aligned}$$

Example: $\phi = \neg \Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R} \cap \mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = \textstyle\bigwedge_{\pi \in \Pi_3} \\
\neg((\mathsf{R}_1 \rightarrow \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \rightarrow \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \rightarrow \mathtt{p}_3(a))) \\
\wedge \\
(((\mathsf{R}_1 \wedge \mathsf{B}_1) \rightarrow \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \rightarrow \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \rightarrow \mathtt{p}_3(a))) \\
\wedge \\
*_1 \wedge *_2 \wedge *_3,
\end{aligned}$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi\in\Pi_n} \mathrm{trans}_n(\pi,\phi) \wedge \bigwedge_{\pi\in\Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1,\ldots,t_k)) &= \mathtt{P}_\pi(t_1,\ldots,t_k)\\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi,\psi)\\
\mathrm{trans}_n(\pi, \psi_1\wedge\psi_2) &= \mathrm{trans}_n(\pi,\psi_1)\wedge\mathrm{trans}_n(\pi,\psi_2)\\
\mathrm{trans}_n(\pi, \forall x\psi) &= \forall x(\mathrm{trans}_n(\pi,\psi))\\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\,\psi) &= \textstyle\bigwedge_{\pi\in\Pi_n}(\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e})\rightarrow\mathrm{trans}_n(\pi,\psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{s}) &= \mathsf{s}_\pi\\
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\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1\setminus\mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1)\wedge\neg\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg\Box_{\mathsf{R}}\,\mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\,\mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = \textstyle\bigwedge_{\pi\in\Pi_3}&\\
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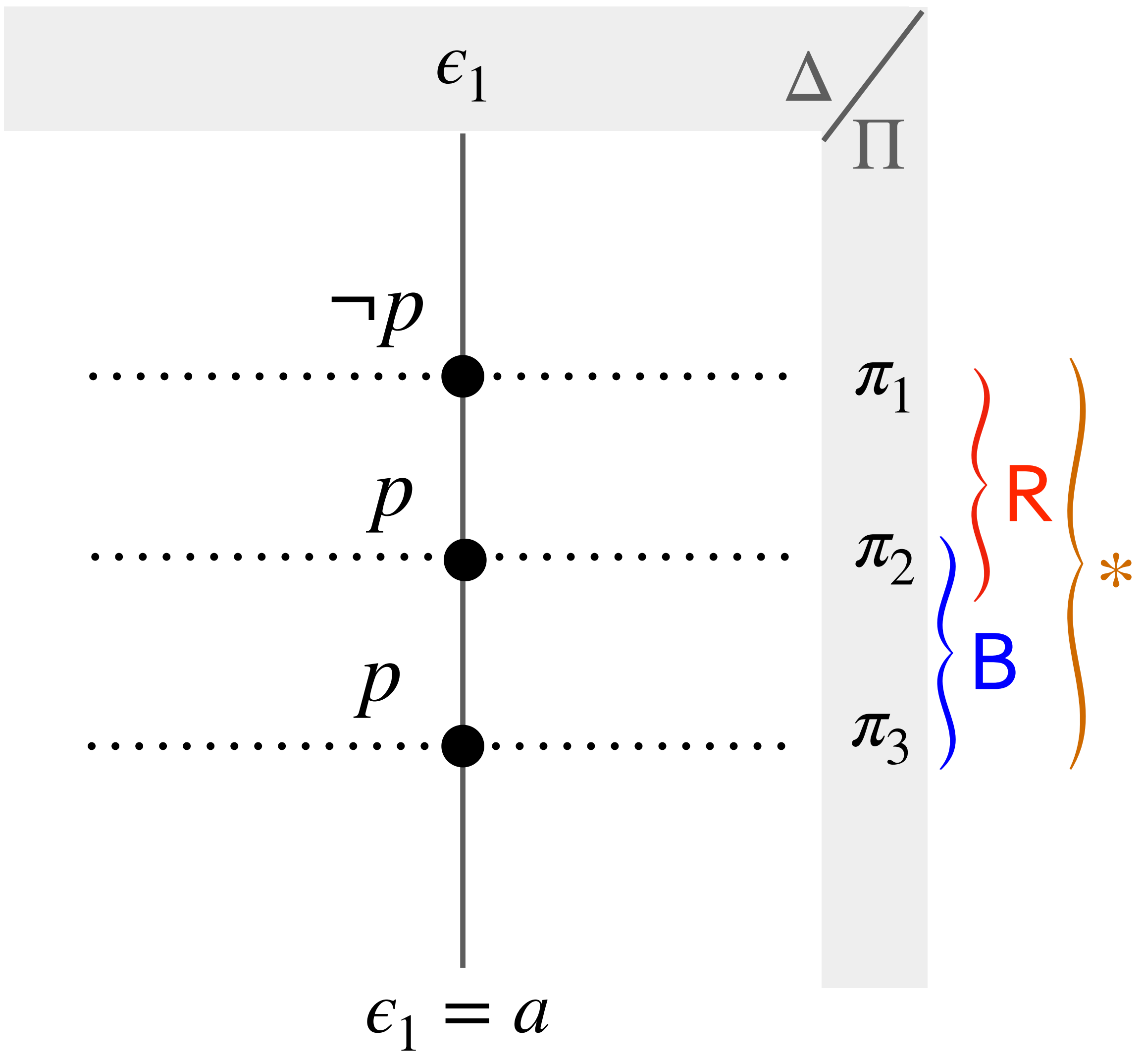
\wedge&\\
(((\mathsf{R}_1\wedge\mathsf{B}_1)\rightarrow\mathtt{p}_1(a))\wedge((\mathsf{R}_2\wedge\mathsf{B}_2)\rightarrow\mathtt{p}_2(a))\wedge((\mathsf{R}_3\wedge\mathsf{B}_3)\rightarrow\mathtt{p}_3(a)))&\\
\wedge&\\
*_1\wedge *_2\wedge *_3,&
\end{aligned}$$



Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

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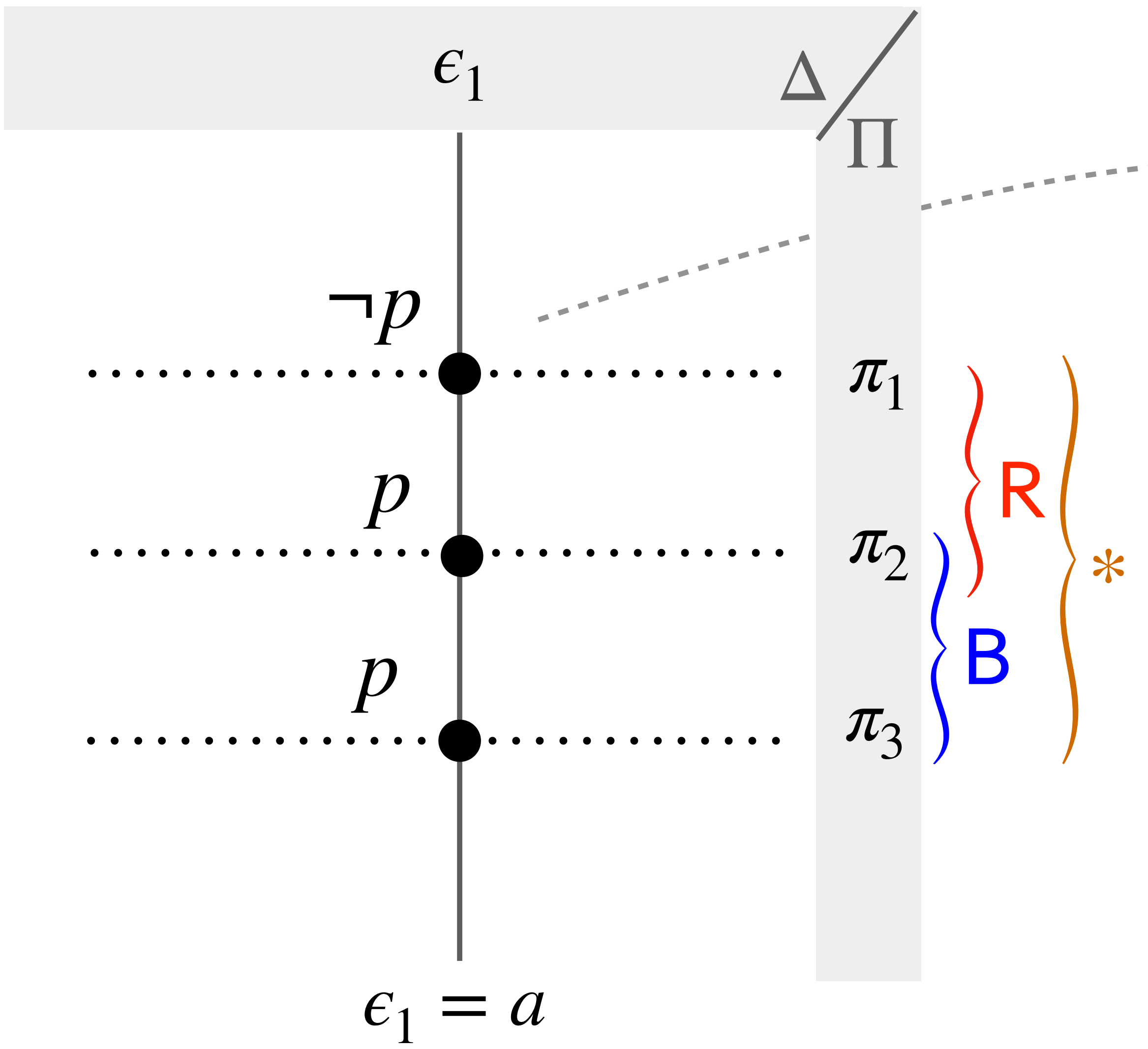
Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}\mathrm{Trans}_3(\phi) = \bigwedge_{\pi\in\Pi_3}\\ \neg((\mathsf{R}_1 \rightarrow \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \rightarrow \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \rightarrow \mathtt{p}_3(a)))\\ \wedge\\ (((\mathsf{R}_1 \wedge \mathsf{B}_1) \rightarrow \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \rightarrow \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \rightarrow \mathtt{p}_3(a)))\\ \wedge\\ *_1 \wedge *_2 \wedge *_3,\end{aligned}$$



$\mathsf{R}_1 \wedge \neg\mathsf{B}_1 \wedge \neg p_1(a)$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
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Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

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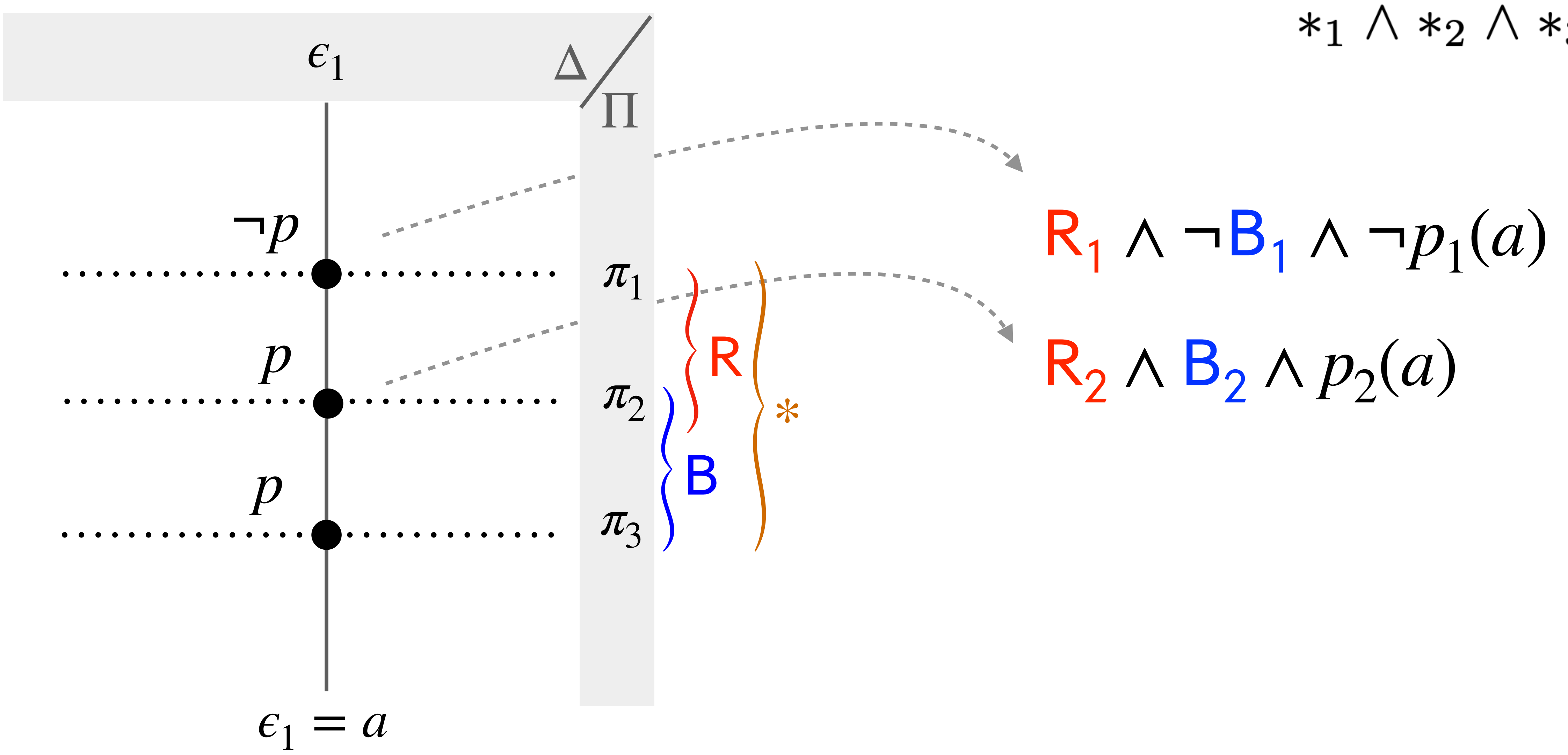
\wedge& \\
(((\mathsf{R}_1 \wedge \mathsf{B}_1) \rightarrow \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \rightarrow \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \rightarrow \mathtt{p}_3(a)))& \\
\wedge& \\
*_1 \wedge *_2 \wedge *_3,&
\end{aligned}$$



Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

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Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

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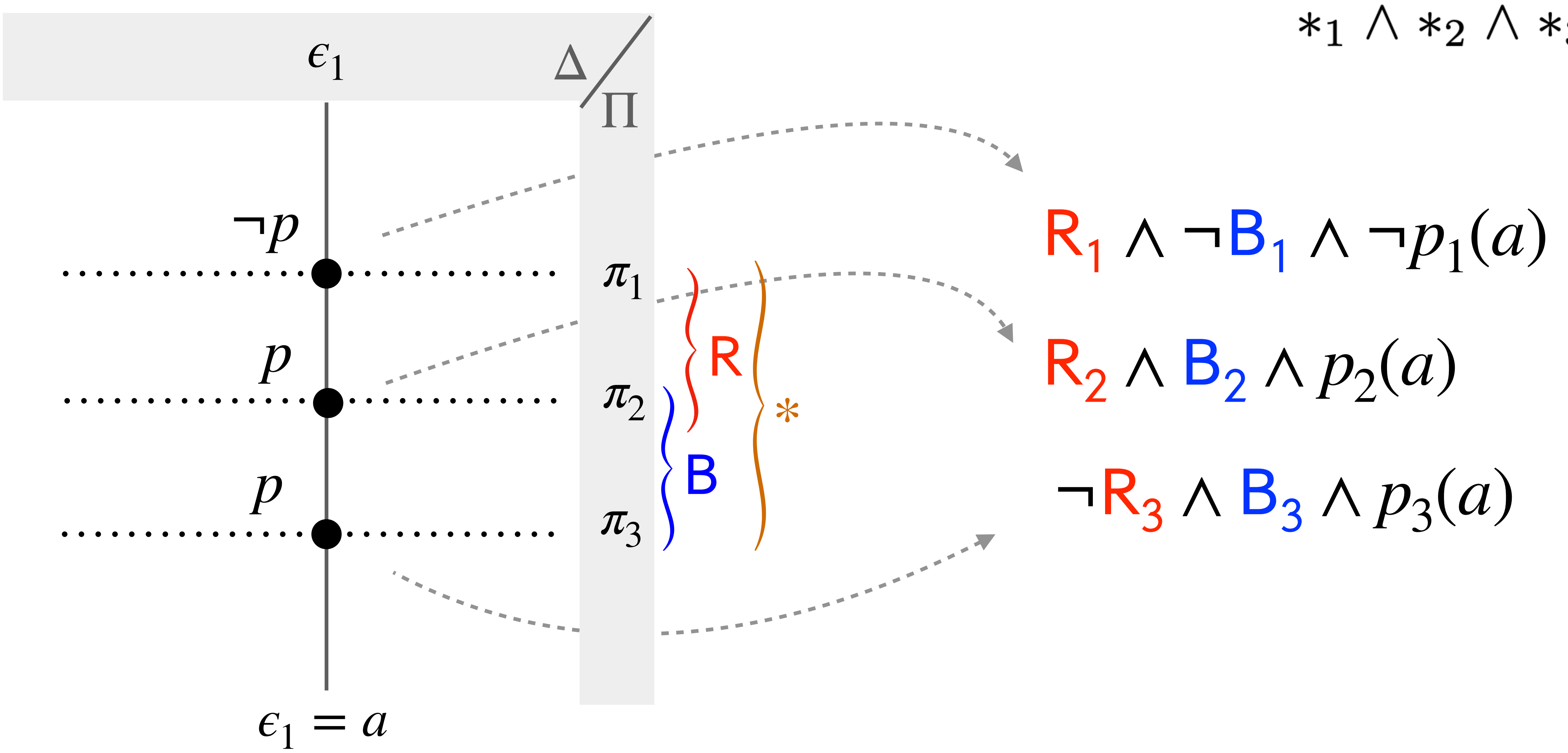

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Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

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\end{aligned}$$

Theorem:
A formula ϕ is n-satisfiable in FOSL if and only if $\mathrm{Trans}_n(\phi)$ is satisfiable in first-order logic.

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

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\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}} \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Theorem:
A formula ϕ is n-satisfiable in FOSL if and only if $\mathsf{Trans}_n(\phi)$ is satisfiable in first-order logic.

Lemma:
Let F be a fragment of FOL. Then the satisfiability of $\mathbb{S}_F$, the sentential standpoint-F fragment of FOSL,

- is decidable iff it is for F, and
- has the same complexity as F (if at least NP)

Reasoning with Standpoint-Enhanced DLs

Standpoint-$\mathcal{SROIQ}b_s$

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[1] SROIQbs is an extension of SROIQ allowing safe Boolean role expressions oversimple roles at no complexity cost (Rudolph, Krotzsch, and Hitzler 2008)

Reasoning with Standpoint-Enhanced DLs

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Theorem: *Given* $\varphi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$, *the set* $\mathrm{Trans}(\varphi)$

1. *is a valid* $\mathcal{SROIQ}b_s$ *knowledge base,*
2. *is equisatisfiable with* φ,
3. *is of polynomial size wrt.* φ, *and*
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Standpoint SROIQbs

$\square_{\mathsf{FR}}[\mathtt{Clearing} \sqcup \mathtt{Woodland} \sqsubseteq \mathtt{Forest}]$

$\square_{\mathsf{GP}}[\mathtt{Woodland} \equiv \mathtt{Forest}]$

$\square_{\mathsf{FR \cup GP}}[\mathtt{Clearing} \sqsubseteq \neg\mathtt{Woodland}]$

$\square_{\mathsf{FR \cup GP}}[\mathtt{Clearing(c)}]$

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$\square_{\mathrm{FR\cup GP}}$[Clearing ⊑ ¬Woodland]
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Trans_n

SROIQbs

⊤⊑¬(∀u.MFR_1)⊔(¬(Clairiere_1⊔Woodland_1)⊔Forest_1)⊓¬(∀u.MFR_2)⊔(¬(Clairiere_2⊔Woodland_2)⊔Forest_2)⊓…⊓¬(∀u.MFR_n)⊔(¬(Clairiere_n⊔Woodland_n)⊔Forest_n)⊓¬(∀u.MGP_1)⊔(¬Woodland_1⊔Forest_1)⊓¬(∀u.MGP_2)⊔(¬Woodland_2⊔Forest_2)⊓…⊓¬(∀u.MGP_n)⊔(¬Woodland_n⊔Forest_n)⊓…⊓∀u.M*1⊓∀u.M*2⊓…⊓∀u.M*n…

Polynomial Equisatisfiable Translation

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Polynomial Equisatisfiable Translation ⟶ OWL2 Reasoners

OWL2 Extension and support in Protégé (current work)

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standpoint *protégé* *(plugin)*

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Future work: production of (Standpoint-SROIQ) KBs for testing and didactic purposes.

Monodic Fragments

Tractable Reasoning in $\mathbb{S}_{\mathcal{EL}}$

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$$\forall x\ \Box_{\mathsf{P}}(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \Diamond_{\mathsf{T}}\mathsf{Tumor}(x)$$

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- We must sacrifice empty standpoints, rigid roles, and nominals.

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- We preserve $\mathcal{EL}$'s favourable PTime standard reasoning.
- We must sacrifice empty standpoints, rigid roles, and nominals.
- We adapt techniques based on “quasi-models” for modal logics.

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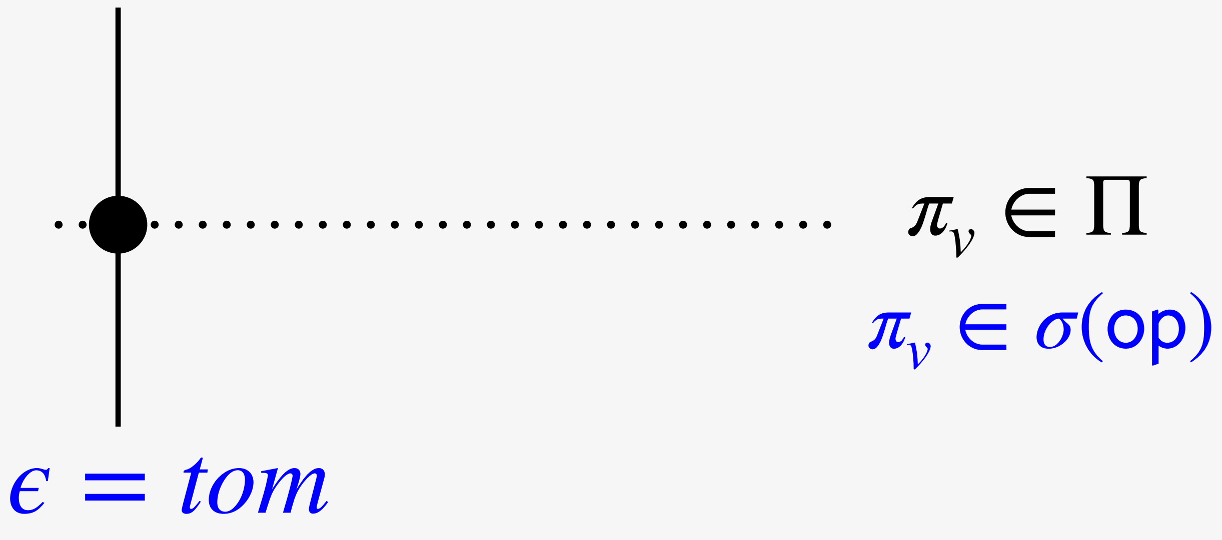

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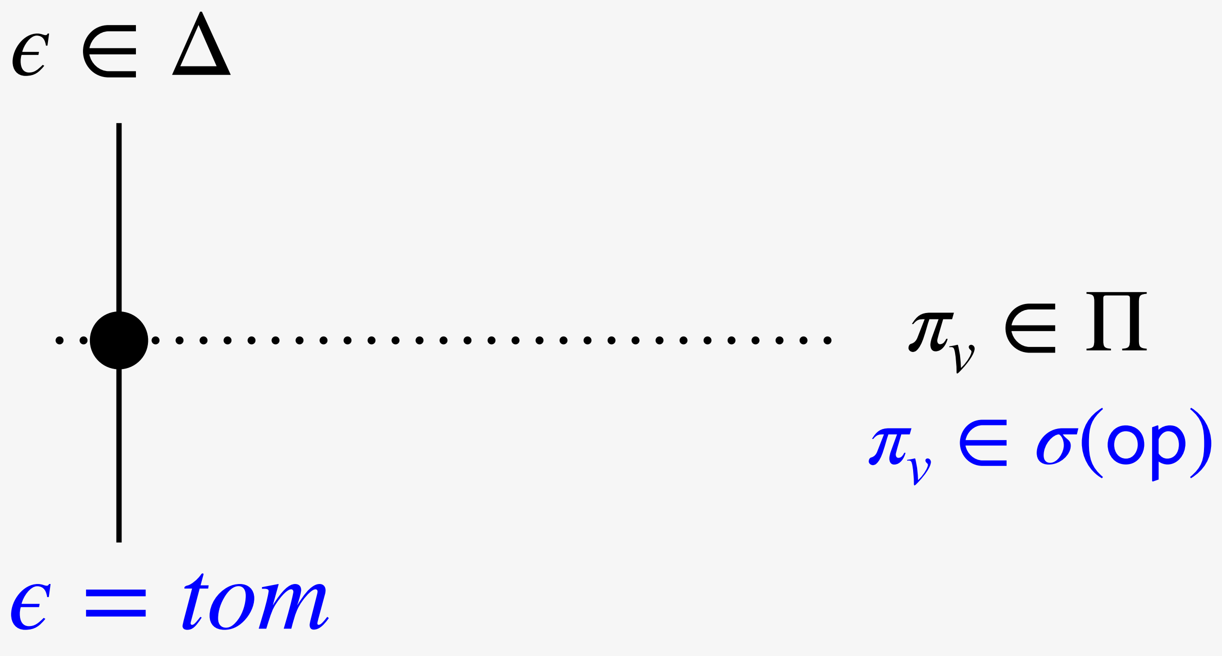

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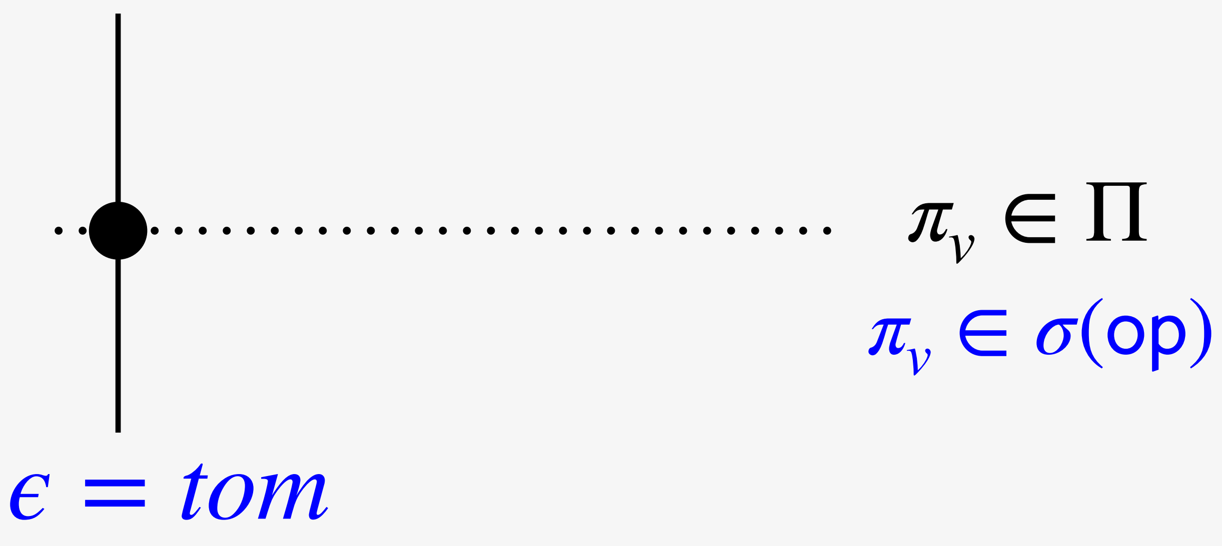

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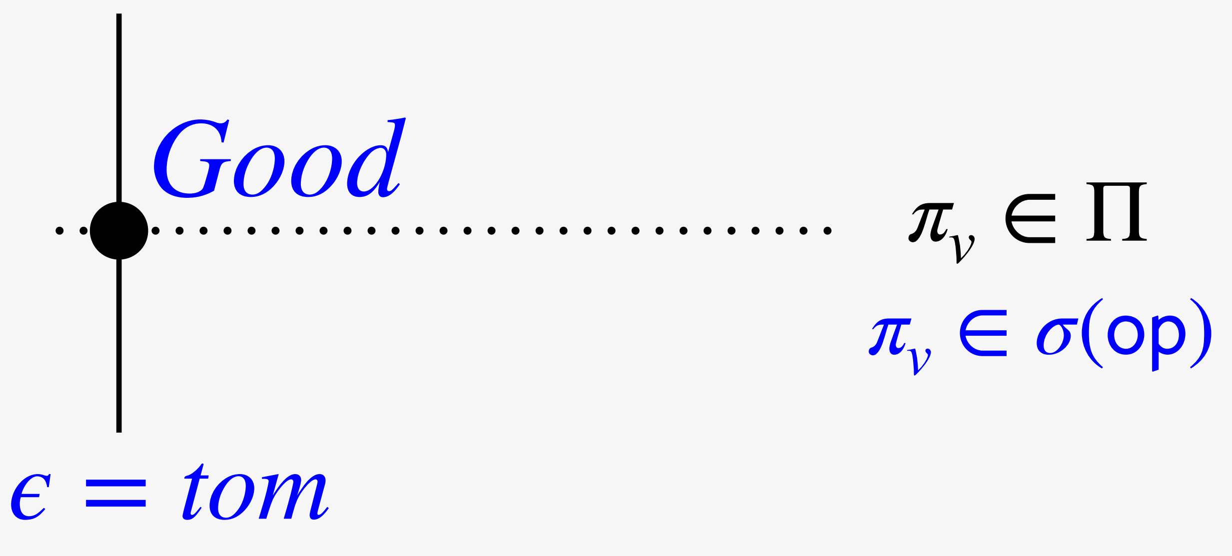

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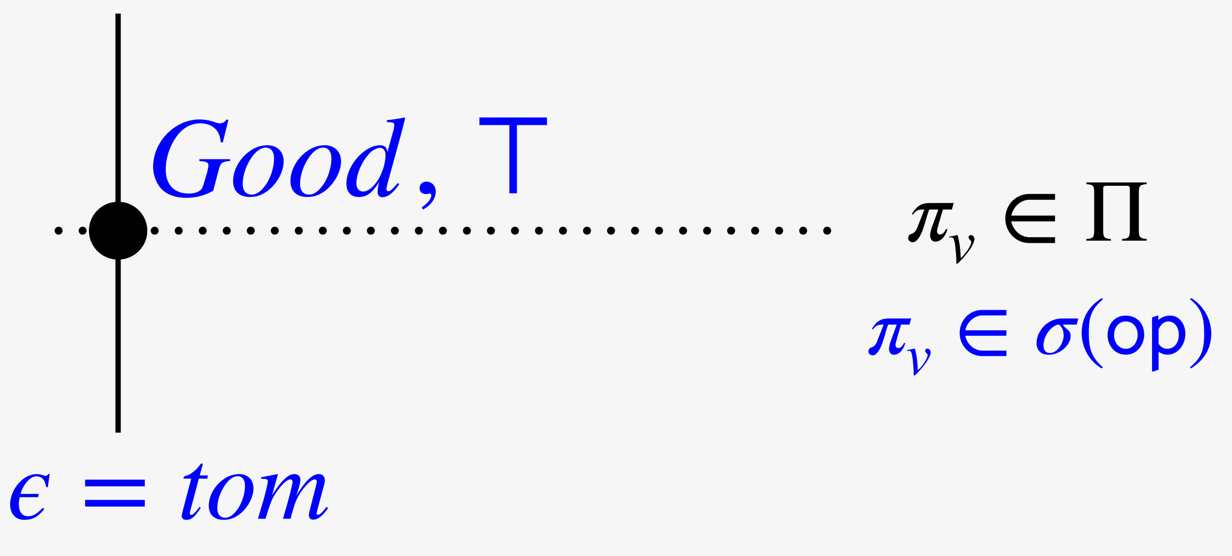

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Local labelling (LL) rule:

$\mathbf{R}_{\preceq}$ If $\{x : \mathsf{s},\ x' : \mathsf{s} \preceq \mathsf{s}'\} \subseteq S$ but $(x : \mathsf{s}') \notin S$, then set $S := S \cup \{x : \mathsf{s}'\}$.

Local content (LC) rules:

$\mathbf{R}_{\sqcap}$ If $\{x : C,\ x : D\} \subseteq S$, $(x : C \sqcap D) \notin S$ and $C \sqcap D \in \mathsf{C}_{\mathcal{K}}$, then set $S := S \cup \{x : C \sqcap D\}$.

$\mathbf{R}_{\sqsubseteq}$ If $\{x : C,\ x : C \sqsubseteq D\} \subseteq S$ but $(x : D) \notin S$, then set $S := S \cup \{x : D\}$.

$\mathbf{R}_{\Box}$ If $\{x : \Box_{\mathsf{s}}\Phi,\ x' : \mathsf{s}\} \subseteq S$ but $(x' : \Phi) \notin S$, then set $S := S \cup \{x' : \Phi\}$.

$\mathbf{R}_g$ If $(x : \mathbf{G}) \in S$ but $(x' : \mathbf{G}) \notin S$, then set $S := S \cup \{x' : \mathbf{G}\}$.

$\mathbf{R}_a$ If $\{x : a,\ x : C(a)\} \subseteq S$ but $(x : C) \notin S$, then set $S := S \cup \{x : C\}$.

$\mathbf{R}_{\Diamond}$ If $(x : \Diamond_{\mathsf{s}} C) \in S$ and $\{x' : \mathsf{s},\ x' : C\} \nsubseteq S$ for all x' in S, then create a fresh variable x' and set $S := S \cup \{x' : C,\ x' : \mathsf{s},\ x' : *,\ x' : \top\}$.

Global non-generating (GN) rules:

$\mathbf{R}_{\downarrow}$ If $(x : C) \in \mathcal{S}(\varepsilon)$, $\langle \varepsilon', x', \varepsilon, x, R\rangle \in \mathcal{R}$, and $\exists R.C \in \mathsf{C}_{\mathcal{K}}$, but $(x' : \exists R.C) \notin \mathcal{S}(\varepsilon')$, then set $\mathcal{S}(\varepsilon') := \mathcal{S}(\varepsilon') \cup \{x' : \exists R.C\}$.

$\mathbf{R}_r$ If $\{x : a,\ x : R(a,b)\} \subseteq \mathcal{S}(\varepsilon)$ and $(x' : b) \in \mathcal{S}(\varepsilon')$, but $\langle \varepsilon, x, \varepsilon', x, R\rangle \notin \mathcal{R}$, then set $\mathcal{S}(\varepsilon') := \mathcal{S}(\varepsilon') \cup \{x : \top\} \cup \{x : \mathsf{s} \mid \mathsf{s} \in \mathsf{st}_{\varepsilon}(x)\}$ and $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x, R\rangle\}$.

$\mathbf{R}_{r'}$ If $\{x : b,\ x : R(a,b)\} \subseteq \mathcal{S}(\varepsilon)$ and $(x' : a) \in \mathcal{S}(\varepsilon')$, but $\langle \varepsilon', x, \varepsilon, x, R\rangle \notin \mathcal{R}$, then set $\mathcal{S}(\varepsilon') := \mathcal{S}(\varepsilon') \cup \{x : \top\} \cup \{x : \mathsf{s} \mid \mathsf{s} \in \mathsf{st}_{\varepsilon}(x)\}$ and $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon', x, \varepsilon, x, R\rangle\}$.

$\mathbf{R}_{\exists'}$ If $(x : \exists R.C) \in \mathcal{S}(\varepsilon)$, $(C, \mathsf{st}_{\varepsilon}(x), x') \in \mathcal{L}(\varepsilon')$ with $\varepsilon \neq \varepsilon'$ or $x = x'$, but $\langle \varepsilon, x, \varepsilon'', x'', R\rangle \notin \mathcal{R}$ whenever $(C, \mathsf{st}_{\varepsilon}(x), x'') \in \mathcal{L}(\varepsilon'')$ and $\varepsilon \neq \varepsilon''$ or $x = x''$, then set $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x', R\rangle\}$.

Global generating (GG) rule:

$\mathbf{R}_{\exists}$ If$(x : \exists R.C) \in \mathcal{S}(\varepsilon)$, but $\langle \varepsilon, x, \varepsilon'', x'', R\rangle \notin \mathcal{R}$ whenever $(C, \mathsf{st}_{\varepsilon}(x), x'') \in \mathcal{L}(\varepsilon'')$ and $\varepsilon \neq \varepsilon''$ or $x = x''$, then create ε' and a fresh variable x', and then set $\mathcal{L}(\varepsilon') := \{(C, \mathsf{st}_{\varepsilon}(x), x')\}$, $\mathcal{S}(\varepsilon') := S_0^{\mathcal{K}} \cup \{x' : C,\ x' : \top\} \cup \{x' : \mathsf{s} \mid \mathsf{s} \in \mathsf{st}_{\varepsilon}(x)\}$, $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x', R\rangle\}$.

Tableau Algorithm for $\mathbb{S}_{\mathscr{EL}}$

Example:

$C \sqsubseteq \exists R.D$

$\Box_{\mathsf{s}}[a : \Diamond_{\mathsf{s}} C]$

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$C \sqsubseteq \exists R.D$

$\Box_s[a : \Diamond_s C]$

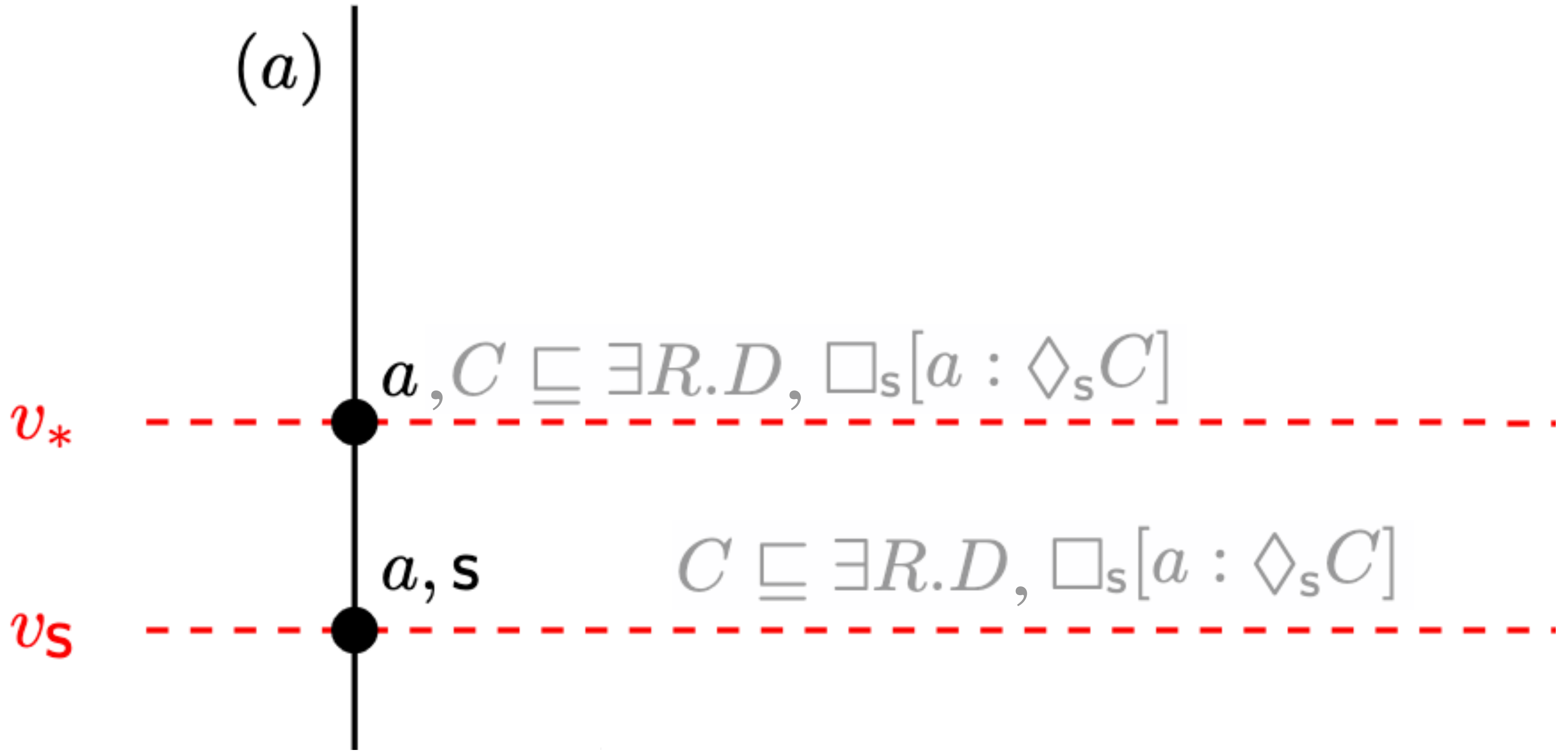

Tableau Algorithm for $\mathbb{S}_{\mathscr{E}\mathscr{L}}$

Example:

$C \sqsubseteq \exists R.D$

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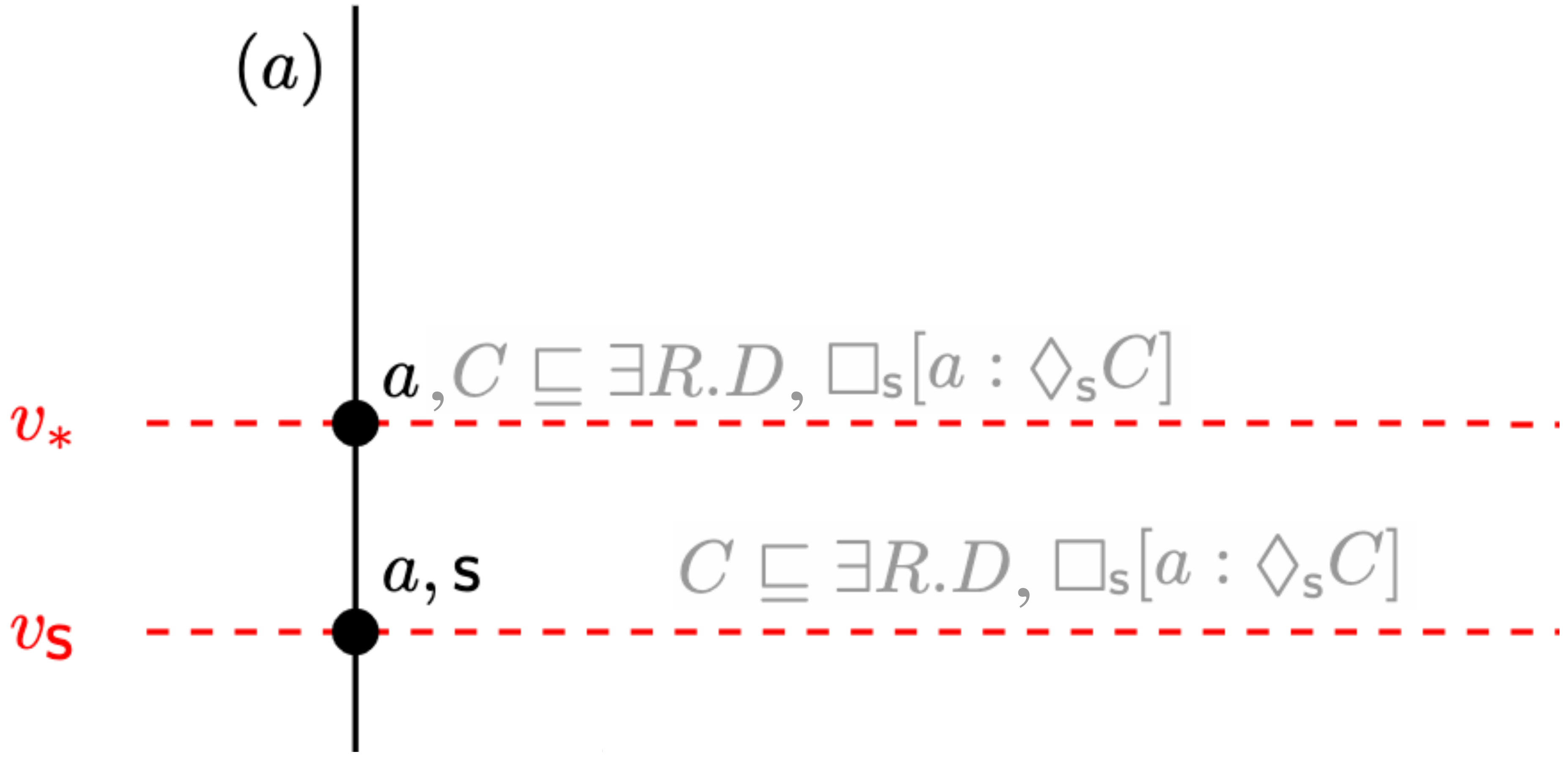
$\mathbf{R}_a$ If $\{x : a,\ x : C(a)\} \subseteq S$ but $(x : C) \notin S$, then set $S := S \cup \{x : C\}$.



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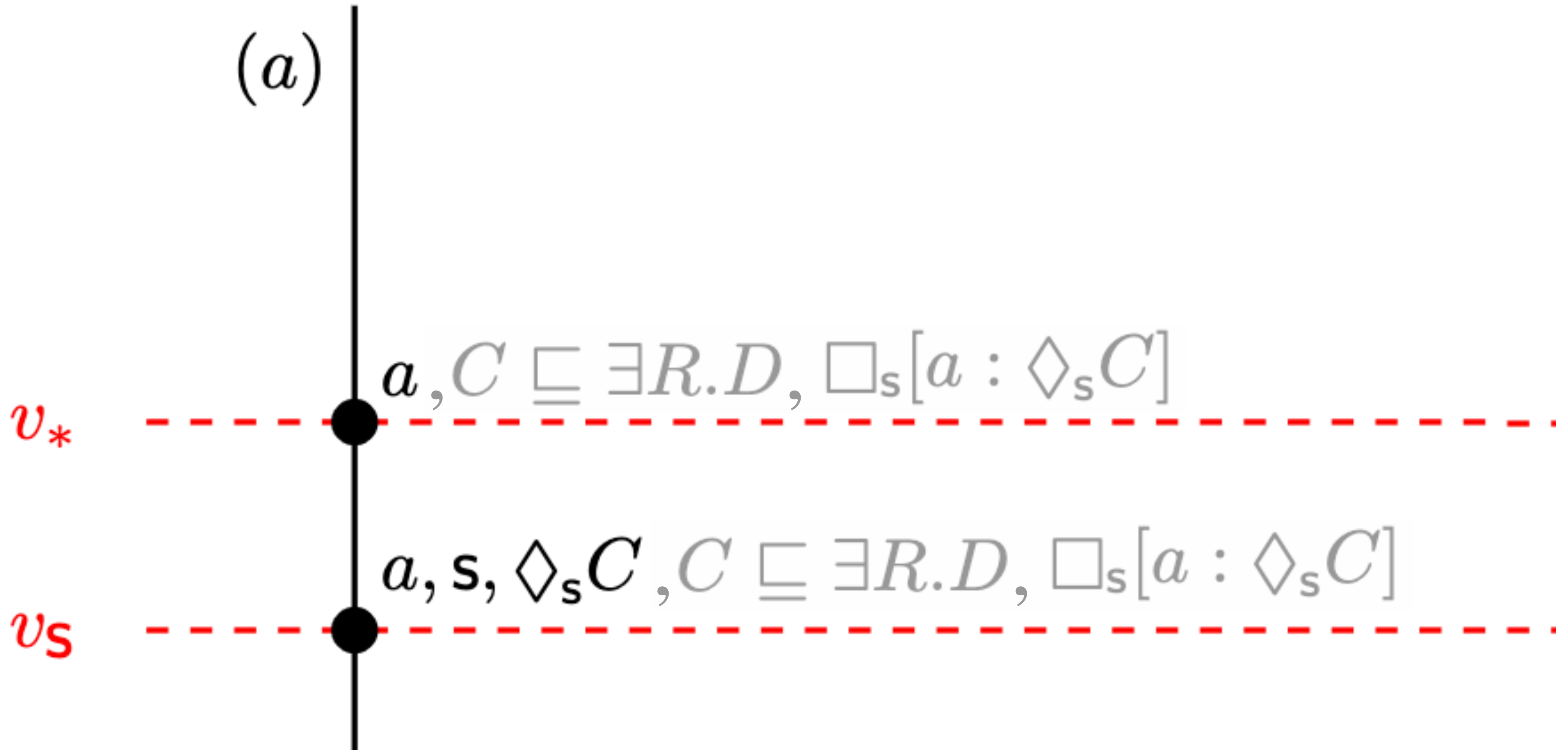

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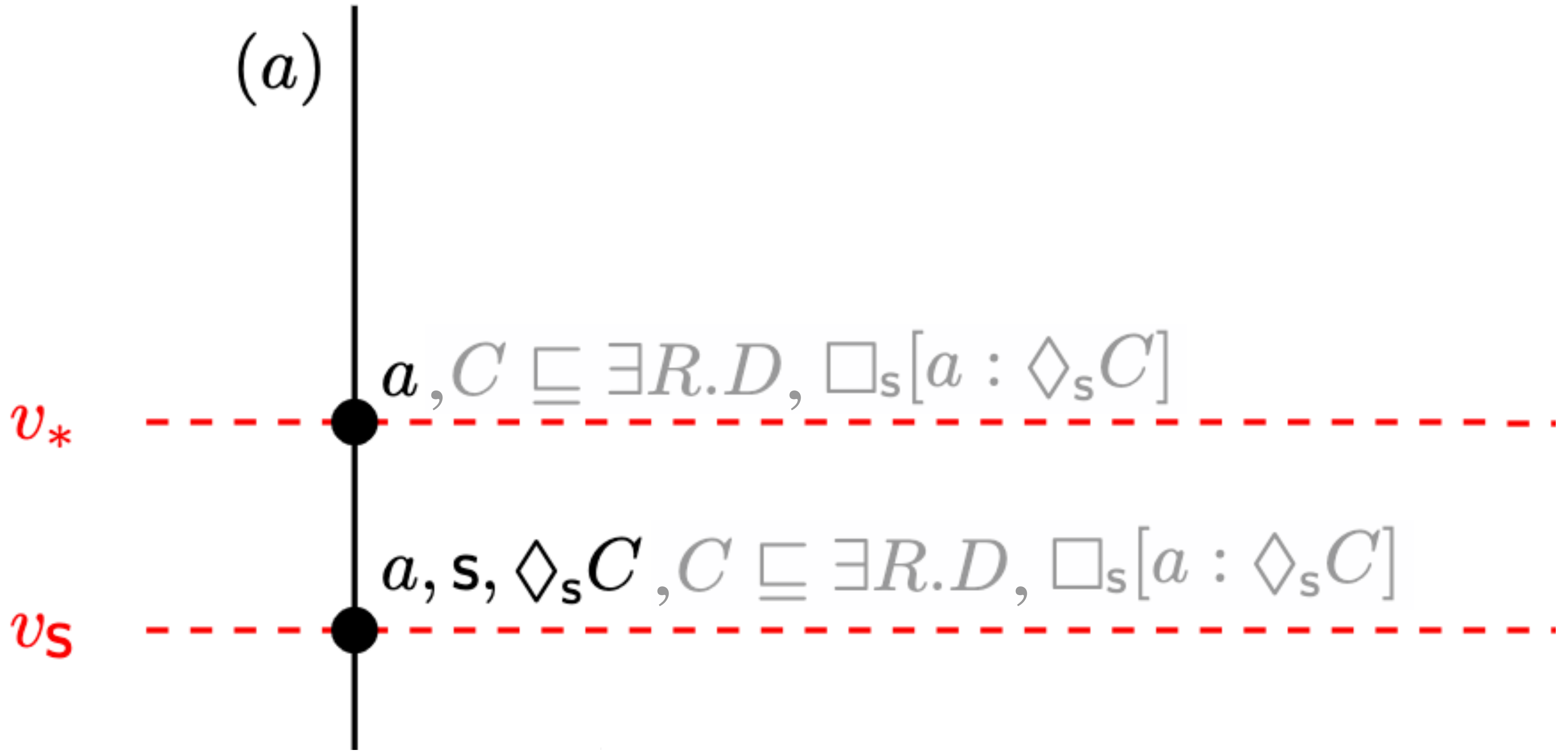

Tableau Algorithm for $\mathbb{S}_{\mathscr{EL}}$

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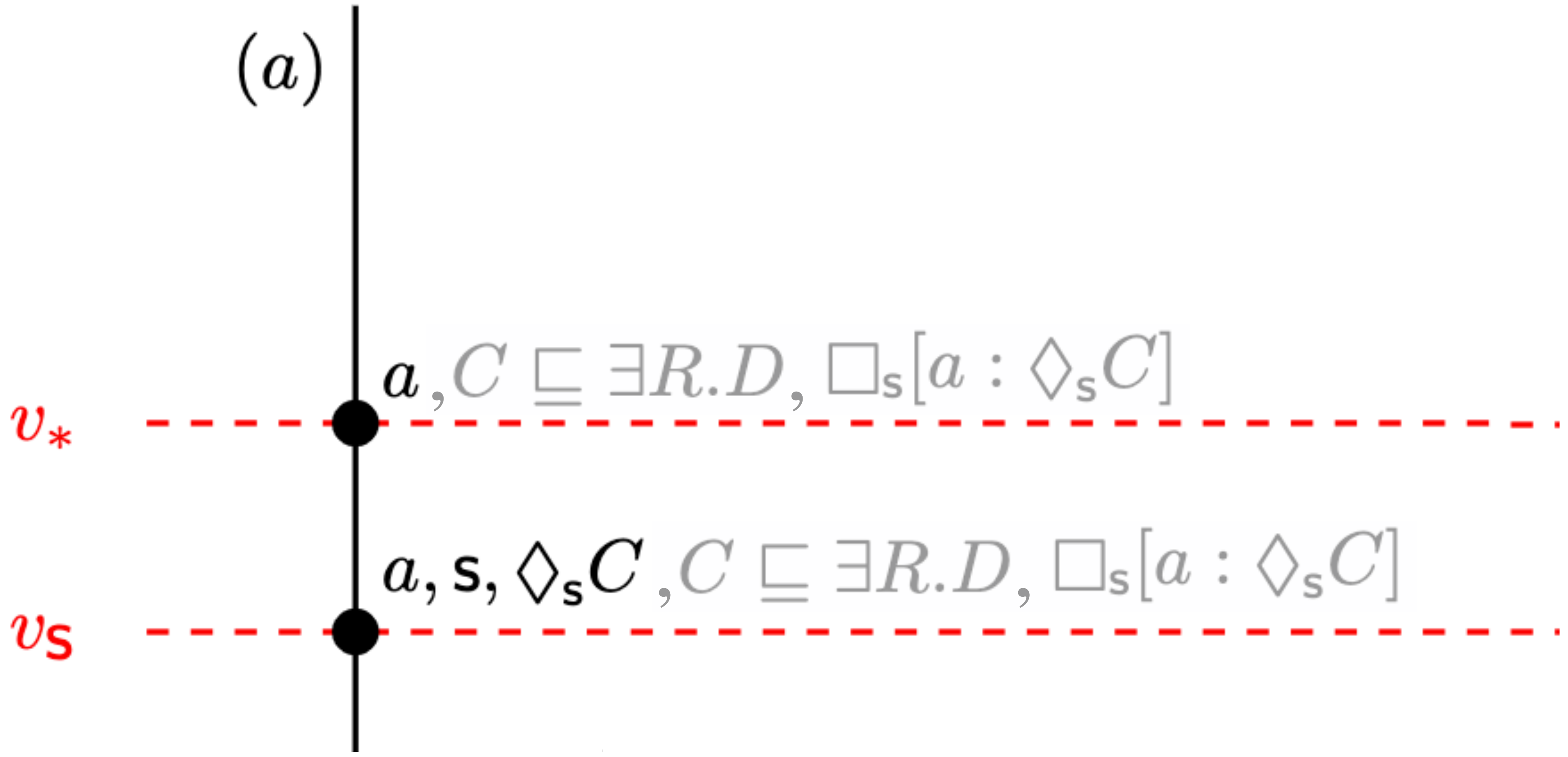

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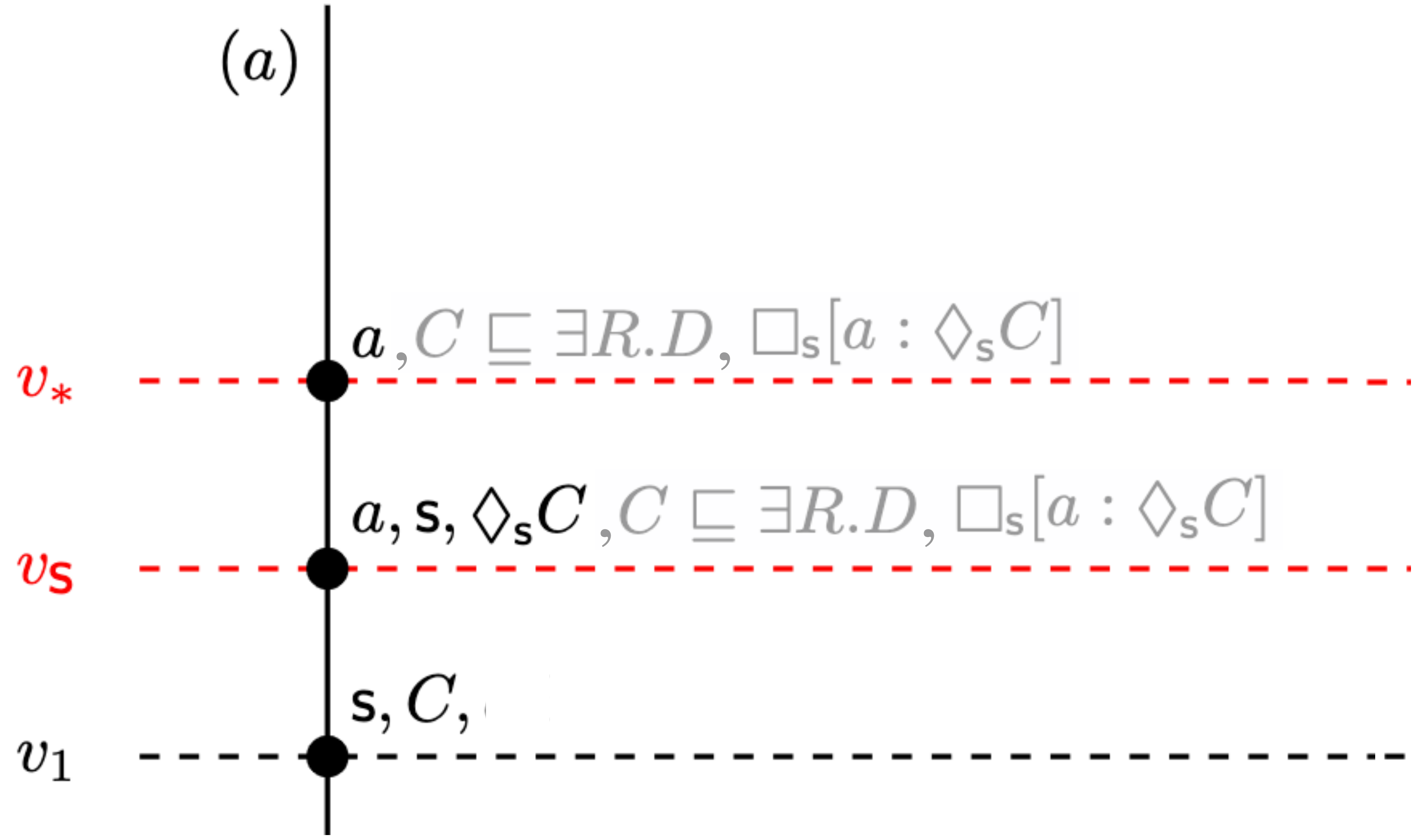

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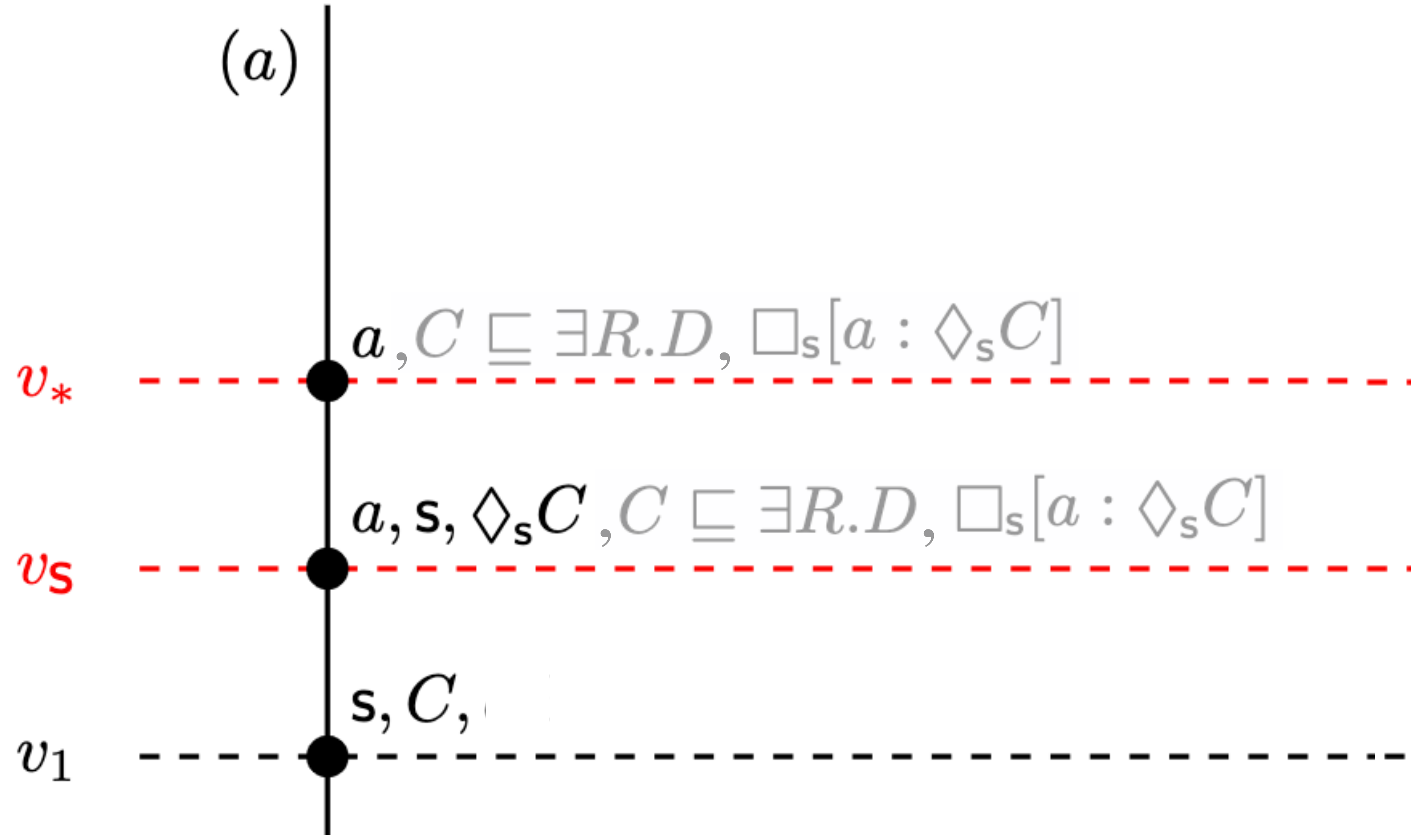

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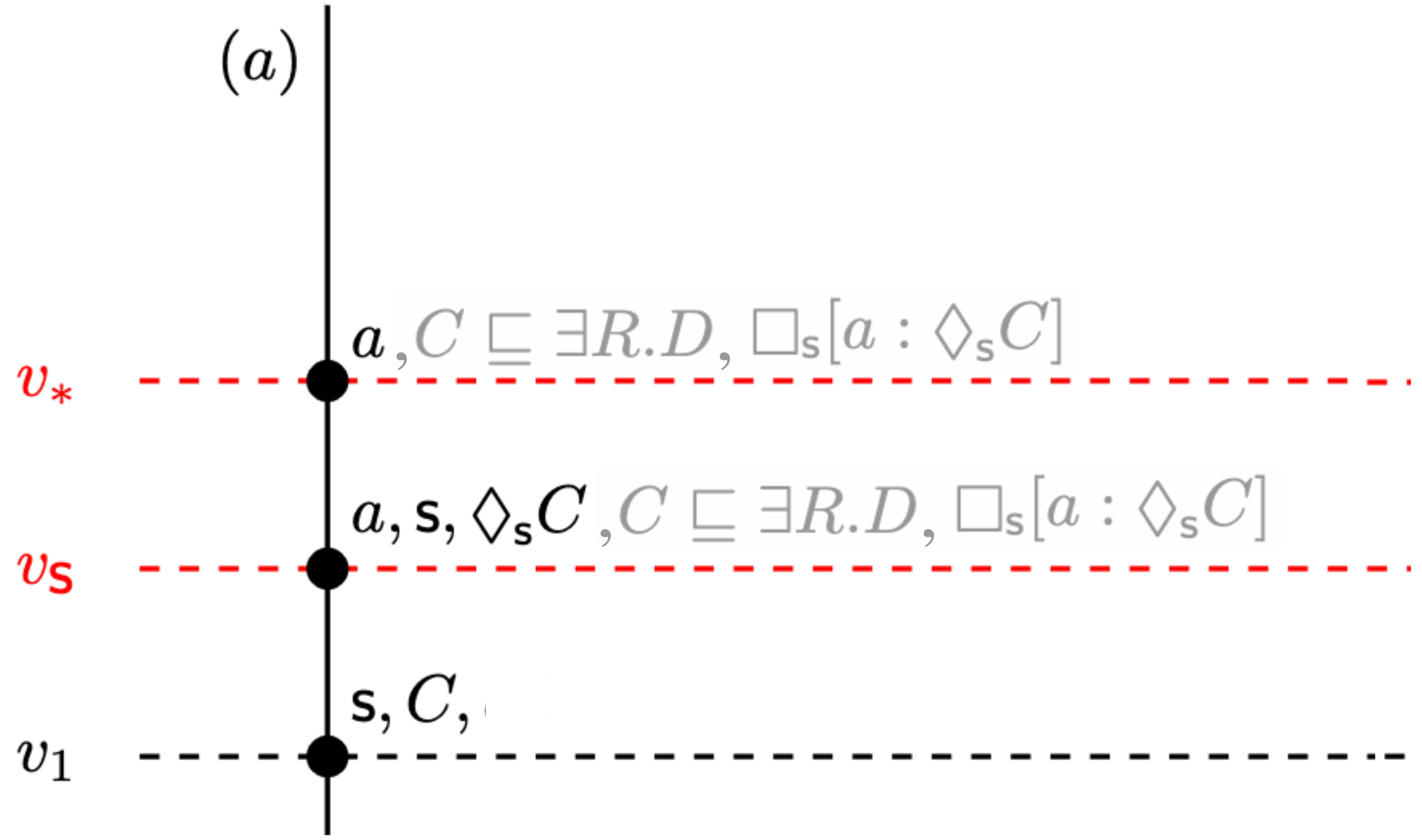

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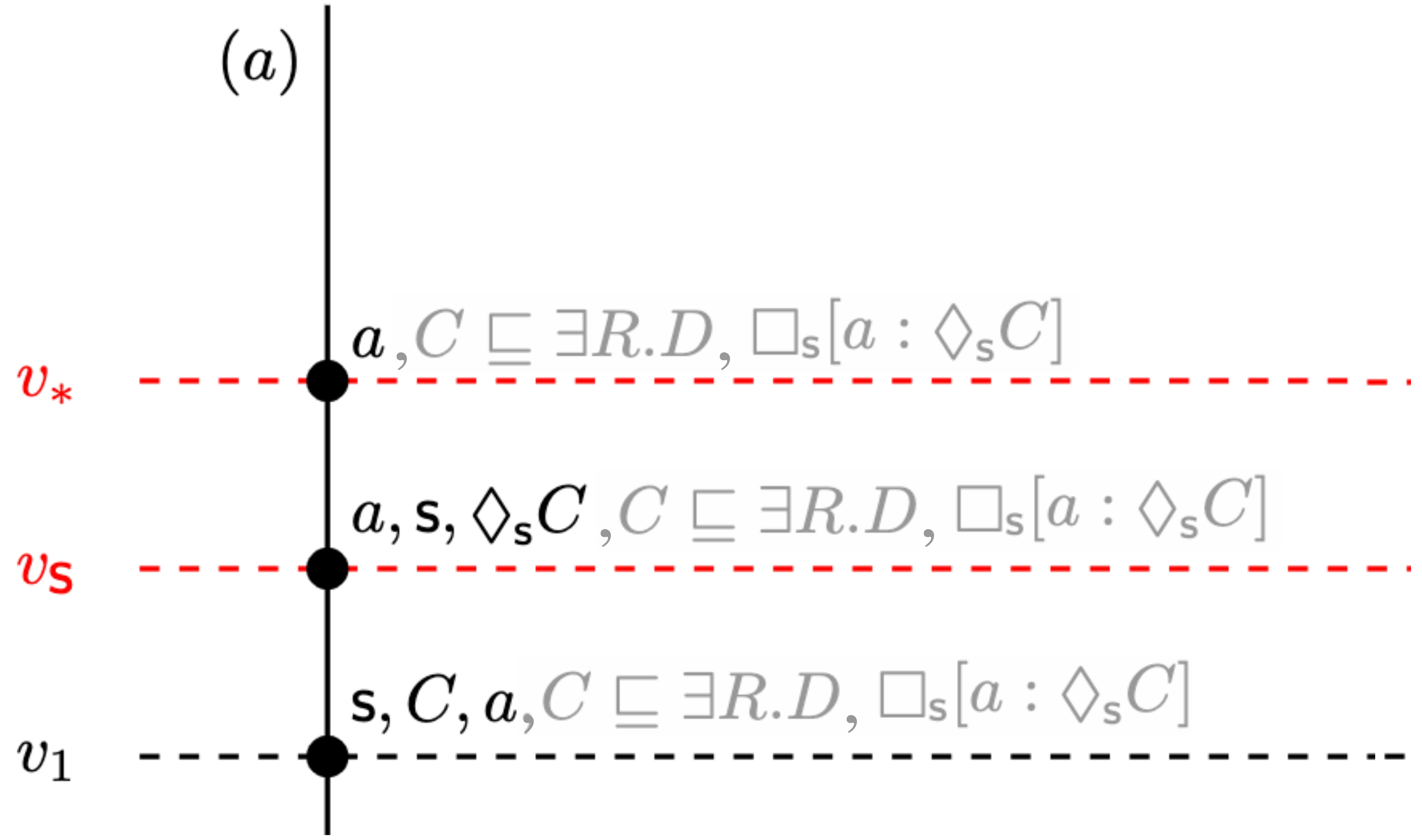

Tableau Algorithm for $\mathbb{S}_{\mathcal{EL}}$

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$C \sqsubseteq \exists R.D$

$\Box_{\mathsf{s}}[a : \Diamond_{\mathsf{s}} C]$

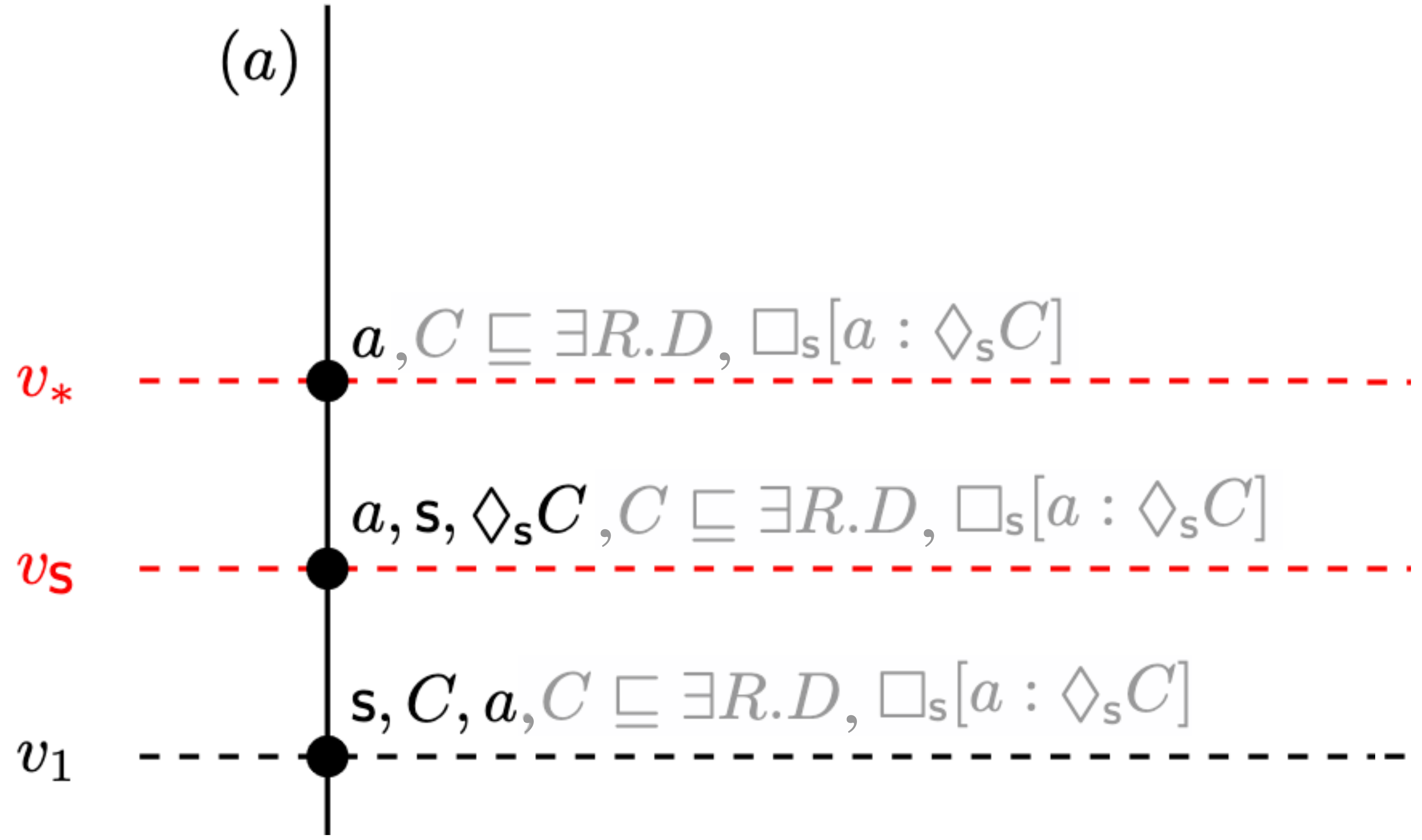

Tableau Algorithm for $\mathbb{S}_{\mathscr{E}\mathscr{L}}$

Example:

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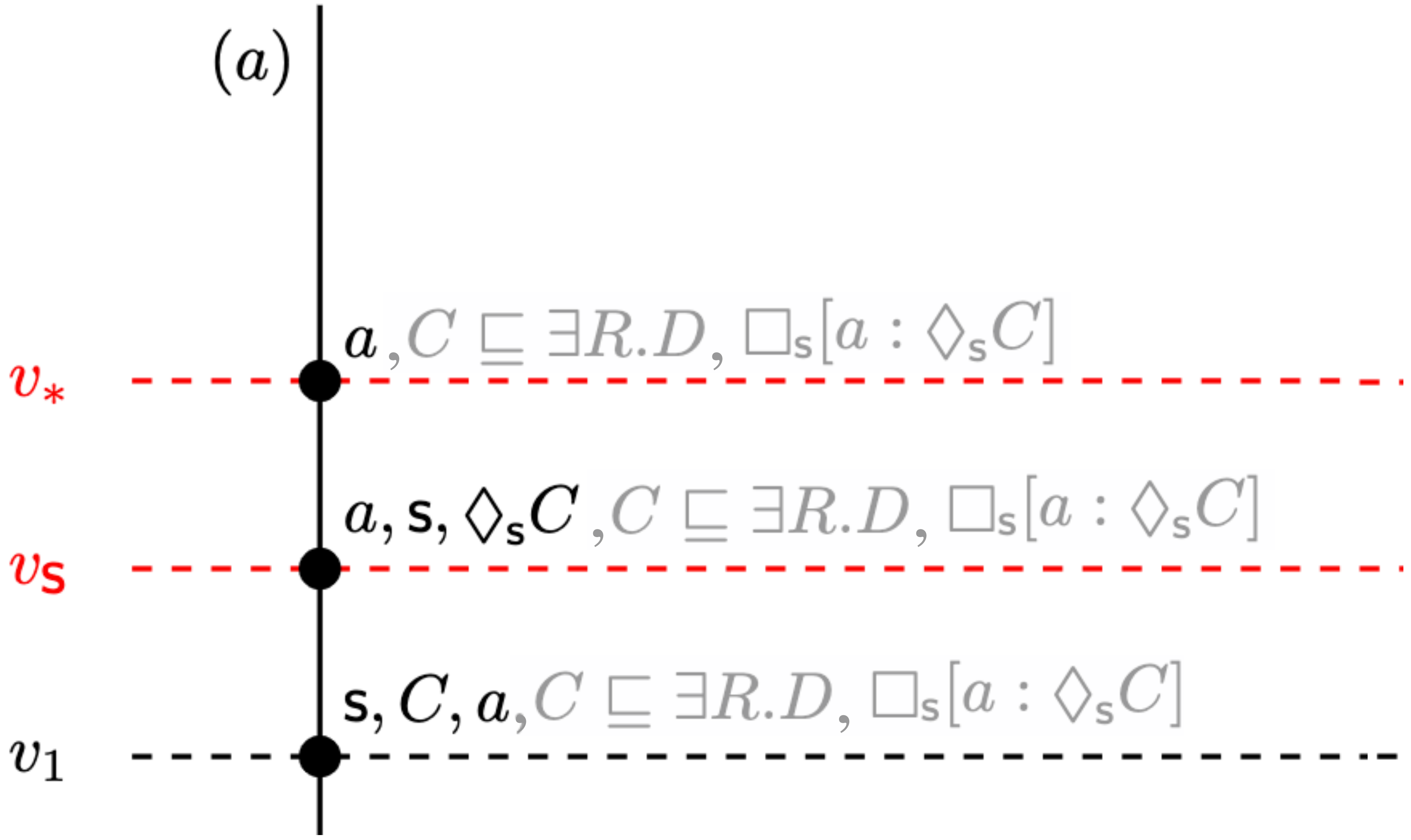
$\mathbf{R}_{\sqsubseteq}$ If $\{x : C,\ x : C \sqsubseteq D\} \subseteq S$ but $(x : D) \notin S$, then set $S := S \cup \{x : D\}$.



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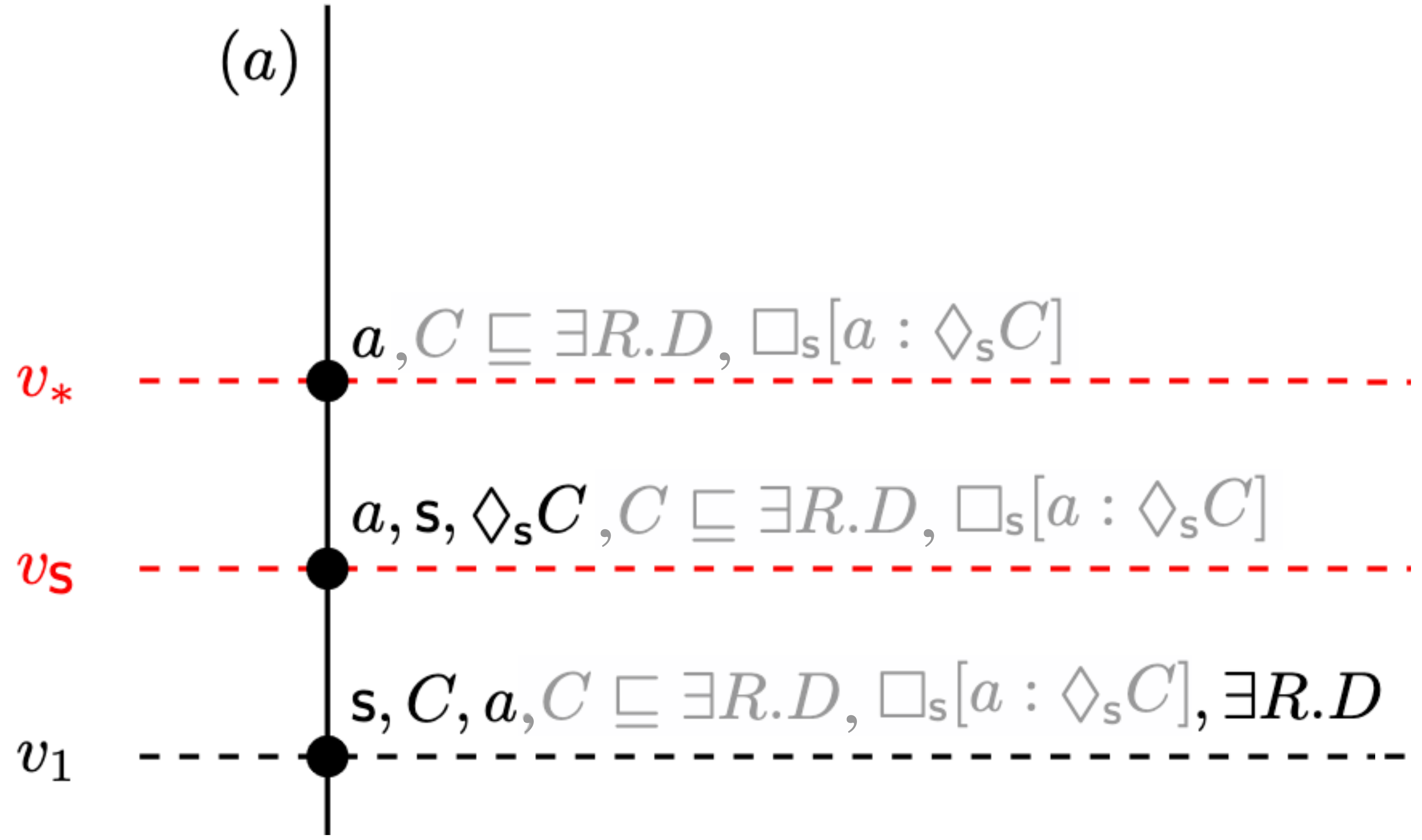

Tableau Algorithm for $\mathbb{S}_{\mathscr{E}\mathscr{L}}$

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$C \sqsubseteq \exists R.D$

$\square_s[a : \lozenge_s C]$

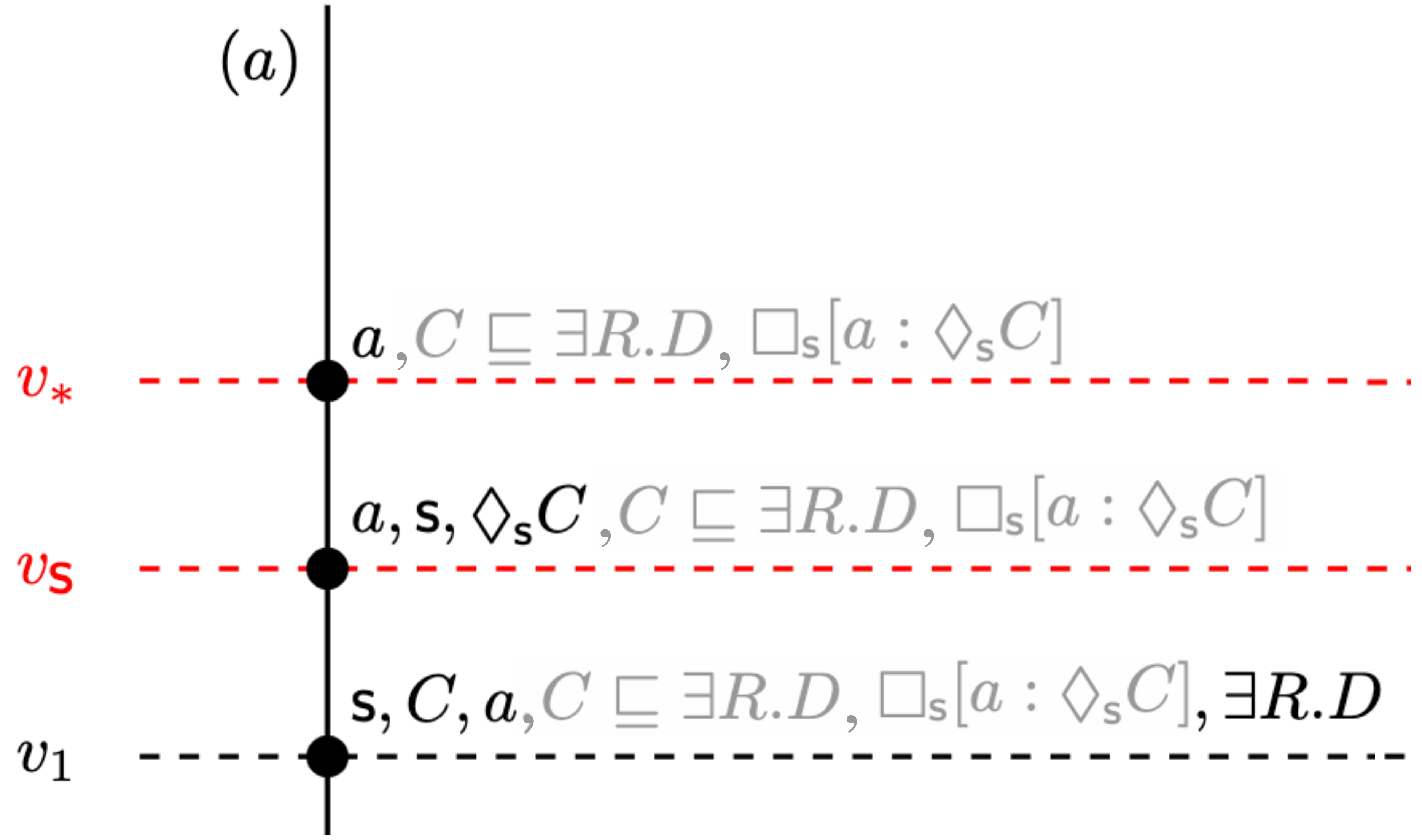

Tableau Algorithm for $\mathbb{S}_{\mathscr{E}\mathscr{L}}$

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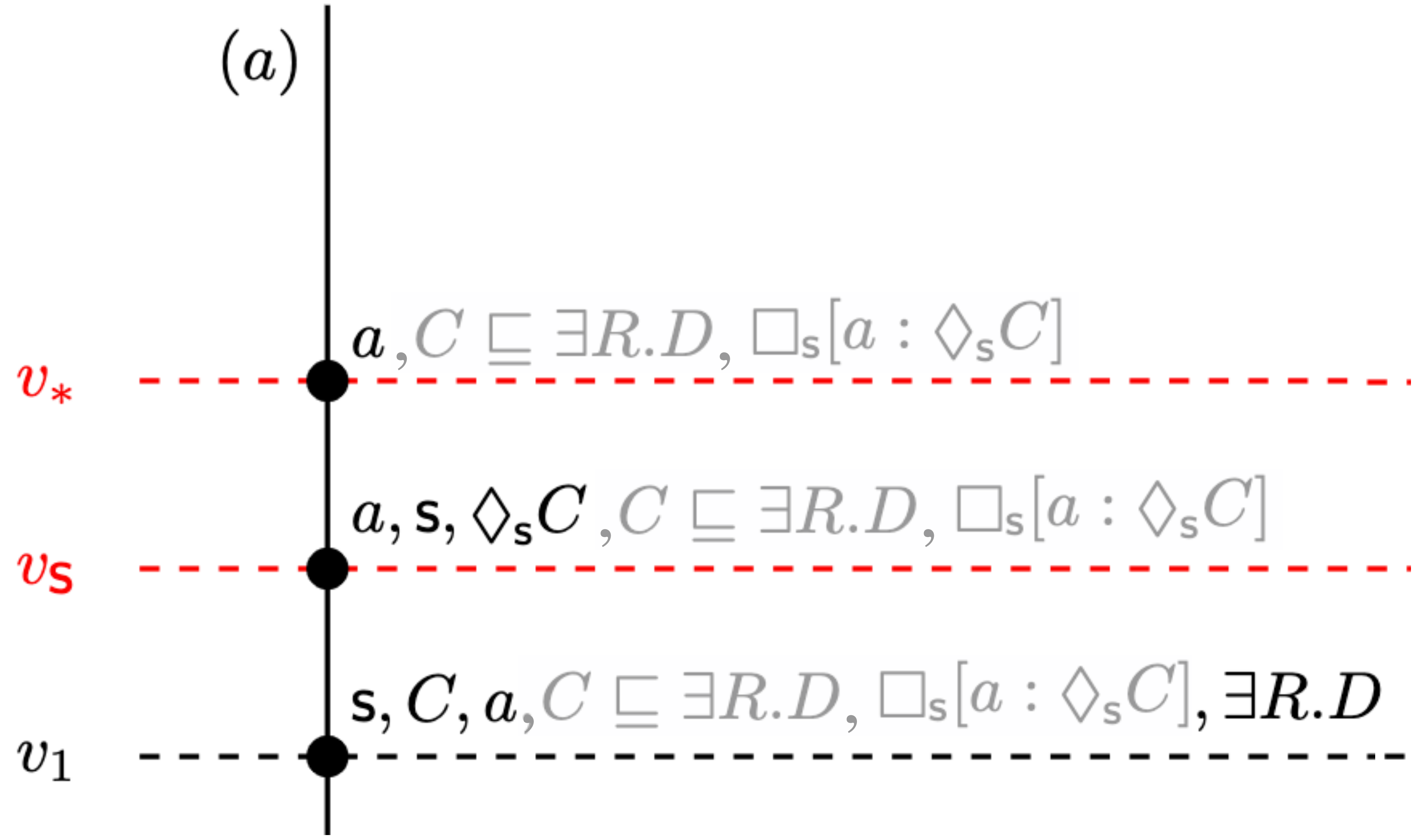

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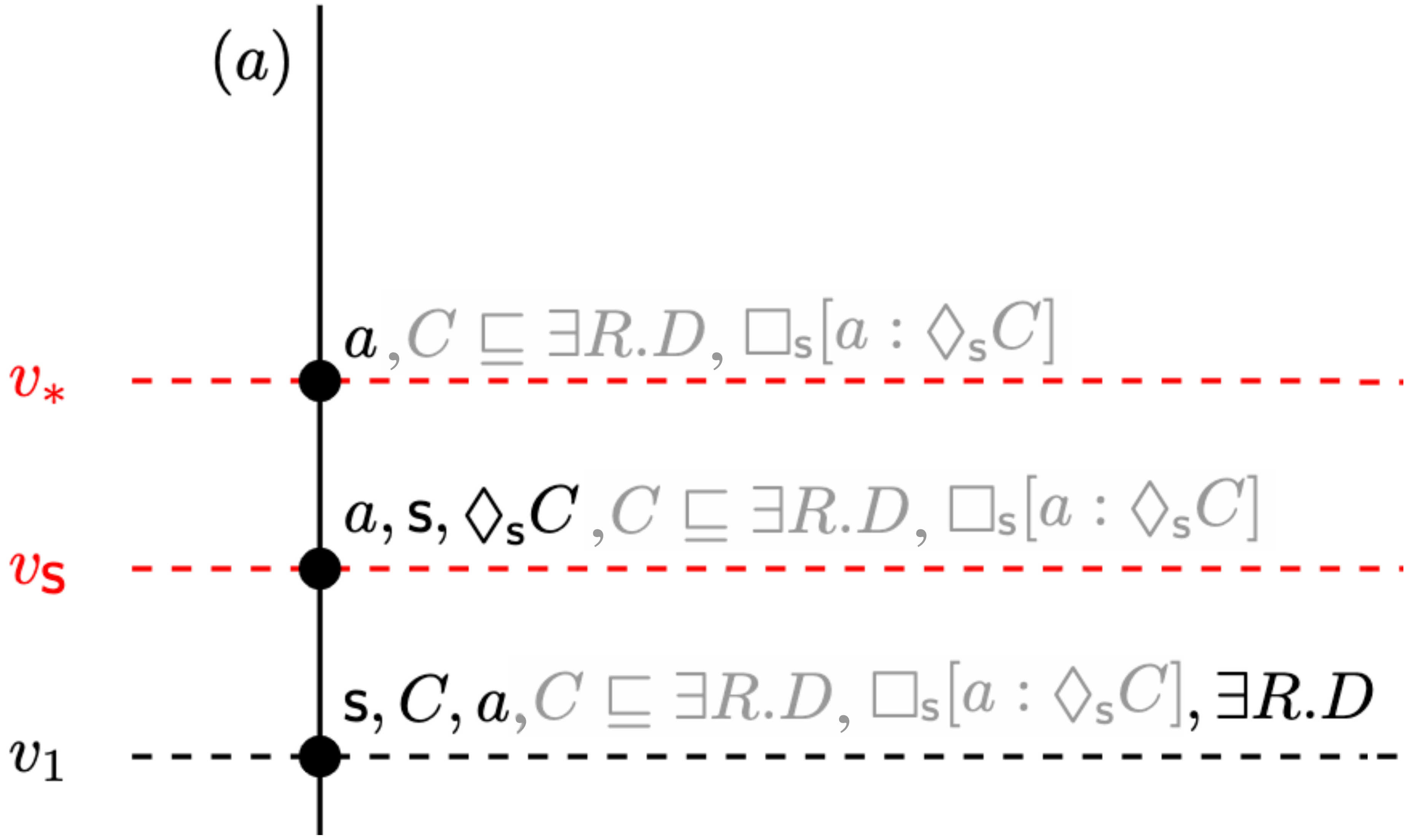

Tableau Algorithm for $\mathbb{S}_{\mathcal{EL}}$

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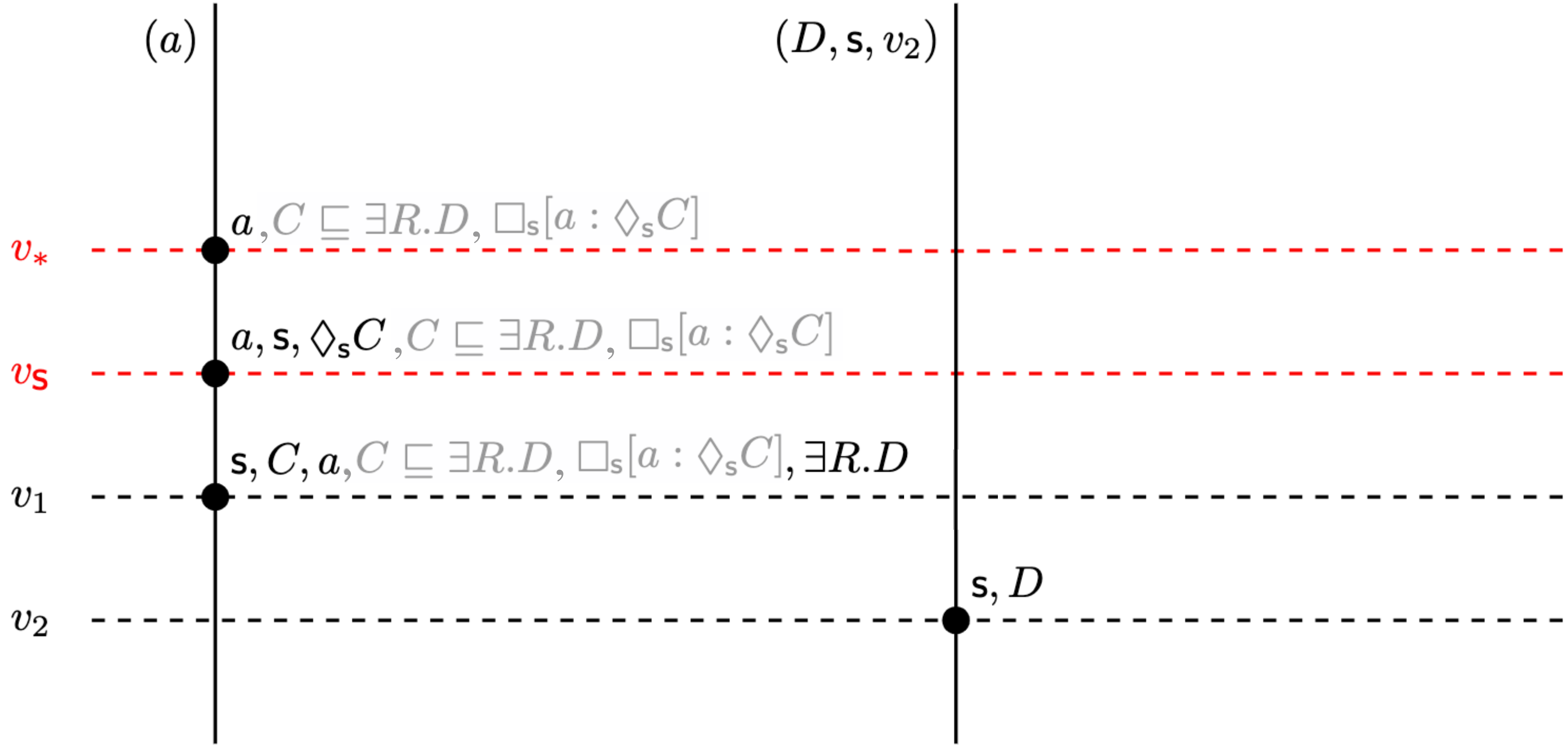

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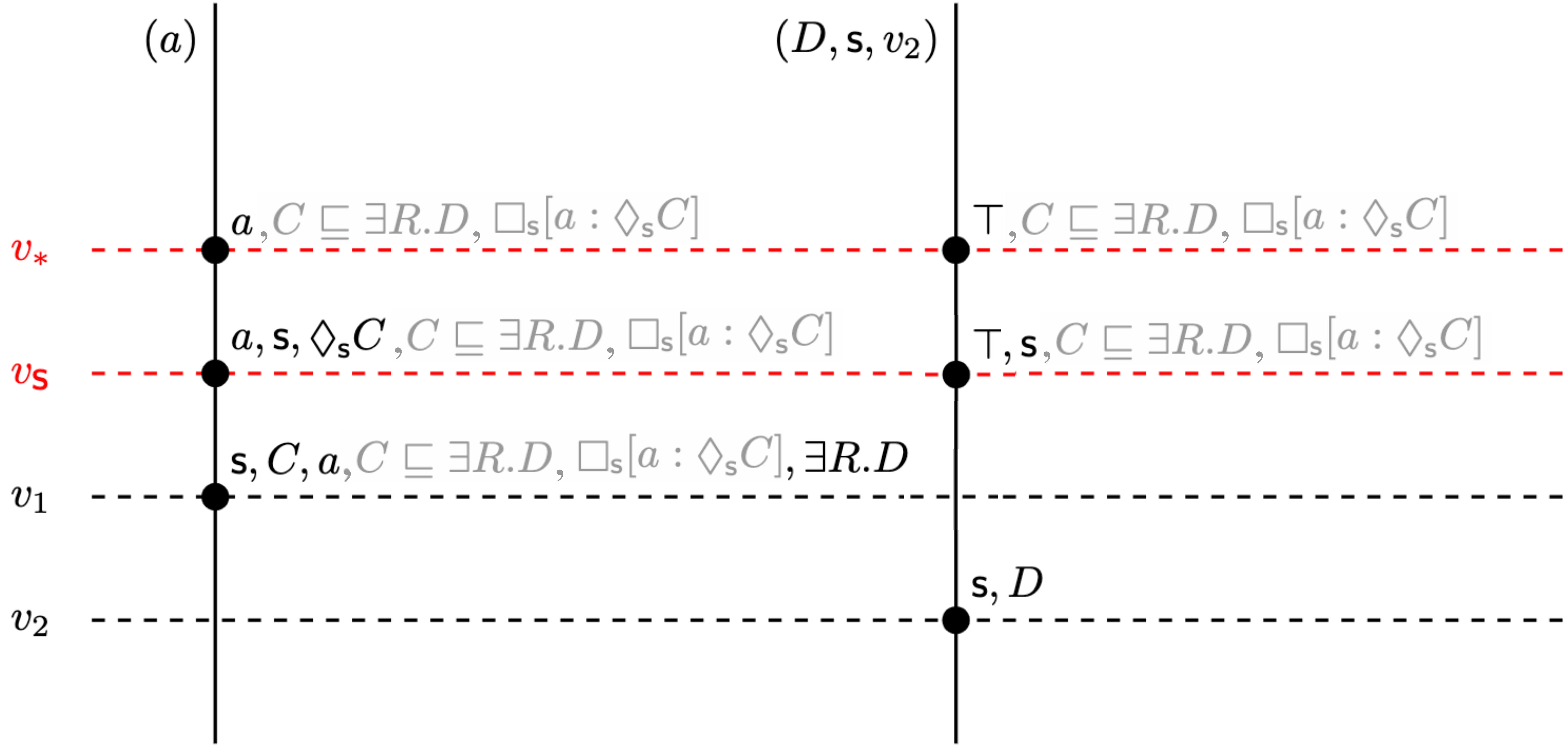

Tableau Algorithm for $\mathbb{S}_{\mathscr{E}\mathscr{L}}$

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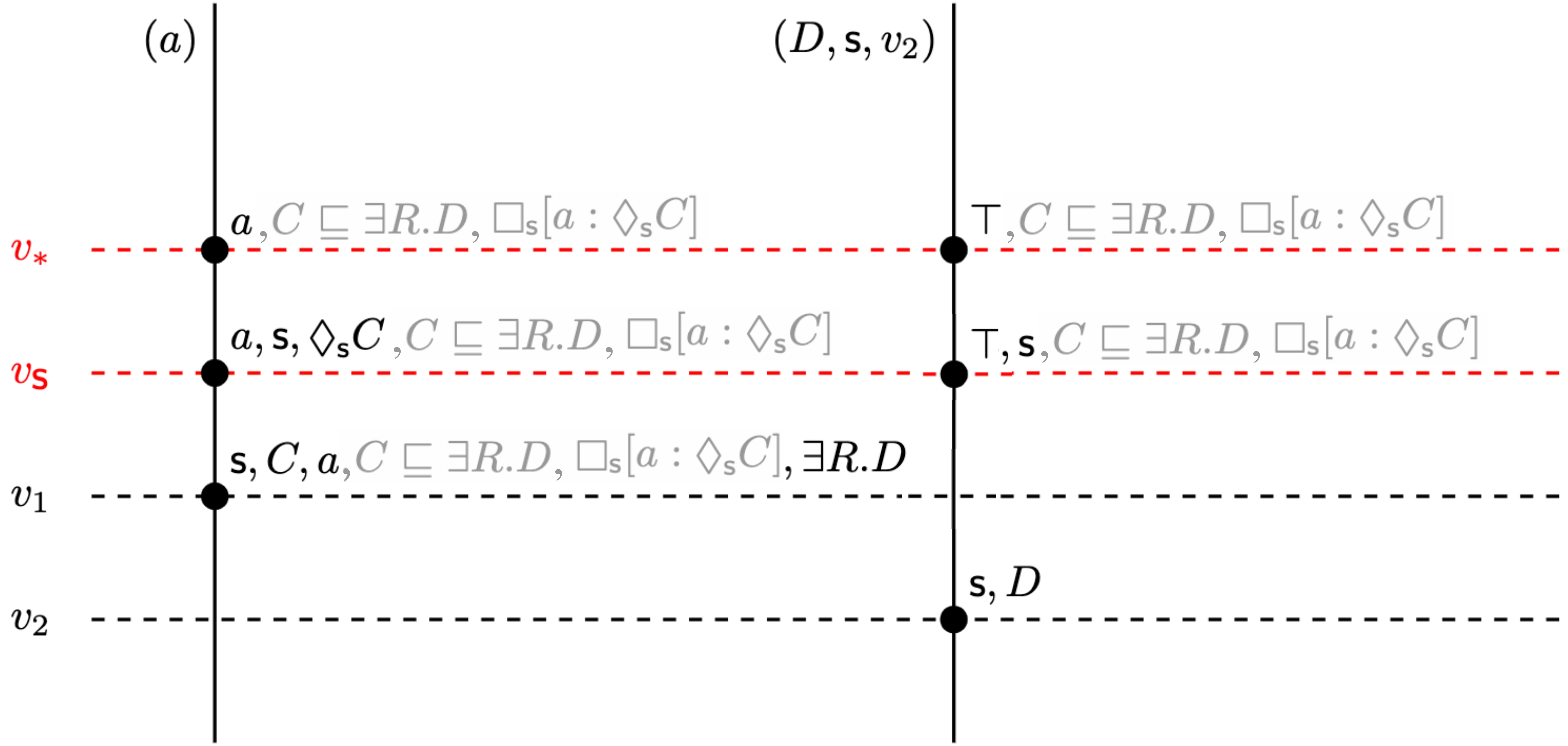

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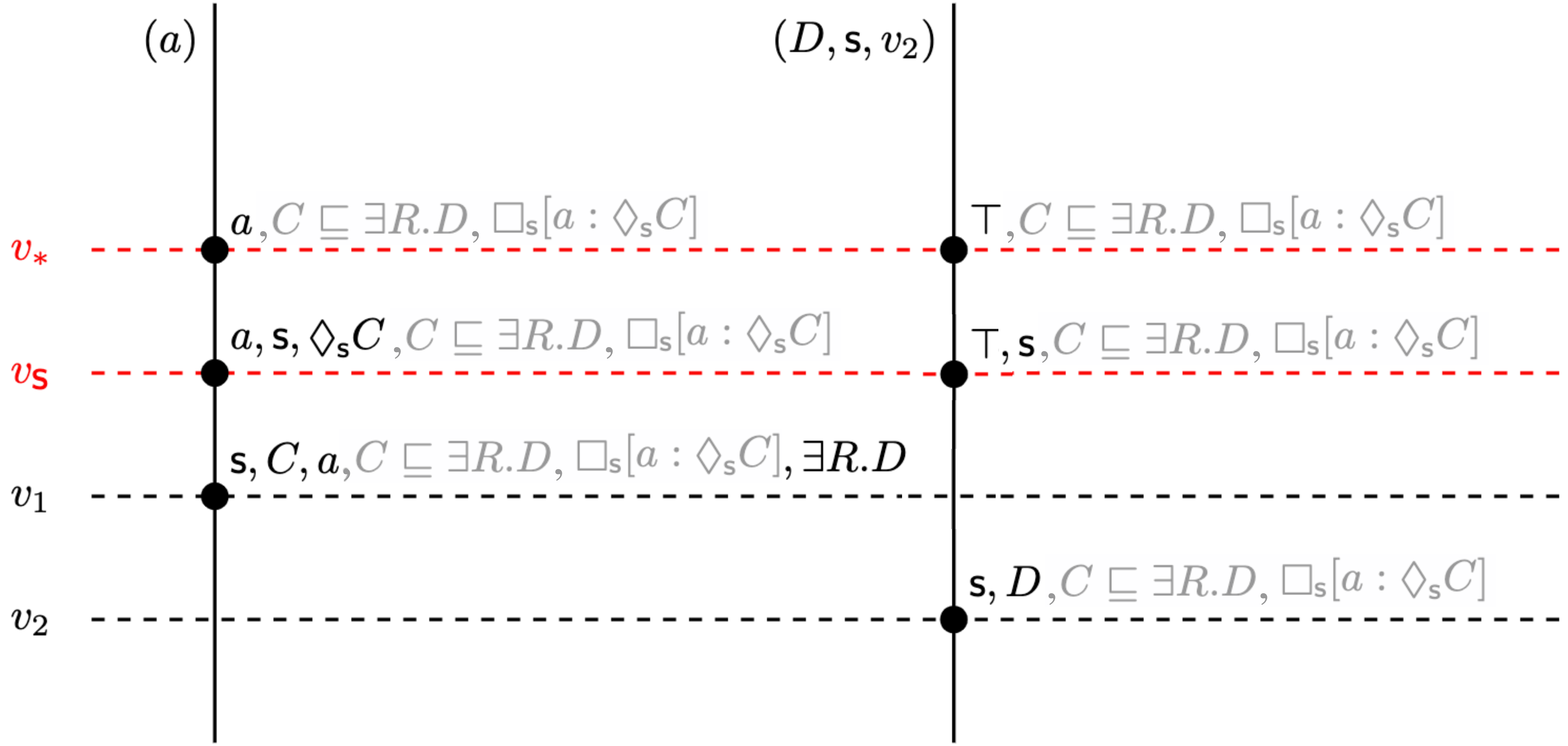

Models and Quasi-Models for $\mathbb{S}_{\mathscr{EL}}$

Example:

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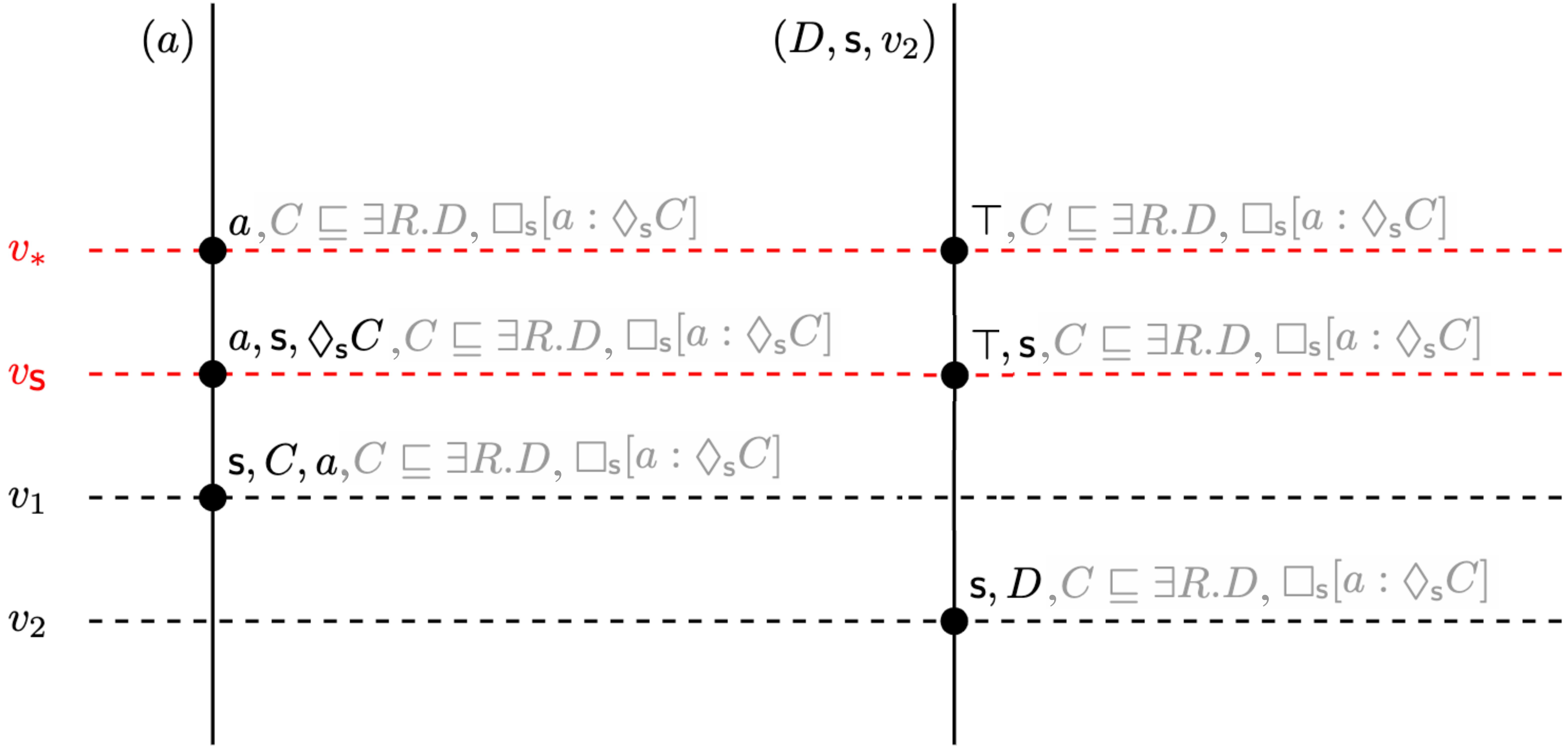
$\Box_s[a : \Diamond_s C]$



Models and Quasi-Models for $\mathbb{S}_{\mathscr{EL}}$

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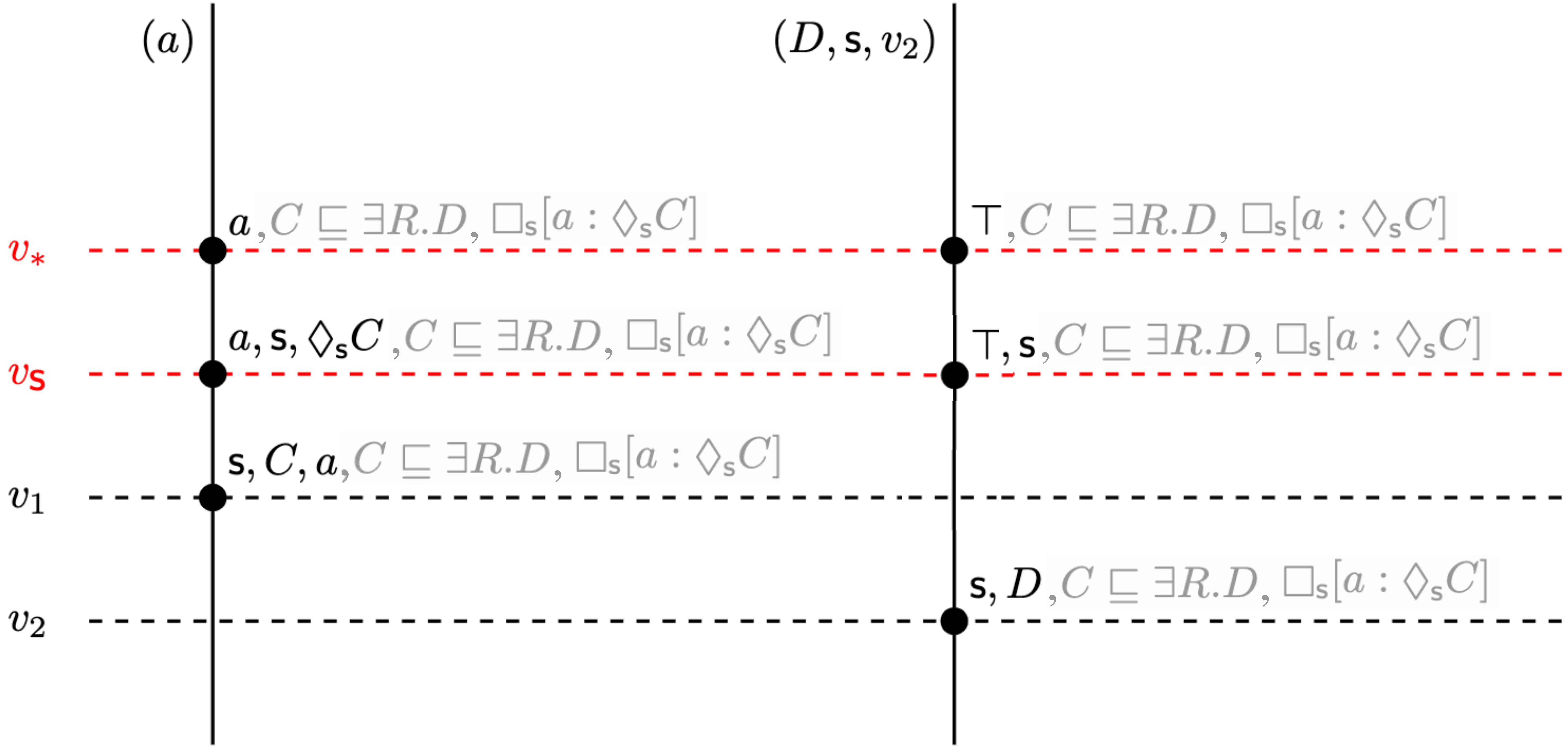
Quasi-model:



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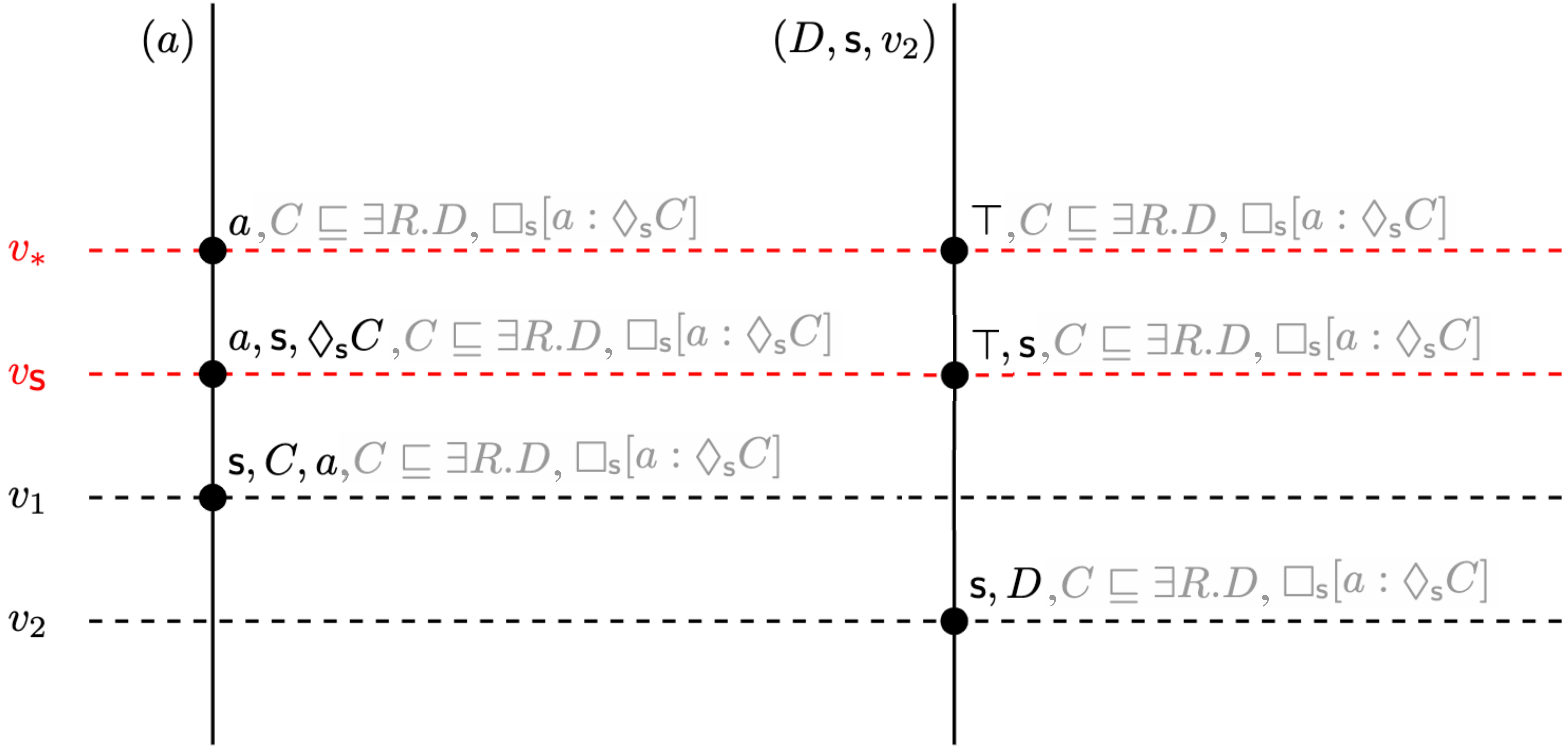

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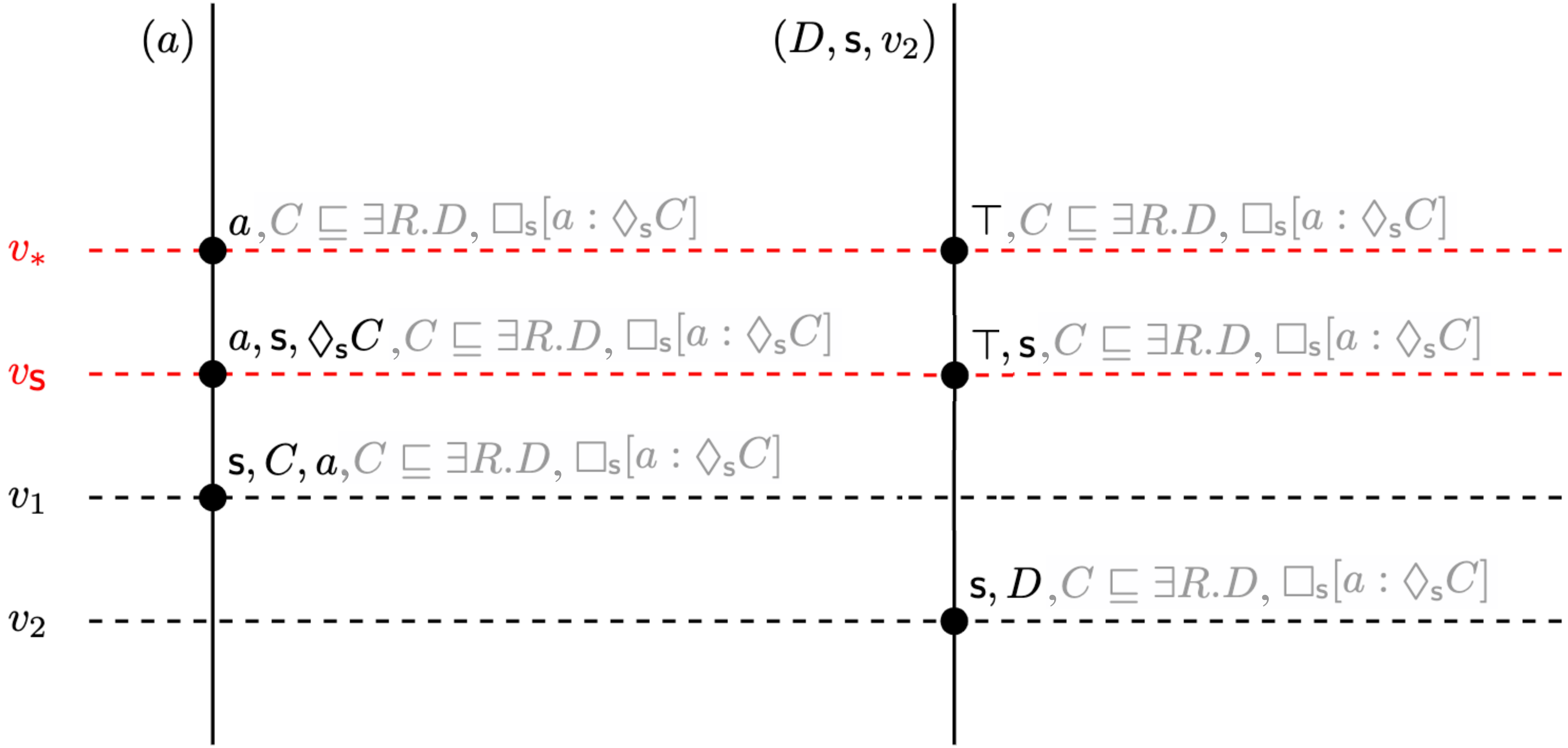

Model:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

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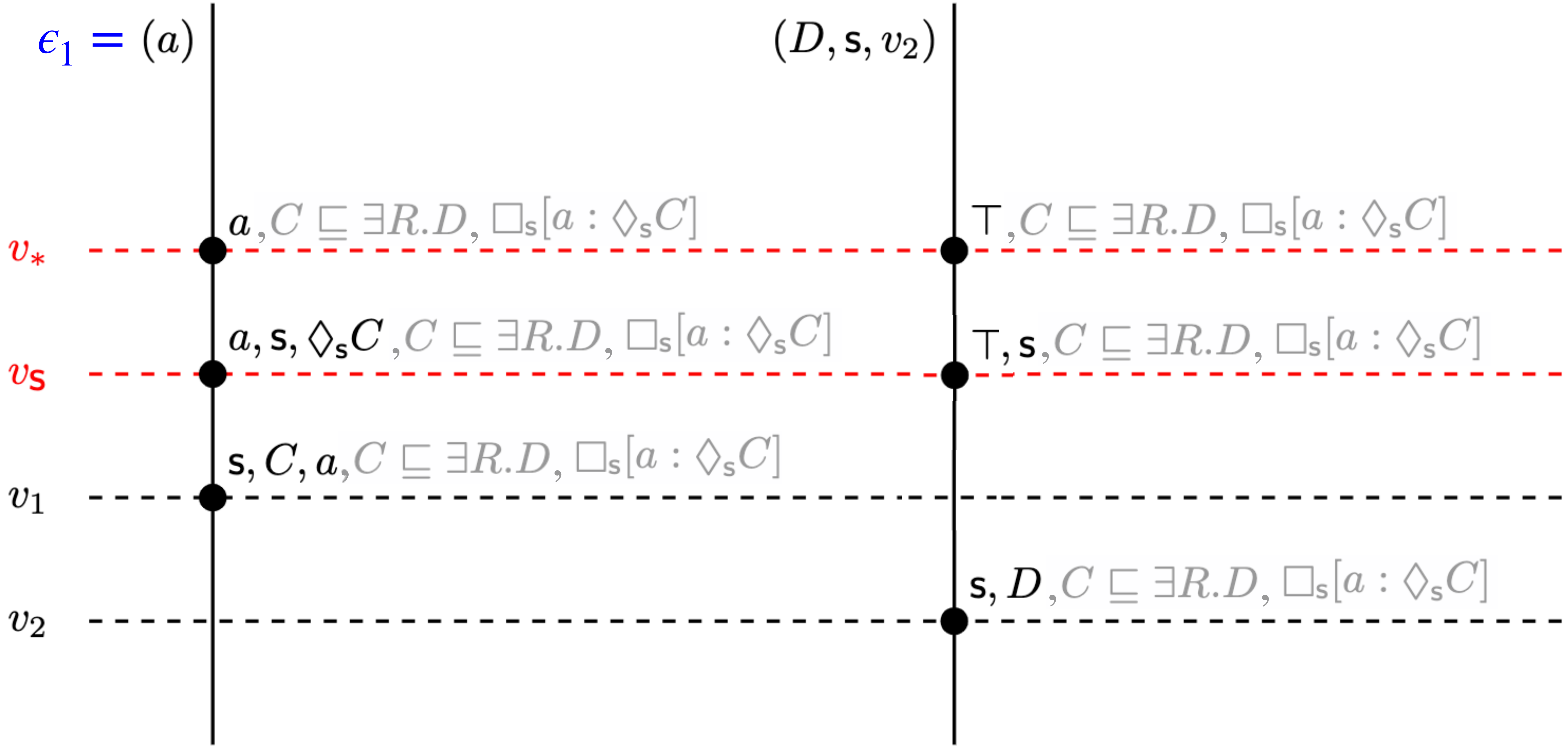

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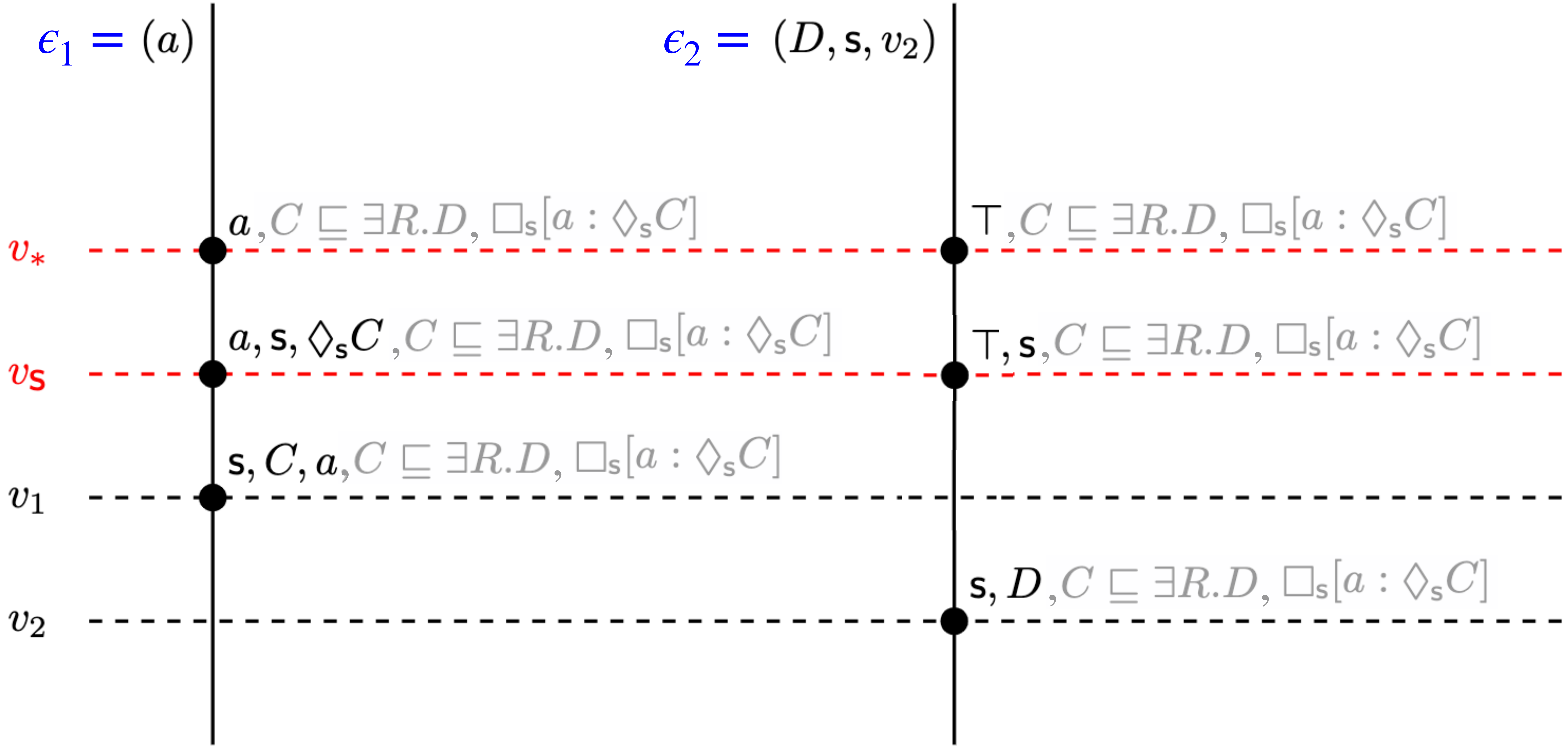

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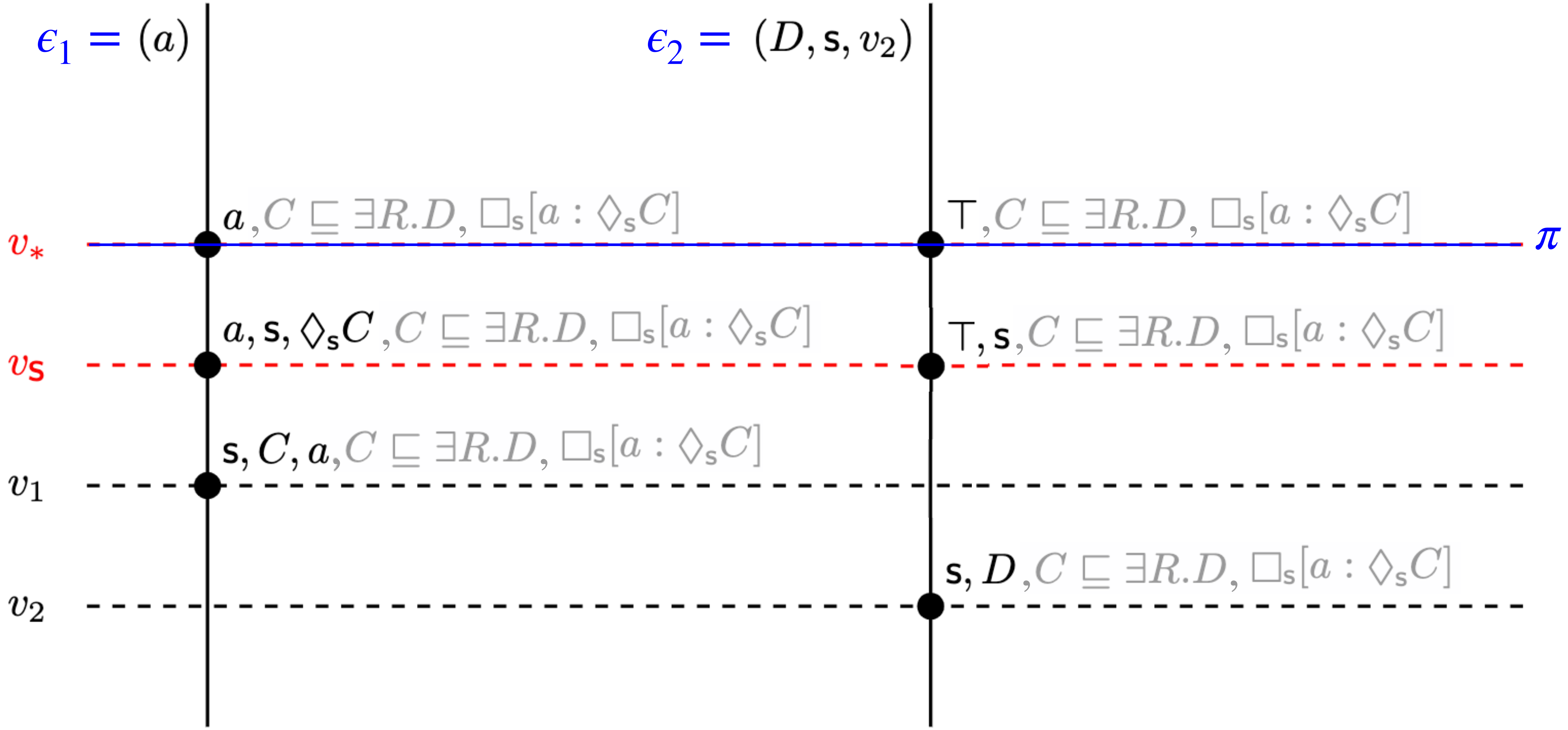

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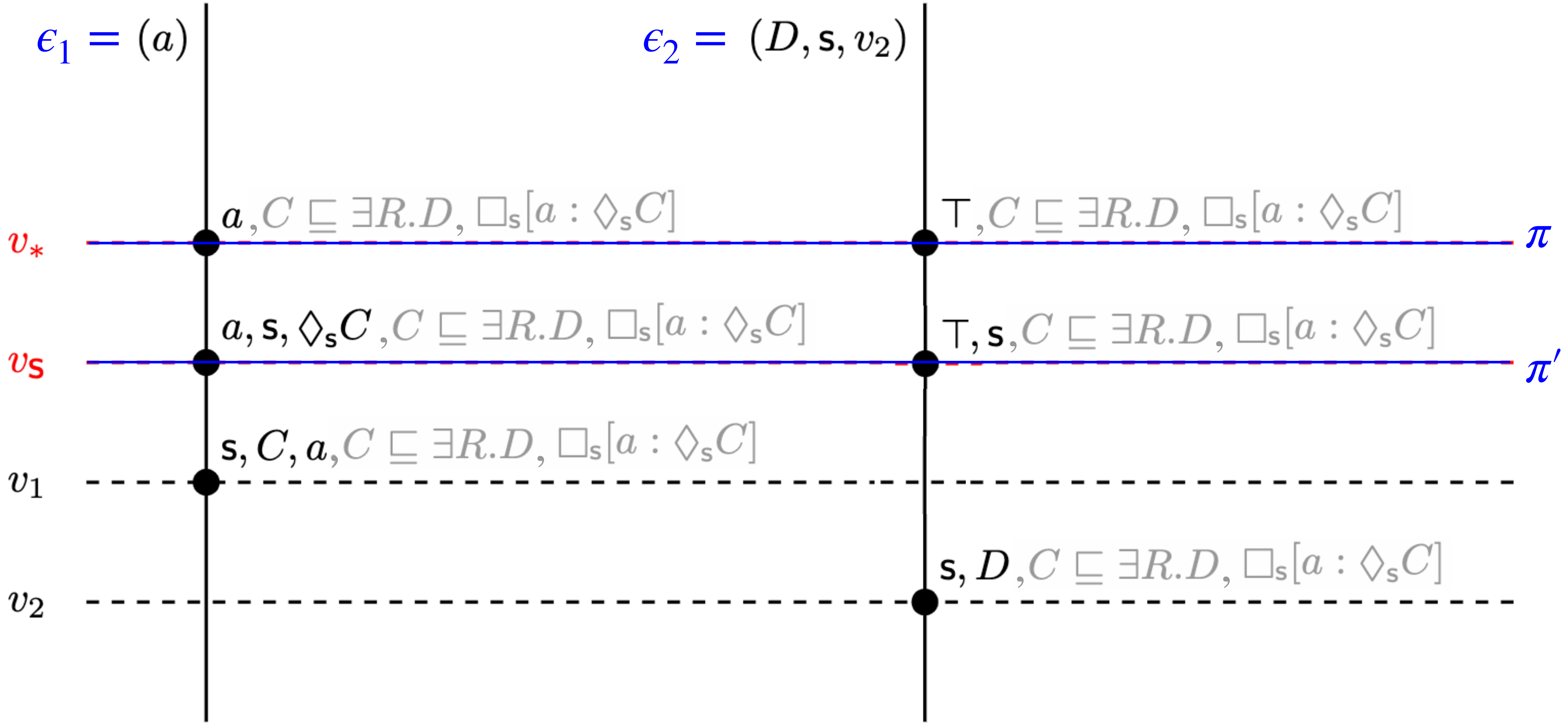

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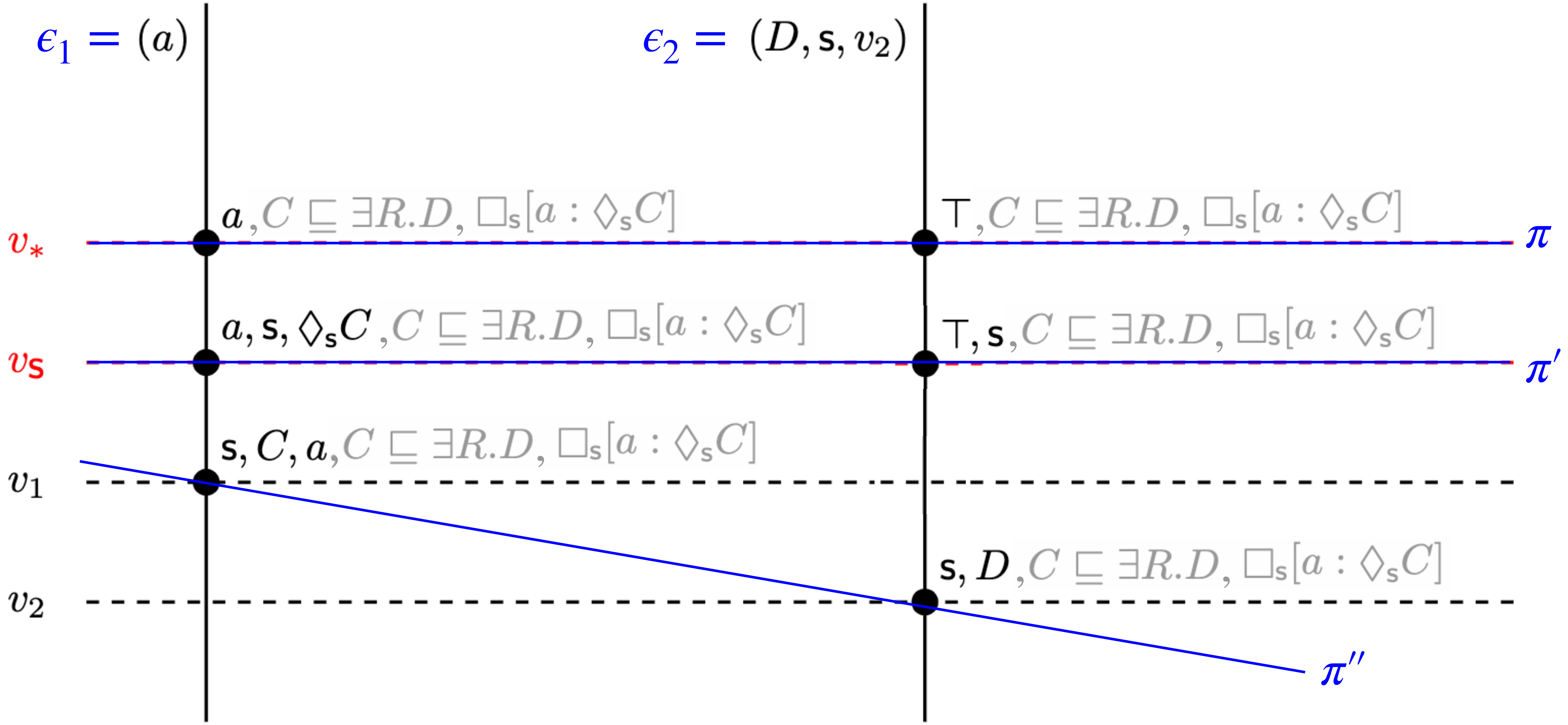

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Models and Quasi-Models for $\mathbb{S}_{\mathscr{EL}}$

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$\square_{\mathsf{s}}[a : \lozenge_{\mathsf{s}} C]$

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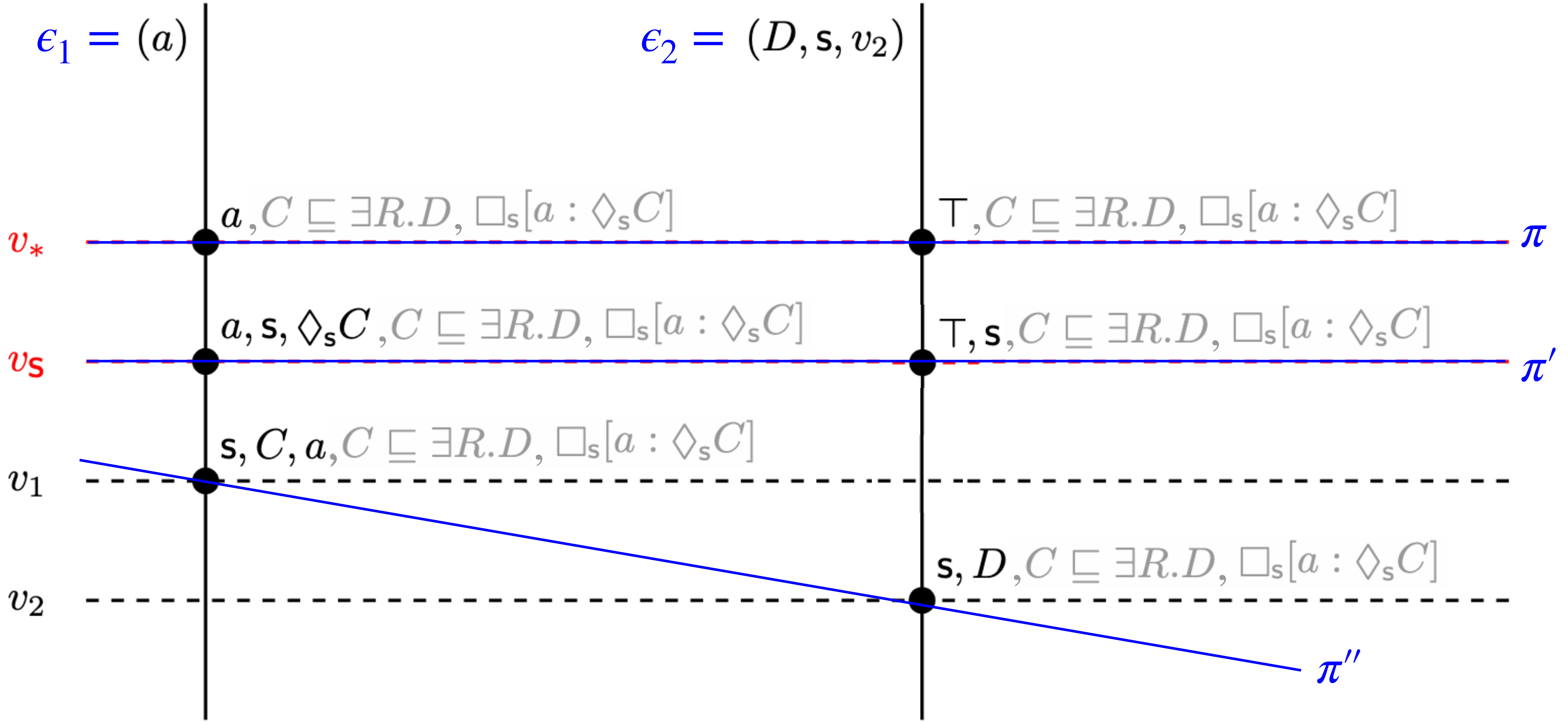

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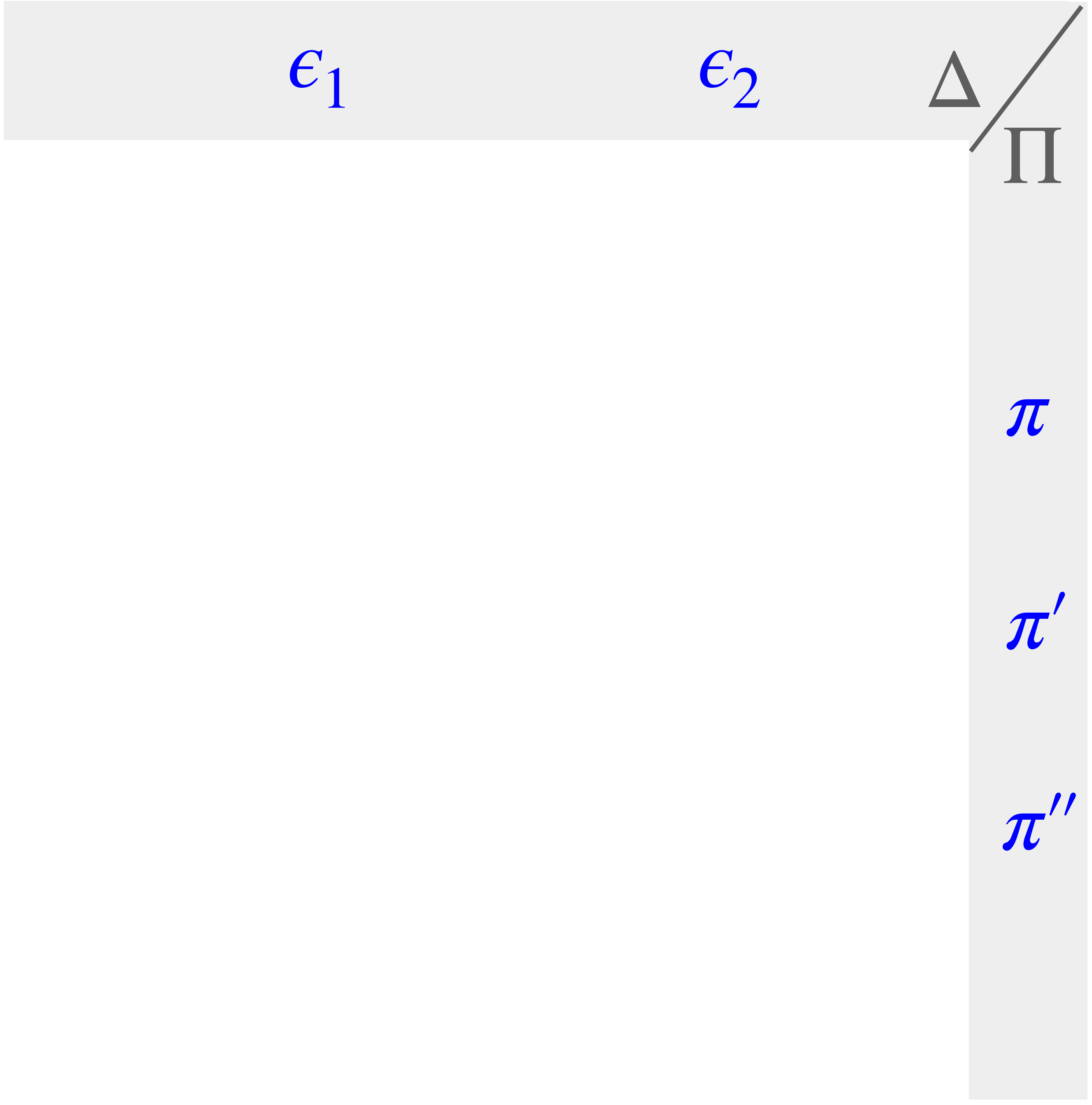

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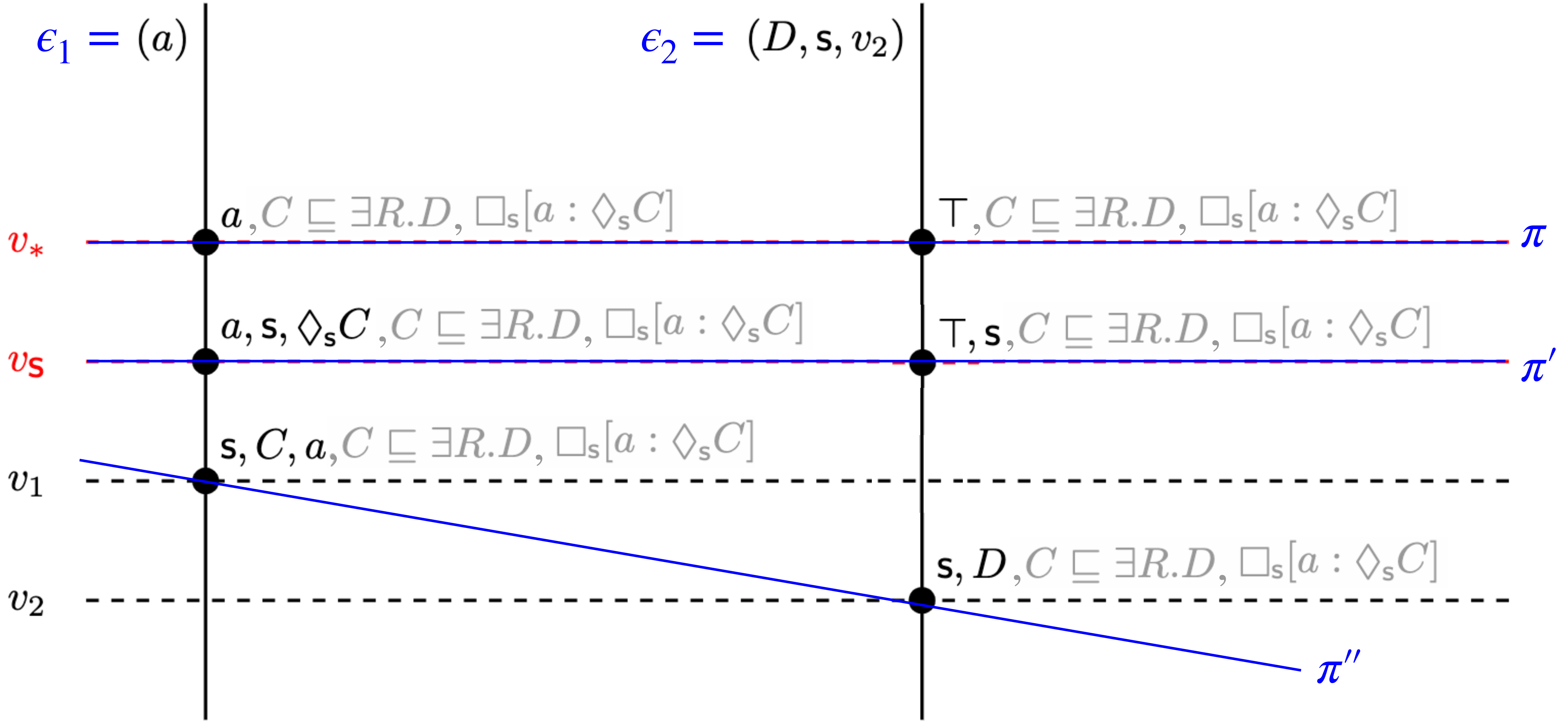

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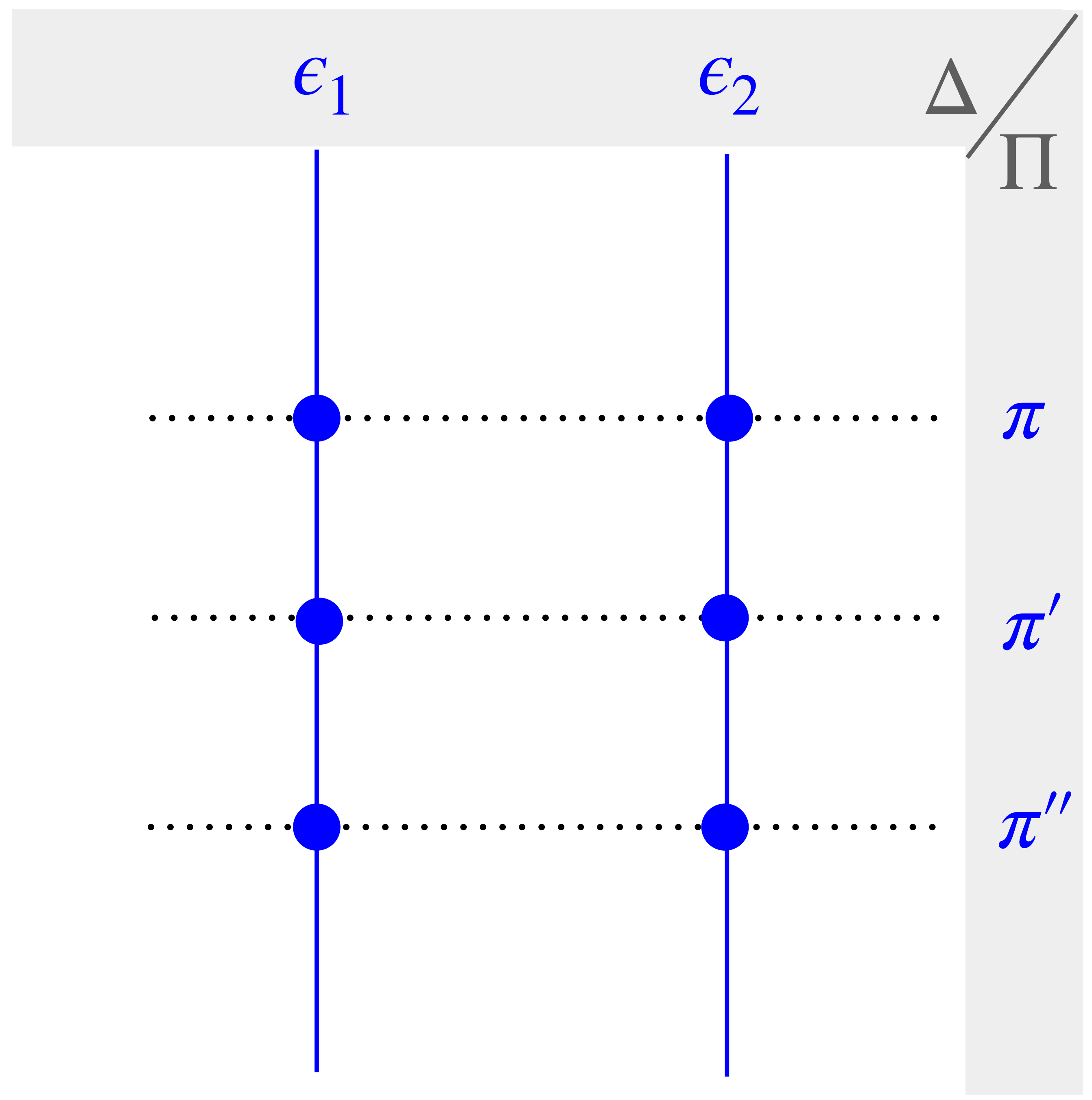

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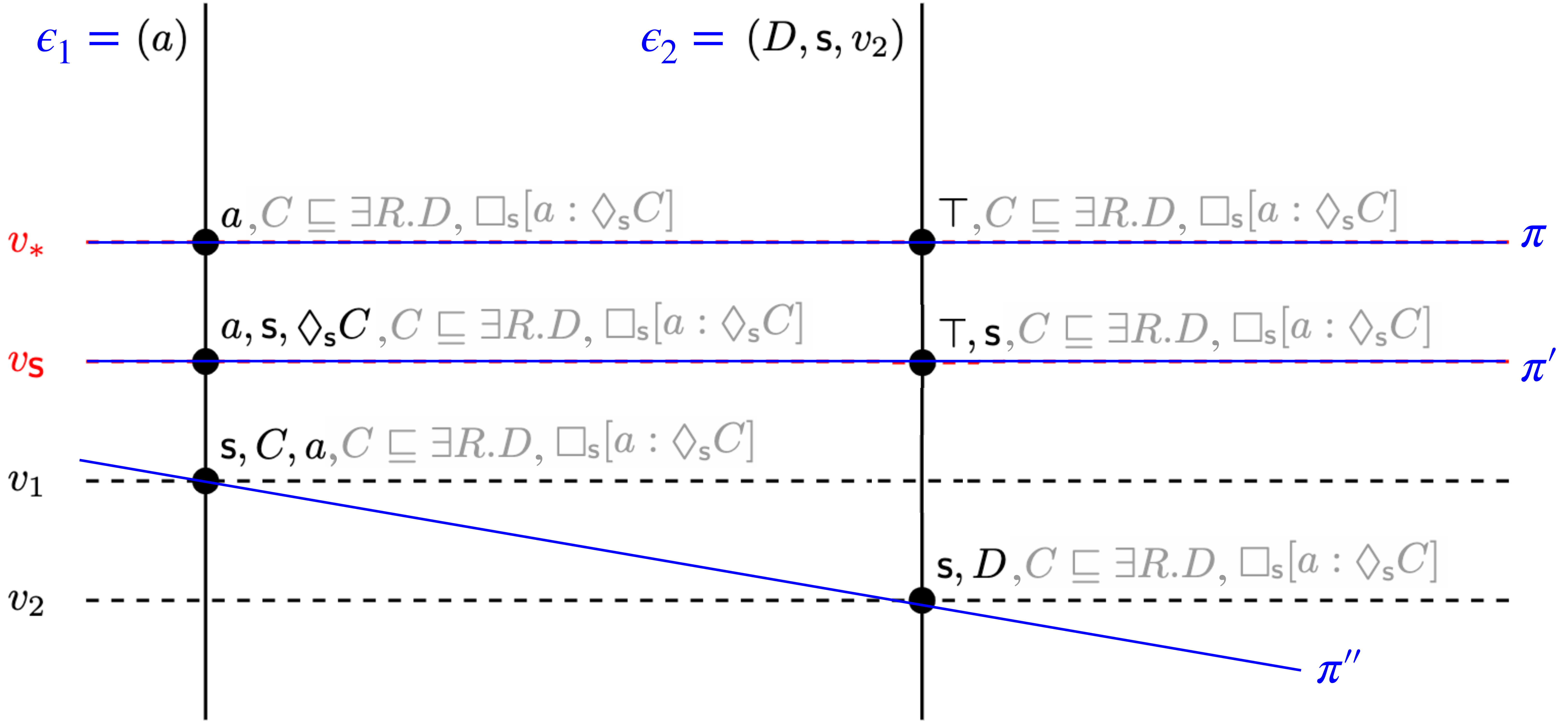

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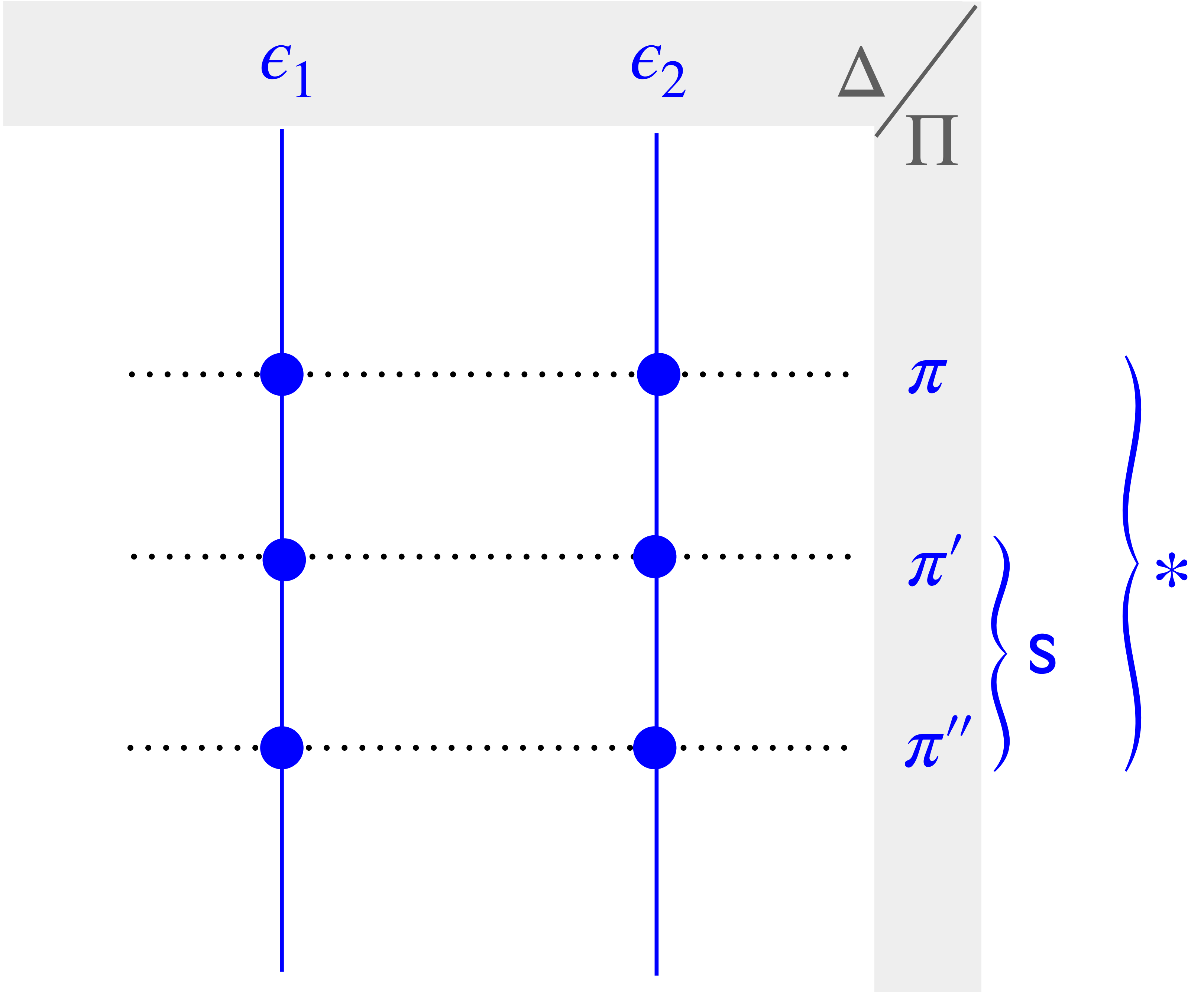

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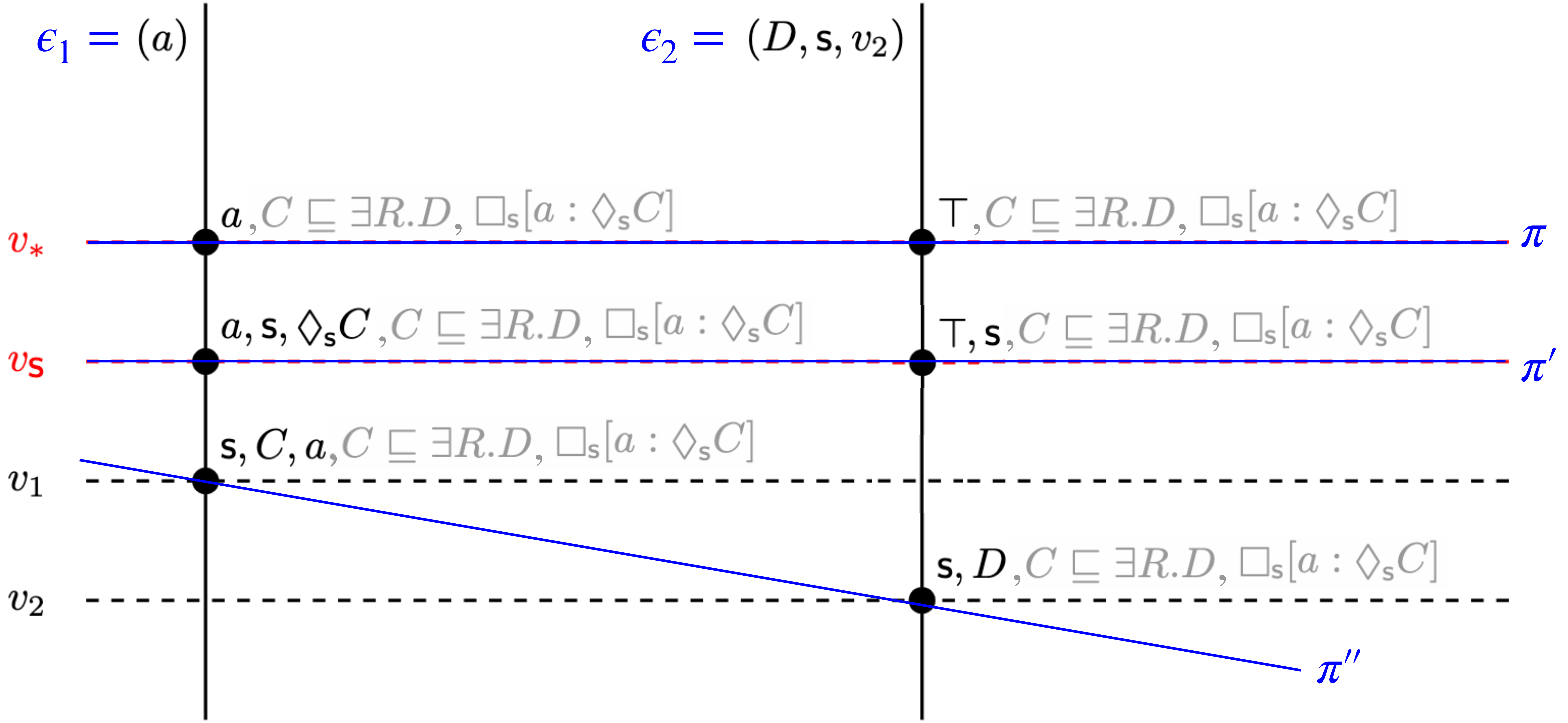

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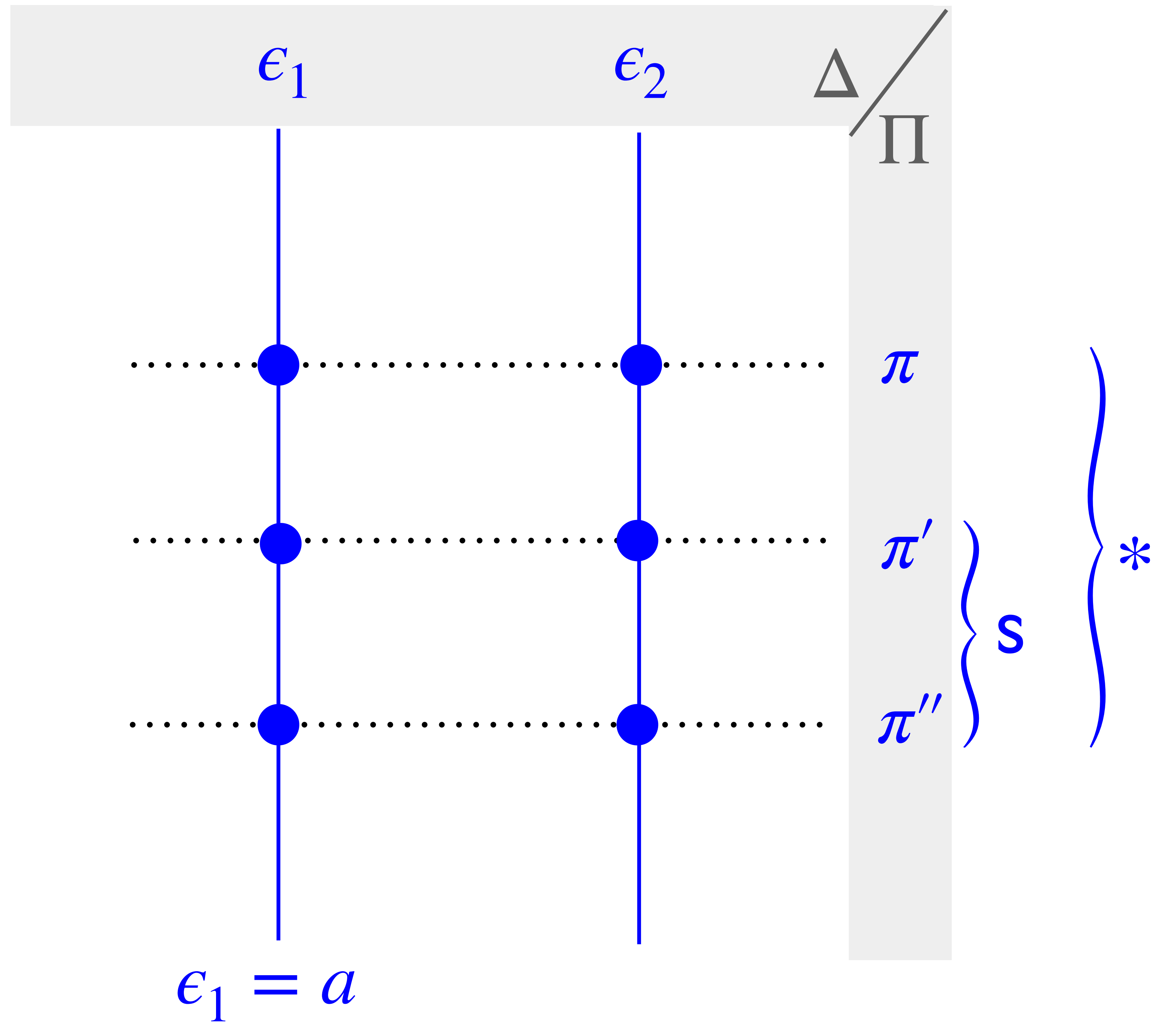


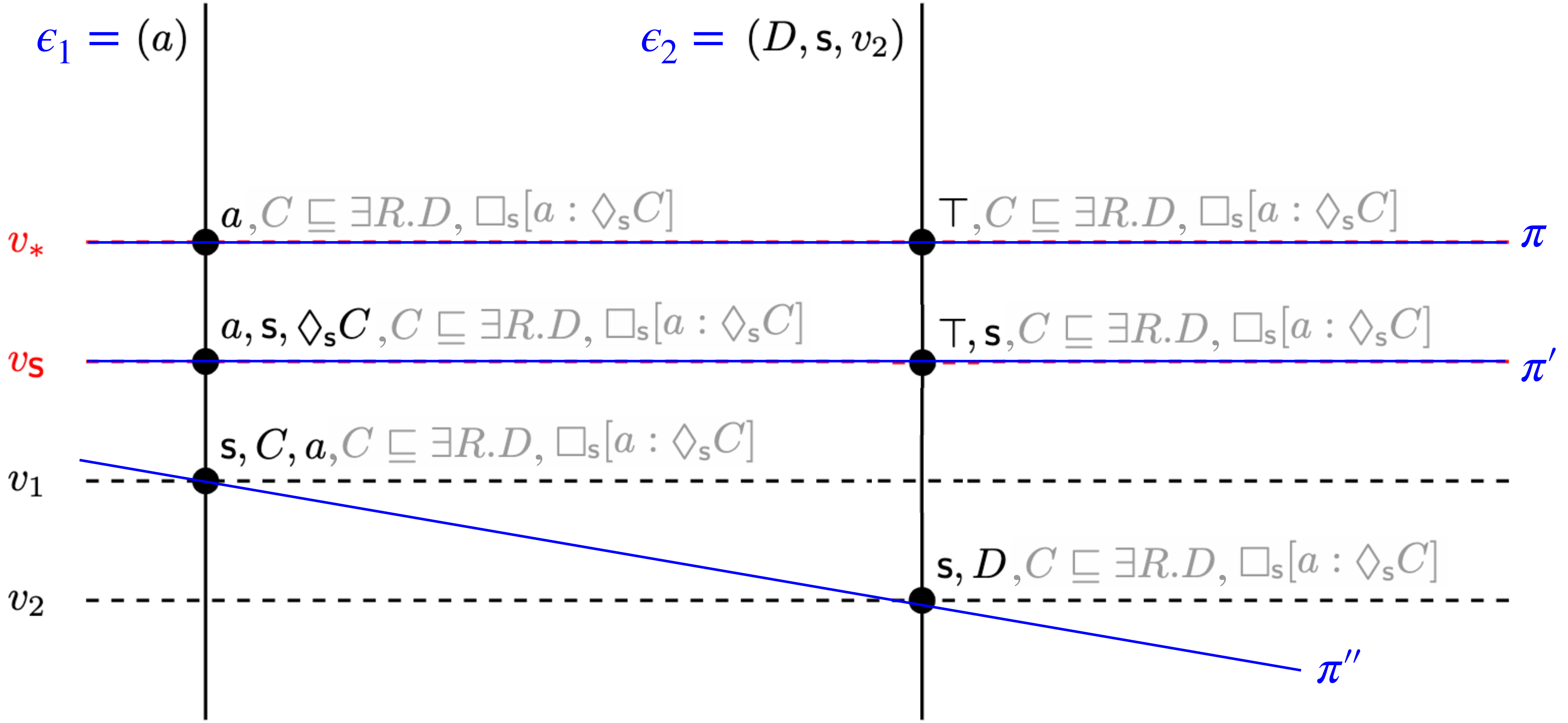
Models and Quasi-Models for $\mathbb{S}_{\mathscr{EL}}$

Example:

$C \sqsubseteq \exists R.D$

$\Box_{\mathsf{s}}[a : \Diamond_{\mathsf{s}} C]$

Quasi-model:



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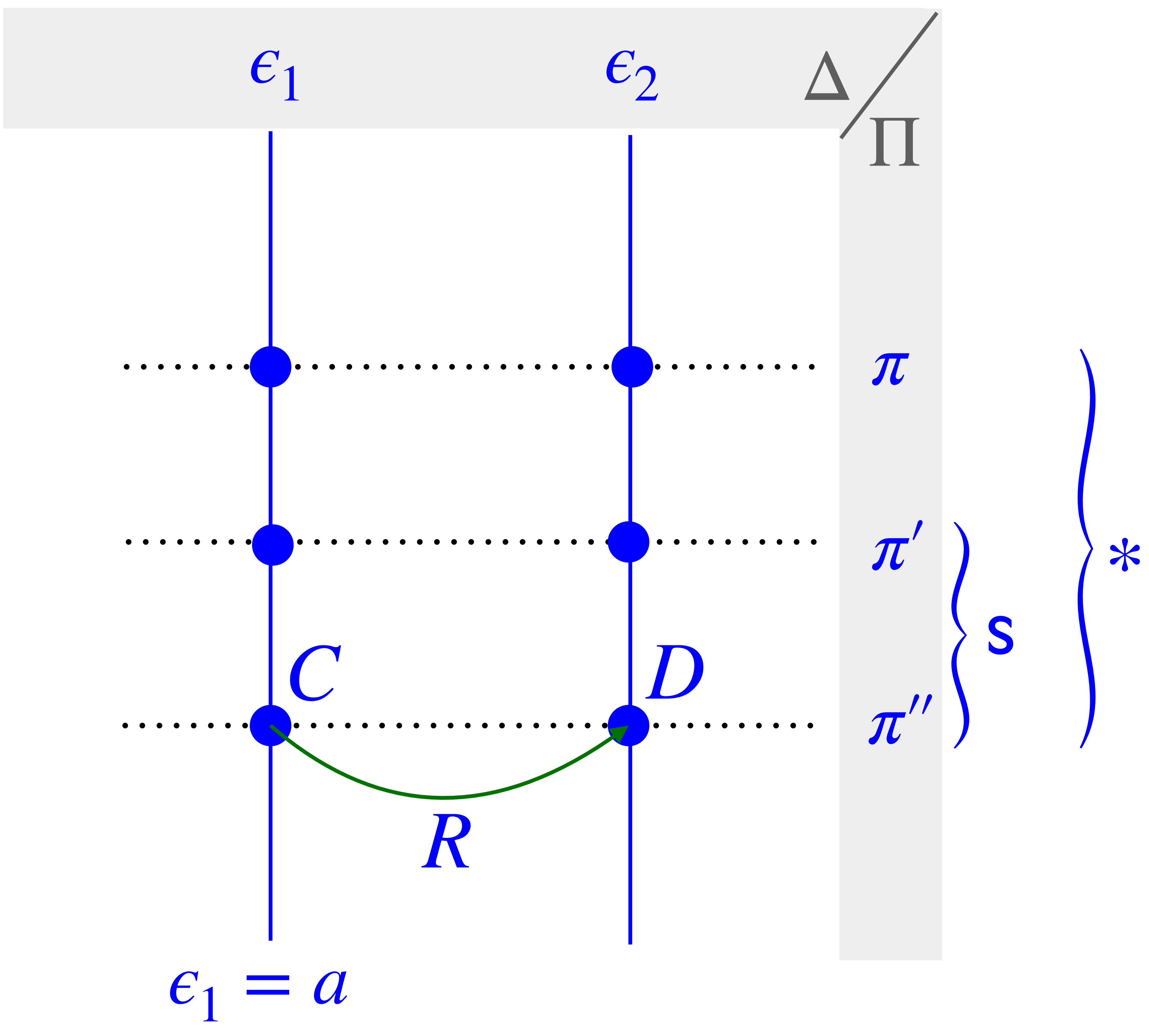

Implementations for $\mathbb{S}_{\mathscr{E}\mathscr{L}}$ (current work)

Implementations for $\mathbb{S}_{\mathcal{EL}}$ (current work)

A Datalog-Based Subsumption Calculus for $\mathbb{S}_{\mathcal{EL}}$

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<u>A Datalog-Based Subsumption Calculus for</u> $\mathbb{S}_{\mathscr{EL}}$

(in collaboration with Sebastian Rudolph and Hannes Strass)

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Future Work: OWL2 EL integration, generation of Standpoint Knowledge Bases

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Supporting LTL+Standpoints in the BLACK Satisfiability Checker

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Notice: Empty standpoints can be introduced at no cost

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- Implementations, tutorials and pedagogical resources
- Generalising results to “set-based” modal logics

The end.