A Journey to the Frontiers of Query Rewritability

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INTRO

The setting: Existential Rules and Boolean Conjunctive Queries

Existential Rules

Rules are first-order formulas of the form

$$\forall \vec{x}, \vec{y}. \phi(\vec{x}, \vec{y}) \rightarrow \exists \vec{z}. \psi(\vec{y}, \vec{z})$$

where ϕ and ψ are conjunctions of atoms.

Boolean Conjunctive Queries

A *Boolean conjunctive query* (BCQ) is a first-order formula of the form $\exists \vec{y}. \phi(\vec{y})$ where ϕ is a conjunction of atoms.

Universal Models and BCQ Entailment

Definition: Knowledge Base

A *knowledge base* is a pair $\langle\mathcal{R},\mathcal{D}\rangle$ with $\mathcal{R}$ a rule set and $\mathcal{D}$ a fact set.

Definition: Universal Model

A model of a KB is *universal* if it can be homomorphically embedded into every other model of this knowledge base.

A universal model $\mathcal{U}$ is the “least” model in terms of BCQ entialment:

Theorem: Universal Models and Query Entailment

Consider a KB $\mathcal{K}$ a a universal model $\mathcal{U}$ of $\mathcal{K}$. Then, $\mathcal{K}$ entails a BCQ q if and only if $\mathcal{U}$ satisfies q.

Remark

Every knowledge base admits a (possibly infinite) universal model.

Definition: BDD

A rule set $\mathcal{R}$ is *BDD* (or simply *FO-rewritable*) if, for every CQ $\mathcal{Q}$, there is a union of BCQs (UCQ) $\mathcal{Q}'$ such that:

$$\langle \mathcal{R}, \mathcal{D} \rangle \models \mathcal{Q} \iff \mathcal{D} \models \mathcal{Q}'$$

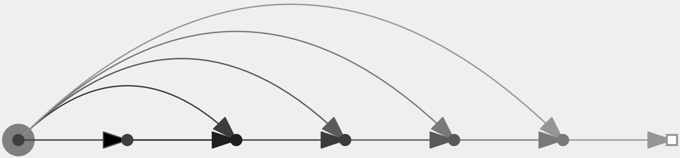
for every database $\mathcal{D}$.

Remark

In the above, BDD stands for “bounded derivation depth”.

The Chase Algorithm

$$E(x, y) \to \exists z. E(y, z)$$
$$A(x) \wedge E(x, y) \wedge E(y, z) \to E(x, z)$$



Alternative Definition: BDD

A ruleset $\mathcal{R}$ is BDD if, for every BCQ $\mathcal{Q}$, there is some $k \geq 0$ such that

$$\langle \mathcal{R}, \mathcal{D} \rangle \models \mathcal{Q} \iff \mathit{Chase}_k(\mathcal{R}, \mathcal{D})$$

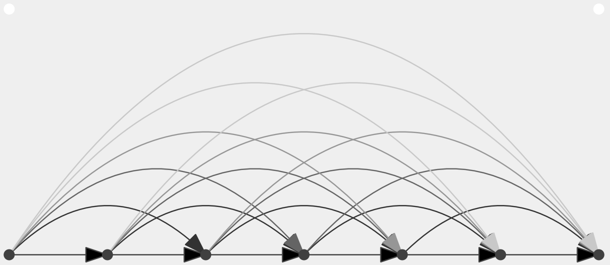
for every fact set $\mathcal{D}$.

Remark

In the above, $\mathit{Chase}_k(\mathcal{R}, \mathcal{D})$ is the k-th step of the parallel chase.

Transitivity is not BDD

$$E(x, y) \wedge E(y, z) \rightarrow E(x, z)$$



Remark

Note that $\exists \mathcal{Q} \,.\, \forall k \,.\, \exists \mathcal{D} \,.\, \langle \mathcal{D}, \mathcal{R} \rangle \models \mathcal{Q} \iff \mathit{Chase}_k(\mathcal{R}, \mathcal{D}) \models \mathcal{Q}$

Intersecting BDD with FES

Conjecture

$$BDD \cap FES = Uniform\ BDD$$

$$\begin{array}{rl} (BDD) & \forall \mathcal{Q} \,.\, \exists \boldsymbol{k} \,.\, \forall \mathcal{D} \,.\, \langle \mathcal{D}, \mathcal{R} \rangle \models \mathcal{Q} \iff Chase_k(\mathcal{R}, \mathcal{D}) \models \mathcal{Q} \\ (FES) & \forall \mathcal{D} \,.\, \exists \boldsymbol{k} \,.\, \forall \mathcal{Q} \quad \langle \mathcal{D}, \mathcal{R} \rangle \models \mathcal{Q} \iff Chase_k(\mathcal{R}, \mathcal{D}) \models \mathcal{Q} \\ (UBDD) & \exists \boldsymbol{k} \,.\, \forall \mathcal{Q} \,.\, \forall \mathcal{D} \quad \langle \mathcal{D}, \mathcal{R} \rangle \models \mathcal{Q} \iff Chase_k(\mathcal{R}, \mathcal{D}) \models \mathcal{Q} \end{array}$$

Definition

A rule set $\mathcal{R}$ is *local* if every atom of the chase can be derived only from a constant number of atoms of the database.

Note that local rule sets are BDD/FUS.

Theorem

Every BDD single-head rule set over a binary signature is local.

Theorem

If a rule set is FES and local, then it is UBDD.

Are there any BDD rule sets that are not local?

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Indeed! Let’s see one that is “extremely” non-local.