Standpoint Logic:

Multi-Perspective Knowledge Representation

Lucía Gómez Álvarez

Presented at Université de Montpellier, 2 Jan 2022

International Center for Computational Logic

TECHNISCHE UNIVERSITÄT DRESDEN

Outline

Motivation

The semantic variability of natural language

A long known problem:

- Most natural language terms do not have *precise universally agreed definitions* that fix their meanings.
- Words are used in a variety of ways that adapt to different *contexts and points of view*.

The semantic variability of natural language

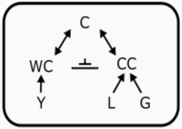
A long known problem:

- Most natural language terms do not have *precise universally agreed definitions* that fix their meanings.
- Words are used in a variety of ways that adapt to different *contexts and points of view*.

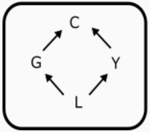
Example

It is globally accepted (thus also by *CT* and *HP*) that ***`Yellow`***, ***`Green`***, and ***`Lime`*** are ***`Colour`***. According to *CT*, ***`Colour`*** is either a warm colour (***WC***) or a cold colour (***CC***) but not both, ***`Yellow`*** is a ***WC***, and ***`Green`*** and ***`Lime`*** are ***CC***. According to *HP*, ***`Lime`*** is ***`Green`*** and ***`Yellow`***.

s_{CT}



s_{HP}



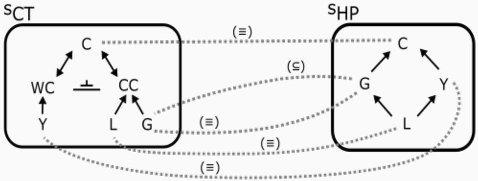
The semantic variability of natural language

A long known problem:

- Most natural language terms do not have *precise universally agreed definitions* that fix their meanings.
- Words are used in a variety of ways that adapt to different *contexts and points of view*.

Example

It is globally accepted (thus also by *CT* and *HP*) that ***Yellow***, ***Green***, and ***Lime*** are ***Colour***. According to *CT*, ***Colour*** is either a warm colour (***WC***) or a cold colour (***CC***) but not both, ***Yellow*** is a ***WC***, and ***Green*** and ***Lime*** are ***CC***. According to *HP*, ***Lime*** is ***Green*** and ***Yellow***.



The semantic variability of natural language

Main strategies:

- Representation of a unified view. E.g. Ontology merging.

The semantic variability of natural language

Main strategies:

- Representation of a unified view. E.g. Ontology merging.
- Representation of the diversity $\longrightarrow$ Standpoint Logic.

The semantic variability of natural language

Main strategies:

- Representation of a unified view. E.g. Ontology merging.

- Representation of the diversity $\longrightarrow$ Standpoint Logic.
  - Represent *standpoints* on the interpretation of a domain.
  - Express *relations* and *hierarchies* between different systems of interpretation.
  - Represent both *precise and borderline facts* at global or local levels.
  - Perform global and local *inferences and consistency checks*.

Overview of the Standpoint Framework

Modal logics are a family of non-classical logics that extend the classical (propositional or first-order) logic with dual modal operators, typically $\Box$ and $\Diamond$.

Language of propositional ML: logical connectives $\neg, \wedge$, a unary modal operator $\Box$, and a set of atomic propositions $p_0, p_1, \ldots$. Formulae:

$$\varphi \ ::= \ p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box\,\varphi.$$

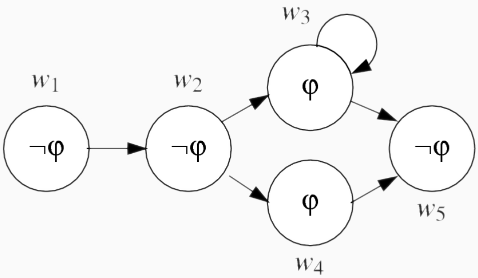
The dual operator : $\Diamond\,\varphi ::= \neg\,\Box\,\neg\varphi$

E.g. Alethic logics, epistemic logics, temporal logics, spatial logics, ...

Kripke semantics

A Kripke model is a tuple $\mathcal{M} = \langle W, R, V \rangle$ where:

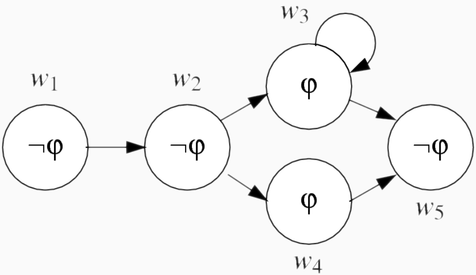
1. W is a set of possible worlds.
2. R is a binary relation on W.
3. V is a valuation function which assigns a truth value to each pair of a world and an atomic formula (i.e. $V : W \times F \to \{0, 1\}$ where F is the set of atomic formulae)



Kripke semantics

The satisfaction function defined as:

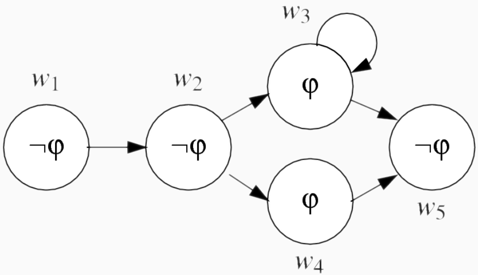
1. $\mathcal{M}, w \models p$ iff $V(w, p) = t$
2. $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
3. $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
4. $\mathcal{M}, w \models \Box\varphi$ iff for every w' of W, if wRw' then $\mathcal{M}, w' \models \varphi$



Kripke semantics

The satisfaction function defined as:

1. $\mathcal{M}, w \models p$ iff $V(w, p) = t$
2. $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
3. $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
4. $\mathcal{M}, w \models \Box\varphi$ iff for every w' of W, if wRw' then $\mathcal{M}, w' \models \varphi$

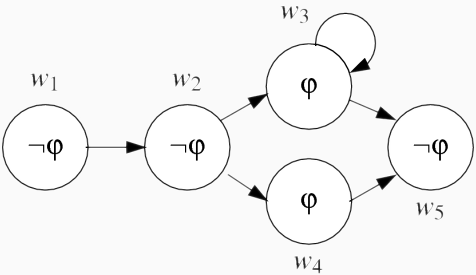


- $\mathcal{M}, w_1 \models \Box\neg\varphi$

Kripke semantics

The satisfaction function defined as:

1. $\mathcal{M}, w \models p$ iff $V(w, p) = t$
2. $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
3. $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
4. $\mathcal{M}, w \models \Box\varphi$ iff for every w' of W, if wRw' then $\mathcal{M}, w' \models \varphi$

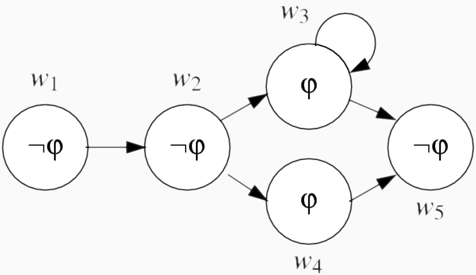


- $\mathcal{M}, w_1 \models \Box\neg\varphi$, $\mathcal{M}, w_2 \models \Box\varphi$

Kripke semantics

The satisfaction function defined as:

1. $\mathcal{M}, w \models p$ iff $V(w, p) = t$
2. $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
3. $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
4. $\mathcal{M}, w \models \Box\varphi$ iff for every w' of W, if wRw' then $\mathcal{M}, w' \models \varphi$

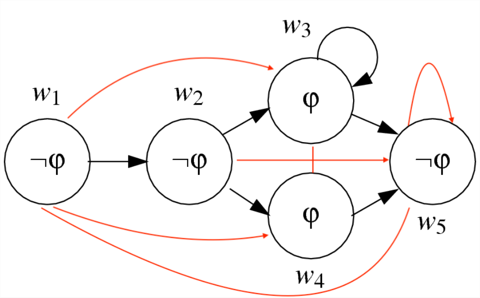


- $\mathcal{M}, w_1 \models \Box\neg\varphi$, $\mathcal{M}, w_2 \models \Box\varphi$, $\mathcal{M}, w_1 \models \Box\Box\varphi$

Kripke semantics

We may have multiple modal operators.

Different restrictions (e.g. symmetry, transitivity, ...) on the relations.
Eg. In doxastic logic relations are serial, transitive and euclidean.



Supervaluationism + Modal logic

Q: *What do we take from supervaluationism?*

A: The core intuition: hyper-ambiguity.

> "Natural language can be interpreted in many different yet equally acceptable ways, commonly referred to as precisifications."

Supervaluationism + Modal logic

Q: *What do we take from supervaluationism?*

A: The core intuition: hyper-ambiguity.

> "Natural language can be interpreted in many different yet equally acceptable ways, commonly referred to as precisifications."

With modal logic, we replace

structure of possible worlds $\longrightarrow$ structure of precisifications.

Supervaluationism + Modal logic

Q: *What do we take from supervaluationism?*

A: The core intuition: hyper-ambiguity.

"Natural language can be interpreted in many different yet equally acceptable ways, commonly referred to as precisifications."

With modal logic, we replace

structure of possible worlds $\longrightarrow$ structure of precisifications.

Q: *What do we not take from supervaluationism?*

A: The study of vagueness and the sorites paradox.

Supervaluationism + Modal logic

Q: *What do we take from supervaluationism?*

A: The core intuition: hyper-ambiguity.

> "Natural language can be interpreted in many different yet equally acceptable ways, commonly referred to as precisifications."

With modal logic, we replace

structure of possible worlds $\longrightarrow$ structure of precisifications.

Q: *What do we not take from supervaluationism?*

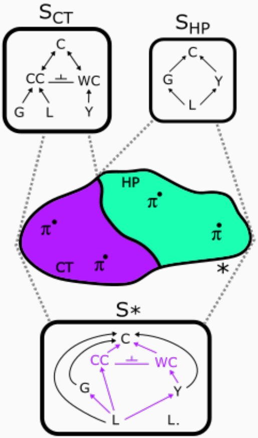
A: The study of vagueness and the sorites paradox.

Semantics of Standpoint Logic

$$\mathcal{M} = \langle \Pi, \sigma, \delta \rangle$$

A precisification is a complete and precise interpretation of the language.
The set of admissible precisifications Π replaces the set of worlds in the Kripke model.
$\delta : \mathcal{P} \to 2^{\Pi}$ assigns sets of precisifications to propositional variables

A standpoint is a partial interpretation or sharpening of the semantics of terms.
It is modelled by a set of precisifications that are consistent with the standpoint, given by $\sigma : \mathcal{S} \to 2^{\Pi}$.



A (multi-modal) logic of standpoints

Standpoints are modelled by means of modal operators:

For a standpoint s, then $\Box_s \varphi$ means that:

"according to standpoint s, it is the case that φ";

i.e. φ is the case for all precisifications which are accessible for the standpoint s.

E.g.

1. $\Box_{\mathrm{CT}}[CC \wedge WC \rightarrow \bot]$
2. $\Box_{\mathrm{CT}}[((\mathtt{Green} \vee \mathtt{Lime}) \rightarrow CC)) \wedge (\mathtt{Yellow} \rightarrow WC)]$
3. $\Box_{\mathrm{HP}}[\mathtt{Lime} \rightarrow (\mathtt{Yellow} \wedge \mathtt{Green})]$

A (multi-modal) logic of standpoints

The special operator $\Box_*$ is the universal standpoint, and it is used to refer to the set of all admissible precisifications. E.g.

$$\Box_*[(\mathtt{Yellow} \vee \mathtt{Green} \vee \mathtt{Lime}) \to \mathtt{Colour}]$$

A (multi-modal) logic of standpoints

The special operator $\Box_*$ is the universal standpoint, and it is used to refer to the set of all admissible precisifications. E.g.

$$\Box_*[(\mathtt{Yellow} \vee \mathtt{Green} \vee \mathtt{Lime}) \rightarrow \mathtt{Colour}]$$

We have the usual definable operator $\Diamond_s$, and $\mathcal{I}_s$ and $\mathcal{D}_s$ (also definable). E.g.

$$\Box_* \mathtt{Lime} \rightarrow \mathcal{I}_* \mathtt{Yellow}$$

A (multi-modal) logic of standpoints

The special operator $\Box_*$ is the universal standpoint, and it is used to refer to the set of all admissible precisifications. E.g.

$$\Box_*[(\mathit{Yellow} \vee \mathit{Green} \vee \mathit{Lime}) \rightarrow \mathit{Colour}]$$

We have the usual definable operator $\Diamond_s$, and $\mathcal{I}_s$ and $\mathcal{D}_s$ (also definable). E.g.

$$\Box_* \mathit{Lime} \rightarrow \mathcal{I}_* \mathit{Yellow}$$

Standpoint expressions can be used. E.g.

$$\Box_{\mathsf{CT} \cup \mathsf{HP}}(\mathit{Lime} \rightarrow \mathit{Yellow}) \vee (\mathit{Yellow} \rightarrow \mathit{WC})$$

A (multi-modal) logic of standpoints

The special operator $\Box_*$ is the universal standpoint, and it is used to refer to the set of all admissible precisifications. E.g.

$$\Box_*[(\mathtt{Yellow} \vee \mathtt{Green} \vee \mathtt{Lime}) \rightarrow \mathtt{Colour}]$$

We have the usual definable operator $\Diamond_s$, and $\mathcal{I}_s$ and $\mathcal{D}_s$ (also definable). E.g.

$$\Box_* \, \mathtt{Lime} \rightarrow \mathcal{I}_* \, \mathtt{Yellow}$$

Standpoint expressions can be used. E.g.

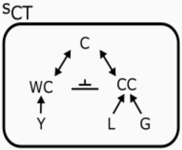
$$\Box_{\mathsf{CT} \cup \mathsf{HP}}(\mathtt{Lime} \rightarrow \mathtt{Yellow}) \vee (\mathtt{Yellow} \rightarrow \mathtt{WC})$$

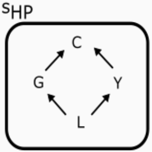
The definable operator $\preceq$ encodes the sharper relation. E.g.

$$CT \preceq *$$

Managing standpoints

Example
It is globally accepted (thus also by *CT* and *HP*) that `Yellow`, `Green`, and `Lime` are `Colour`. According to *CT*, `Colour` is either a warm colour (*WC*) or a cold colour (*CC*) but not both, `Yellow` is a *WC*, and `Green` and `Lime` are *CC*. According to *HP*, `Lime` is `Green` and `Yellow`.





- $\square_{CT}[CC \leftrightarrow \neg WC]$
- $\square_{CT}[((\mathtt{Green} \vee \mathtt{Lime}) \rightarrow CC)) \wedge (\mathtt{Yellow} \rightarrow WC)]$
- $\square_{HP}[\mathtt{Lime} \rightarrow (\mathtt{Yellow} \wedge \mathtt{Green})]$
- ...

Managing standpoints

Representation of correspondences as complex as allowed by the base logic language. **Example:**

$$\Box_{\mathrm{HP}}[\mathit{Green}] \leftrightarrow (\Box_{\mathrm{CT}}[\mathit{Green}] \vee \Box_{\mathrm{HP}}[\mathit{Lime}])$$

Managing standpoints

Representation of correspondences as complex as allowed by the base logic language. **Example:**

$$\square_{\mathrm{HP}}[\mathit{Green}] \leftrightarrow (\square_{\mathrm{CT}}[\mathit{Green}] \vee \square_{\mathrm{HP}}[\mathit{Lime}])$$

Representation of Standpoint Hierarchies and Combinations.
Example:
An Ink brand with the standpoint IB merges and reuses the categorisation of HP and a certain "ColourBase" (CB), and in addition specifies that ***Ochre*** and ***Gold*** are (types of) ***Yellow***.

$$\square_{\mathrm{IB}}[(\mathit{Gold} \vee \mathit{Ochre}) \rightarrow \mathit{Yellow}]$$
$$IB \preceq HP \wedge IB \preceq CB$$

Complexity of standpoint logic

The simpler semantics are linked to a small-model property.

The satisfiability problem for a propositional base language $\mathbb{S}$ is NP-complete.

Complexity of standpoint logic

The simpler semantics are linked to a small-model property.

The satisfiability problem for a propositional base language $\mathbb{S}$ is NP-complete.

We can prove this with a Translation into One-Variable FOL.

Complexity of standpoint logic

The simpler semantics are linked to a small-model property.

The satisfiability problem for a propositional base language $\mathbb{S}$ is NP-complete.

We can prove this with a Translation into One-Variable FOL.

Definition

The function $\mathrm{trans} : \mathcal{L}_{\mathbb{S}} \to \mathcal{L}_{FO1}$ is recursively defined as follows (with symbols from $\mathcal{P}$ and $\mathcal{S}$ repurposed as unary predicates, $\mathsf{s}', \mathsf{s} \in \mathcal{S}$ and $p \in \mathcal{P}$):

$$
\begin{aligned}
\mathrm{trans}(p) &= p(x) & \mathrm{trans}(\varphi_1 \land \varphi_2) &= \mathrm{trans}(\varphi_1) \land \mathrm{trans}(\varphi_2) \\
\mathrm{trans}(\neg\varphi) &= \neg\mathrm{trans}(\varphi) & \mathrm{trans}(\square_{\mathsf{s}}\, \varphi) &= \forall x.(\mathsf{s}(x) \to \mathrm{trans}(\varphi)) \\
& & \mathrm{trans}(\mathsf{s}' \preceq \mathsf{s}) &= \forall x.(\mathsf{s}'(x) \to \mathsf{s}(x))
\end{aligned}
$$

For an $\mathbb{S}$ formula φ with standpoint constants $s_1, \ldots, s_k$, let

$$Trans(\varphi) := \forall x.(\mathrm{trans}(\varphi)) \land \forall x.(*(x))$$

Towards First-Order Standpoint Logic (FOSL)

First-Order Standpoint Logic (FOSL)

Natural extension into modal FOL:

The set $\mathcal{E}_{\mathcal{S}}$ of *standpoint expressions* is defined as follows:

$$\mathrm{e}_1, \mathrm{e}_2 ::= * \mid \mathrm{s} \mid \mathrm{e}_1 \cup \mathrm{e}_2 \mid \mathrm{e}_1 \cap \mathrm{e}_2 \mid \mathrm{e}_1 \setminus \mathrm{e}_2$$

The set $\mathbb{S}_{\mathrm{FO}}$ of FOSL *formulas* is then given by

$$\varphi, \psi ::= P(t_1, \ldots, t_n) \mid \neg\varphi \mid \varphi \wedge \psi \mid \forall x \varphi \mid \Box_{\mathrm{e}}\, \varphi,$$

where $P \in \mathcal{P}$ is an n-ary predicate symbol, $t_1, \ldots, t_n \in \mathcal{T}$ are terms, $x \in \mathcal{V}$, and $\mathrm{e} \in \mathcal{E}_{\mathcal{S}}$.

- We adopt the rigid domain assumption and
- enforce rigid constants.

Semantics of FOSL

Given a signature $\langle \mathcal{P}, \mathcal{C}, \mathcal{S} \rangle$, a *first-order standpoint structure* $\mathfrak{M}$ is a tuple $\langle \Delta, \Pi, \sigma, \gamma \rangle$ where:

- Δ is a non-empty set, the *domain* of $\mathfrak{M}$;
- Π is the set of *precisifications*;
- σ is a function mapping each standpoint symbol from $\mathcal{S}$ to a set of precisifications (i.e., a subset of Π);
- γ is a function mapping each precisification from Π to an ordinary first-order structure $\mathcal{I}$ over the domain Δ, whose interpretation function $\cdot^{\mathcal{I}}$ maps:
    - each predicate symbol $P \in \mathcal{P}$ of arity n to an n-ary relation $P^{\mathcal{I}} \subseteq \Delta^n$,
    - each constant symbol $a \in \mathcal{C}$ to a domain element $a^{\mathcal{I}} \in \Delta$.

  Moreover, for any two $\pi_1, \pi_2 \in \Pi$ and every $a \in \mathcal{C}$ we require $a^{\gamma(\pi_1)} = a^{\gamma(\pi_2)}$.

Small Model Property and Translation of Sentential Formulas

Small Model Property of Sentential Formulas

One interesting aspect of standpoint logic is that its simplified Kripke semantics brings about convenient model-theoretic properties that do not hold for arbitrary (multi)modal logics.

This carries over to some fragments of FOSL.

Definition
Let φ be a formula of FOSL. We say that φ is *sentential* iff for all subformulas of φ that are of the form $\square_{\mathsf{e}}\,\psi$, all variables occurring in ψ are bound by a quantifier.

Theorem

A sentential FOSL formula φ is satisfiable iff it has a model with at most $|\varphi|$ precisifications. That is, for sentential FOSL, satisfiability and $|\varphi|$-satisfiability coincide.

Translation to plain FOL

The translation maps $SSNF(\varphi)$ into a formula of (standpoint-free) FOL.

We “emulate” standpoint structures $\langle \Delta, \Pi_n, \sigma, \gamma \rangle$ by means of a “superposition” of $\gamma(\pi)$, introducing n “copies” of the original set of predicates.

The top-level translation is then defined to set:

$$\mathrm{Trans}_n(\varphi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \varphi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi ,$$

where trans_n is inductively defined by

$$\begin{aligned}
\mathrm{trans}_n(\pi, P(t_1, \dots, t_n)) &= P_\pi(t_1, \dots, t_n) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg \mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_e\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, e) \to \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

Translation to plain FOL

The top-level translation is then defined to set:

$$\mathrm{Trans}_n(\varphi) = \bigwedge_{\pi\in\Pi_n} \mathrm{trans}_n(\pi,\varphi) \wedge \bigwedge_{\pi\in\Pi_n} *_\pi,$$

where trans_n and $\mathrm{trans}_{\mathcal{E}}$ are inductively defined by

$$\begin{aligned}
\mathrm{trans}_n(\pi, P(t_1,\ldots,t_n)) &= P_\pi(t_1,\ldots,t_n)\\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi,\psi)\\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi,\psi_1) \wedge \mathrm{trans}_n(\pi,\psi_2)\\
\mathrm{trans}_n(\pi, \forall x\psi) &= \forall x(\mathrm{trans}_n(\pi,\psi))\\
\mathrm{trans}_n(\pi', \Box_e\,\psi) &= \bigwedge_{\pi\in\Pi_n}(\mathrm{trans}_{\mathcal{E}}(\pi, e) \to \mathrm{trans}_n(\pi,\psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, s) &= s_\pi\\
\mathrm{trans}_{\mathcal{E}}(\pi, e_1 \cup e_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, e_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, e_2)\\
\mathrm{trans}_{\mathcal{E}}(\pi, e_1 \cap e_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, e_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, e_2)\\
\mathrm{trans}_{\mathcal{E}}(\pi, e_1 \setminus e_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, e_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, e_2)
\end{aligned}$$

Expressive Decidable FOSL Fragments

Expressive Decidable FOSL Fragments

We will next look into some popular decidable FO fragments and establish decidability and complexity results for reasoning in their sentential standpoint versions.

Expressive Decidable FOSL Fragments

We will next look into some popular decidable FO fragments and establish decidability and complexity results for reasoning in their sentential standpoint versions.

Definition
Let $\mathcal{F}$ denote some FO fragment. Then the logic *sentential Standpoint-$\mathcal{F}$*, denoted $\mathbb{S}_{[\mathcal{F}]}$ contains the formulas φ, where

- φ is a sentential FOSL formula,
- all variables inside φ are bound by some quantifier,
- for every subformula $\psi \in \mathit{Sub}(\mathrm{SSNF}(\varphi))$ holds $\psi \in \mathcal{F}$.

$\mathcal{F}$ is called *standpoint-friendly* if every $\varphi \in \mathbb{S}_{[\mathcal{F}]}$ satisfies $\mathrm{Trans}_{|\varphi|}(\mathrm{SSNF}(\varphi)) \in \mathcal{F}$.

Expressive Decidable FOSL Fragments

Lemma
Let $\mathcal{F}$ be a standpoint-friendly fragment of FOL. Then the following hold:

1. *Satisfiability for $\mathbb{S}_{[\mathcal{F}]}$ is decidable if and only if it is for $\mathcal{F}$.*
2. *If the satisfiability problem in $\mathcal{F}$ is at least NP-hard, then its complexity coincides with that of $\mathbb{S}_{[\mathcal{F}]}$.*

Standpoint-friendly fragments of FOL:

- The propositional fragment PF
- The counting 2-variable fragment C^2
- The guarded fragment GF
- The triguarded fragment TGF

Therefore these four decidable fragments of FOL allow for accommodating standpoints without any increase in complexity.

Sentential Standpoint-$\mathcal{SROIQ}b_s$

We next present the highly expressive yet decidable logic Standpoint-$\mathcal{SROIQ}b_s$[1].

The $\mathcal{SROIQ}$ family serves as the logical foundation of popular ontology languages like OWL 2 DL.

$\mathcal{SROIQ}$ is a semantic fragment of FOL, so we can leverage the previously established results. We get

- favorable and tight complexity results for reasoning in Standpoint-$\mathcal{SROIQ}b_s$.
- practical reasoning in "Standpoint-OWL" with the translation & highly optimized OWL 2 DL reasoners off the shelf.

[1] $\mathcal{SROIQ}b_s$ is an extension of $\mathcal{SROIQ}$ allowing safe Boolean role expressions oversimple roles at no complexity cost (Rudolph, Krotzsch, and Hitzler 2008).

Sentential Standpoint-$\mathcal{SROIQ}b_s$

We next present the highly expressive yet decidable logic Standpoint-$\mathcal{SROIQ}b_s$

The translation is inspired by the one presented for FOSL. $\mathcal{SROIQ}b_s$ comes with diverse syntactic restrictions that we need to cope with by good deal of "paraphrasing".

These restrictions are:

- the absence of a native way to form Boolean combinations of axioms,
- the absence of nullary predicates (i.e., propositional symbols),
- complex role inclusion axioms and the regularity restrictions imposed on them.

$\mathcal{SROIQ}b_s$ role and concept expressions

Name	Syntax	Semantics
inverse role	$\boldsymbol{s}^-$	$\{(x,y) \in \boldsymbol{\Delta} \times \boldsymbol{\Delta} \mid (y,x) \in \boldsymbol{s}^{\mathcal{I}}\}$
role union	$r_1 \cup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$
role intersection	$r_1 \cap r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$
role difference	$r_1 \setminus r_2$	$r_1^{\mathcal{I}} \setminus r_2^{\mathcal{I}}$
universal role	$\boldsymbol{u}$	$\boldsymbol{\Delta}^{\mathcal{I}} \times \boldsymbol{\Delta}^{\mathcal{I}}$
nominal	$\{a\}$	$\{a^{\mathcal{I}}\}$
top	$\top$	$\boldsymbol{\Delta}^{\mathcal{I}}$
bottom	$\bot$	$\emptyset$
negation	$\neg C$	$\boldsymbol{\Delta}^{\mathcal{I}} \setminus C^{\mathcal{I}}$
conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
univ. restriction	$\forall r.C$	$\{x \mid \forall y.(x,y) \in r^{\mathcal{I}} \to y \in C^{\mathcal{I}}\}$
exist. restriction	$\exists r.C$	$\{x \mid \exists y.(x,y) \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
Self concept	$\exists r.\mathit{Self}$	$\{x \mid (x,x) \in r^{\mathcal{I}}\}$
qualified number	$\leqslant n\, r.C$	$\{x \mid \#\{y \in C^{\mathcal{I}} \mid (x,y) \in r^{\mathcal{I}}\} \leq n\}$
restrictions	$\geqslant n\, r.C$	$\{x \mid \#\{y \in C^{\mathcal{I}} \mid (x,y) \in r^{\mathcal{I}}\} \geq n\}$

Syntax and semantics of $\mathcal{SROIQ}b_s$ axioms.

For RIAs, $\boldsymbol{r} \in \mathcal{P}_2^{\mathrm{ns}}$, while $r_i \in \mathcal{R}$ and $r_i \prec \boldsymbol{r}$ for all $i \in \{1, \dots, n\}$.

Name	Syntax	Semantics
role inclusion axioms (RIAs)	$r_1 \circ \ldots \circ r_n \sqsubseteq \boldsymbol{r}$	$r_1^{\mathcal{I}} \circ \ldots \circ r_n^{\mathcal{I}} \subseteq \boldsymbol{r}^{\mathcal{I}}$
	$\boldsymbol{r} \circ r_1 \circ \ldots \circ r_n \sqsubseteq \boldsymbol{r}$	$\boldsymbol{r}^{\mathcal{I}} \circ r_1^{\mathcal{I}} \circ \ldots \circ r_n^{\mathcal{I}} \subseteq \boldsymbol{r}^{\mathcal{I}}$
	$r_1 \circ \ldots \circ r_n \circ \boldsymbol{r} \sqsubseteq \boldsymbol{r}$	$r_1^{\mathcal{I}} \circ \ldots \circ r_n^{\mathcal{I}} \circ \boldsymbol{r}^{\mathcal{I}} \subseteq \boldsymbol{r}^{\mathcal{I}}$
	$\boldsymbol{r} \circ \boldsymbol{r} \sqsubseteq \boldsymbol{r}$	$\boldsymbol{r}^{\mathcal{I}} \circ \boldsymbol{r}^{\mathcal{I}} \subseteq \boldsymbol{r}^{\mathcal{I}}$
general concept inclusion (GCI)	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
concept essertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$r(a, b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in \boldsymbol{r}^{\mathcal{I}}$
equality	$a \doteq b$	$a^{\mathcal{I}} = b^{\mathcal{I}}$
inequality	$a \dot{\neq} b$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$

The set $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ of *sentential Standpoint-$\mathcal{SROIQ}$ sentences* is defined inductively as follows:

- if Ax is a $\mathcal{SROIQ}b_s$ axiom then $\mathsf{Ax} \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$,
- if $\varphi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$ then $\neg\varphi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$,
- if $\varphi, \psi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$ then $\varphi\wedge\psi,\ \varphi\vee\psi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$,
- if $\varphi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$ and $\mathsf{e} \in \mathcal{E}_\mathcal{S}$ then $\square_\mathsf{e}\,\varphi,\ \lozenge_\mathsf{e}\,\varphi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$.

We say a $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ sentence φ is in *negation normal form* (NNF), if negation occurs only inside or directly in front of $\mathcal{SROIQ}$ axioms.

Translation of Sentential Standpoint-$\mathcal{SROIQ}b_s$

As before, we fix $\Pi_{|\varphi|}$ and let our translation's vocabulary $\mathbb{V}_{[\mathcal{SROIQ}b_s]}(\varphi)$ consist of all individual names inside φ, plus, for each $\pi \in \Pi_{|\varphi|}$, the following symbols:

- a concept name A^π for each $A \in \mathcal{P}_1$;
- a simple role name s^π for each $s \in \mathcal{P}_2^{\mathrm{s}}$;
- non-simple role names r^π and $\underline{r}^\pi$ for each $r \in \mathcal{P}_2^{\mathrm{ns}} \setminus \{u\}$,
- a simple role name s_ρ^π for each unnegated RIA ρ inside φ;
- a fresh constant name a_ρ^π for each negated RIA ρ inside φ;
- a concept name M_π^{s} for each $\mathrm{s} \in \mathcal{S}$.

Translation of Sentential Standpoint-$\mathcal{SROIQ}b_s$

The translation $\mathrm{Trans}(\varphi)$ of φ is then a set of $\mathcal{SROIQ}$ axioms defined as follows:

1 - $\mathrm{Trans}(\varphi)$ contains the RIA $\underline{\boldsymbol{r}}^{\pi} \sqsubseteq \boldsymbol{r}^{\pi}$ for every $\boldsymbol{r} \in \mathcal{P}_2^{\mathrm{ns}} \setminus \{u\}$ and each $\pi \in \Pi_{|\varphi|}$.

2 - For every unnegated RIA ρ inside φ and each $\pi \in \Pi_{|\varphi|}$, $\mathrm{Trans}(\varphi)$ contains the RIA $BG_{\pi}(\rho)$, with BG_{π} defined by

$$
\begin{aligned}
r_1 \circ \ldots \circ r_n \sqsubseteq \boldsymbol{r} \quad &\mapsto \quad r_1^{\pi} \circ \ldots \circ r_n^{\pi} \circ \boldsymbol{s}_{\rho}^{\pi} \sqsubseteq \underline{\boldsymbol{r}}^{\pi} \\
\boldsymbol{r} \circ r_1 \circ \ldots \circ r_n \sqsubseteq \boldsymbol{r} \quad &\mapsto \quad \underline{\boldsymbol{r}}^{\pi} \circ r_1^{\pi} \circ \ldots \circ r_n^{\pi} \circ \boldsymbol{s}_{\rho}^{\pi} \sqsubseteq \underline{\boldsymbol{r}}^{\pi} \\
r_1 \circ \ldots \circ r_n \circ \boldsymbol{r} \sqsubseteq \boldsymbol{r} \quad &\mapsto \quad \boldsymbol{s}_{\rho}^{\pi} \circ r_1^{\pi} \circ \ldots \circ r_n^{\pi} \circ \underline{\boldsymbol{r}}^{\pi} \sqsubseteq \underline{\boldsymbol{r}}^{\pi} \\
\boldsymbol{r} \circ \boldsymbol{r} \sqsubseteq \boldsymbol{r} \quad &\mapsto \quad \boldsymbol{s}_{\rho}^{\pi} \circ \underline{\boldsymbol{r}}^{\pi} \circ \boldsymbol{r}^{\pi} \sqsubseteq \boldsymbol{r}^{\pi},
\end{aligned}
$$

whereby the role expression r^{π} is obtained from r by substituting every role name $\boldsymbol{s}$ with $\boldsymbol{s}^{\pi}$ (except $\boldsymbol{u}$ which remains unaltered).

Translation of Sentential Standpoint-$\mathcal{SROIQ}b_s$

3 - $\mathrm{Trans}(\varphi)$ contains the GCI

$$\top \sqsubseteq \textstyle\bigsqcap_{\pi \in \Pi_{|\varphi|}} \mathrm{trans}(\pi, \varphi) \sqcap \bigsqcap_{\pi \in \Pi_{|\varphi|}} \forall u.M_\pi^*$$

where, by inductive definition,

$$\begin{aligned}
\mathrm{trans}(\pi, \mathsf{Ax}) &= \mathrm{trans}^+(\pi, \mathsf{Ax}) \\
\mathrm{trans}(\pi, \neg\mathsf{Ax}) &= \mathrm{trans}^-(\pi, \mathsf{Ax}) \\
\mathrm{trans}(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}(\pi, \psi_1) \sqcap \mathrm{trans}(\pi, \psi_2) \\
\mathrm{trans}(\pi, \psi_1 \vee \psi_2) &= \mathrm{trans}(\pi, \psi_1) \sqcup \mathrm{trans}(\pi, \psi_2) \\
\mathrm{trans}(\pi', \Box_{\mathsf{e}}\, \psi) &= \textstyle\bigsqcap_{\pi \in \Pi_{|\varphi|}} (\neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \sqcup \mathrm{trans}(\pi, \psi)) \\
\mathrm{trans}(\pi', \Diamond_{\mathsf{e}}\, \psi) &= \textstyle\bigsqcup_{\pi \in \Pi_{|\varphi|}} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \sqcap \mathrm{trans}(\pi, \psi))
\end{aligned}$$

Translation of Sentential Standpoint-$\mathcal{SROIQ}b_s$

As before, $\mathrm{trans}_{\mathcal{E}}$ implements the semantics of standpoint expressions, but now adjusted to the new framework: Each expression $\mathsf{e} \in \mathcal{E}_{\mathcal{S}}$ is transformed into a concept expression $\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e})$ over vocabulary $\left\{ M_{\pi}^{\mathsf{s}} \mid \mathsf{s} \in \mathcal{S}, \pi \in \Pi_{|\varphi|} \right\}$ thus:

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \forall u. M_{\pi}^{\mathsf{s}} \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \sqcup \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \sqcap \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \sqcap \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Translation of Sentential Standpoint-$\mathcal{SROIQ}b_s$

We finish the description, by providing the translation of unnegated and negated $\mathcal{SROIQ}$ axioms (ρ stands for an arbitrary RIA $r_1 \circ \ldots \circ r_m \sqsubseteq \boldsymbol{r}$):

$$
\begin{aligned}
\mathrm{trans}^+(\pi, \rho) &= \forall \boldsymbol{u}.\exists \boldsymbol{s}_\rho^\pi.\mathit{Self} \\
\mathrm{trans}^+(\pi, C \sqsubseteq D) &= \forall \boldsymbol{u}.(\neg C \sqcup D)^\pi \\
\mathrm{trans}^+(\pi, C(a)) &= \exists \boldsymbol{u}.\big(\{a\} \sqcap C^\pi\big) \\
\mathrm{trans}^+(\pi, r(a,b)) &= \exists \boldsymbol{u}.\big(\{a\} \sqcap \exists \underline{r}^\pi.\{b\}\big) \\
\mathrm{trans}^+(\pi, a \doteq b) &= \exists \boldsymbol{u}.\big(\{a\} \sqcap \{b\}\big)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{trans}^-(\pi, \rho) &= \exists \boldsymbol{u}.\big((\forall \boldsymbol{r}^\pi.\neg\{a_\rho^\pi\}) \sqcap (\exists \underline{r}_1^\pi \ldots \exists \underline{r}_m^\pi.\{a_\rho^\pi\})\big) \\
\mathrm{trans}^-(\pi, C \sqsubseteq D) &= \exists \boldsymbol{u}.(C \sqcap \neg D)^\pi \\
\mathrm{trans}^-(\pi, C(a)) &= \exists \boldsymbol{u}.\big(\{a\} \sqcap (\neg C)^\pi\big) \\
\mathrm{trans}^-(\pi, r(a,b)) &= \exists \boldsymbol{u}.\big(\{a\} \sqcap \forall r^\pi.\neg\{b\}\big) \\
\mathrm{trans}^-(\pi, a \doteq b) &= \forall \boldsymbol{u}.\big(\neg\{a\} \sqcup \neg\{b\}\big)
\end{aligned}
$$

With all definitions in place, we obtain the desired result:

Theorem
Given $\varphi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$, *the set* $\mathrm{Trans}(\varphi)$

1. *is a valid* $\mathcal{SROIQ}b_s$ *knowledge base,*
2. *is equisatisfiable with* φ,
3. *is of polynomial size wrt.* φ, *and*
4. *can be computed in polynomial time.*

Conclusions & Future Work

Conclusions & Future Work

Representing multiple, possibly conflicting perspectives or standpoints may be interesting in a variety of cases.

The standpoint framework

- Is versatile enough to represent partial truths and and complex relations between standpoints.
- Is aligned with an established theory of language.
- Has simple and relatable syntax and semantics.
- We can show a (very) small-model property for some fragments, with
- possibility to use base language reasoners with translations.

Conclusions & Future Work

Future work:

- Explore the complexity of fragments allowing
  - rigidity and
  - a more liberal use of modal operators.
- Implementations and experiments to test actual runtimes.
- Conceptual modelling with standpoints for common Knowledge Integration challenges.