How to Agree to Disagree: Managing Conceptual Diversity using Standpoint Logic

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Inria, Université Grenoble Alpes

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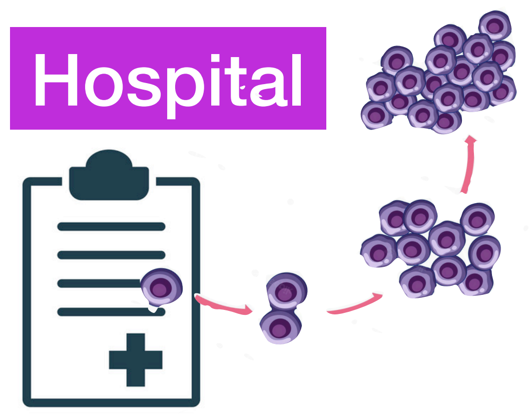
SNOMED CT

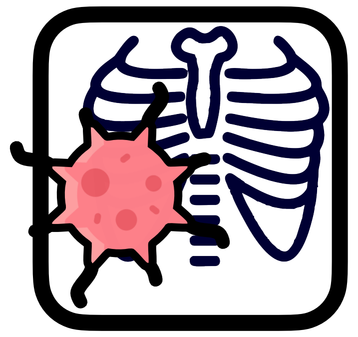
Hospital



yago
select knowledge

WIKIDATA

Private Lab



DBpedia

Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

SNOMED CT

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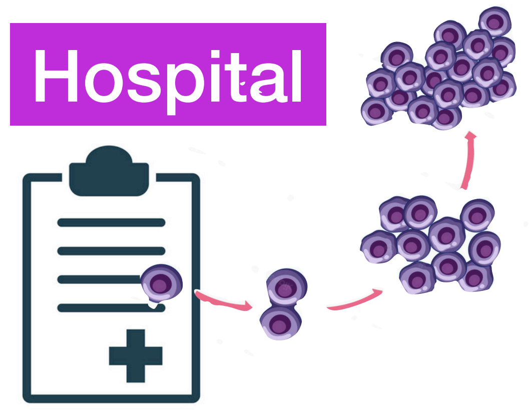

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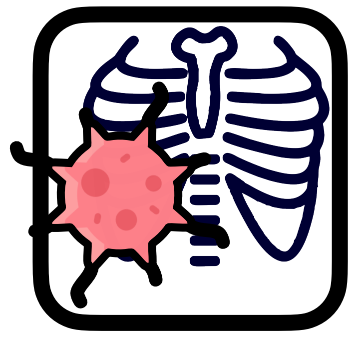

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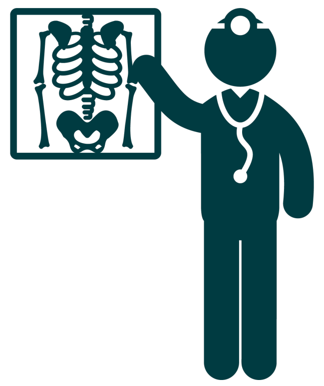

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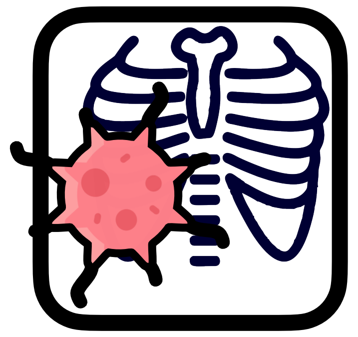

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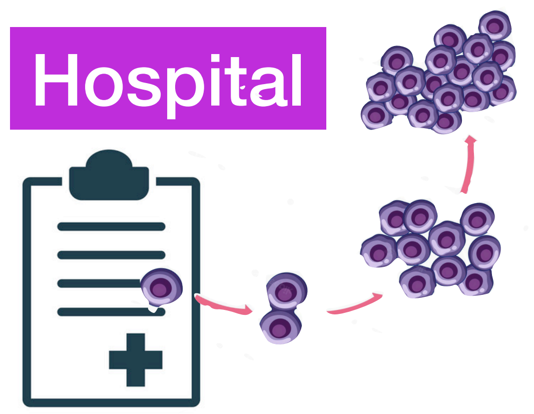

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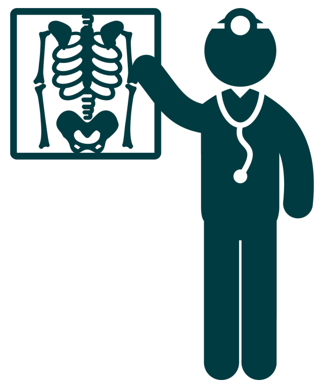

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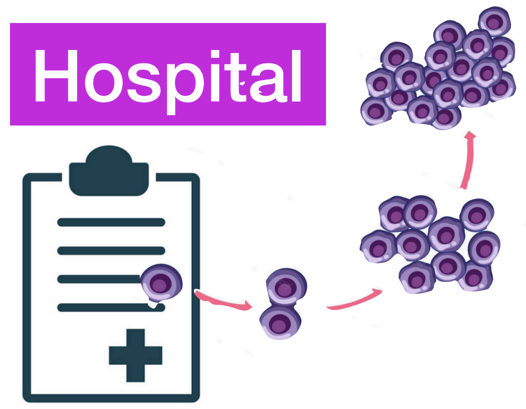

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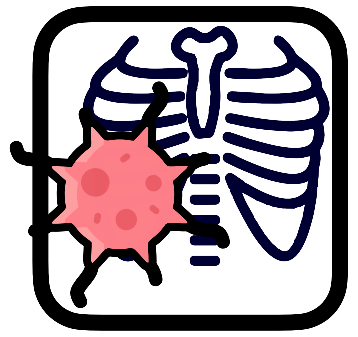

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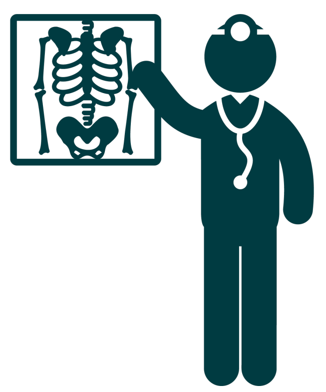

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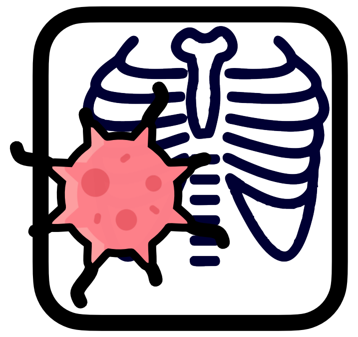

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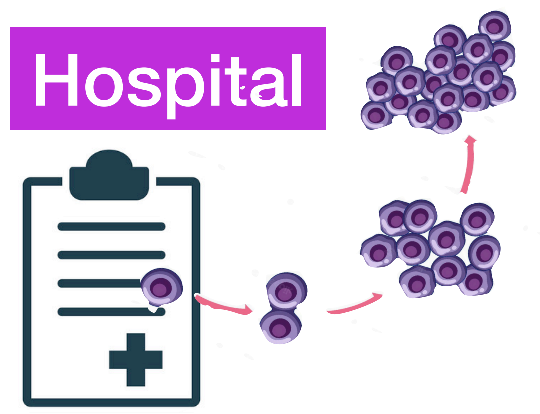

$\mathsf{HighRisk}(p)$

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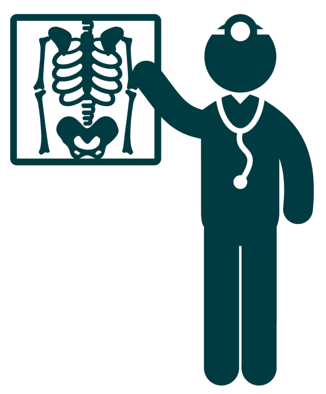

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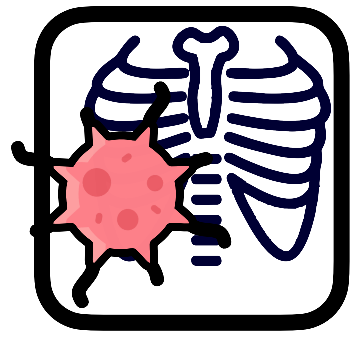

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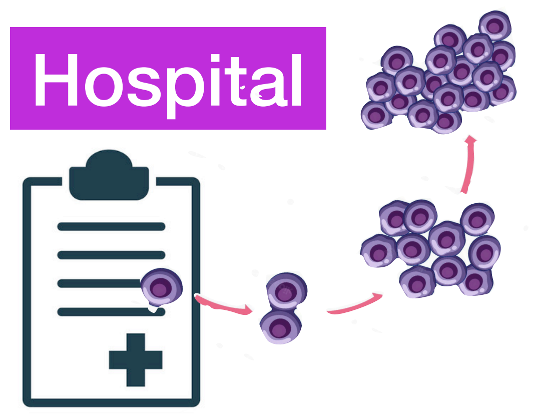
Process $\sqcap$ Tissue $\sqsubseteq \bot$

Hospital



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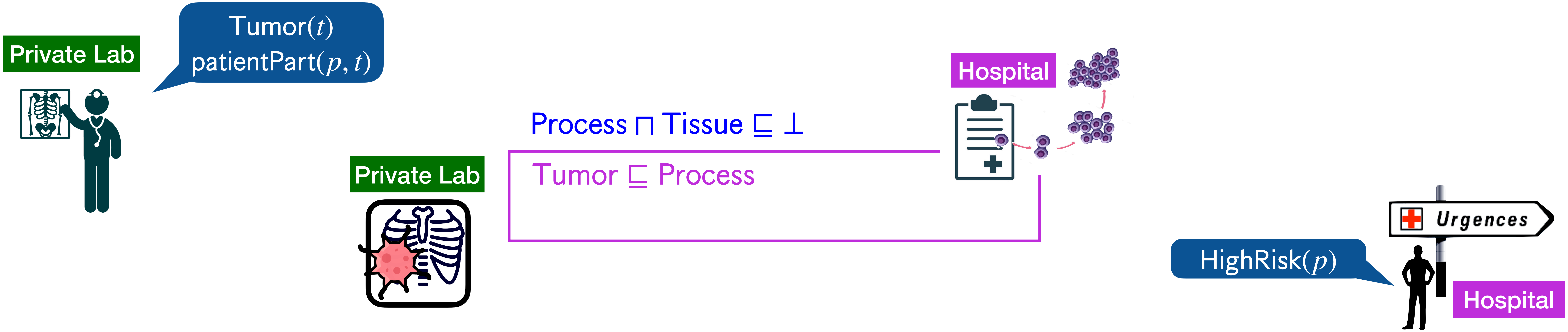

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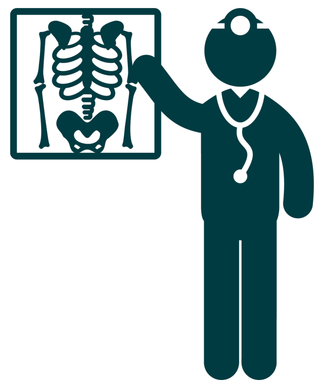

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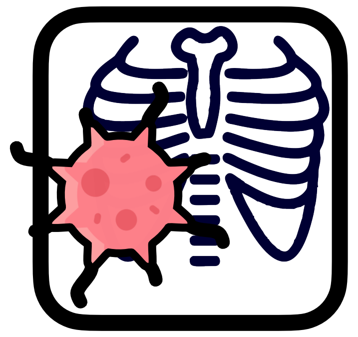
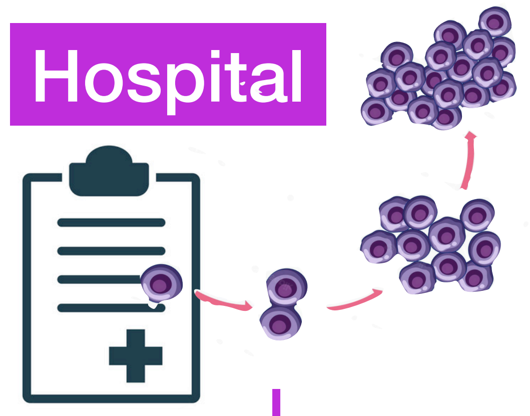


$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$
$\exists \mathsf{hasProcess}.\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

Hospital



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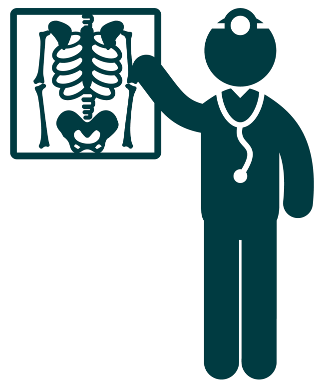

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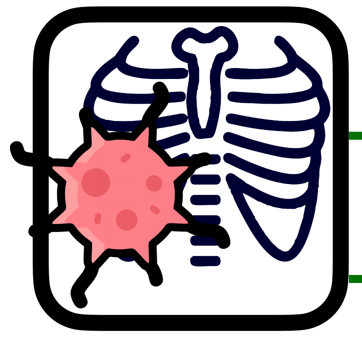

$\boxed{\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}}$

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Hospital

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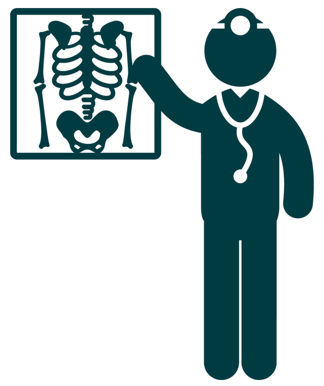

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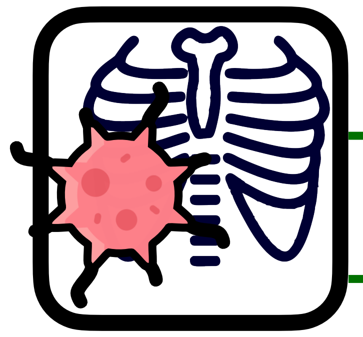

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HighRisk(p)

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Challenge: Integration

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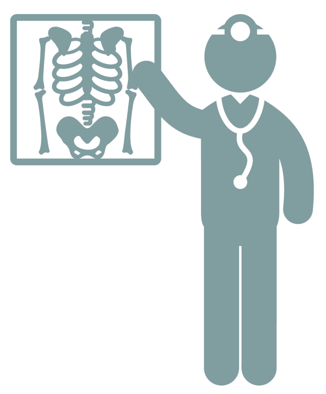

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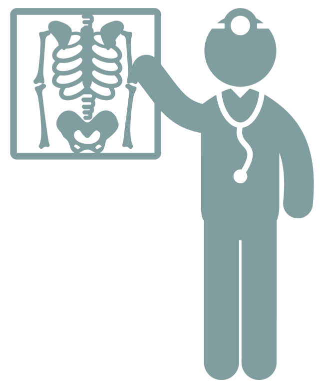

$\mathsf{Tumor}(t)$
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Knowledge Fusion

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

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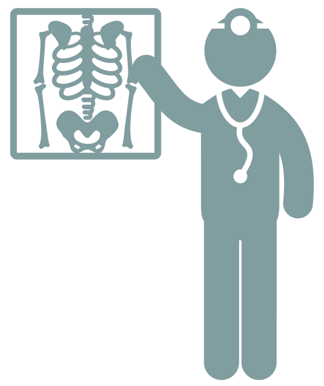

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$\mathsf{Tumor}(t)$

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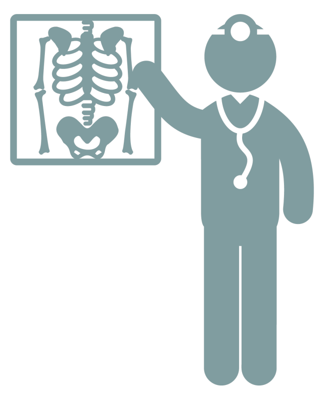

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$\mathsf{Tumor}(t)$

$\mathsf{Process}(t)$ $\mathsf{Tissue}(t)$

Private Lab

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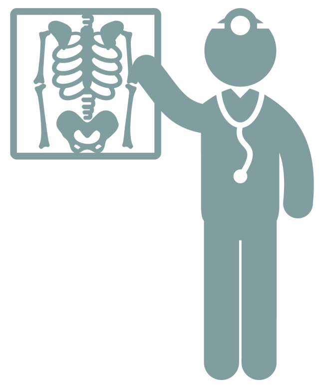

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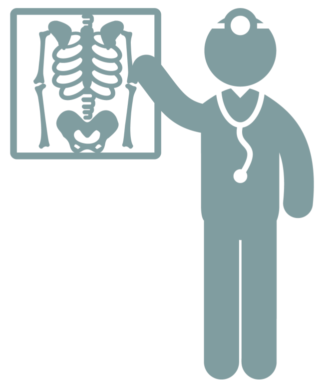
$$\begin{tikzcd} & \mathsf{Tumor}(t) \arrow[dl] \arrow[dr] & \\ \mathsf{Process}(t) \arrow[r] & \bot & \mathsf{Tissue}(t) \arrow[l] \end{tikzcd}$$

Private Lab



Tumor(t)
patientPart(p, t)

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Process $\sqcap$ Tissue $\sqsubseteq \bot$

Tumor $\sqsubseteq$ Process

$\exists$hasProcess . Tumor $\sqsubseteq$ HighRisk

Tumor $\sqsubseteq$ Tissue

HighRisk(p)

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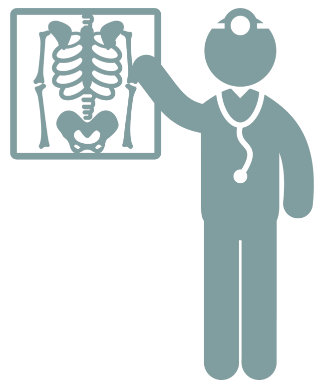

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Multiperspective Ontology Management

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Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

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➡ **Multimodal logic** characterised by simplified Kripke semantics

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$\square_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\lozenge_L [\text{Tumor}] \sqsubseteq \square_L [\text{Tissue}]$

$\lozenge_H [\text{Tumor}] \sqsubseteq \square_H [\text{Process}]$

$\square_H [\exists \text{hasProcess}.\text{Tumor} \sqsubseteq \text{HighRisk}]$

CONSISTENT

$\square_e$ Unequivocal to e

$\lozenge_e$ Conceivable to e

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$\square_H\,[\exists \mathrm{hasProcess}\,.\,\mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$

$(\,L \cup H\,) \preceq S$

(L and H inherit the axioms of S)

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$$(L \cup H) \preceq S$$

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$$\Diamond_L \left[\exists \mathrm{patientPart} . \mathrm{Tumor}\right] \sqsubseteq \square_H \left[\exists \mathrm{hasProcess} . \mathrm{Tumor}\right]$$

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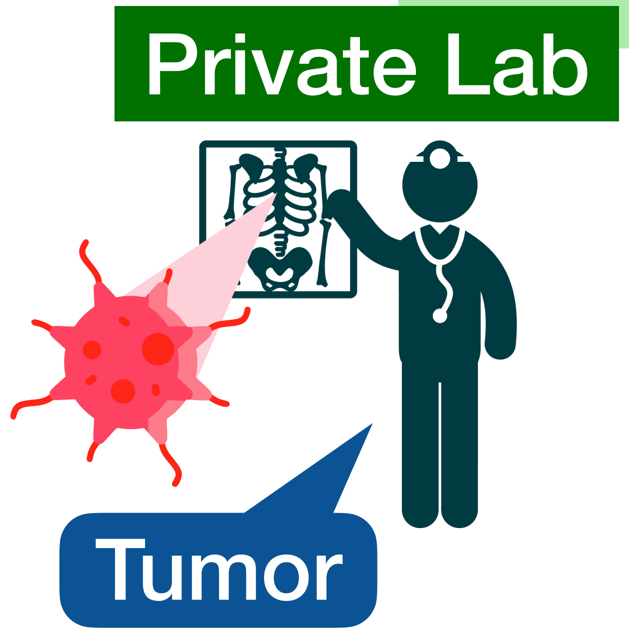
$\Diamond_L [\exists \mathrm{patientPart} . \mathrm{Tumor}] \sqsubseteq \Box_H [\exists \mathrm{hasProcess} . \mathrm{Tumor}]$

$\Diamond_L \mathrm{Tumor}(t)$ $\Box_L \mathrm{patientPart}(p, t)$



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SNOMED CT

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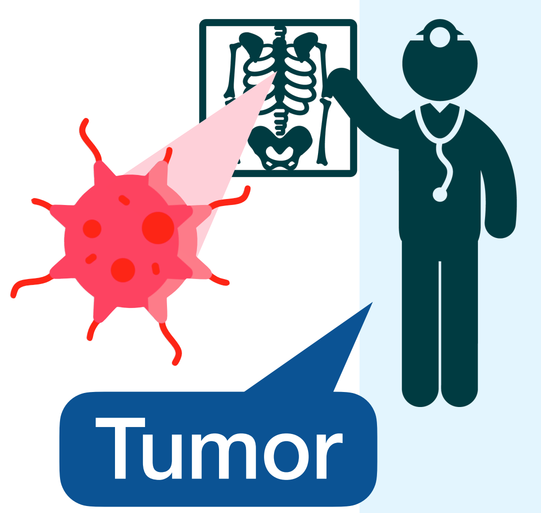
Private Lab



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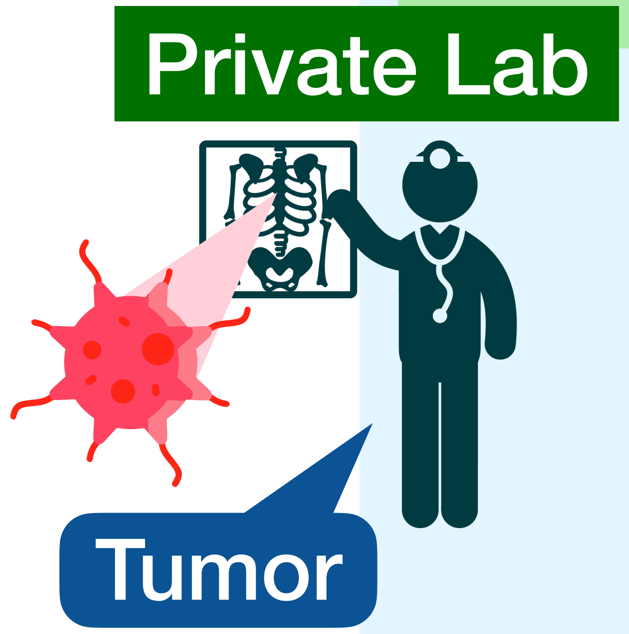
SNOMED CT

$\Diamond_L \mathsf{Tumor}(t)$ $\square_L\,\mathsf{patientPart}(p, t)$



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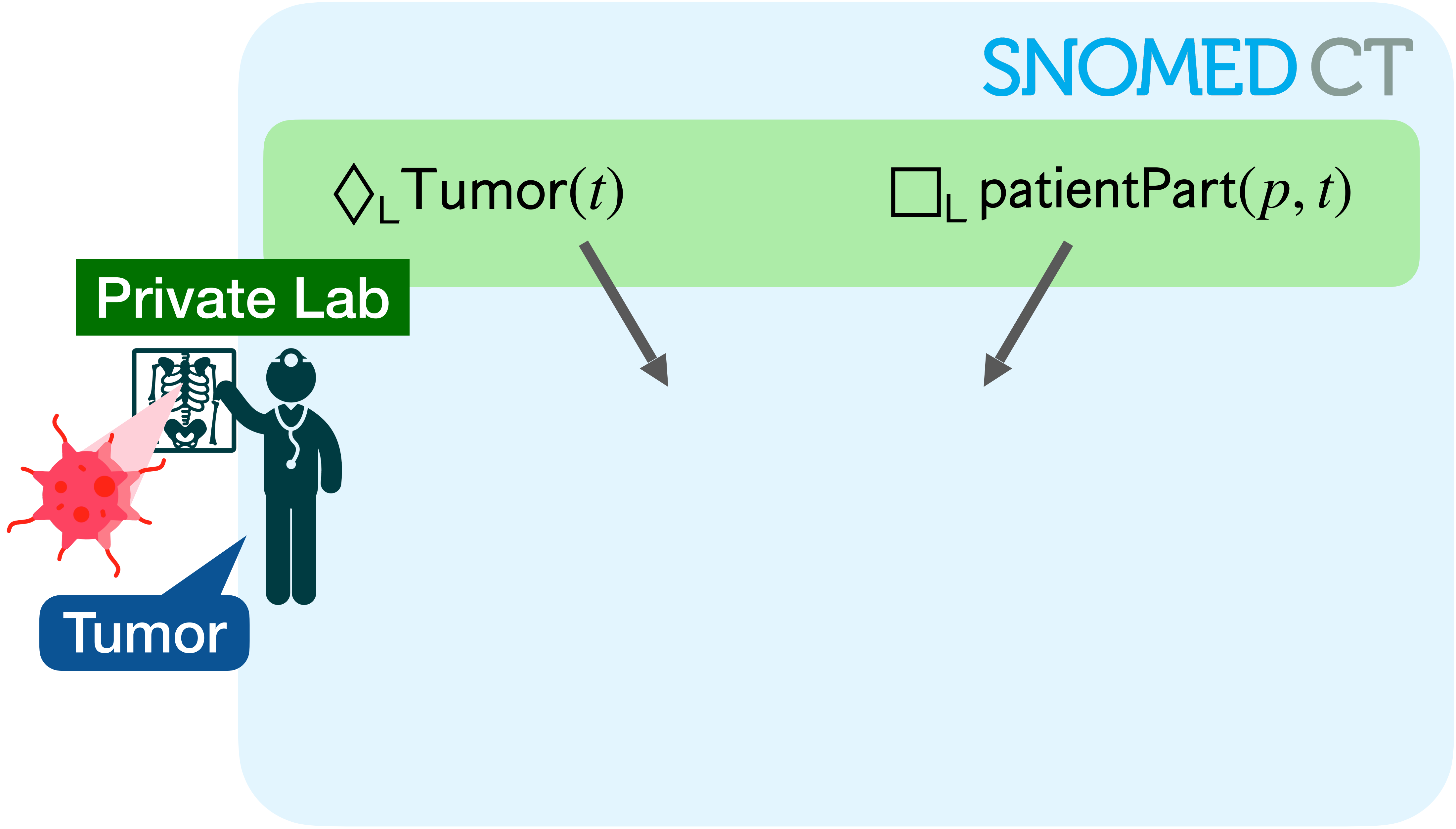

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to “points of view” (standpoints)

CONSISTENT

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$$\Box_{\mathrm{S}} [\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$$

$$\Diamond_{\mathrm{L}} [\mathrm{Tumor}] \sqsubseteq \Box_{\mathrm{L}} [\mathrm{Tissue}]$$

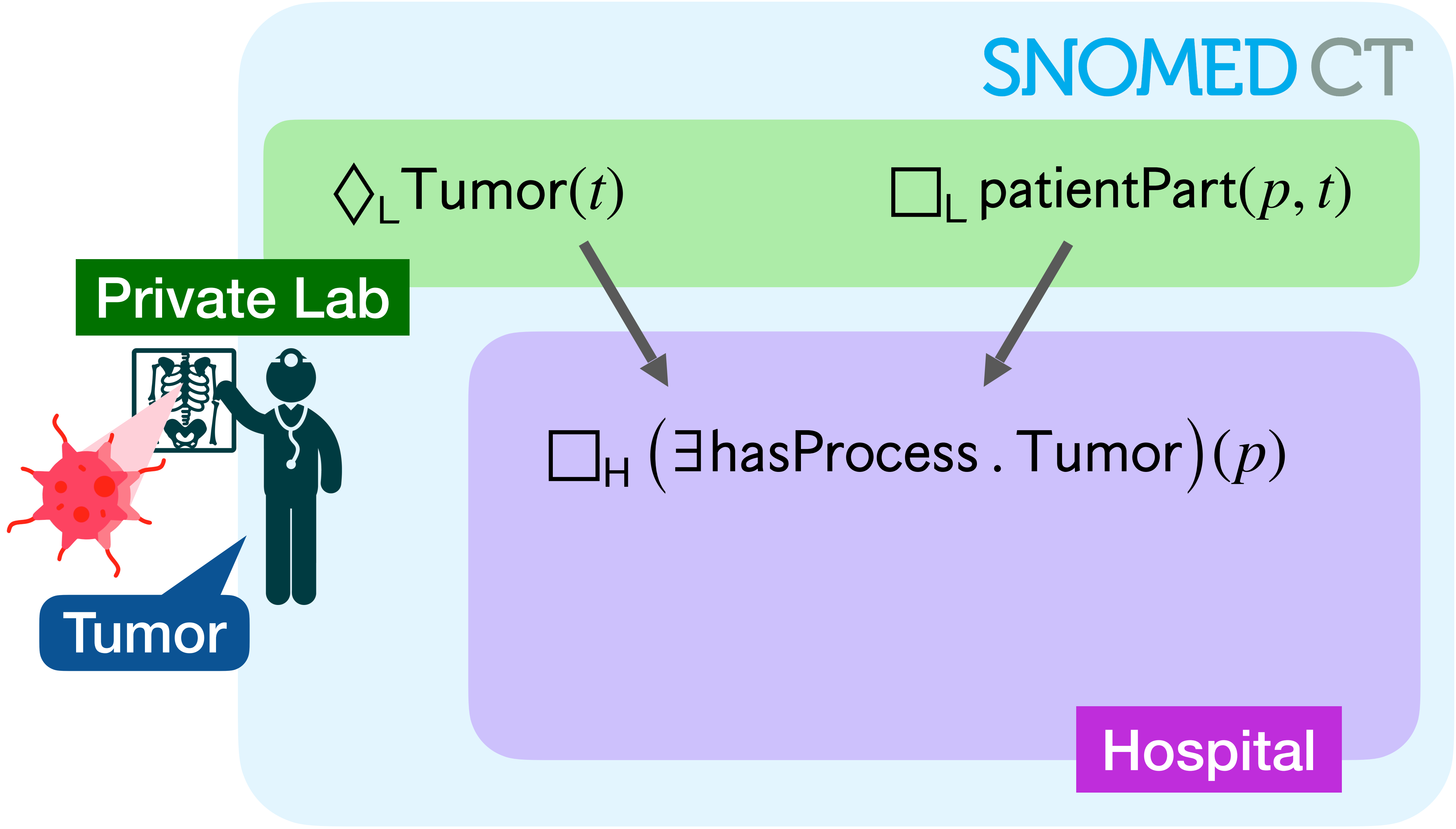
$$\Diamond_{\mathrm{H}} [\mathrm{Tumor}] \sqsubseteq \Box_{\mathrm{H}} [\mathrm{Process}]$$

$$\Box_{\mathrm{H}} [\exists \mathrm{hasProcess} . \mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$$

$$(\mathrm{L} \cup \mathrm{H}) \preceq \mathrm{S}$$

(L and H inherit the axioms of S)

$$\Diamond_{\mathrm{L}} [\exists \mathrm{patientPart} . \mathrm{Tumor}] \sqsubseteq \Box_{\mathrm{H}} [\exists \mathrm{hasProcess} . \mathrm{Tumor}]$$



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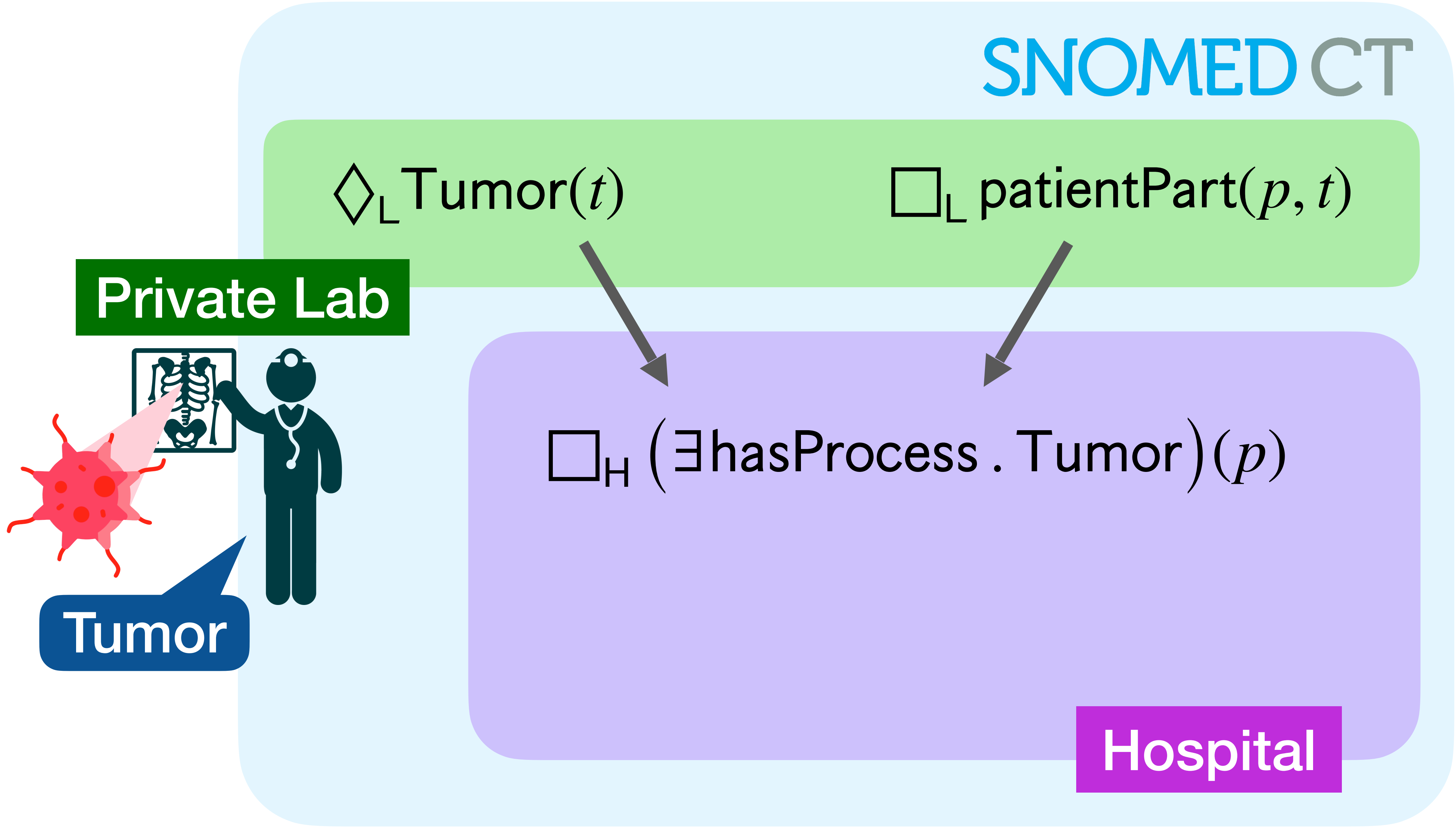

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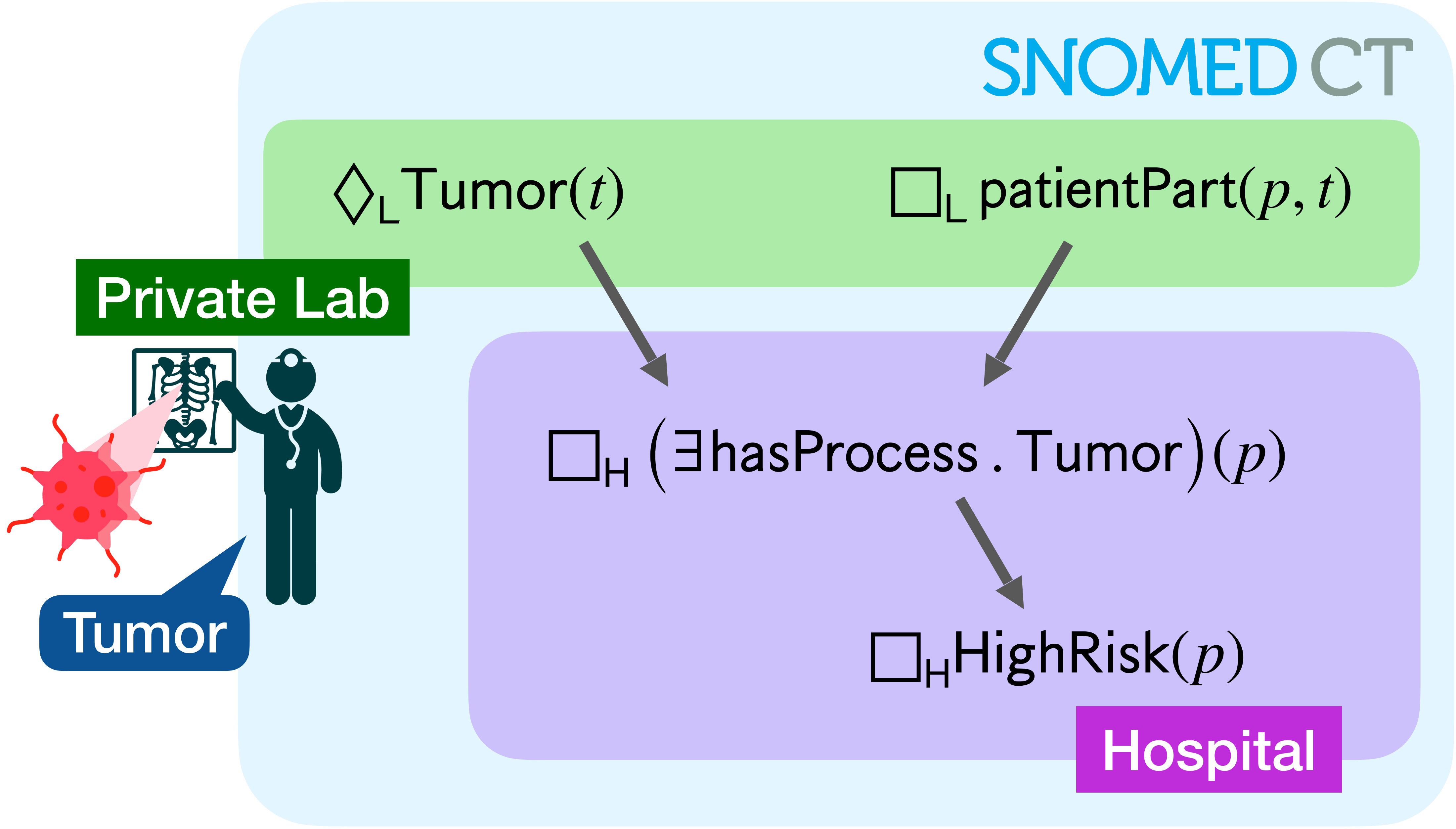

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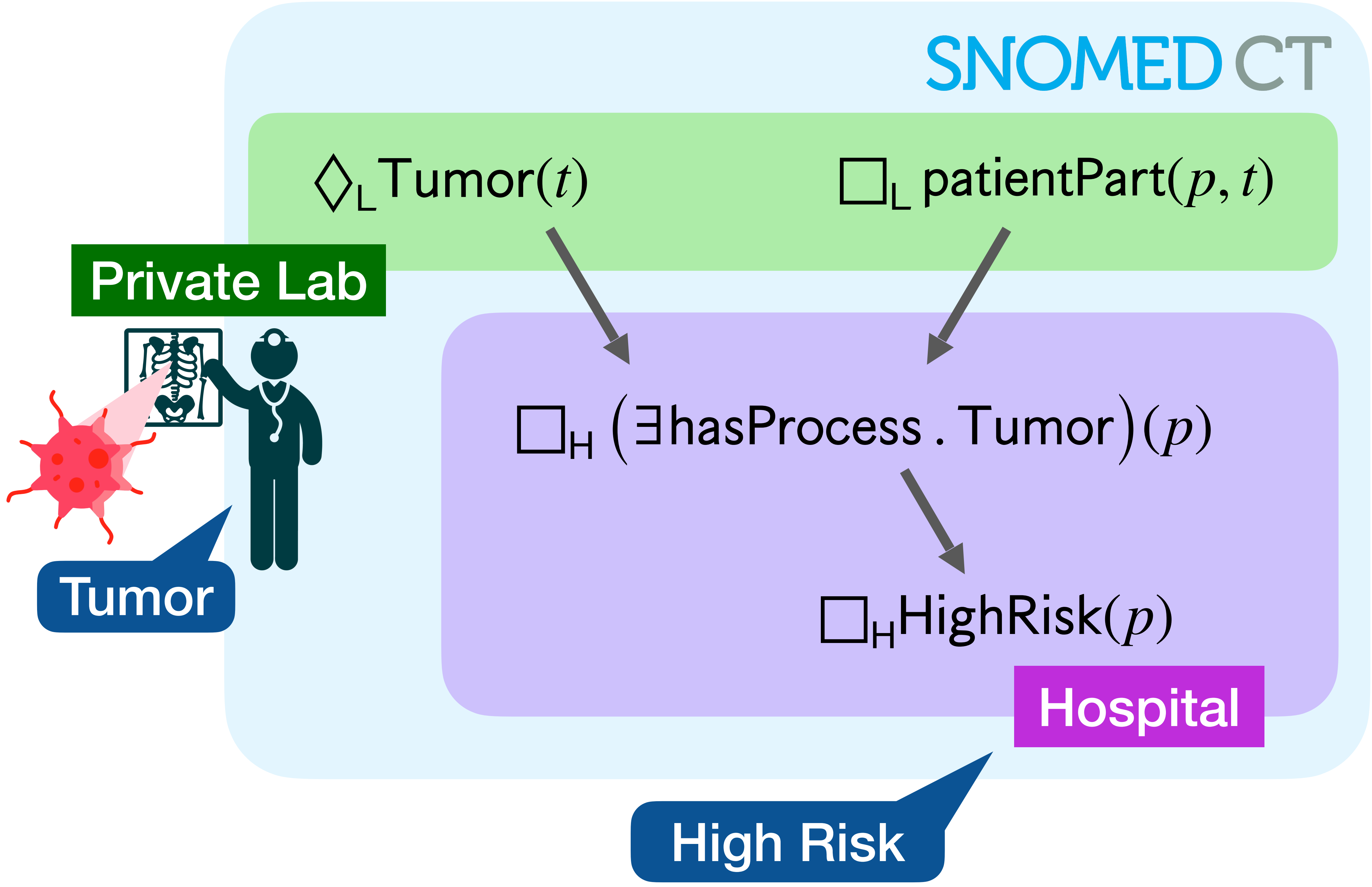

Standpoint FOL – Syntax and Semantics

First-Order Standpoint Logic: Syntax

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

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The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \square_e \phi$$

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$$\square_e \neg\phi \equiv \neg\lozenge_e \phi \text{ (dual)}$$

- $\square_e \phi \longrightarrow$ “it is **unequivocal**, according to e, that ϕ”
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The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

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- $\Box_{e \cap e'} \phi \longrightarrow$ "it is unequivocal, according to the fusion of e and e', that ϕ"

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- $e \preceq e' \longrightarrow$ "*e* inherits or **extends** *e'*"

First-Order Standpoint Logic: Semantics

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Semantics of $\mathbb{S}$

Relational semantics:

First-Order Standpoint Logic: Semantics

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Δ / Π	ϵ_1	ϵ_2
π_1		
π_2		
π_3		

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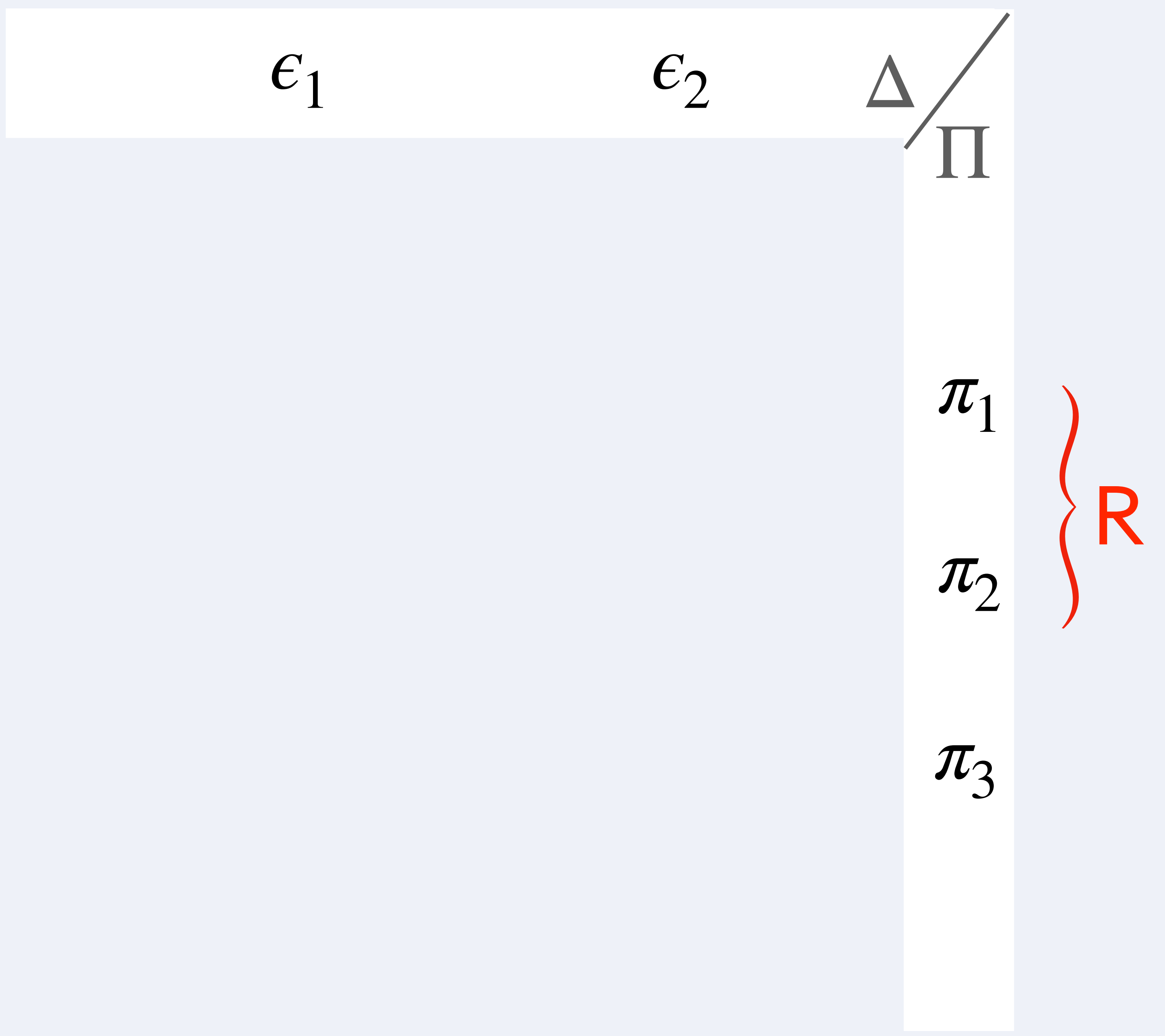

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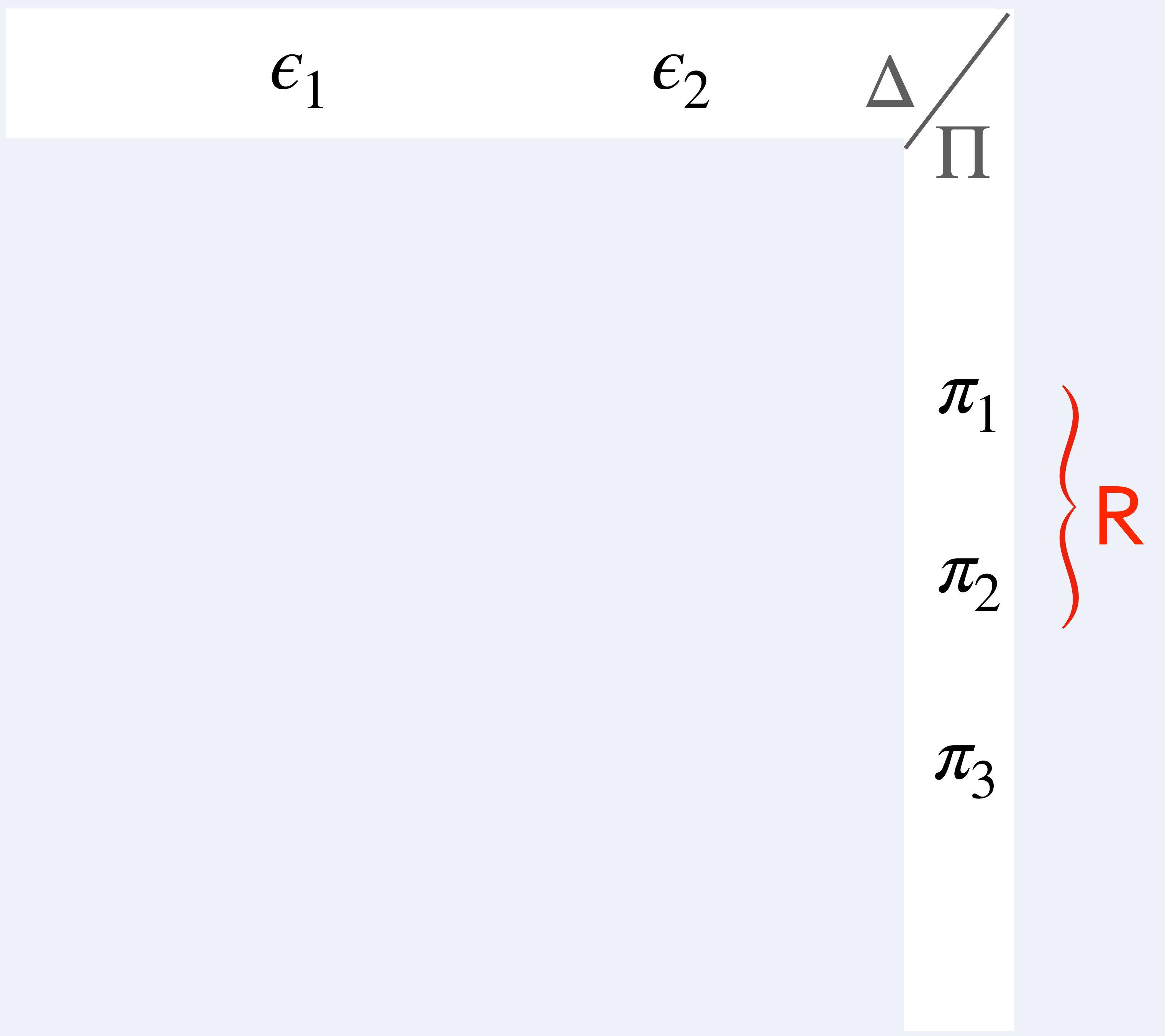

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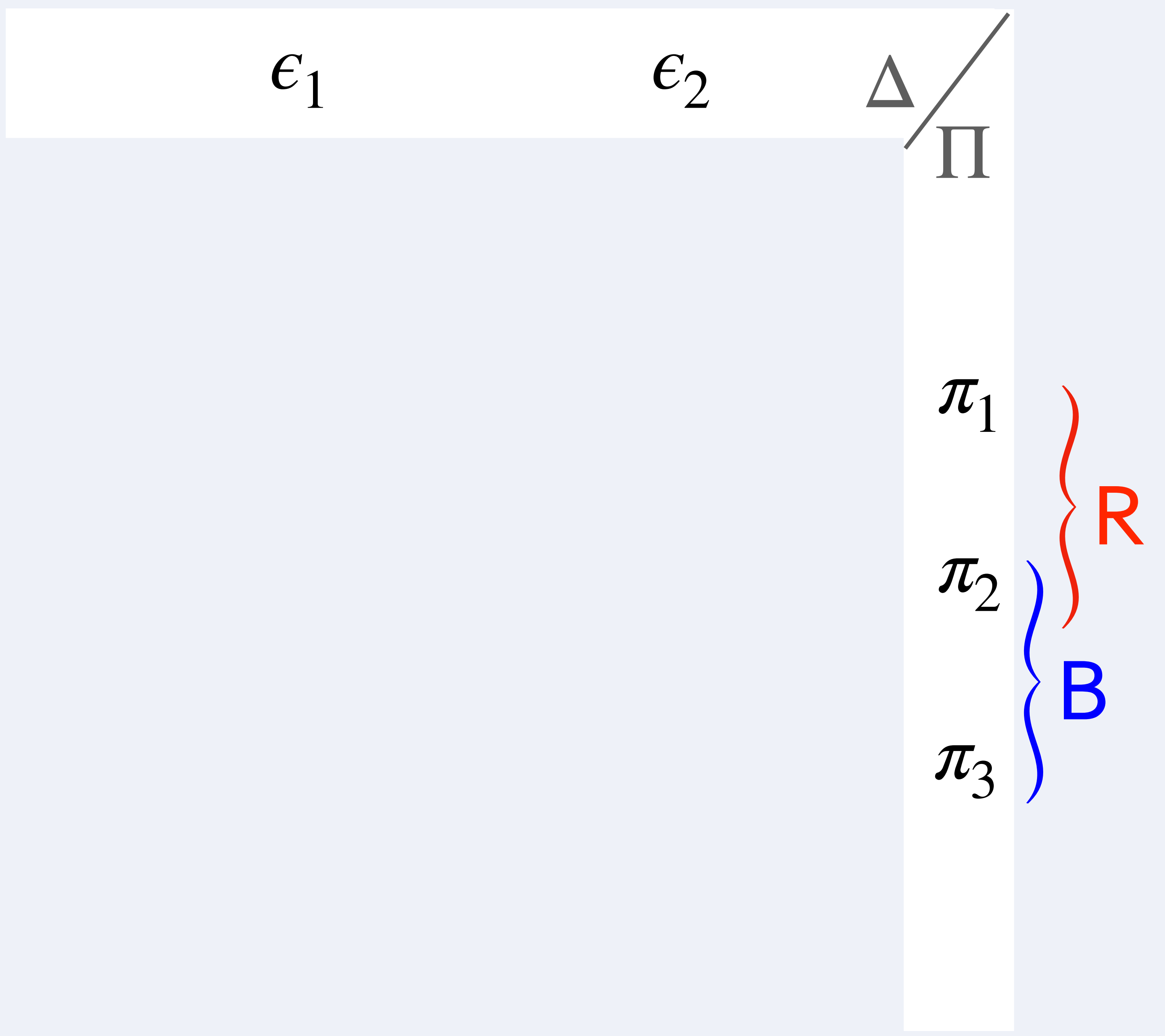

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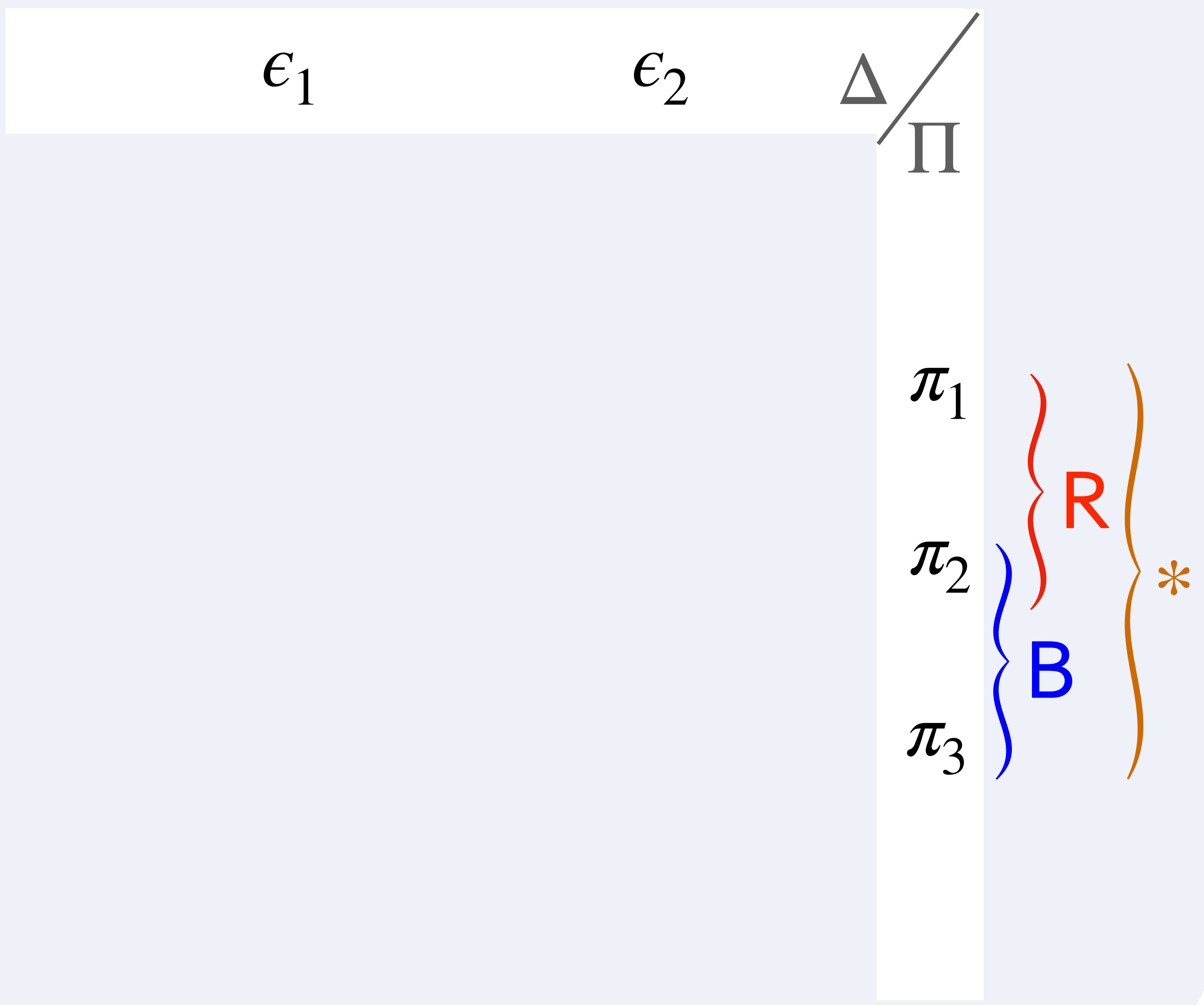

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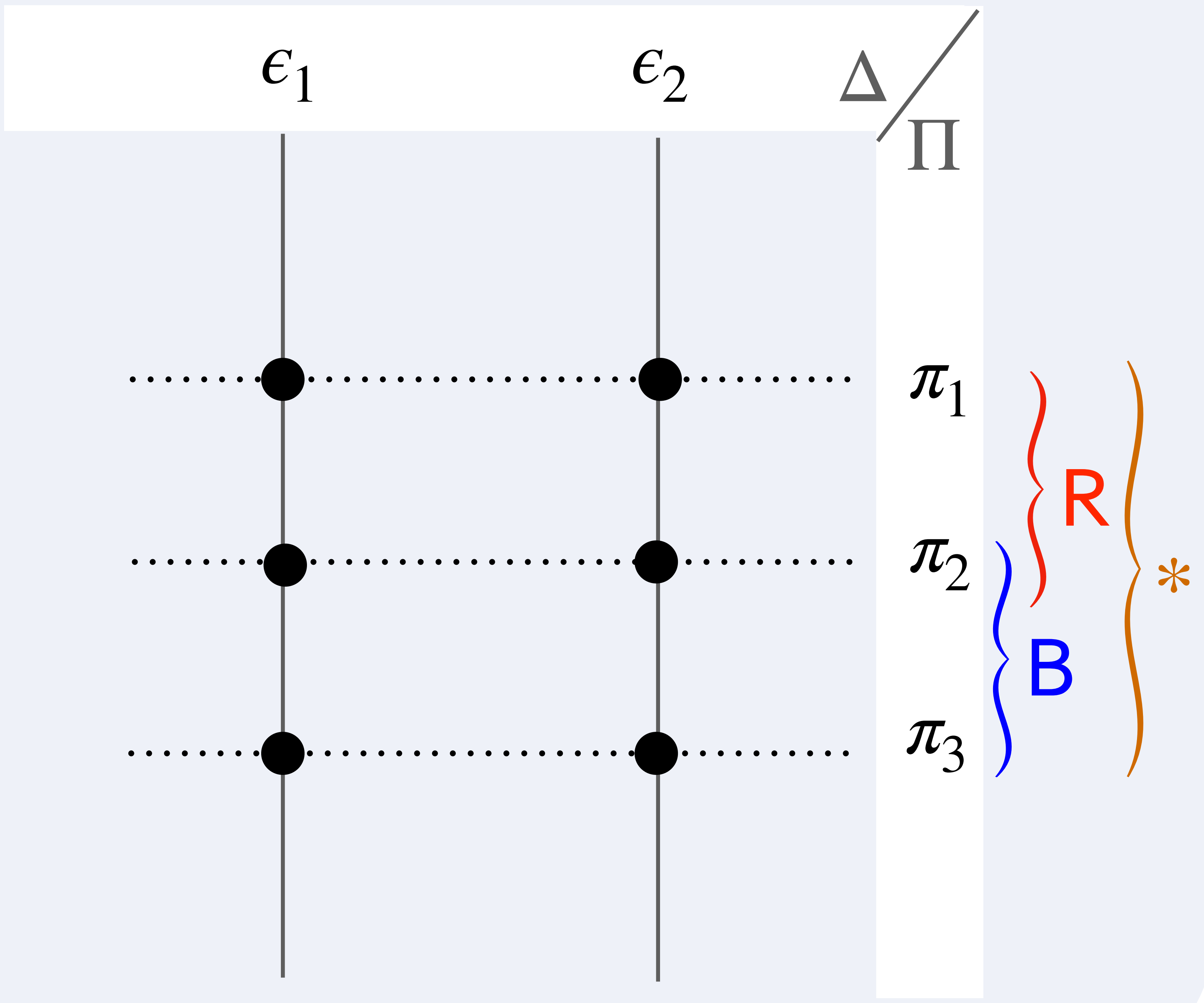

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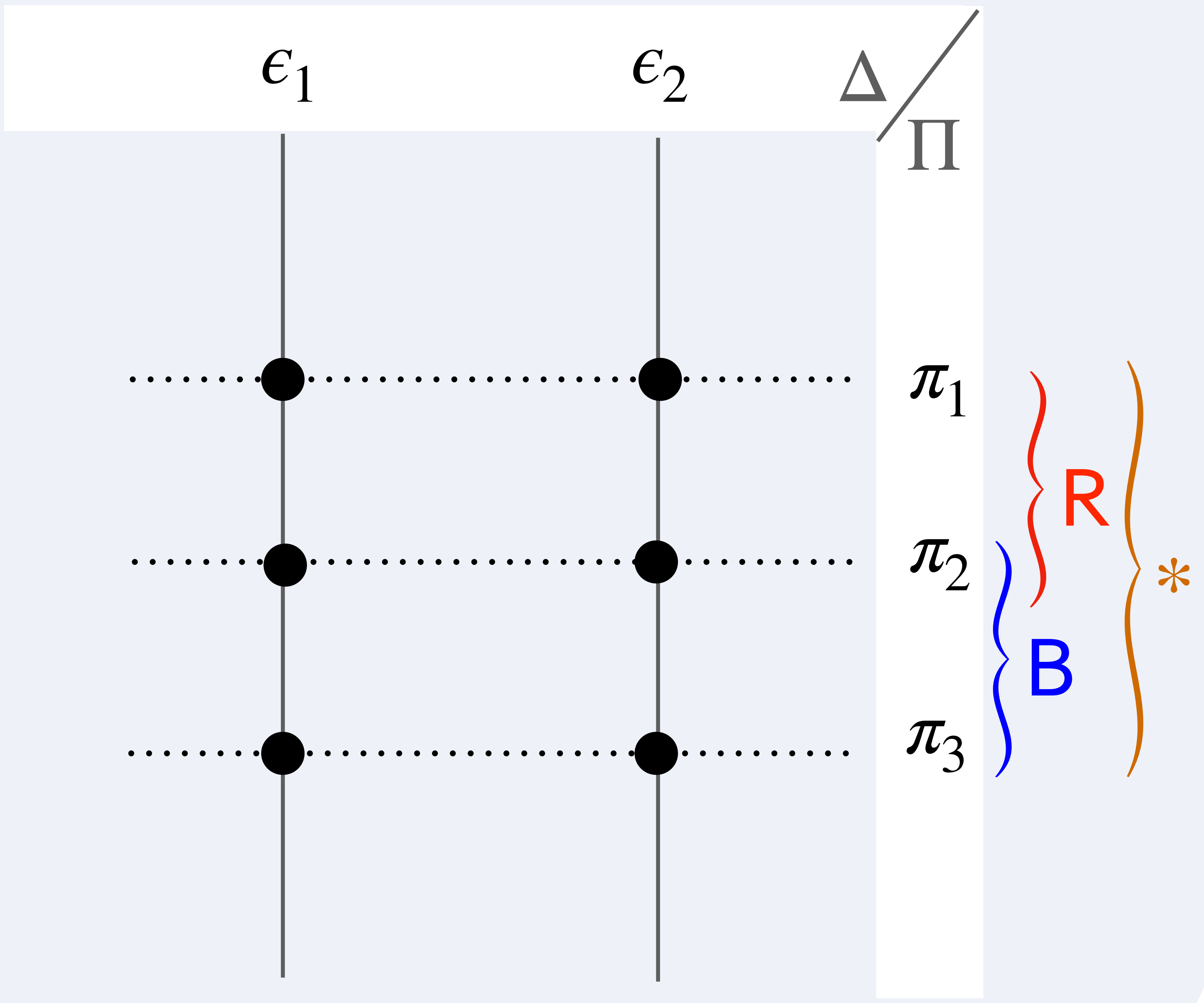

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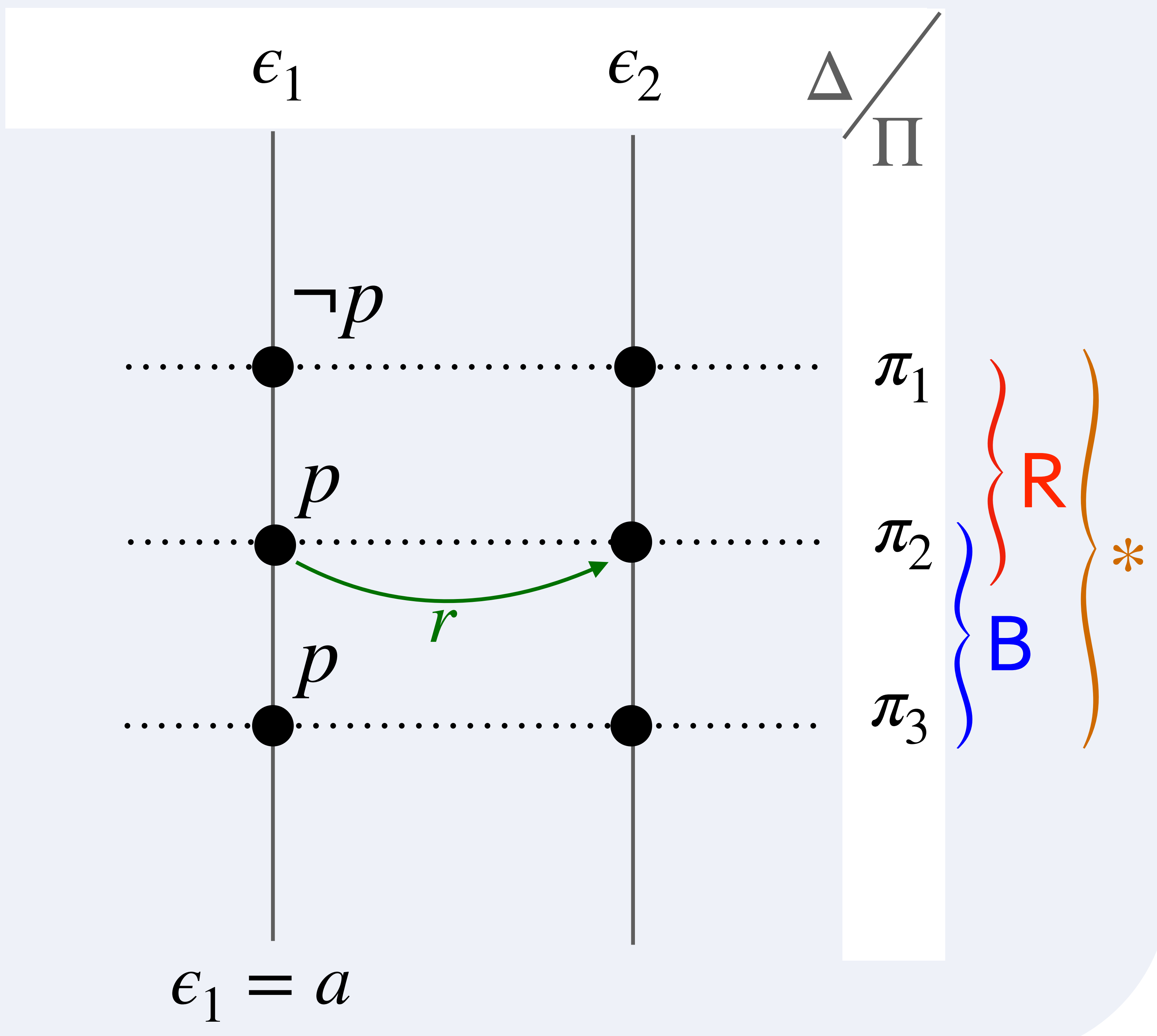

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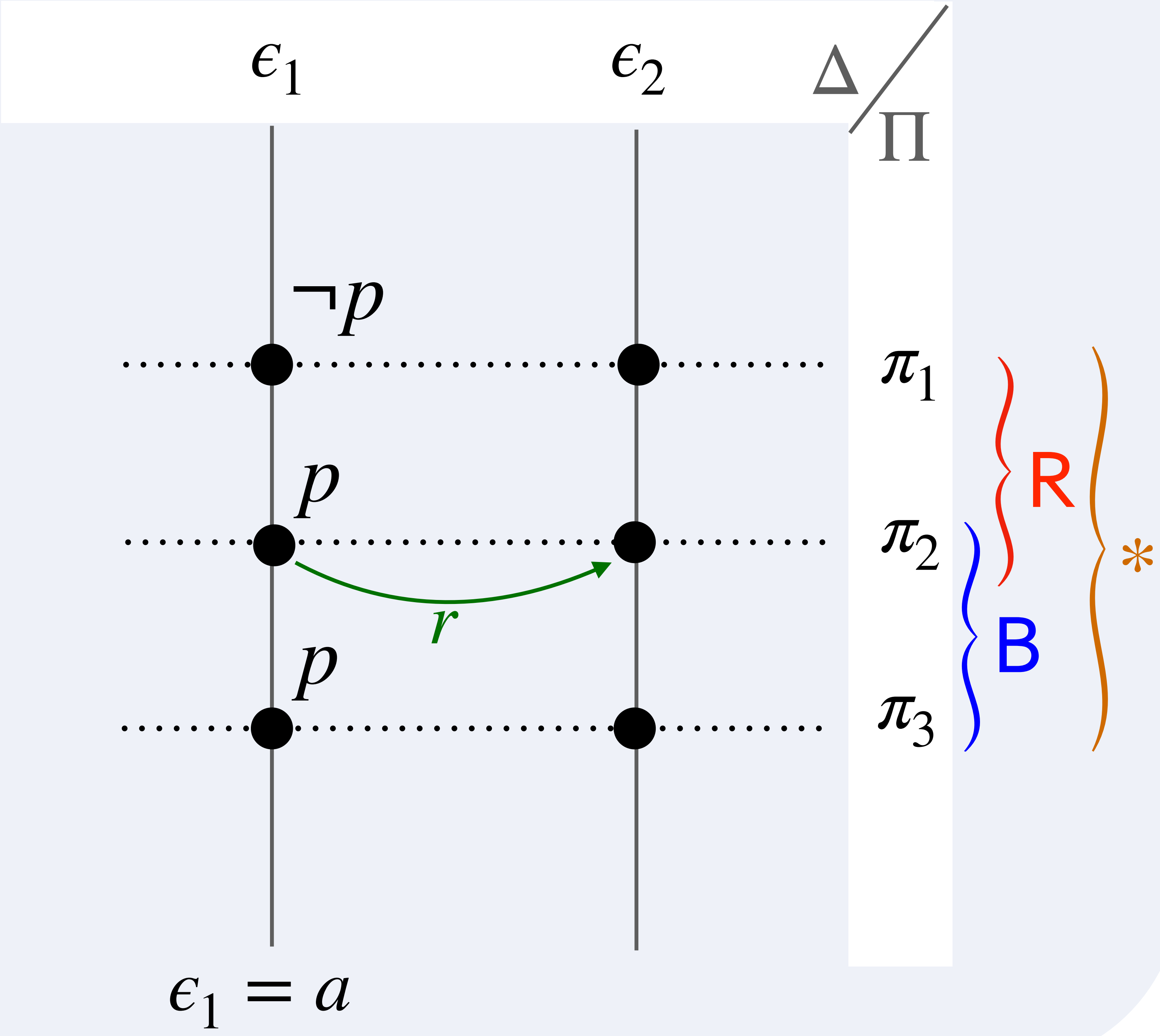

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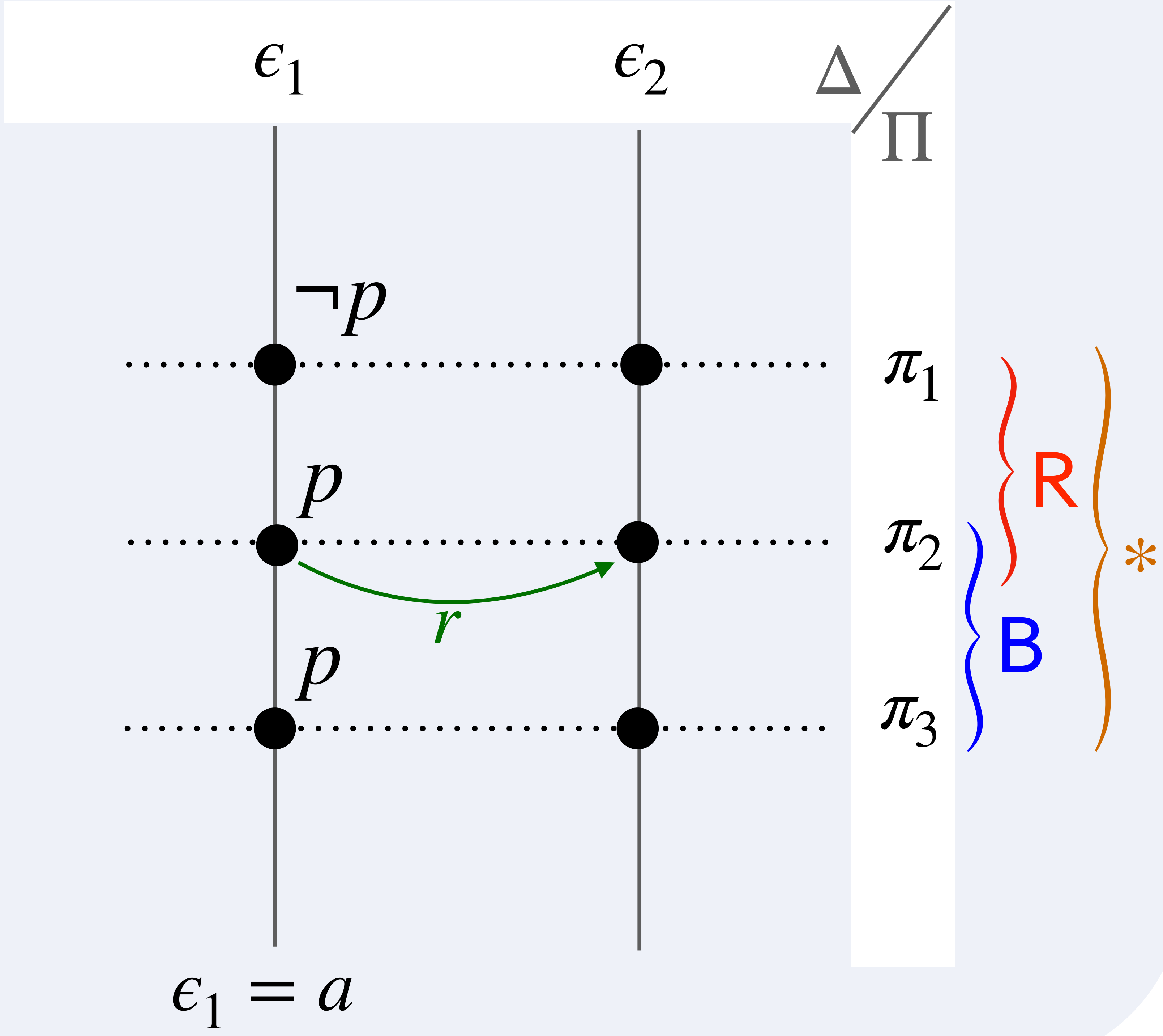

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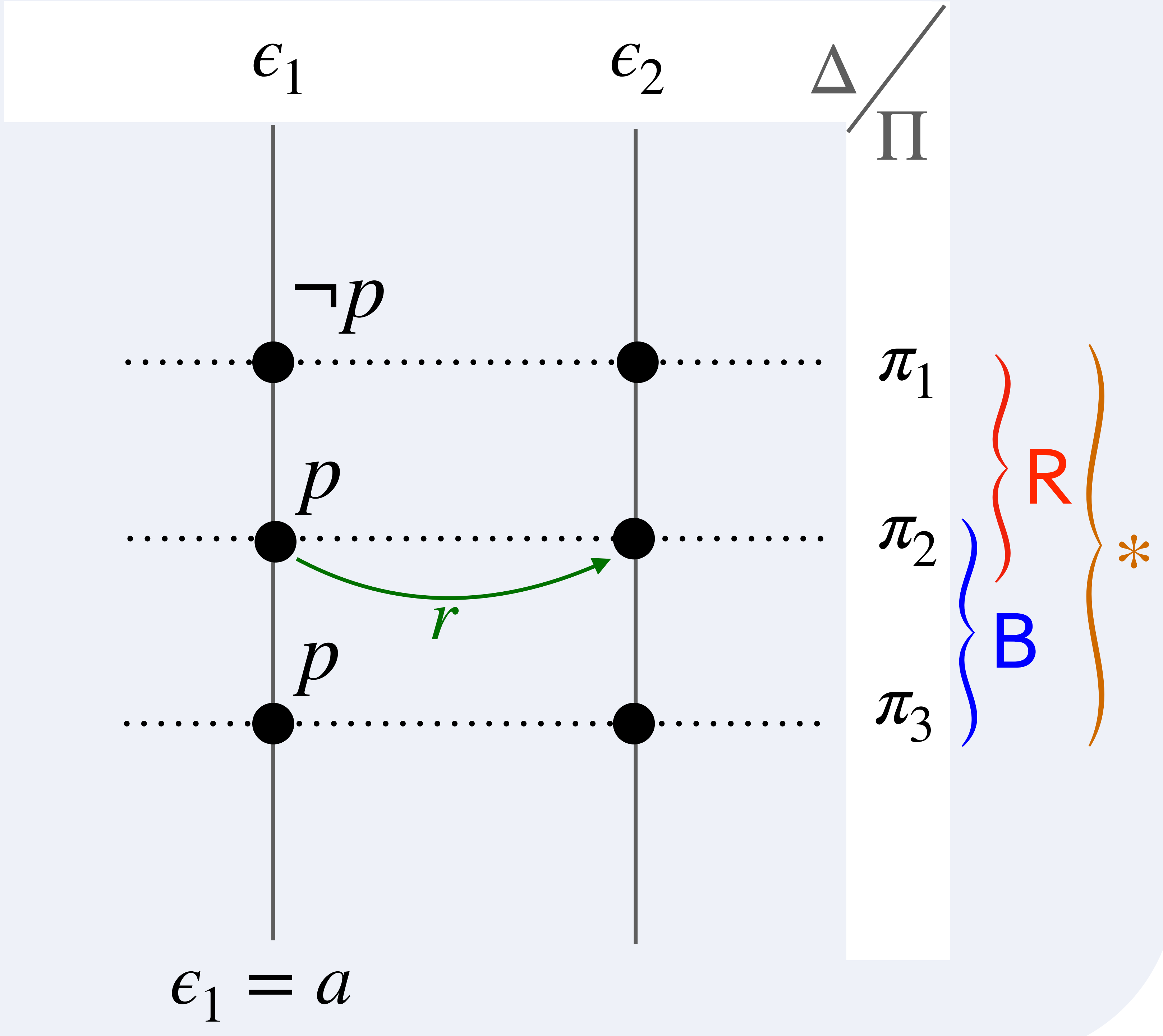

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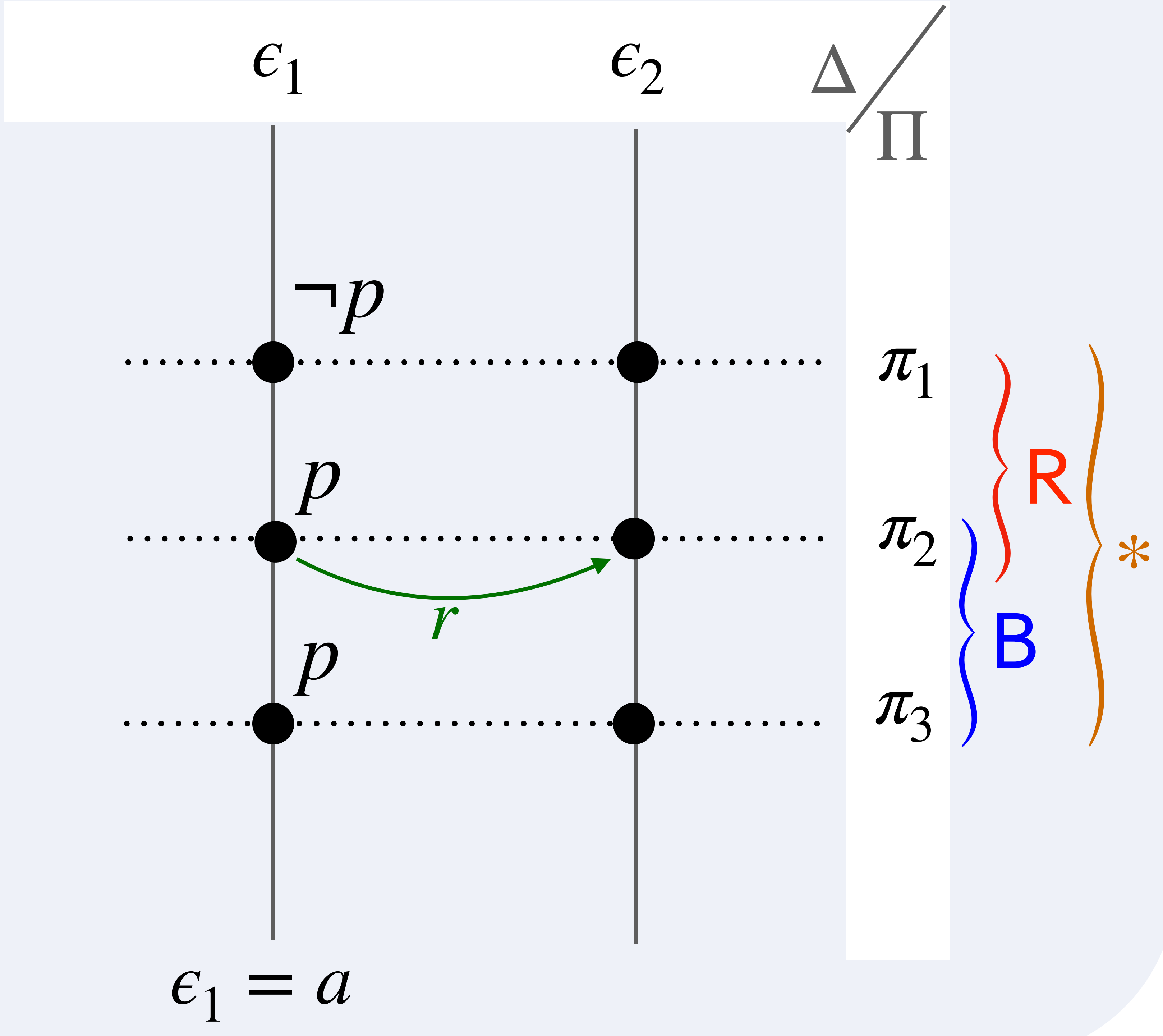

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- $\mathcal{M} \models \square_{\mathsf{B}}\, p(a)$
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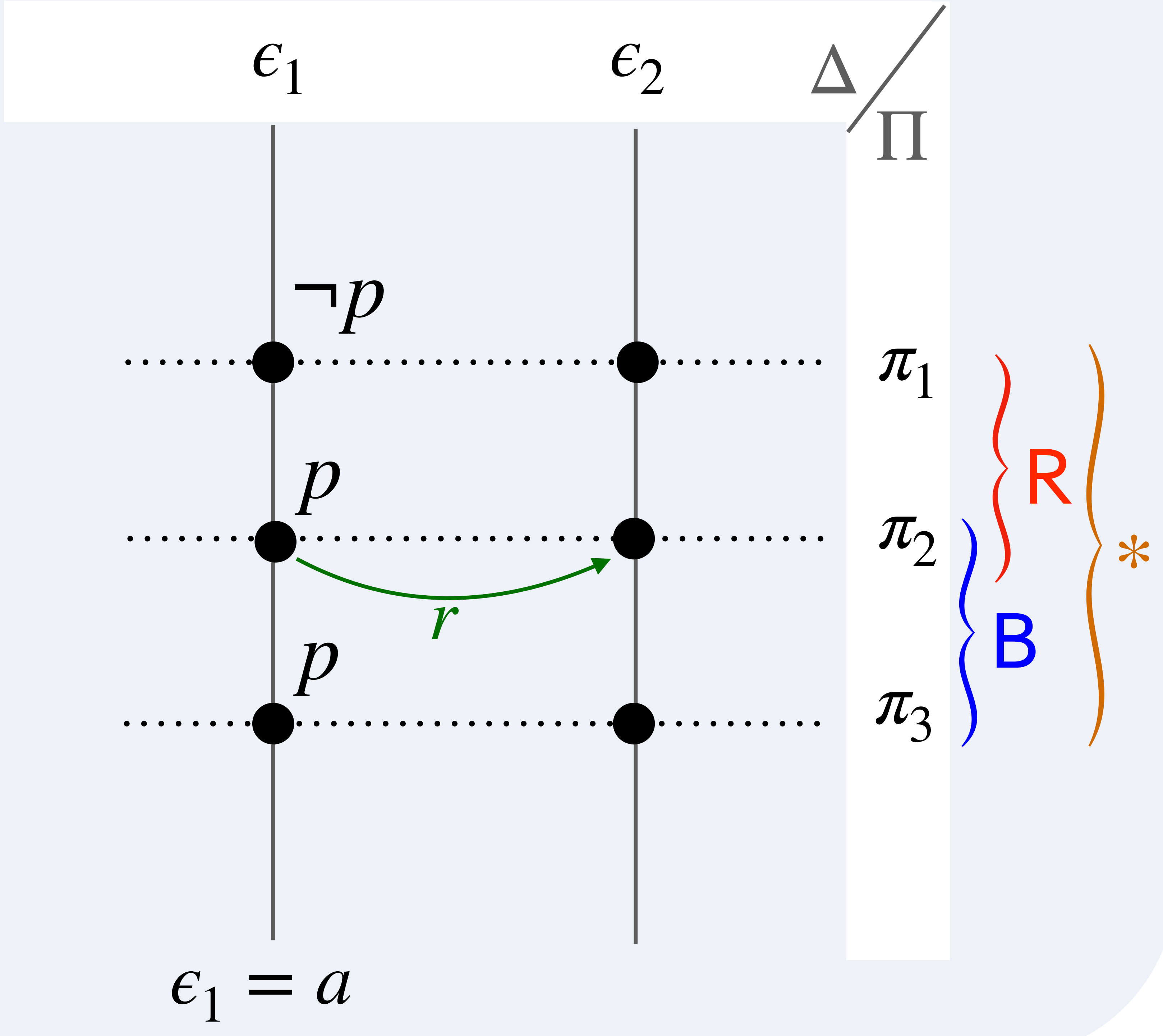

Remark: The semantics of standpoint logic can also be expressed in standard Kripke (relational) semantics.

Extended Example

Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg \exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$

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1. $\square_{\mathsf{S}}(\neg \exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
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4. $\mathsf{L} \preceq \mathsf{S} \ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow\ \square_{\mathsf{L}}\ \mathsf{Tumor}(x)$

Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $\text{L} \preceq \text{S} \ \wedge\ \text{H} \preceq \text{S}$
5. $\forall x\ \lozenge_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_L\ \text{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \lozenge_L \text{Tumor}(x)$

Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg \exists x \; \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\Box_H(\forall x \; \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_L(\forall x \; \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \;\wedge\; \mathsf{H} \preceq \mathsf{S}$
5. $\forall x \; \Diamond_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \Box_L \mathsf{Tumor}(x)$
6. $\forall x \; \Box_H (\exists y \; \mathsf{productOf}(x, y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L \mathsf{Tumor}(x)$
7. $\forall x \; \Box_L \mathsf{Tissue}(x) \rightarrow \Box_H \mathsf{Tissue}(x)$

Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \text{Process}(x) \land \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $\text{L} \preceq \text{S} \ \land\ \text{H} \preceq \text{S}$
5. $\forall x\ \lozenge_S(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \square_L\, \text{Tumor}(x)$
6. $\forall x\ \square_H\,(\exists y\ \text{productOf}(x, y) \land \text{Tumor}(y)) \leftrightarrow \lozenge_L \text{Tumor}(x)$
7. $\forall x\ \square_L\, \text{Tissue}(x) \rightarrow \square_H\, \text{Tissue}(x)$

8. $\lozenge_S(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

Knowledge Integration - Standpoint Logic

1. $\square_{S}(\neg \exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\square_{H}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\square_{L}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $\mathrm{L} \preceq \mathrm{S} \ \wedge \ \mathrm{H} \preceq \mathrm{S}$
5. $\forall x\ \lozenge_{S}(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_{L}\ \mathrm{Tumor}(x)$
6. $\forall x\ \square_{H}(\exists y\ \mathrm{productOf}(x, y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \lozenge_{L}\mathrm{Tumor}(x)$
7. $\forall x\ \square_{L}\ \mathrm{Tissue}(x) \rightarrow \square_{H}\ \mathrm{Tissue}(x)$

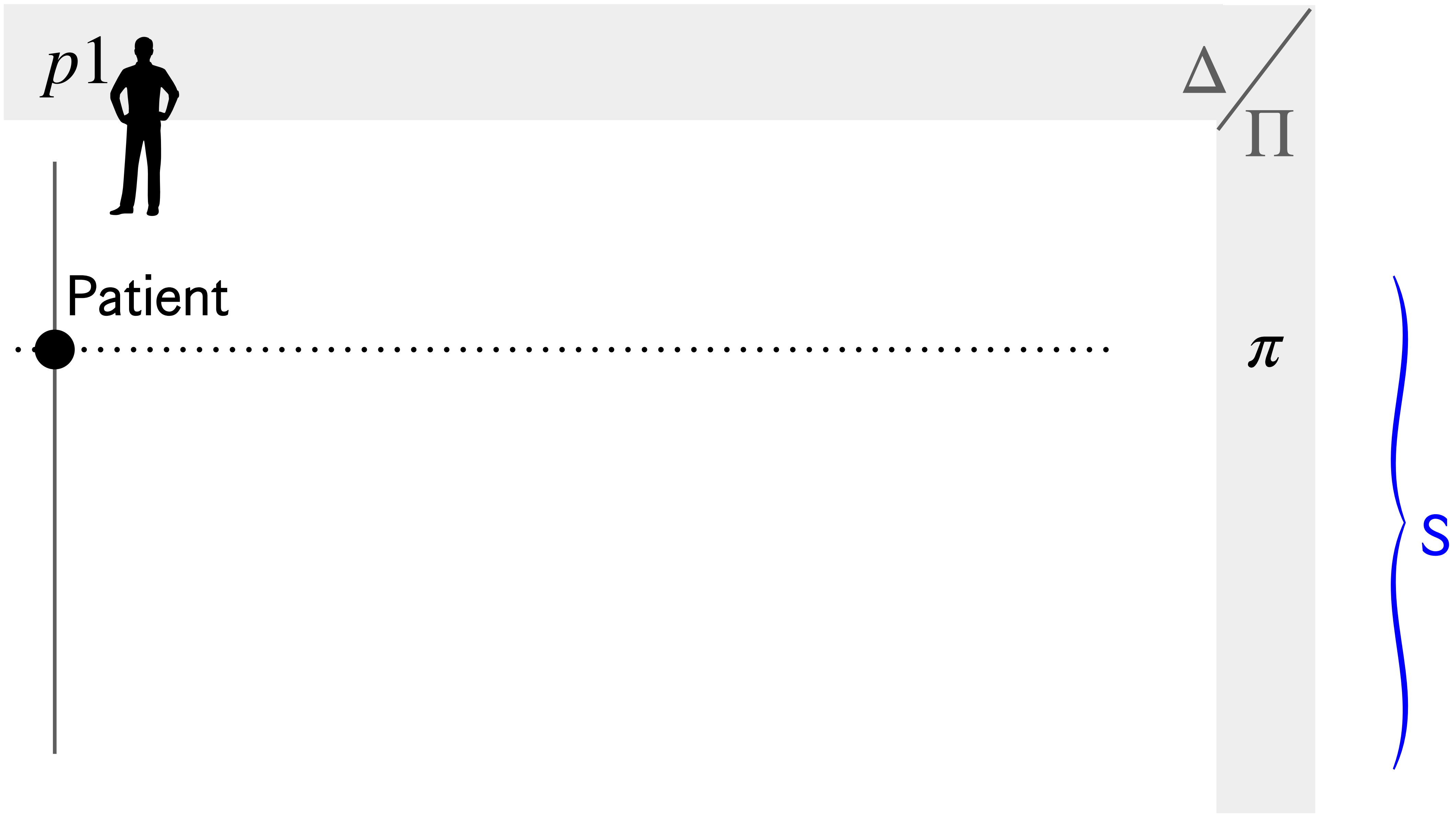
8. $\lozenge_{S}(\mathrm{Patient}(p1) \wedge \mathrm{HasPart}(p1,a) \wedge \mathrm{Tumor}(a) \wedge \mathrm{Object}(a))$

Δ / Π

Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $L \preceq S \ \wedge\ H \preceq S$
5. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_L\,\text{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \text{productOf}(x,y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_L\text{Tumor}(x)$
7. $\forall x\ \square_L\,\text{Tissue}(x) \rightarrow \square_H\,\text{Tissue}(x)$

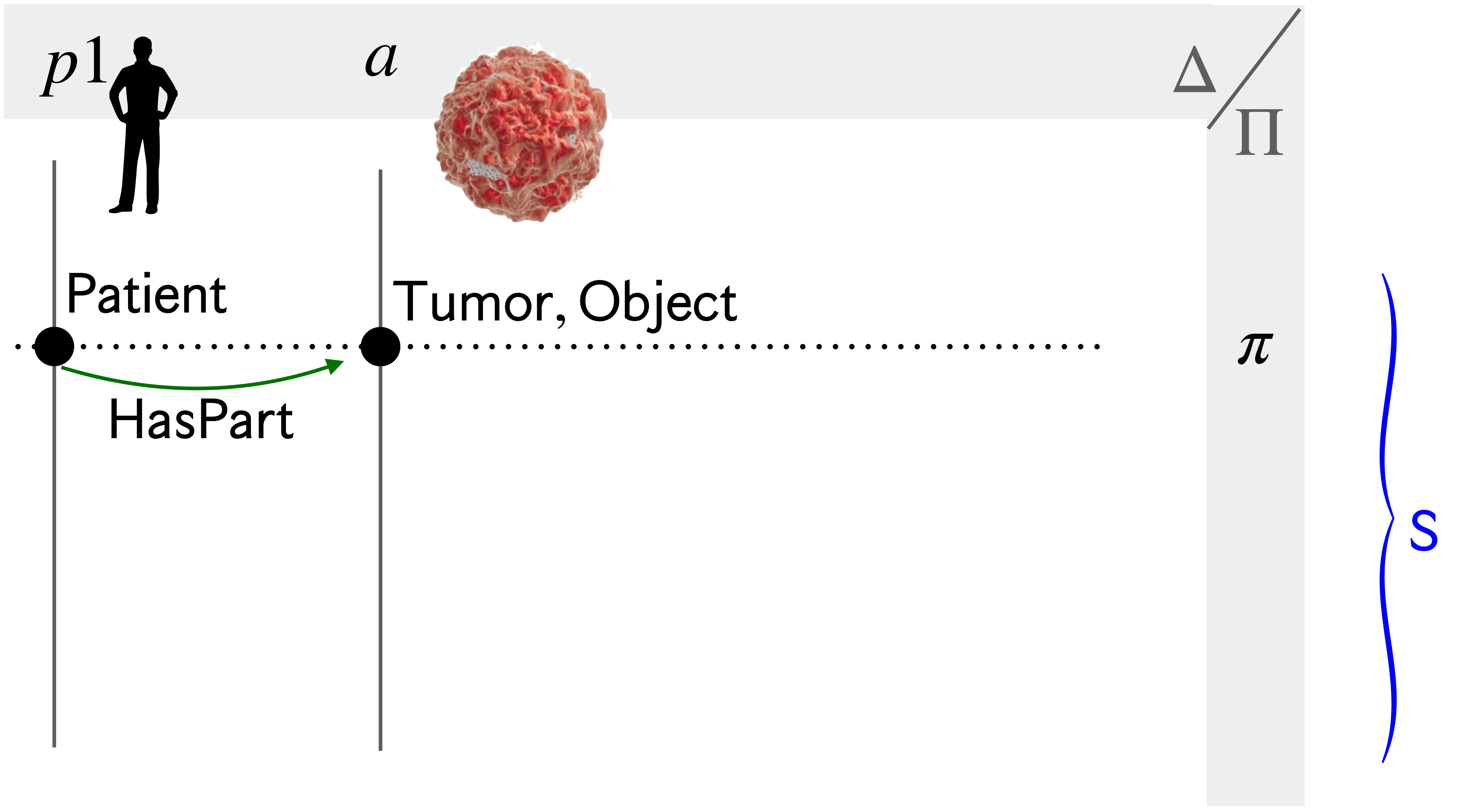
8. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$



Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \text{Process}(x) \land \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $L \preceq S \ \land\ H \preceq S$
5. $\forall x\ \lozenge_S(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \square_L \text{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \text{productOf}(x, y) \land \text{Tumor}(y)) \leftrightarrow \lozenge_L \text{Tumor}(x)$
7. $\forall x\ \square_L \text{Tissue}(x) \rightarrow \square_H \text{Tissue}(x)$

8. $\lozenge_S(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

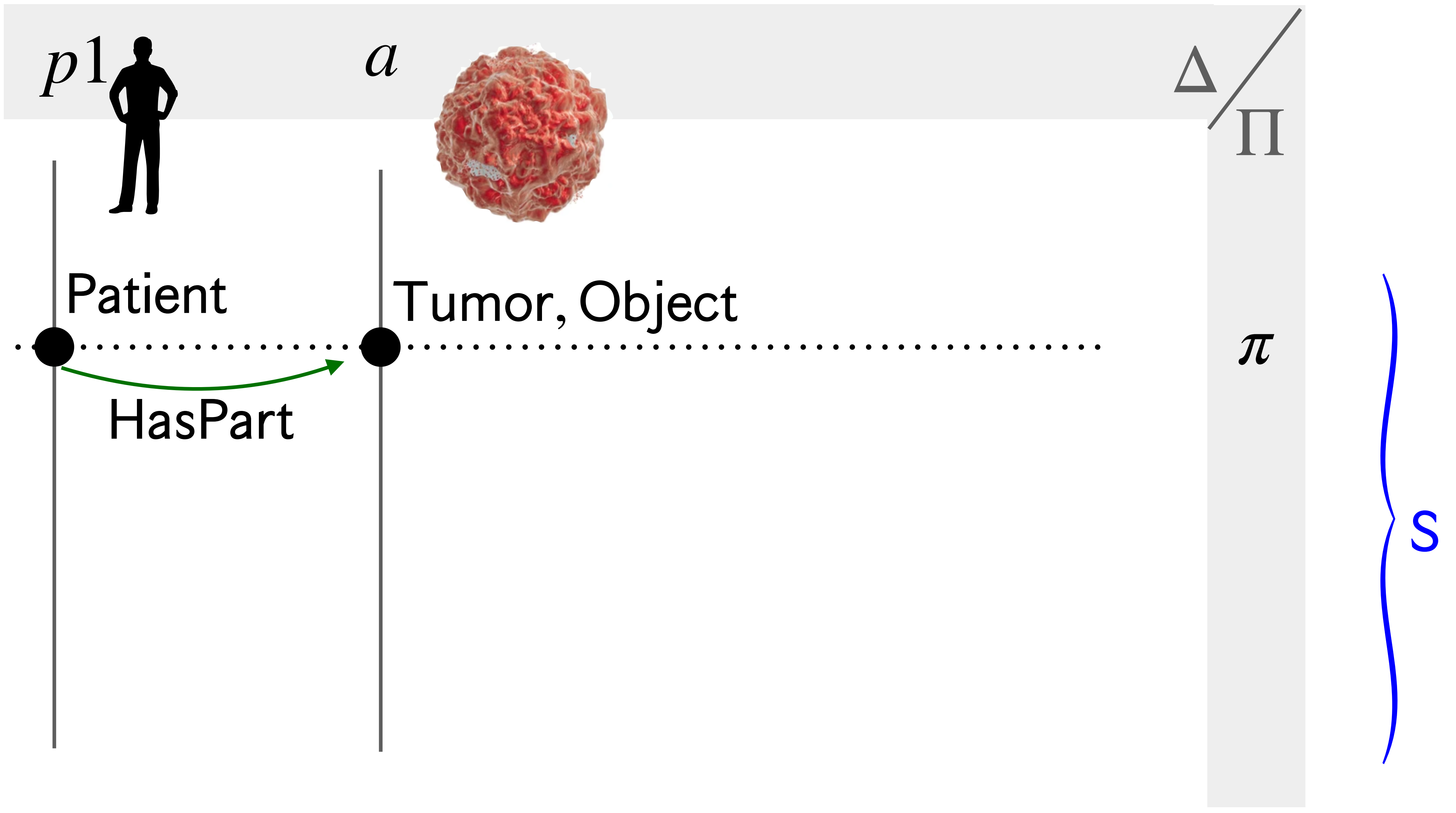


Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_L\ \mathsf{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_L \mathsf{Tumor}(x)$
7. $\forall x\ \square_L\ \mathsf{Tissue}(x) \rightarrow \square_H\ \mathsf{Tissue}(x)$

8. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:



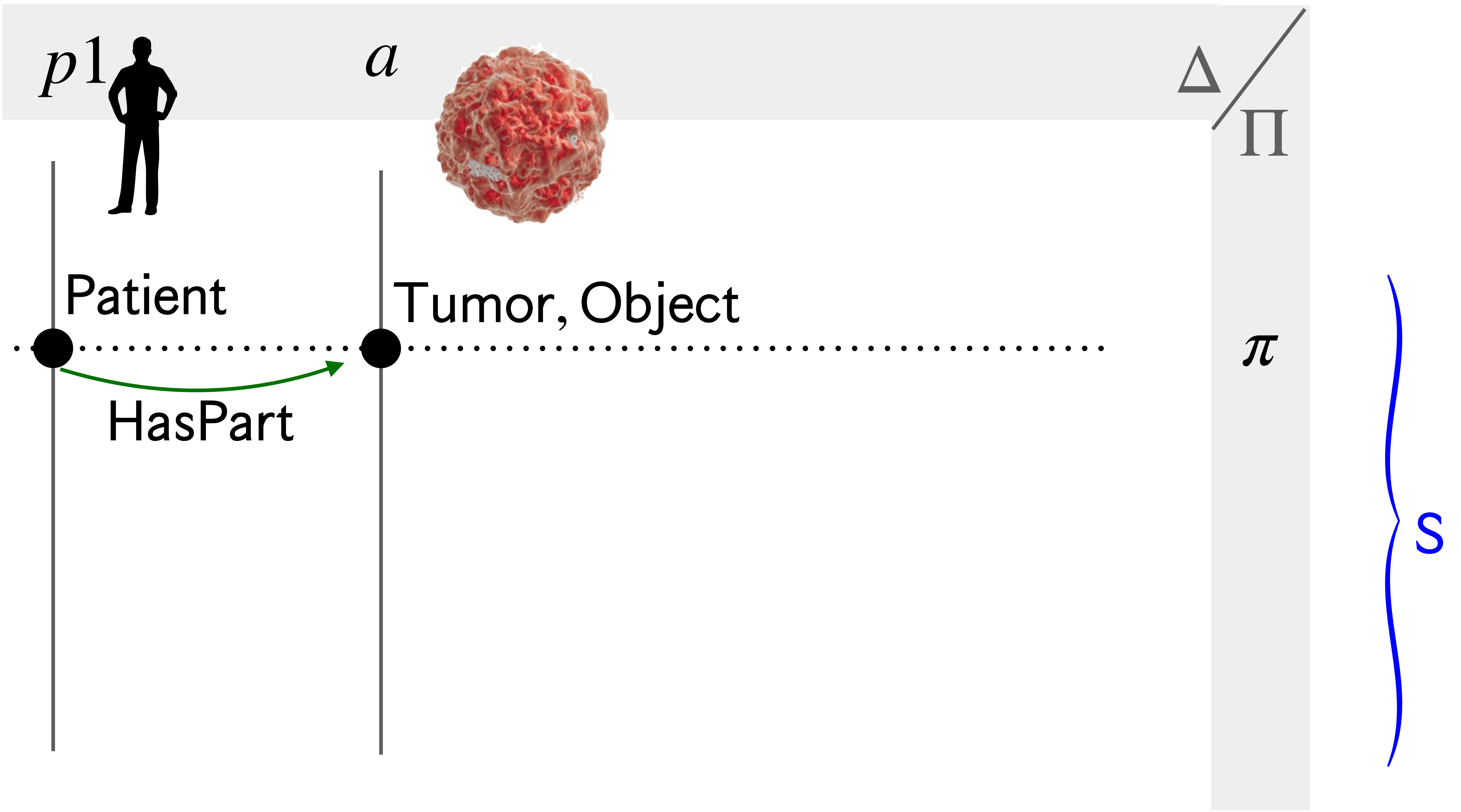
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\square_H(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\square_L(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $\mathrm{L} \preceq \mathrm{S}\ \wedge\ \mathrm{H} \preceq \mathrm{S}$
5. $\forall x\ \Diamond_S(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_L\ \mathrm{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \mathrm{productOf}(x, y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \Diamond_L \mathrm{Tumor}(x)$
7. $\forall x\ \square_L\ \mathrm{Tissue}(x) \rightarrow \square_H\ \mathrm{Tissue}(x)$

8. $\Diamond_S(\mathrm{Patient}(p1) \wedge \mathrm{HasPart}(p1,a) \wedge \mathrm{Tumor}(a) \wedge \mathrm{Object}(a))$

Inferences:

9. $\square_L \mathrm{Tumor}(a)$ (5,8)



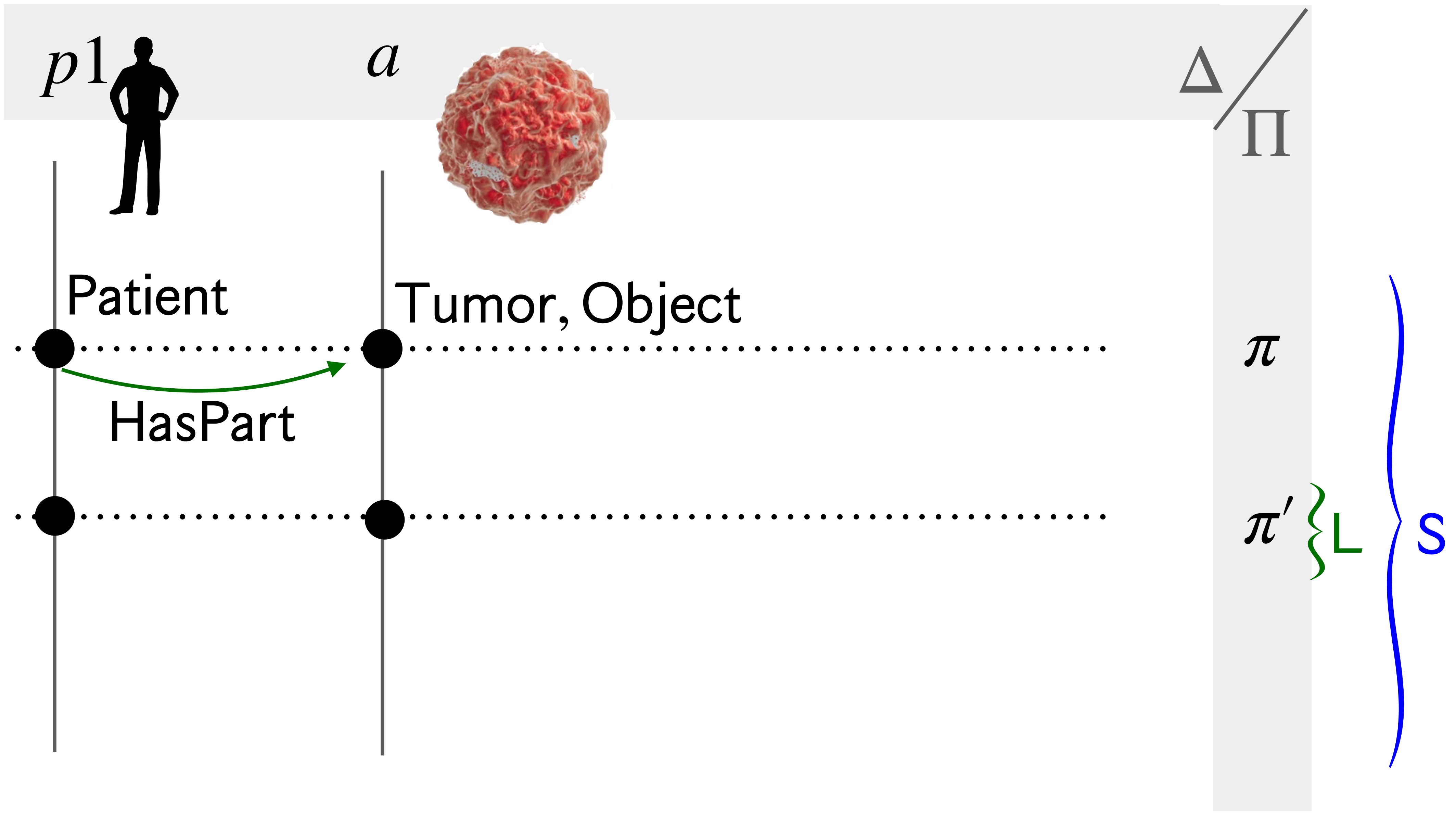
Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_{\mathsf{L}}\, \mathsf{Tumor}(x)$
6. $\forall x\ \square_{\mathsf{H}}(\exists y\ \mathsf{productOf}(x, y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_{\mathsf{L}}\mathsf{Tumor}(x)$
7. $\forall x\ \square_{\mathsf{L}}\, \mathsf{Tissue}(x) \rightarrow \square_{\mathsf{H}}\, \mathsf{Tissue}(x)$

8. $\lozenge_{\mathsf{S}}(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\square_{\mathsf{L}}\mathsf{Tumor}(a)$ (5,8)



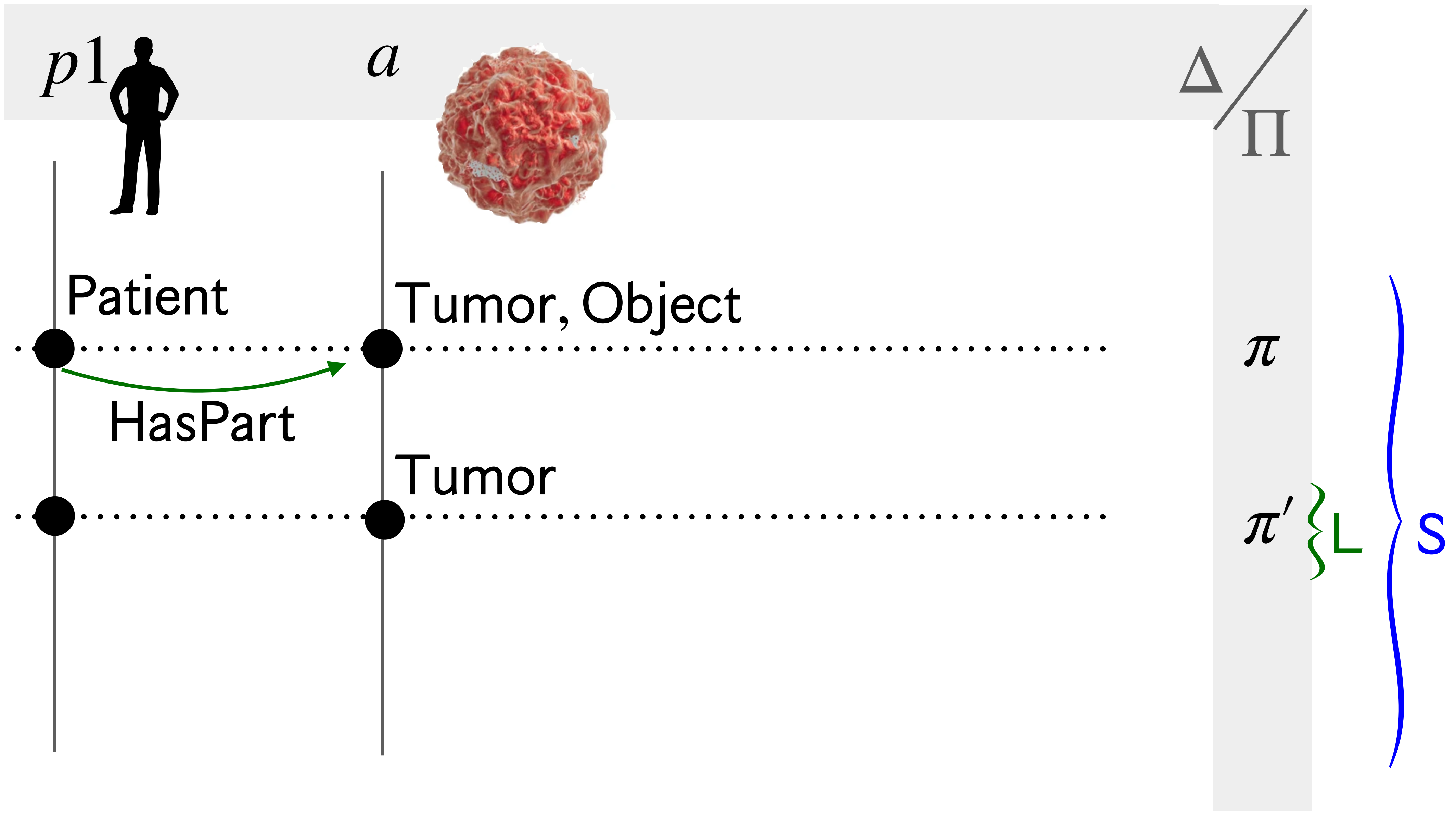
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_L\, \mathsf{Tumor}(x)$
6. $\forall x\ \square_H\, (\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_L \mathsf{Tumor}(x)$
7. $\forall x\ \square_L\, \mathsf{Tissue}(x) \rightarrow \square_H\, \mathsf{Tissue}(x)$

8. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\square_L \mathsf{Tumor}(a)$ (5,8)



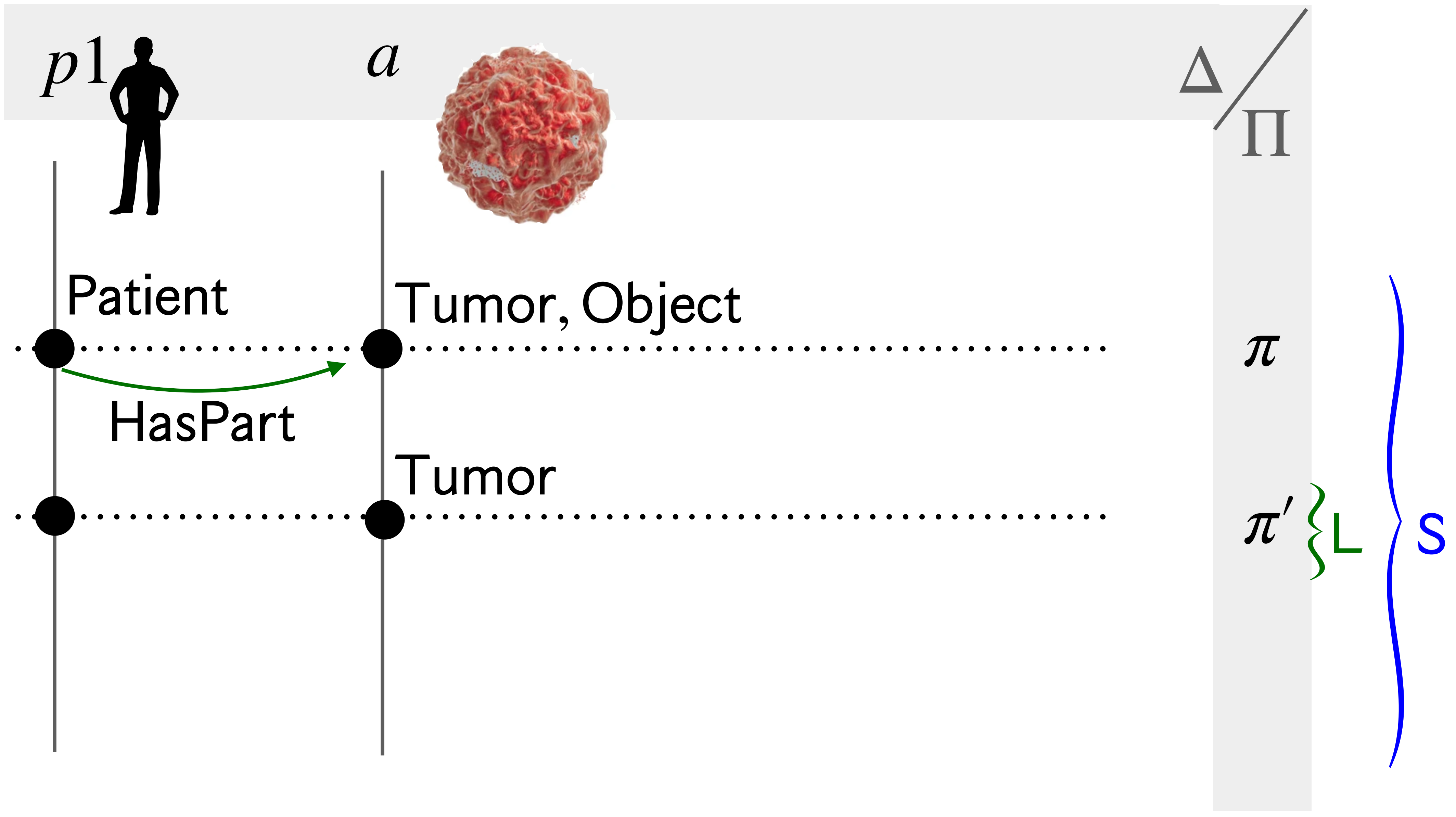
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathrm{Process}(x) \land \mathrm{Tissue}(x))$
2. $\Box_H(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\Box_L(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $L \preceq S \ \land \ H \preceq S$
5. $\forall x\ \Diamond_S(\mathrm{Tumor}(x) \land \mathrm{Object}(x)) \rightarrow \Box_L\ \mathrm{Tumor}(x)$
6. $\forall x\ \Box_H\ (\exists y\ \mathrm{productOf}(x, y) \land \mathrm{Tumor}(y)) \leftrightarrow \Diamond_L \mathrm{Tumor}(x)$
7. $\forall x\ \Box_L\ \mathrm{Tissue}(x) \rightarrow \Box_H\ \mathrm{Tissue}(x)$

8. $\Diamond_S(\mathrm{Patient}(p1) \land \mathrm{HasPart}(p1,a) \land \mathrm{Tumor}(a) \land \mathrm{Object}(a))$

Inferences:

9. $\Box_L \mathrm{Tumor}(a)$ (5,8)
10. $\Box_L \mathrm{Tissue}(a)$ (9,3)



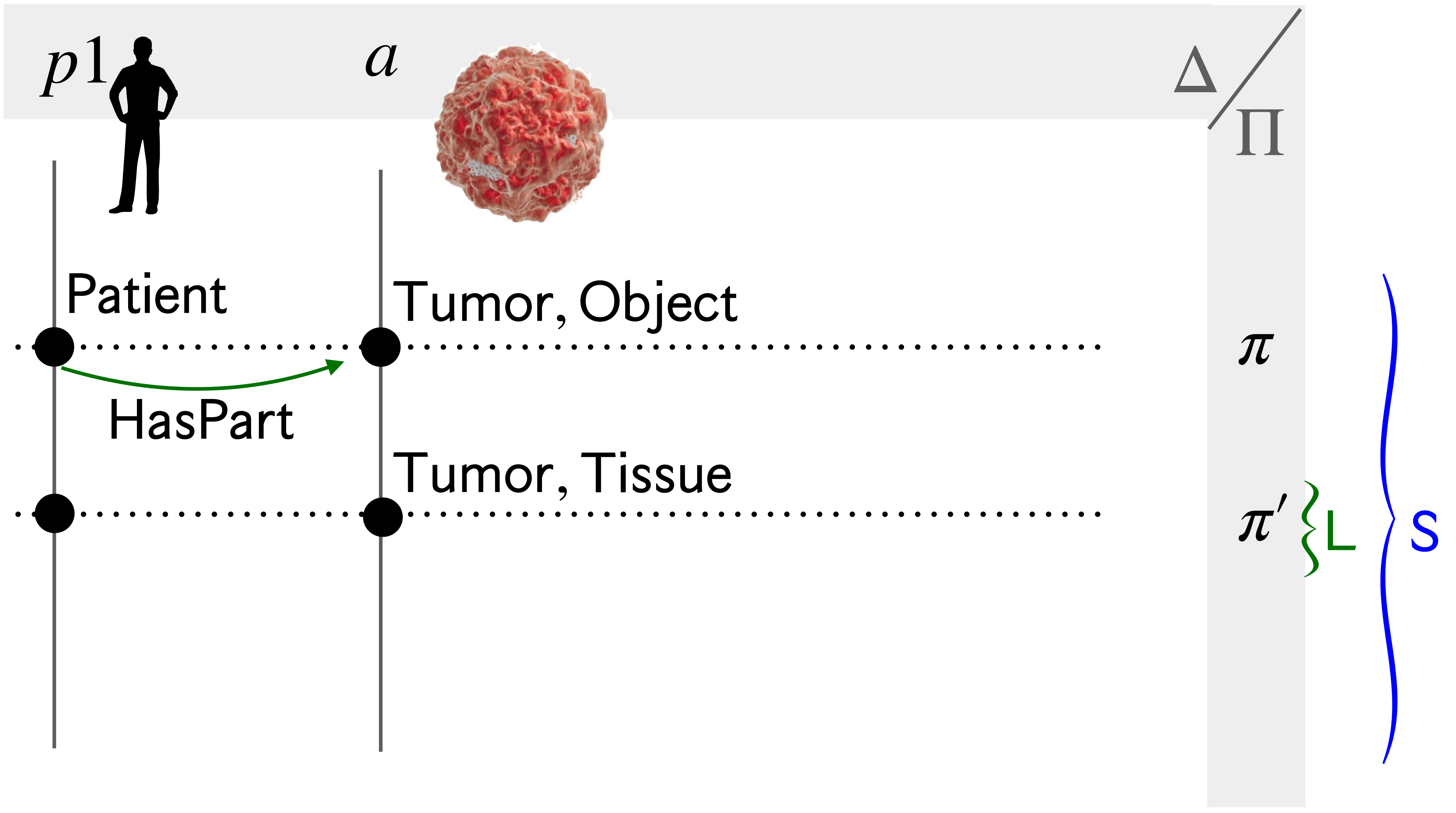
Knowledge Integration - Standpoint Logic

1. $\Box_{\mathsf{S}}(\neg \exists x \; \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\Box_{\mathsf{H}}(\forall x \; \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_{\mathsf{L}}(\forall x \; \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \;\wedge\; \mathsf{H} \preceq \mathsf{S}$
5. $\forall x \; \Diamond_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \Box_{\mathsf{L}} \mathsf{Tumor}(x)$
6. $\forall x \; \Box_{\mathsf{H}} (\exists y \; \mathsf{productOf}(x, y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \Diamond_{\mathsf{L}} \mathsf{Tumor}(x)$
7. $\forall x \; \Box_{\mathsf{L}} \mathsf{Tissue}(x) \rightarrow \Box_{\mathsf{H}} \mathsf{Tissue}(x)$

8. $\Diamond_{\mathsf{S}}(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\Box_{\mathsf{L}} \mathsf{Tumor}(a)$ (5,8)
10. $\Box_{\mathsf{L}} \mathsf{Tissue}(a)$ (9,3)



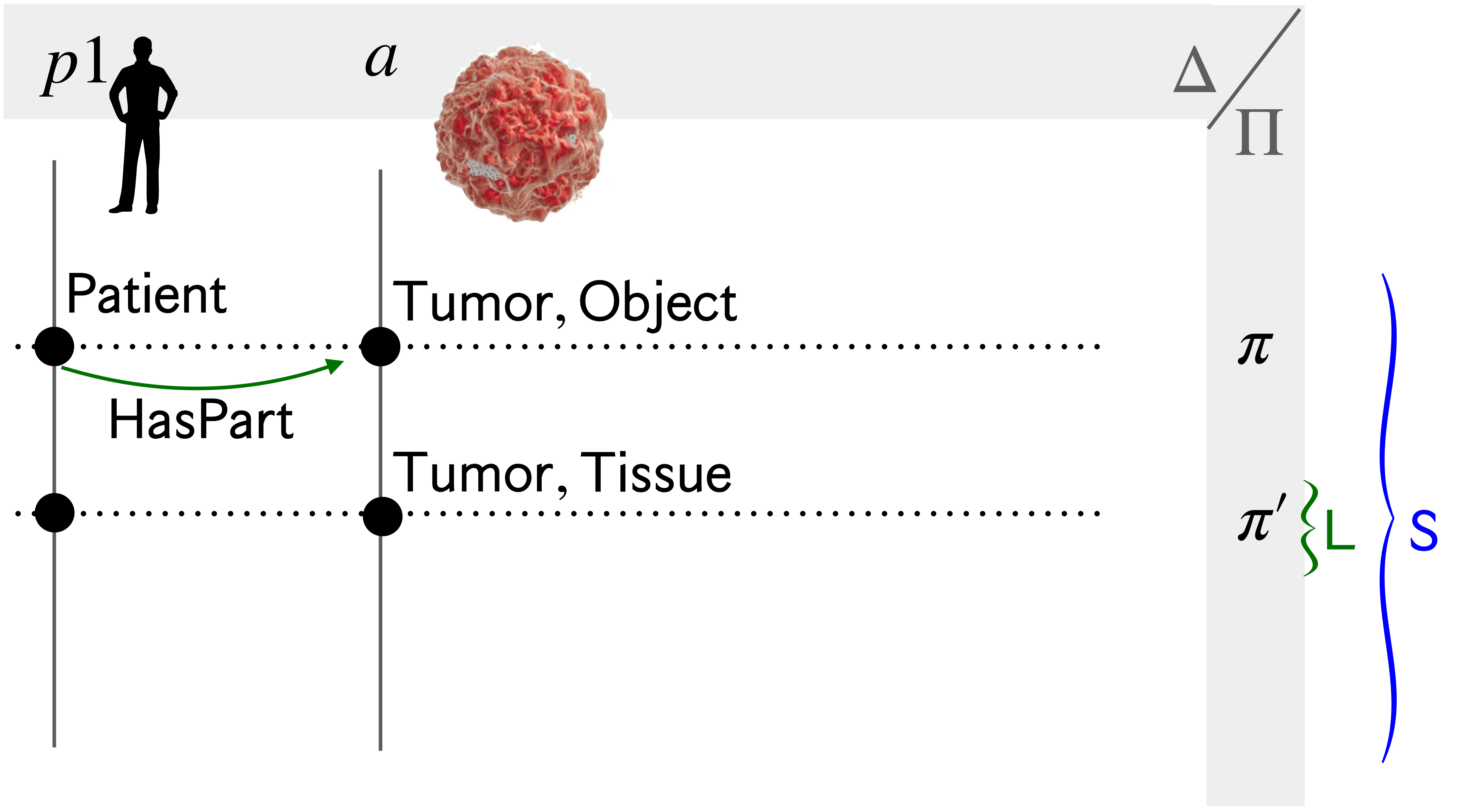
Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_{\mathsf{L}}\,\mathsf{Tumor}(x)$
6. $\forall x\ \square_{\mathsf{H}}(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_{\mathsf{L}}\mathsf{Tumor}(x)$
7. $\forall x\ \square_{\mathsf{L}}\,\mathsf{Tissue}(x) \rightarrow \square_{\mathsf{H}}\,\mathsf{Tissue}(x)$

8. $\lozenge_{\mathsf{S}}(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\square_{\mathsf{L}}\mathsf{Tumor}(a)$ (5,8)
10. $\square_{\mathsf{L}}\mathsf{Tissue}(a)$ (9,3)
11. $\square_{\mathsf{H}}\mathsf{Tissue}(a)$ (10,7)



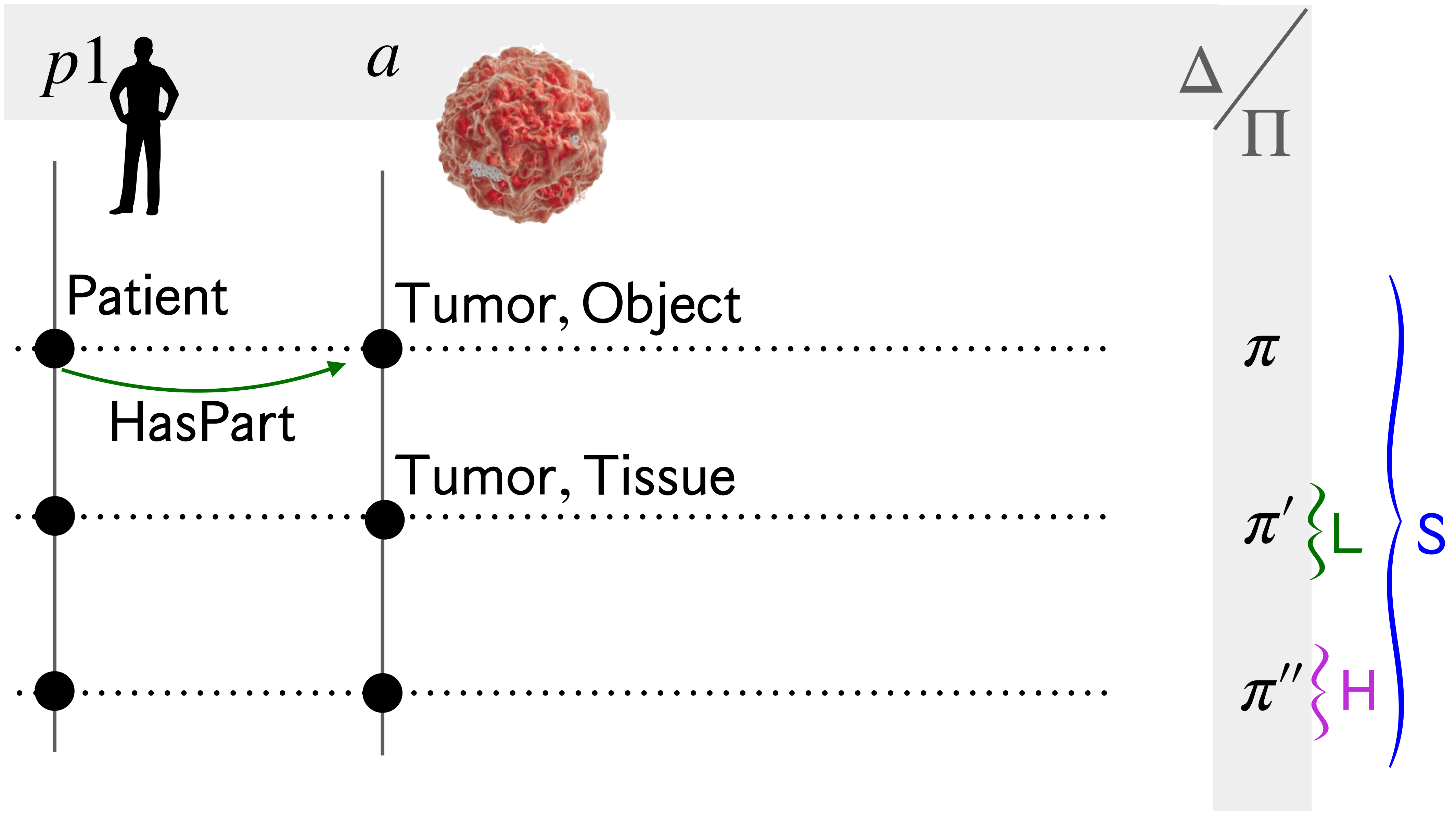
Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg \exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_{\mathsf{L}}\ \mathsf{Tumor}(x)$
6. $\forall x\ \square_{\mathsf{H}}\ (\exists y\ \mathsf{productOf}(x, y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_{\mathsf{L}}\mathsf{Tumor}(x)$
7. $\forall x\ \square_{\mathsf{L}}\ \mathsf{Tissue}(x) \rightarrow \square_{\mathsf{H}}\ \mathsf{Tissue}(x)$

8. $\lozenge_{\mathsf{S}}(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\square_{\mathsf{L}}\mathsf{Tumor}(a)$ (5,8)
10. $\square_{\mathsf{L}}\mathsf{Tissue}(a)$ (9,3)
11. $\square_{\mathsf{H}}\mathsf{Tissue}(a)$ (10,7)



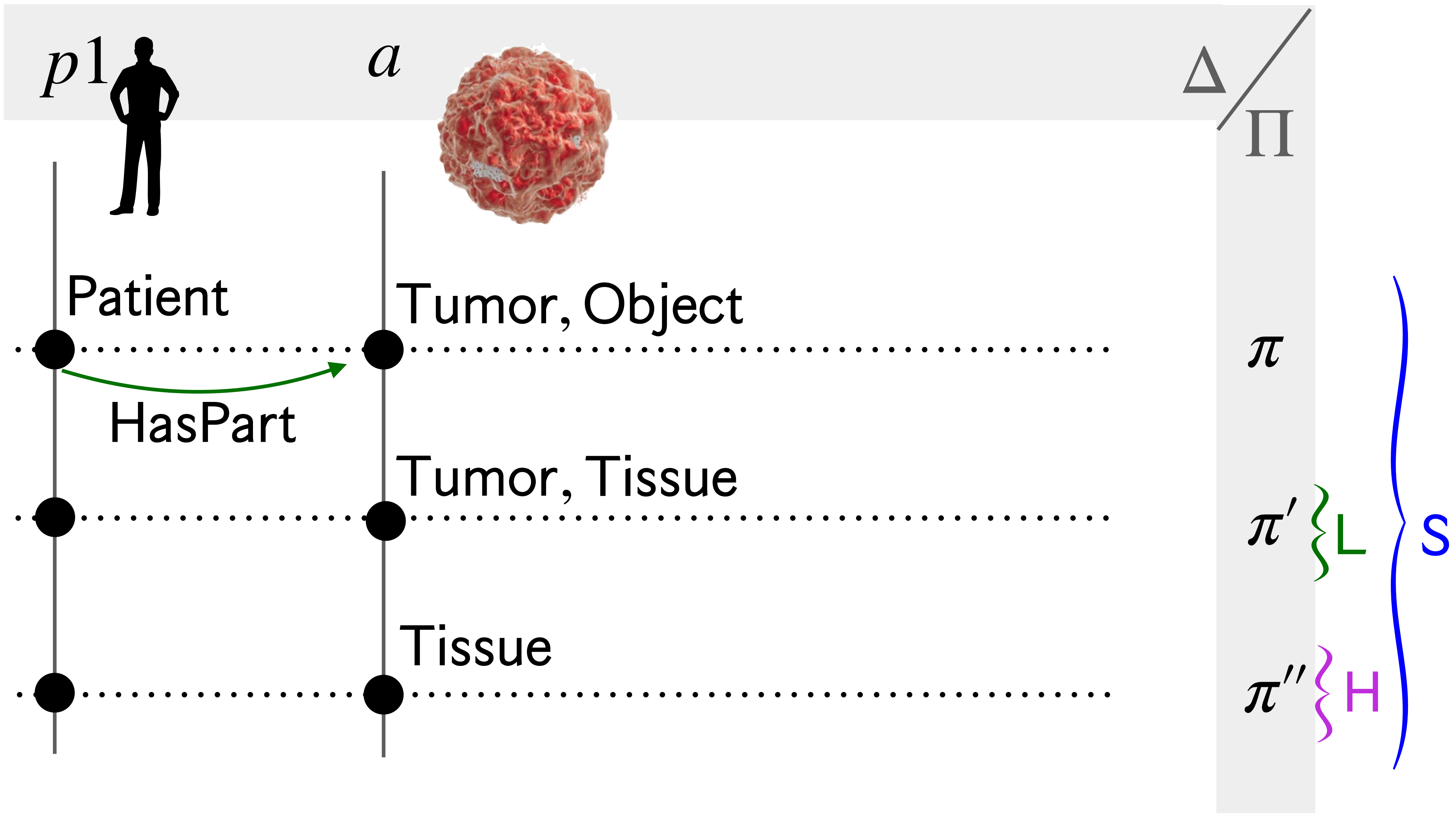
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \text{Process}(x) \land \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $L \preceq S \ \land \ H \preceq S$
5. $\forall x\ \Diamond_S(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \square_L \text{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \text{productOf}(x, y) \land \text{Tumor}(y)) \leftrightarrow \Diamond_L \text{Tumor}(x)$
7. $\forall x\ \square_L \text{Tissue}(x) \rightarrow \square_H \text{Tissue}(x)$

8. $\Diamond_S(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

Inferences:

9. $\square_L \text{Tumor}(a)$ (5,8)
10. $\square_L \text{Tissue}(a)$ (9,3)
11. $\square_H \text{Tissue}(a)$ (10,7)



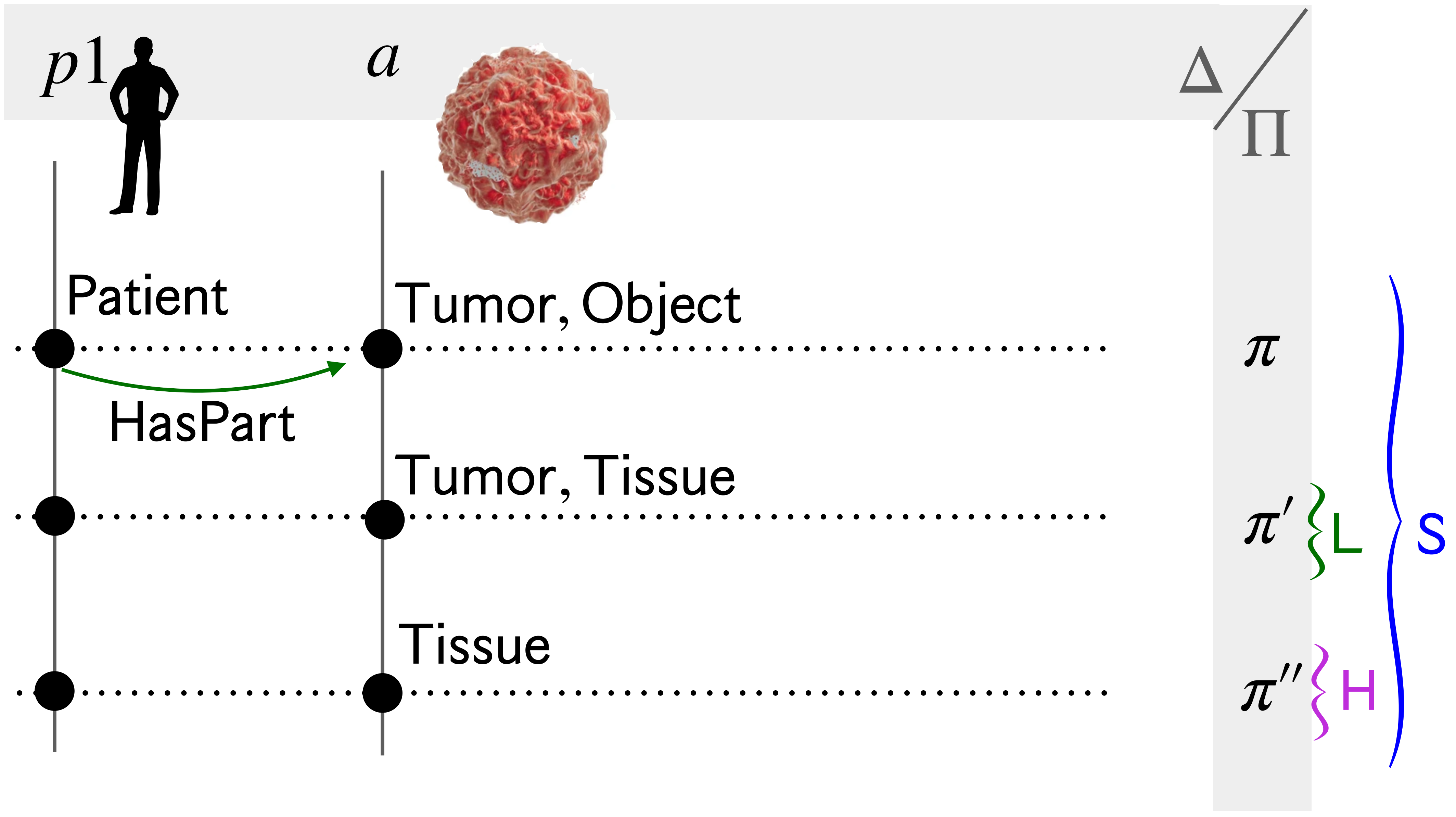
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow\ \square_L\ \mathsf{Tumor}(x)$
6. $\forall x\ \square_H\ (\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow\ \lozenge_L \mathsf{Tumor}(x)$
7. $\forall x\ \square_L\ \mathsf{Tissue}(x) \rightarrow\ \square_H\ \mathsf{Tissue}(x)$

8. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\square_L \mathsf{Tumor}(a)$ (5,8)
10. $\square_L \mathsf{Tissue}(a)$ (9,3)
11. $\square_H \mathsf{Tissue}(a)$ (10,7)
12. $\square_H\ \exists y\ \mathsf{productOf}(a,y) \wedge \mathsf{Tumor}(y)$ (9,6)



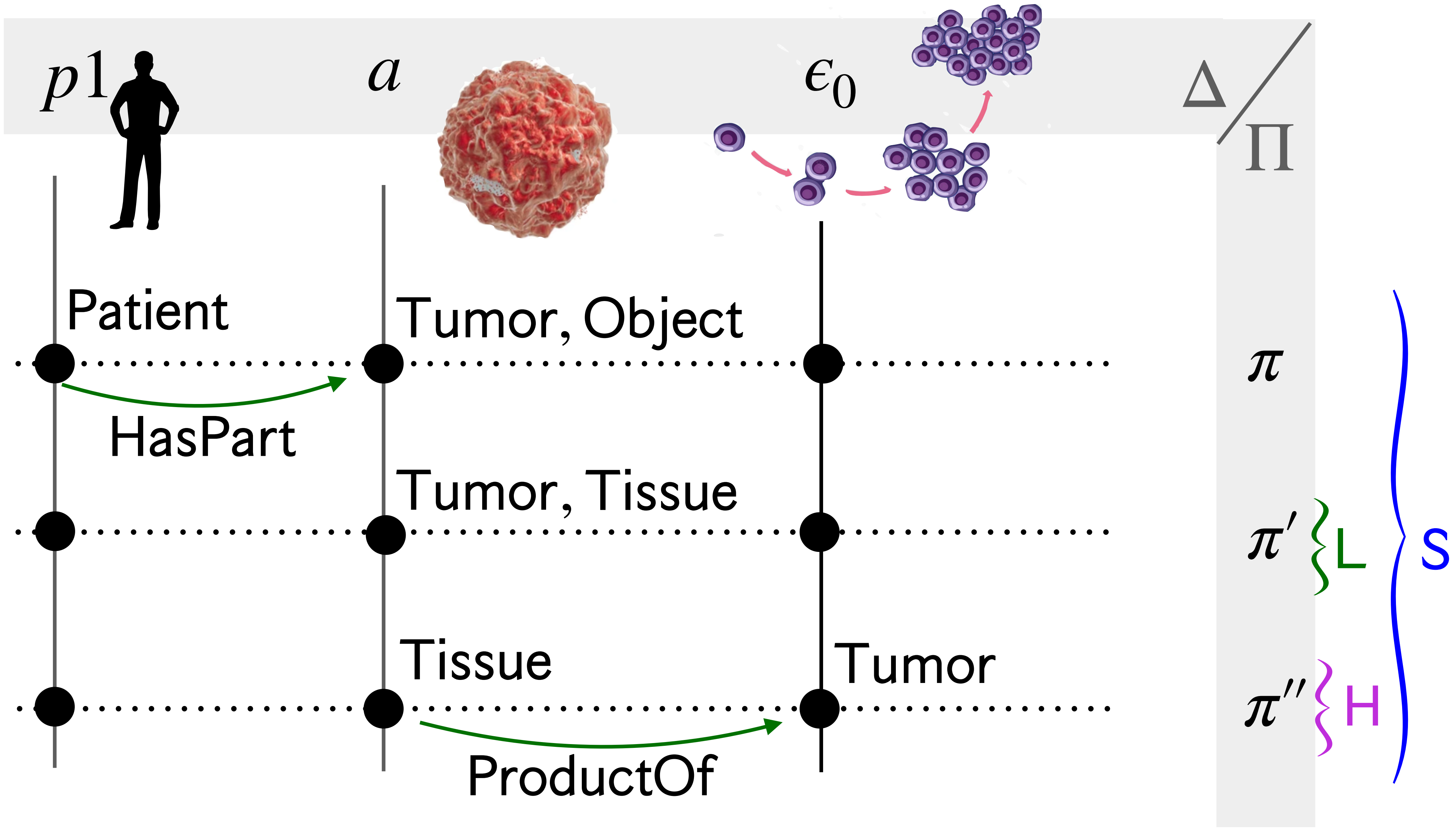
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\square_H(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\square_L(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $\mathrm{L} \preceq \mathrm{S}\ \wedge\ \mathrm{H} \preceq \mathrm{S}$
5. $\forall x\ \lozenge_S(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_L\ \mathrm{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \mathrm{productOf}(x,y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \lozenge_L \mathrm{Tumor}(x)$
7. $\forall x\ \square_L\ \mathrm{Tissue}(x) \rightarrow \square_H\ \mathrm{Tissue}(x)$

8. $\lozenge_S(\mathrm{Patient}(p1) \wedge \mathrm{HasPart}(p1,a) \wedge \mathrm{Tumor}(a) \wedge \mathrm{Object}(a))$

Inferences:

9. $\square_L \mathrm{Tumor}(a)$ (5,8)
10. $\square_L \mathrm{Tissue}(a)$ (9,3)
11. $\square_H \mathrm{Tissue}(a)$ (10,7)
12. $\square_H\ \exists y\ \mathrm{productOf}(a,y) \wedge \mathrm{Tumor}(y)$ (9,6)



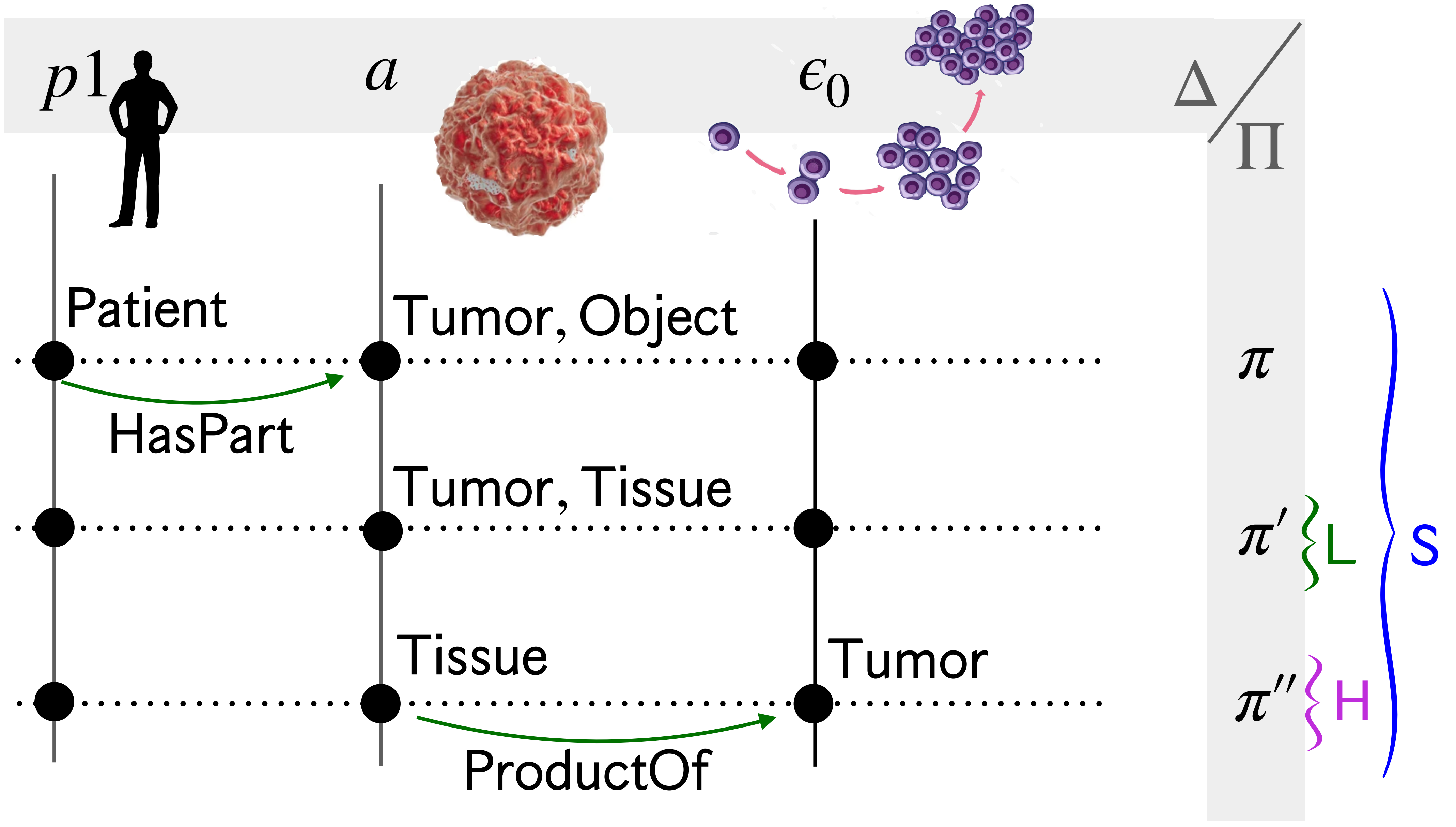
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $\text{L} \preceq \text{S} \ \wedge\ \text{H} \preceq \text{S}$
5. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_L\ \text{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \text{productOf}(x,y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_L \text{Tumor}(x)$
7. $\forall x\ \square_L\ \text{Tissue}(x) \rightarrow \square_H\ \text{Tissue}(x)$

8. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

Inferences:

9. $\square_L \text{Tumor}(a)$ (5,8)
10. $\square_L \text{Tissue}(a)$ (9,3)
11. $\square_H \text{Tissue}(a)$ (10,7)
12. $\square_H\ \exists y\ \text{productOf}(a,y) \wedge \text{Tumor}(y)$ (9,6)
13. $\square_H\ \exists y\ \text{Process}(y)$ (12,2)



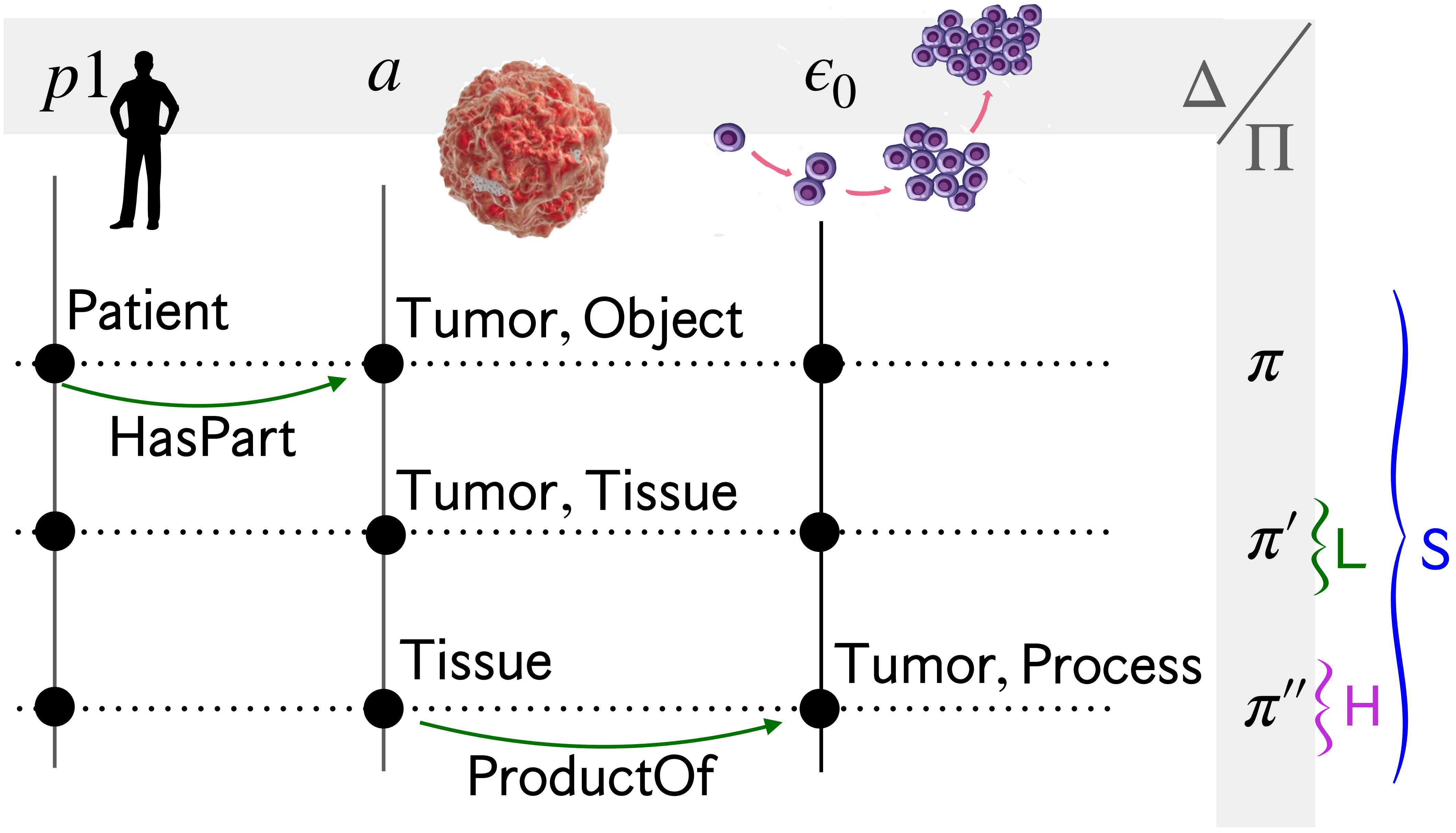
Knowledge Integration - Standpoint Logic

1. $\Box_{\mathsf{S}}(\neg \exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\Box_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \land\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_{\mathsf{S}}(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \Box_{\mathsf{L}}\ \mathsf{Tumor}(x)$
6. $\forall x\ \Box_{\mathsf{H}}\,(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \Diamond_{\mathsf{L}}\mathsf{Tumor}(x)$
7. $\forall x\ \Box_{\mathsf{L}}\ \mathsf{Tissue}(x) \rightarrow \Box_{\mathsf{H}}\ \mathsf{Tissue}(x)$

8. $\Diamond_{\mathsf{S}}(\mathsf{Patient}(p1) \land \mathsf{HasPart}(p1,a) \land \mathsf{Tumor}(a) \land \mathsf{Object}(a))$

Inferences:

9. $\Box_{\mathsf{L}}\mathsf{Tumor}(a)$ (5,8)
10. $\Box_{\mathsf{L}}\mathsf{Tissue}(a)$ (9,3)
11. $\Box_{\mathsf{H}}\mathsf{Tissue}(a)$ (10,7)
12. $\Box_{\mathsf{H}}\ \exists y\ \mathsf{productOf}(a,y) \land \mathsf{Tumor}(y)$ (9,6)
13. $\Box_{\mathsf{H}}\ \exists y\ \mathsf{Process}(y)$ (12,2)



Decidability and Complexity

Decidability and Complexity

Goal: Understanding computational cost of reasoning with standpoint-KR languages

We consider fragments of FOSL.

Important distinction: how much do standpoint operators and quantifiers interleave?

* Sentential fragments often preserve complexity, but have limitations
* Monodic fragments have important applications in knowledge integration
* Beyond monodic, modal logics easily become undecidable

+ liberal use of modal operators

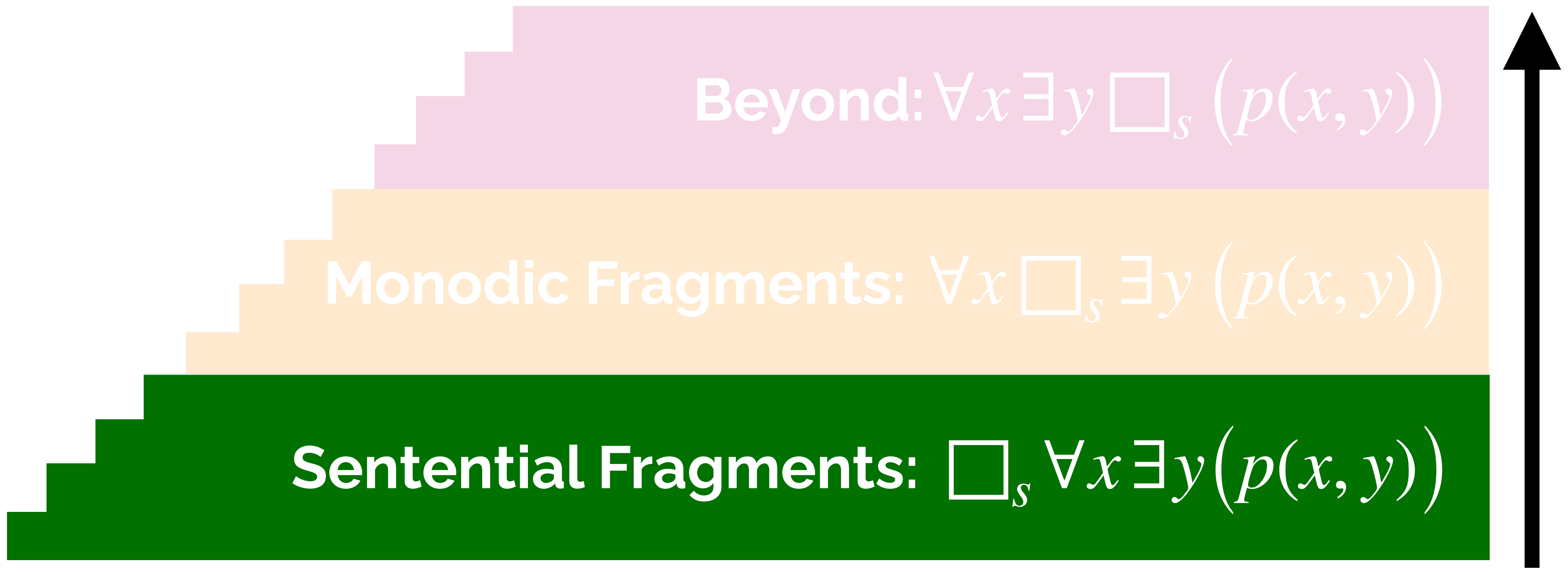
+ technically challenging

Beyond: $\forall x \, \exists y \, \Box_s \big(p(x,y)\big)$

Monodic Fragments: $\forall x \, \Box_s \, \exists y \, \big(p(x,y)\big)$

Sentential Fragments: $\Box_s \, \forall x \, \exists y \big(p(x,y)\big)$

Sentential Fragments



Sentential Fragments

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Sentential Fragments

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Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\Box_e \psi$, all variables occurring in ψ are bound by a quantifier.

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Eg.:

Sentential Fragments

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Eg.: 1. $\square_S$[∀x Tumor(x)→(Process(x) ∨ Tissue(x))] ✓

Sentential Fragments

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Eg.:

1. $\square_S[\forall x\ \mathrm{Tumor}(x) \rightarrow (\mathrm{Process}(x) \lor \mathrm{Tissue}(x))]$ ✓
2. $\Diamond_T[\top] \land \Diamond_P[\top]$ ✓

Sentential Fragments

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Eg.:

1. $\Box_S[\forall x\ \mathrm{Tumor}(x) \rightarrow (\mathrm{Process}(x) \vee \mathrm{Tissue}(x))]$ ✓
2. $\Diamond_T[\top] \wedge \Diamond_P[\top]$ ✓
3. $\forall x \Box_P[\mathrm{Tissue}(x)] \rightarrow \Box_T[\mathrm{Tissue}(x)]$ ✗

Sentential Fragments

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Eg.:

1. $\square_S[\forall x\ \mathrm{Tumor}(x) \rightarrow (\mathrm{Process}(x) \vee \mathrm{Tissue}(x))]$ ✓
2. $\lozenge_T[\top] \wedge \lozenge_P[\top]$ ✓
3. $\forall x \square_P[\mathrm{Tissue}(x)] \rightarrow \square_T[\mathrm{Tissue}(x)]$ ✗

Theorem (Small Model Property):
A sentential FOSL formula ϕ is satisfiable iff it has a model with at most $|\phi|$ precisifications. That is, for sentential FOSL, satisfiability and $|\phi|$-satisfiability coincide.

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\text{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \text{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\text{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
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Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

Sentential Fragments

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FOL model

Sentential Fragments

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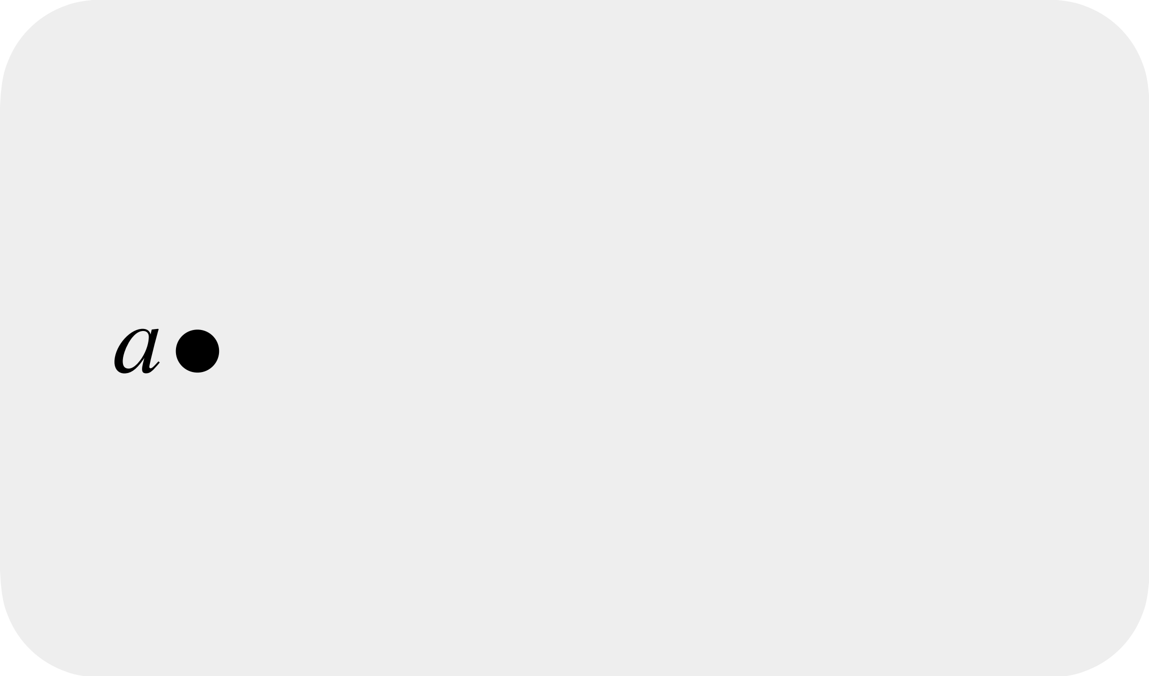

FOL model

Sentential Fragments

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	$\neg p_1$	$*_1$	R_1	$\neg\mathsf{B}_1$
$a\bullet$	p_2	$*_2$	R_2	B_2
	p_3	$*_3$	$\neg\mathsf{R}_3$	B_3

FOL model

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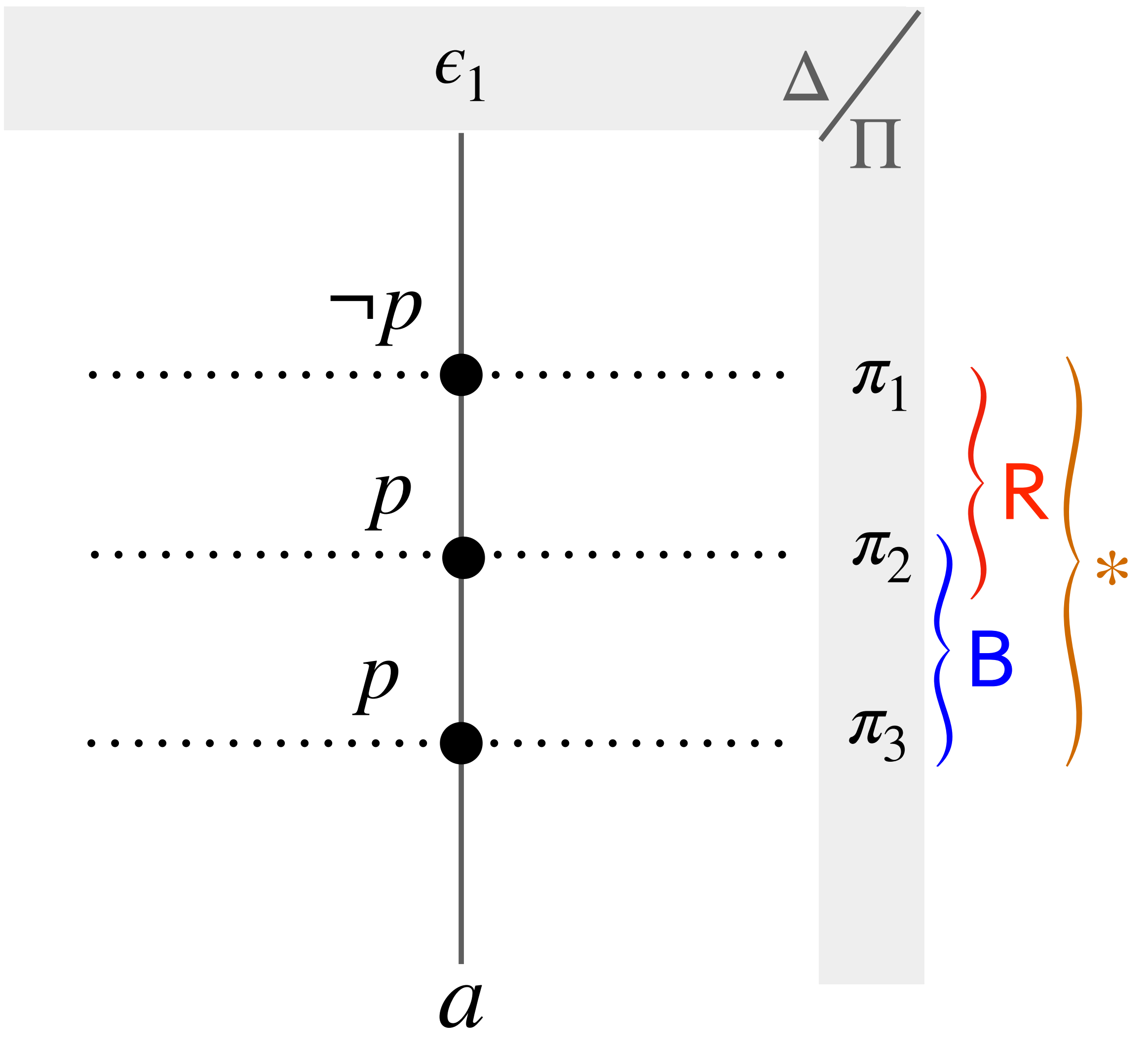
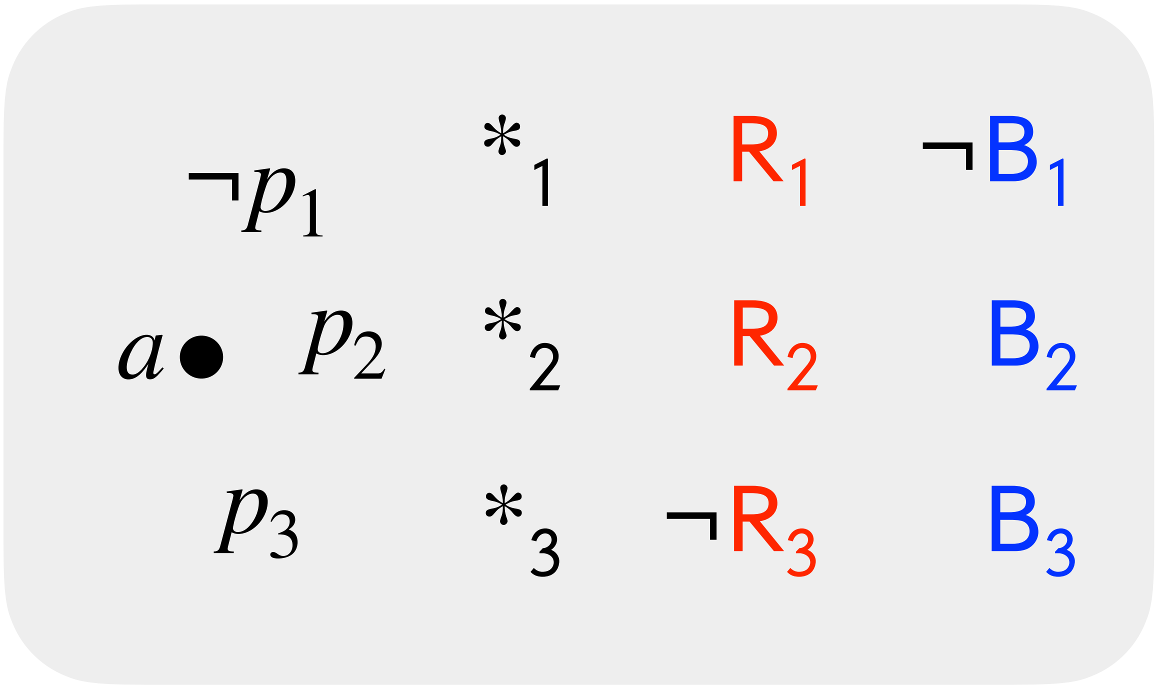


FOSL model



FOL model

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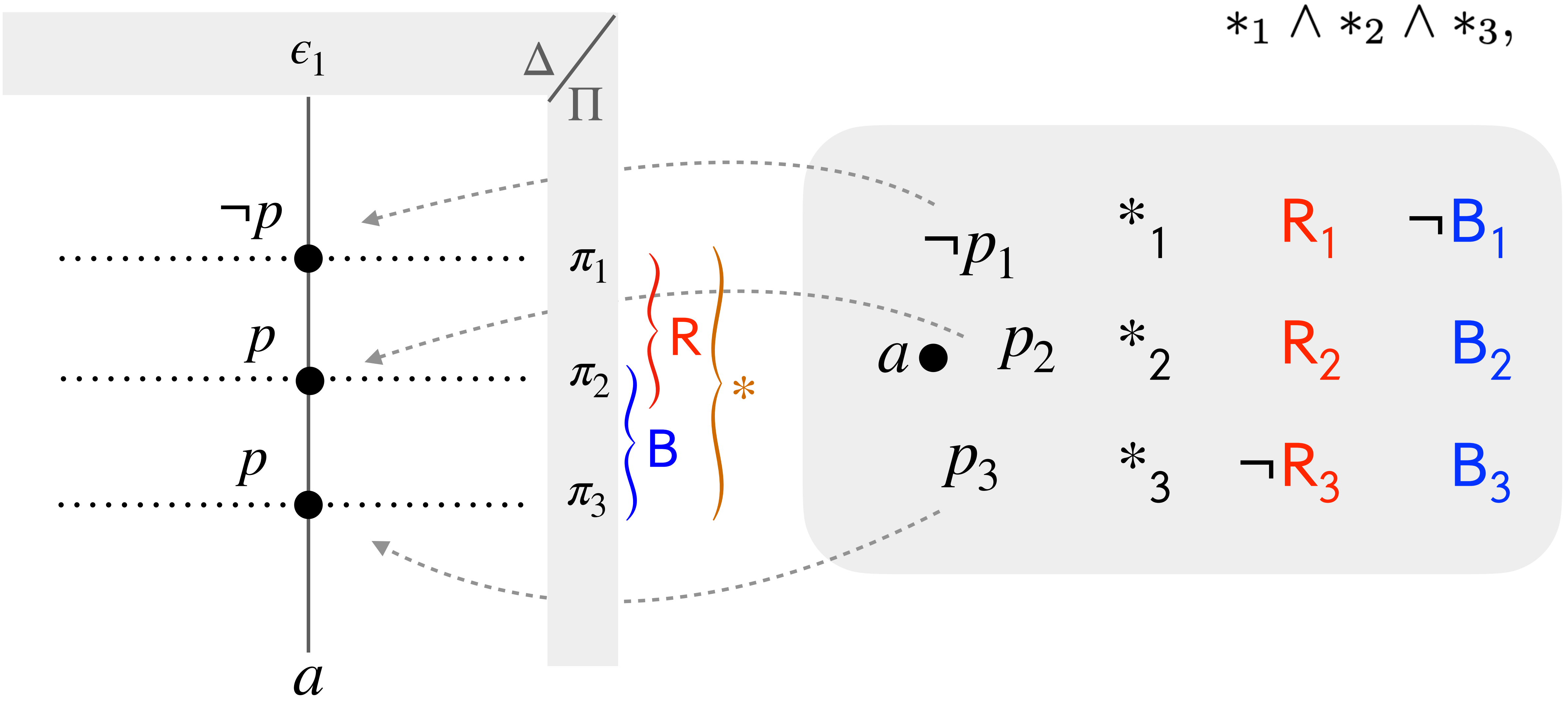

FOSL model **FOL model**

Sentential Fragments

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\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}} \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

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\end{aligned}$$

Lemma:
A formula ϕ is n-satisfiable in FOSL if and only if $\mathrm{Trans}_n(\phi)$ is satisfiable in first-order logic.

Sentential Fragments

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Lemma:
A formula ϕ is n-satisfiable in FOSL if and only if $\text{Trans}_n(\phi)$ is satisfiable in first-order logic.

Theorem:
Let F be a "translation-friendly" fragment of FOL. Then the satisfiability of the sentential standpoint-F fragment of FOSL,

- is decidable iff it is for F, and
- has the same complexity as F (if at least NP)

Decidable Sentential Framgments

- By small model property and generic translation, complexity of decidable fragments is preserved:
  - ➡ S Guarded fragment (GF) → 2ExpTime
  - ➡ S Triguarded fragment (TGF) → 2NExpTime
  - ➡ S Counting 2-variable fragment (C^2) → NExpTime
  - ➡ Standpoint OWL 2 → 2NExpTime (some extra tricks required)

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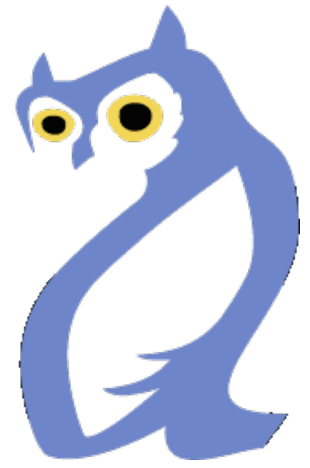
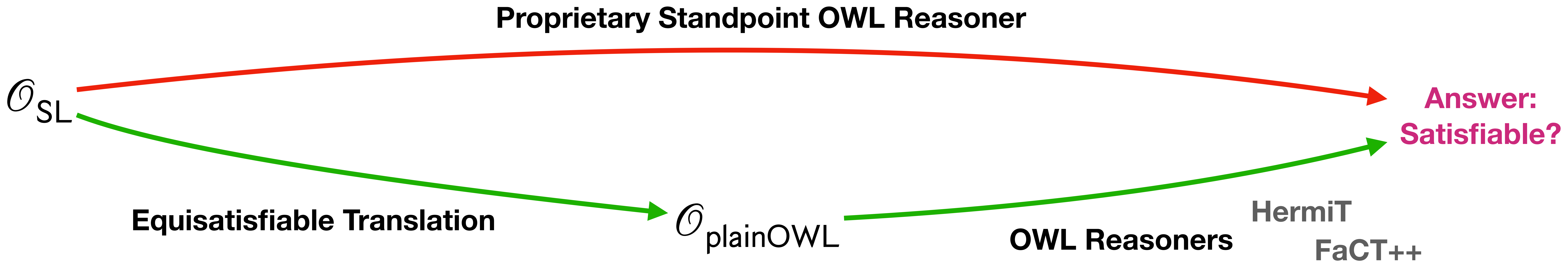
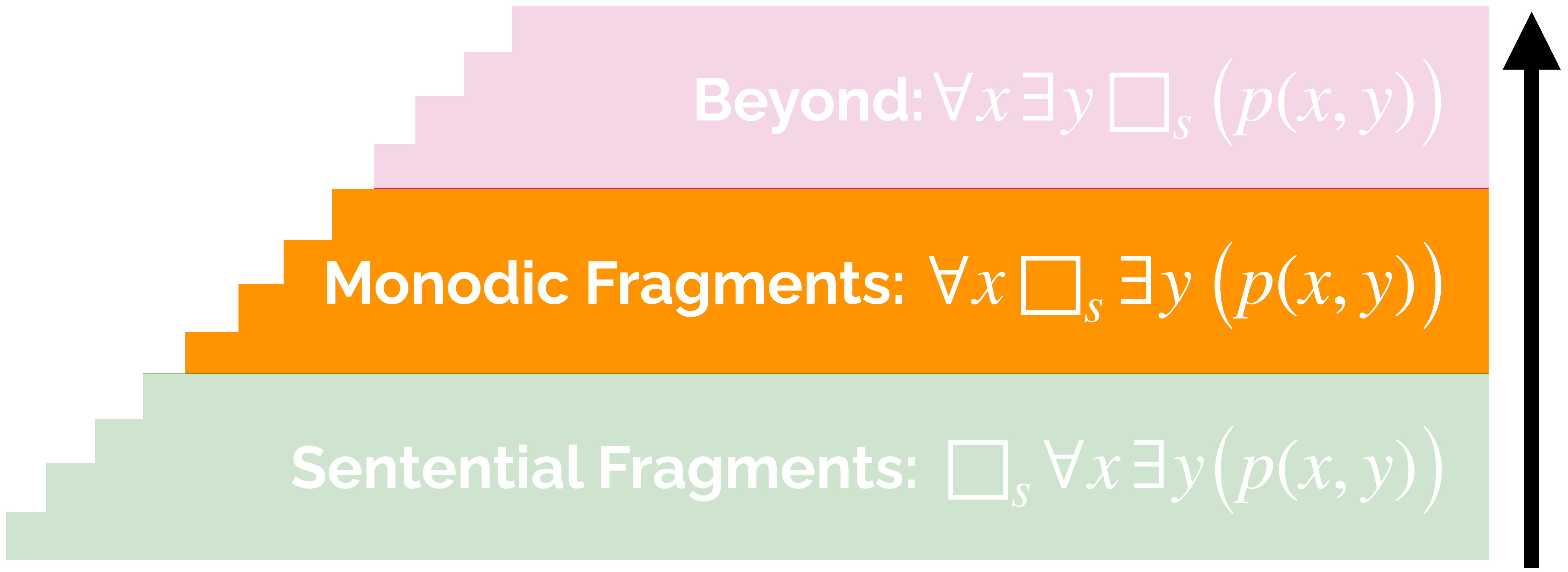


- Result via polynomial equisatisfiable translation → practical implementations



Monodic Fragments



Monodic Standpoint Description Logics

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Monodic modal extensions of DLs can lead to a blowup in complexity.

Tractable Diversity: Scalable Multiperspective Ontology Management via Standpoint $\mathscr{EL}$
Lucía Gómez Álvarez, Sebastian Rudolph, Hannes Straß; *(IJCAI 2023)*

Pushing the Boundaries of Tractable Multiperspective Reasoning: A deduction calculus for Standpoint $\mathscr{EL}$+
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➡ Complexity of the satisfiability of Standpoint-$\mathscr{EL}$+ $\rightarrow$ PTime (deduction calculus)

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➡ Tractability is easily lost:
- Empty standpoints → NP-hard
- Rigid roles → CoNP-hard

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- Empty standpoints $\rightarrow$ NP-hard
- Rigid roles $\rightarrow$ CoNP-hard
- Nominal Concepts $\rightarrow$ ExpTime-hard

Monodic Standpoint Description Logics

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- ➡ Complexity of the satisfiability of Standpoint-$\mathcal{SHIQ}$+ $\rightarrow$ ExpTime-complete
- ➡ Small Model Property (lost with nominal concepts)

Standpoint $\mathcal{SHIQ}$

The description logic $\mathcal{SHIQ}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid A \mid \neg C \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists{\leq}nr\,.\,C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

$\mathsf{Substance}$ $\quad$ $\exists\mathsf{sensitiveTo}\,.\,\mathsf{Nut}$

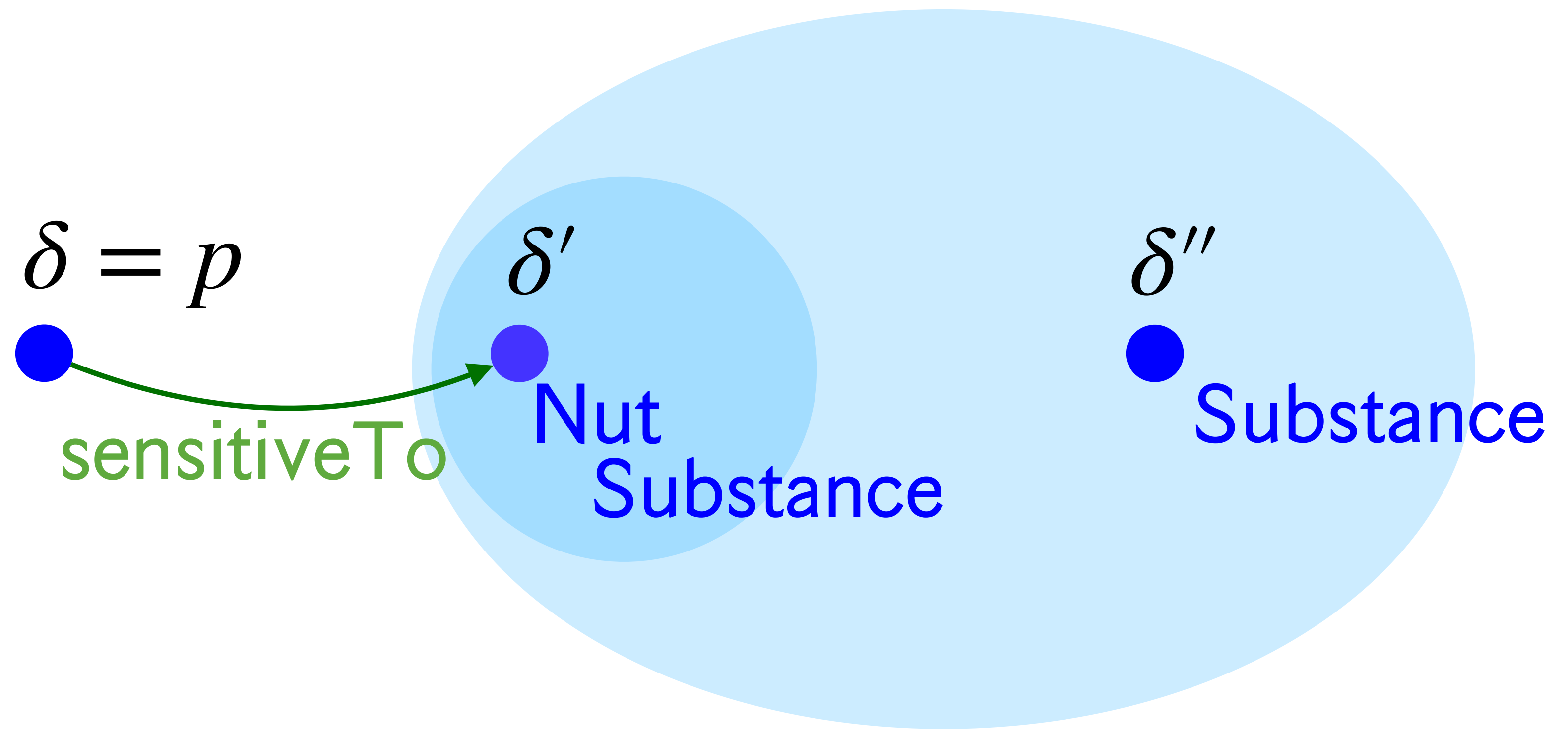
$\mathsf{Substance} \sqcap \neg\mathsf{Dangerous}$ $\quad$ $\exists \leq 1\mathsf{sensitiveTo}\,.\,\mathsf{Substance}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D, \quad R \sqsubseteq R', \quad R \circ R \sqsubseteq R$
- Assertions: $C(a), \quad r(a,b)$

$$\left(\mathsf{Peanut} \sqsubseteq \mathsf{Substance}\right) \wedge \neg \left(\exists\mathsf{sensitiveTo}\,.\,\mathsf{Peanut}\right)(p)$$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual

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Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names

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Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid A \mid \neg C \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists {\leq} n r \,.\, C$

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Towards Standpoint-$\mathcal{SHIQ}$

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Formulas are $\odot_s (\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

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$\Box_\mathsf{L}(\mathsf{Peanut} \sqsubseteq \mathsf{Substance})$ $\quad \Box_\mathsf{L}(\exists\mathsf{sensitiveTo}\,.\,\Diamond_\mathsf{P}\mathsf{Nut})(p)$

Towards Standpoint-$\mathcal{SHIQ}$

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Syntax:

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Substance $\exists$sensitiveTo . Nut $\Diamond_\mathsf{S}$ Dangerous

Substance $\sqcap \neg$Dangerous $\exists \leq 1$sensitiveTo . Substance

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$\Box_\mathsf{L}(\text{Peanut} \sqsubseteq \text{Substance})$ $\Box_\mathsf{L}(\exists\text{sensitiveTo}\,.\,\Diamond_\mathsf{P}\text{Nut})(p)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma \rangle$

Towards Standpoint-$\mathcal{SHIQ}$

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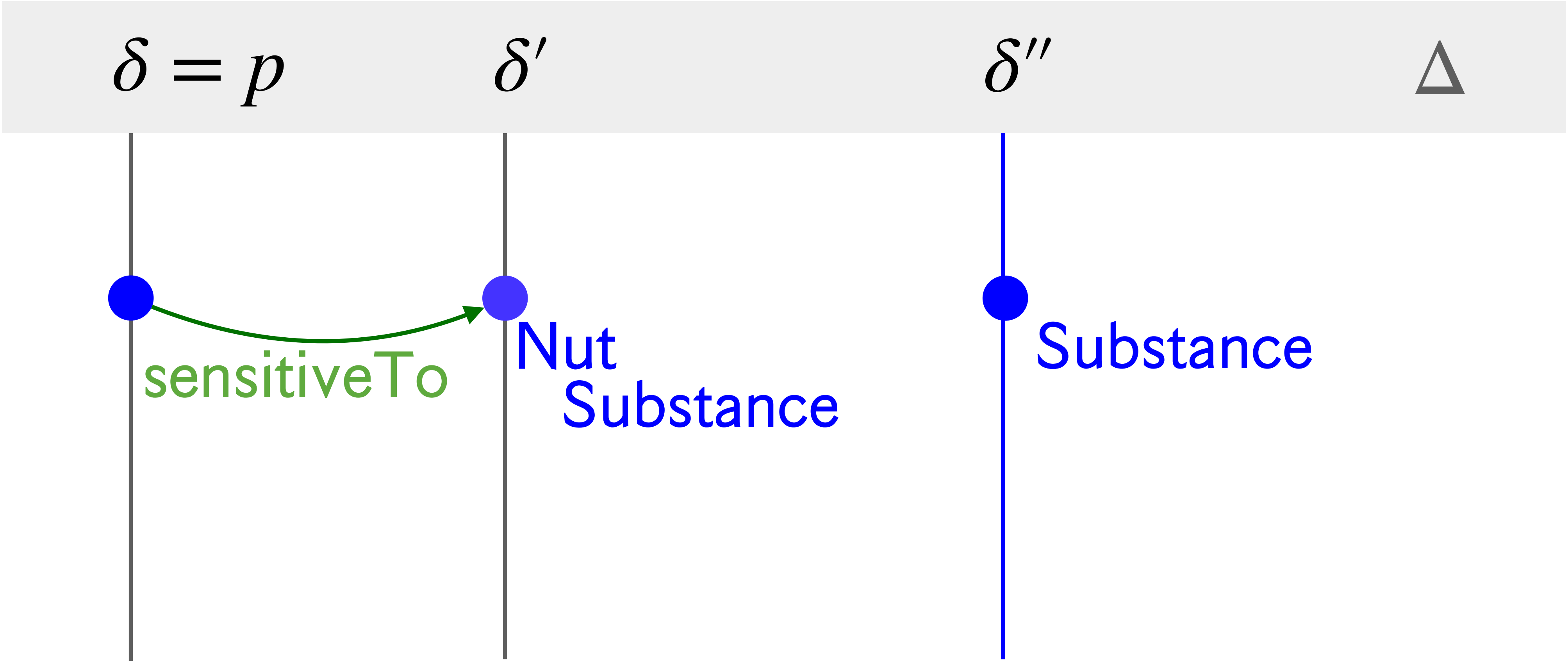

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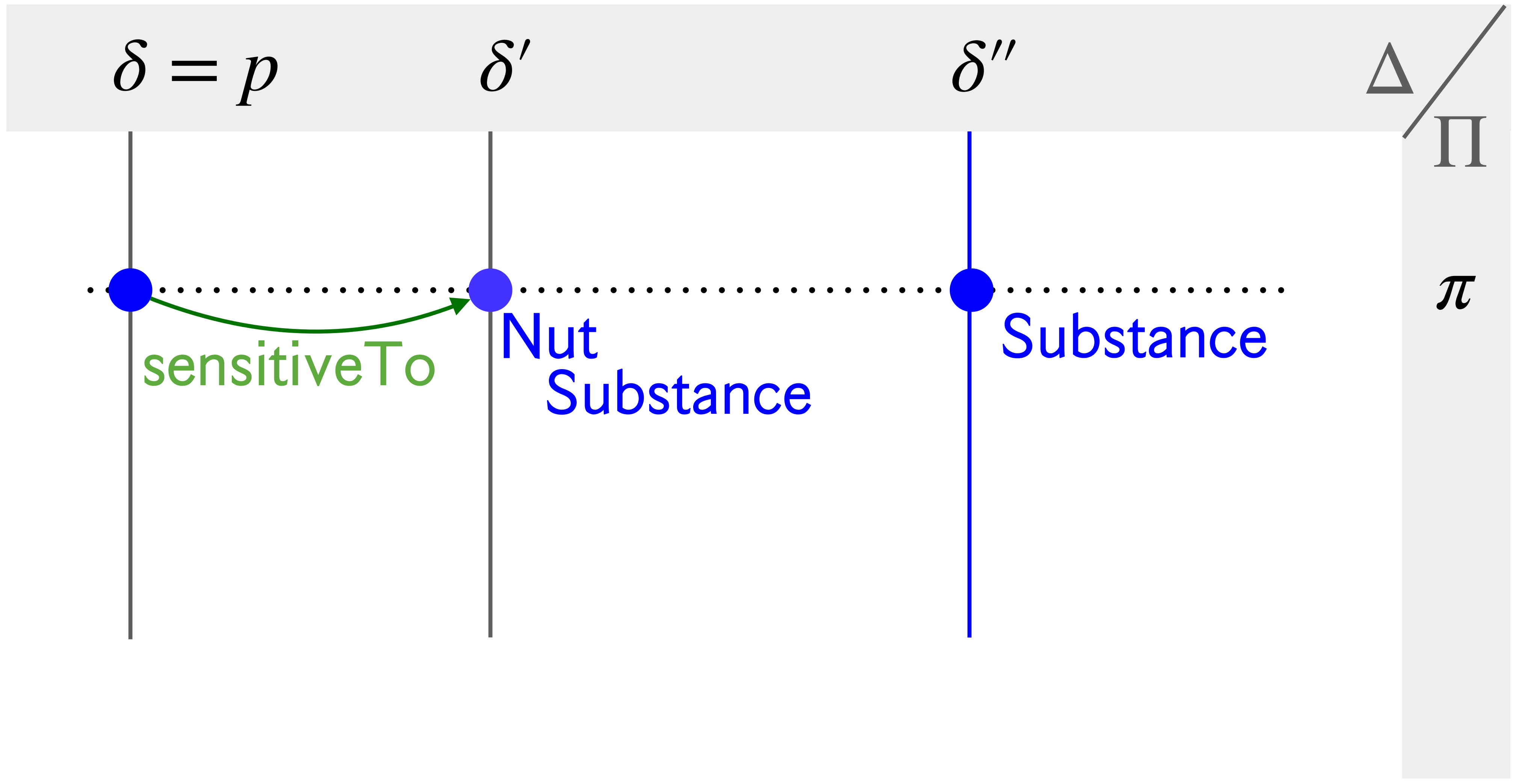

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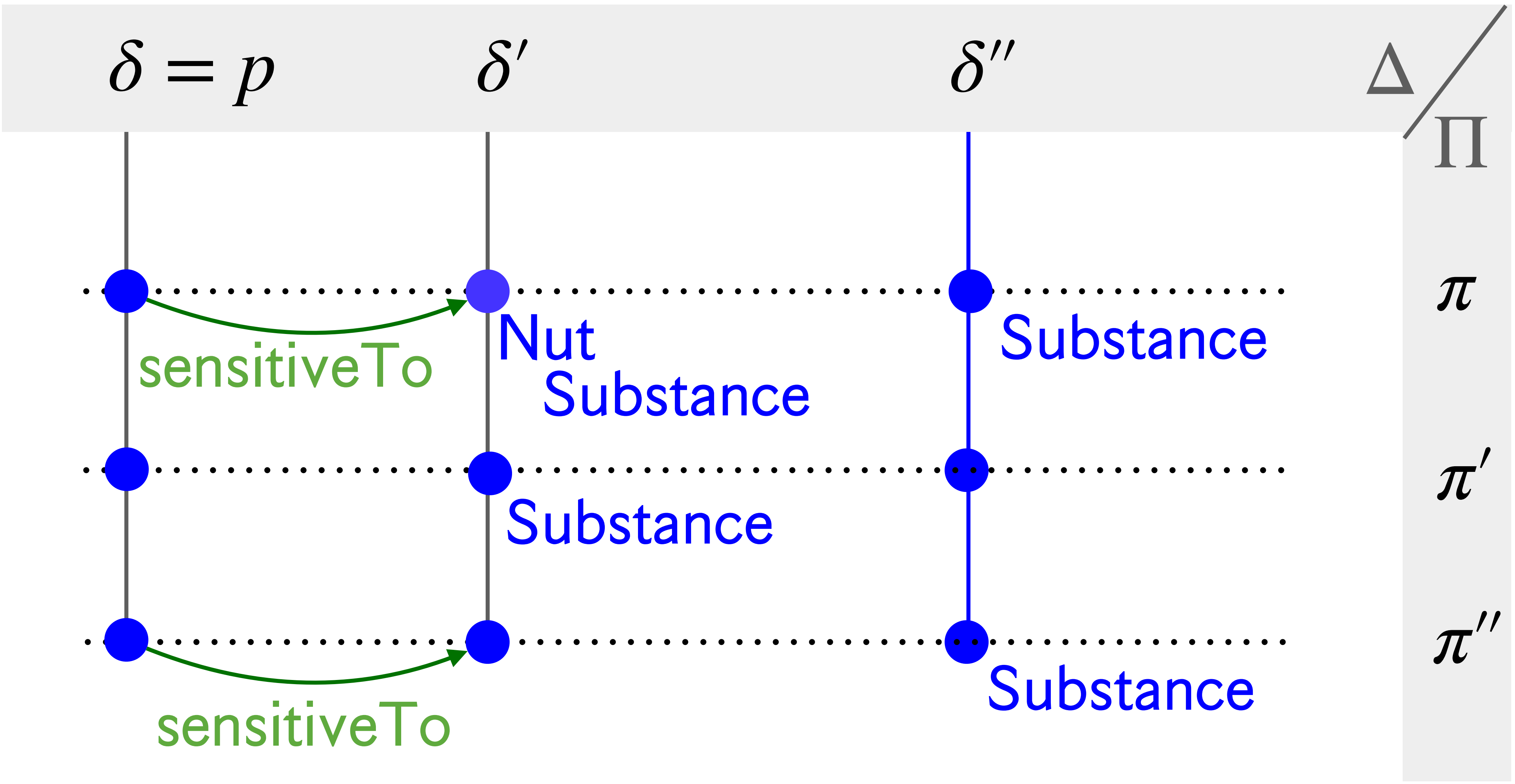

Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S}\rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid A \mid \neg C \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists{\leq}nr\,.\,C \mid \odot_s C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}, s \in N_\mathsf{S}, \odot \in \{\Box, \Diamond\}$.

Substance $\quad \exists$sensitiveTo . Nut $\quad \Diamond_\mathsf{S}$ Dangerous

Substance $\sqcap \neg$Dangerous $\quad \exists \leq 1$sensitiveTo . Substance

Formulas are $\odot_s(\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

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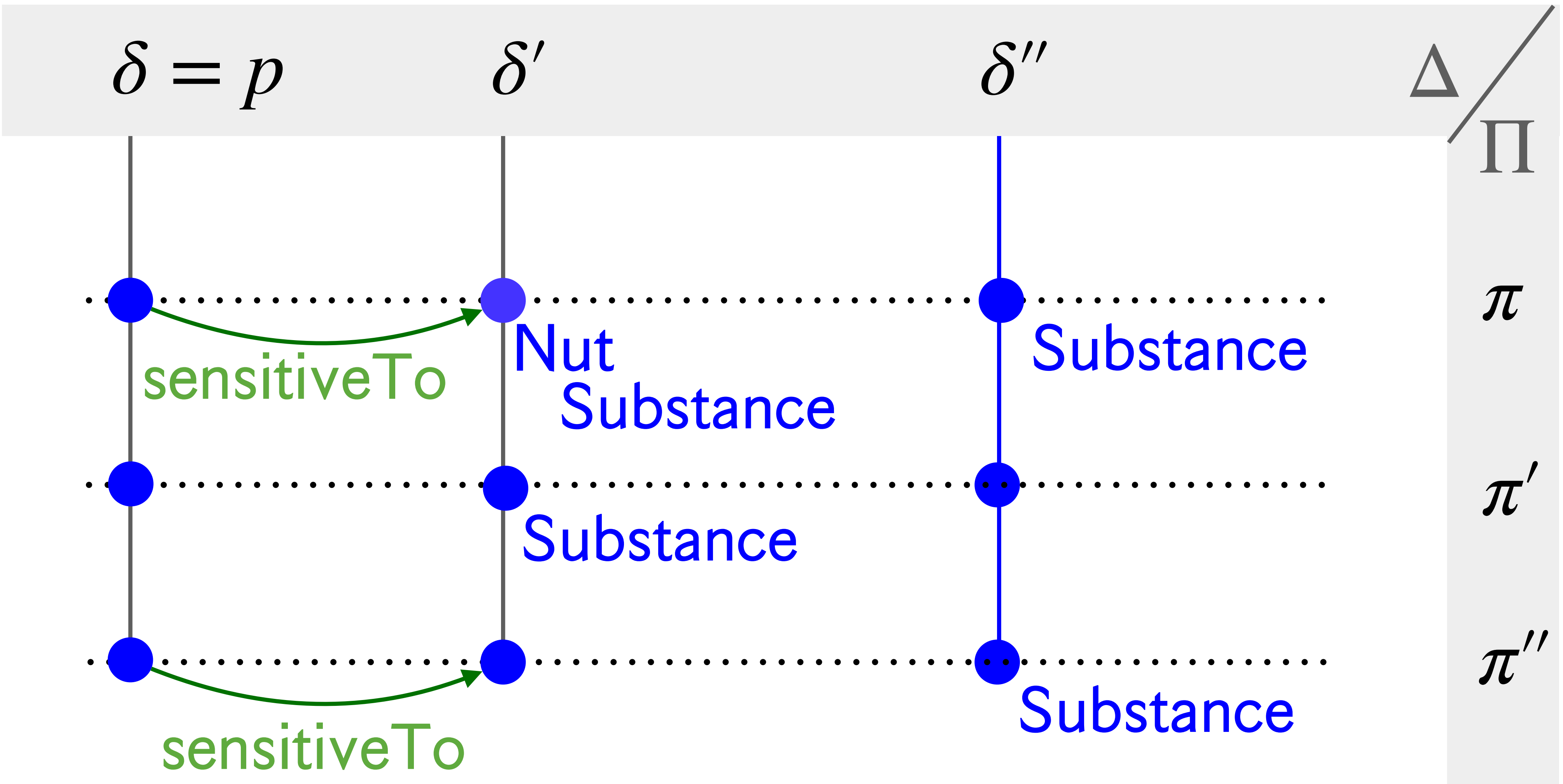

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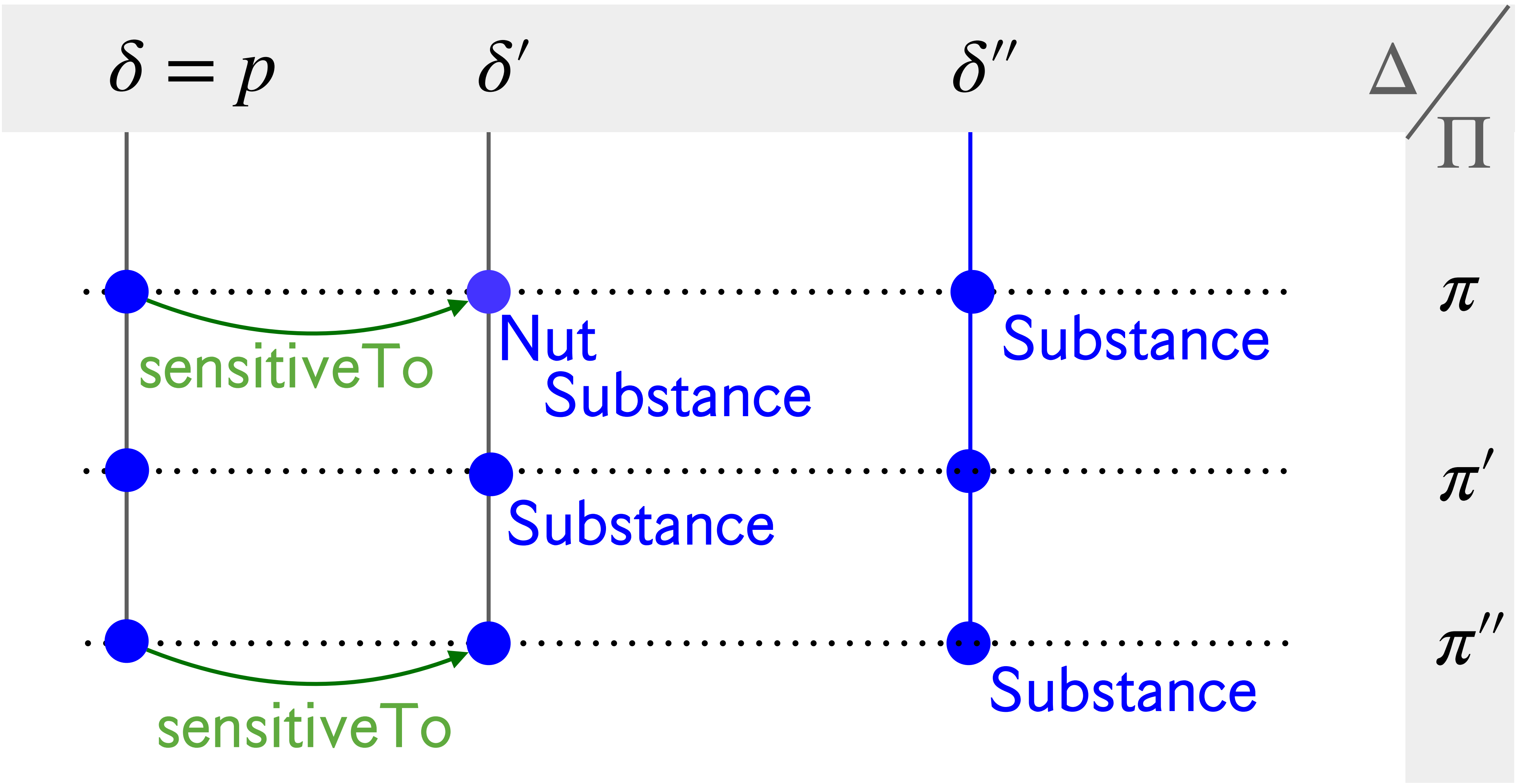

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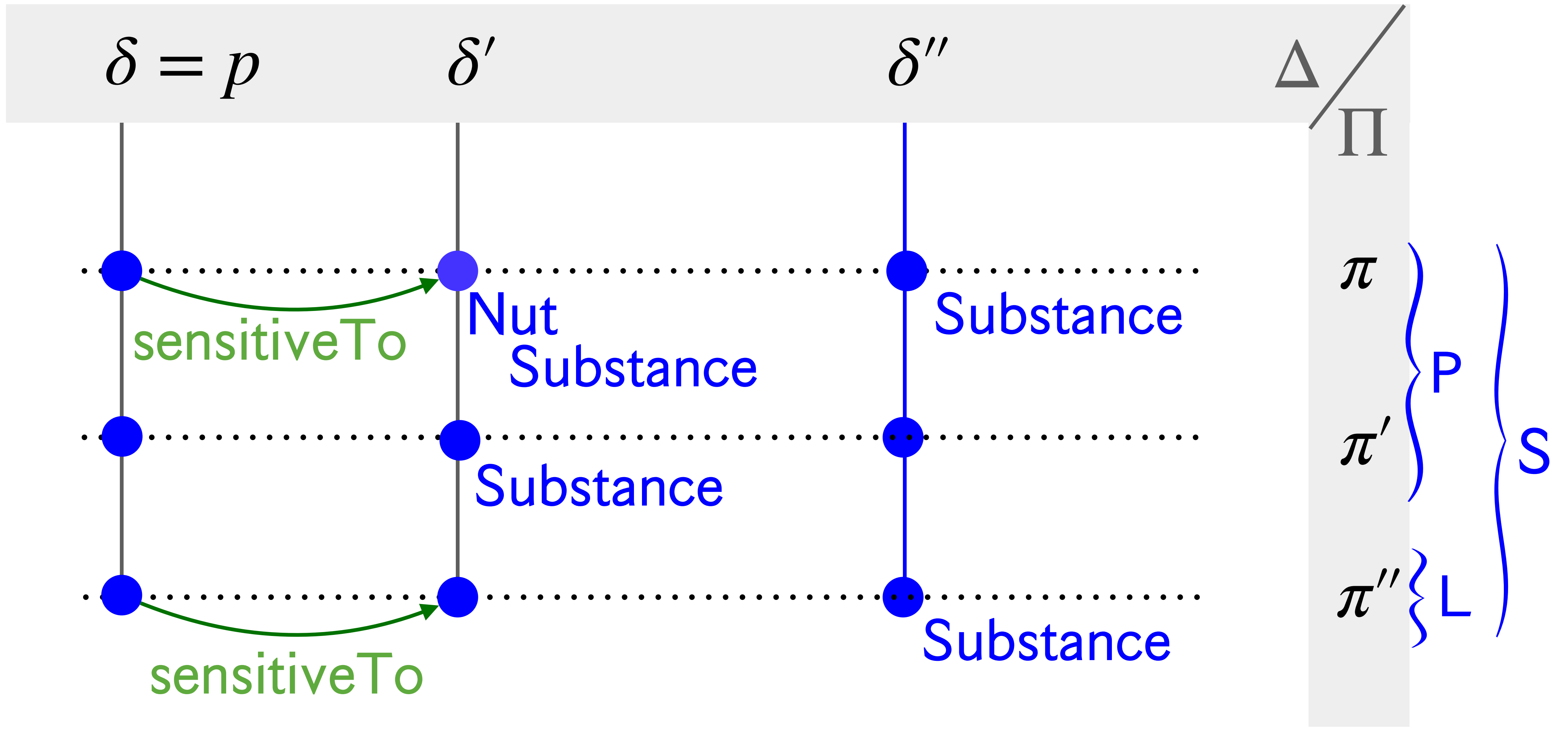

Small Models for Standpoint $\mathcal{SHIQ}$

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

Normalisation:

- Sharpenings not using 0:
  - $s' \preceq s$ $\quad s_1 \cap s_2 \preceq s$
- GCIs:
  - $\Box_s(\top \sqsubseteq C)$ with C in NNF
- Other modalised axioms :
  - $\Box_s \xi$ with ξ any RI, transitivity axiom, role assertion, or concept assertion $C(a)$ with C in NNF

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Lemma 1. *Any $\mathbb{S}_{\mathcal{SHIQ}}$ KB $\mathcal{K}$ can be transformed into a KB in normal form $\mathcal{K}'$ such that*

- *$\mathcal{K}$ and $\mathcal{K}'$ are equisatisfiable*
- *$\mathcal{K}'$ is linear in the size of $\mathcal{K}$*
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Notice that $\Diamond_s$ only occurs at the concept level

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

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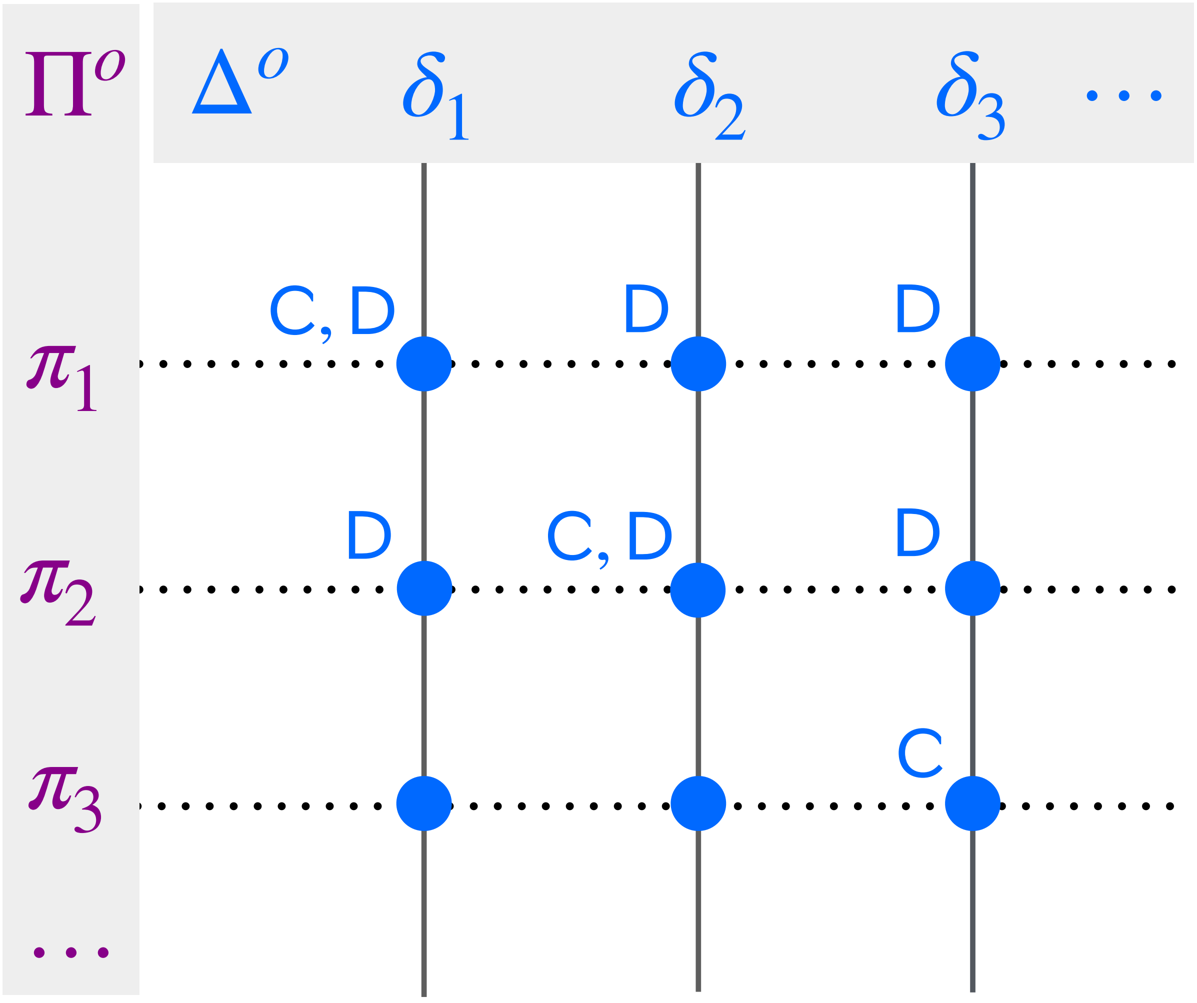
Tidy models

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

Tidy models

$$\mathscr{K} = \{ \Box_\star(\top \sqsubseteq D), \Box_\star(\top \sqsubseteq \Diamond_\star C) \}$$



Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

Tidy models

Intuitively, we want to pick the necessary elements from $\mathscr{D}^o$:

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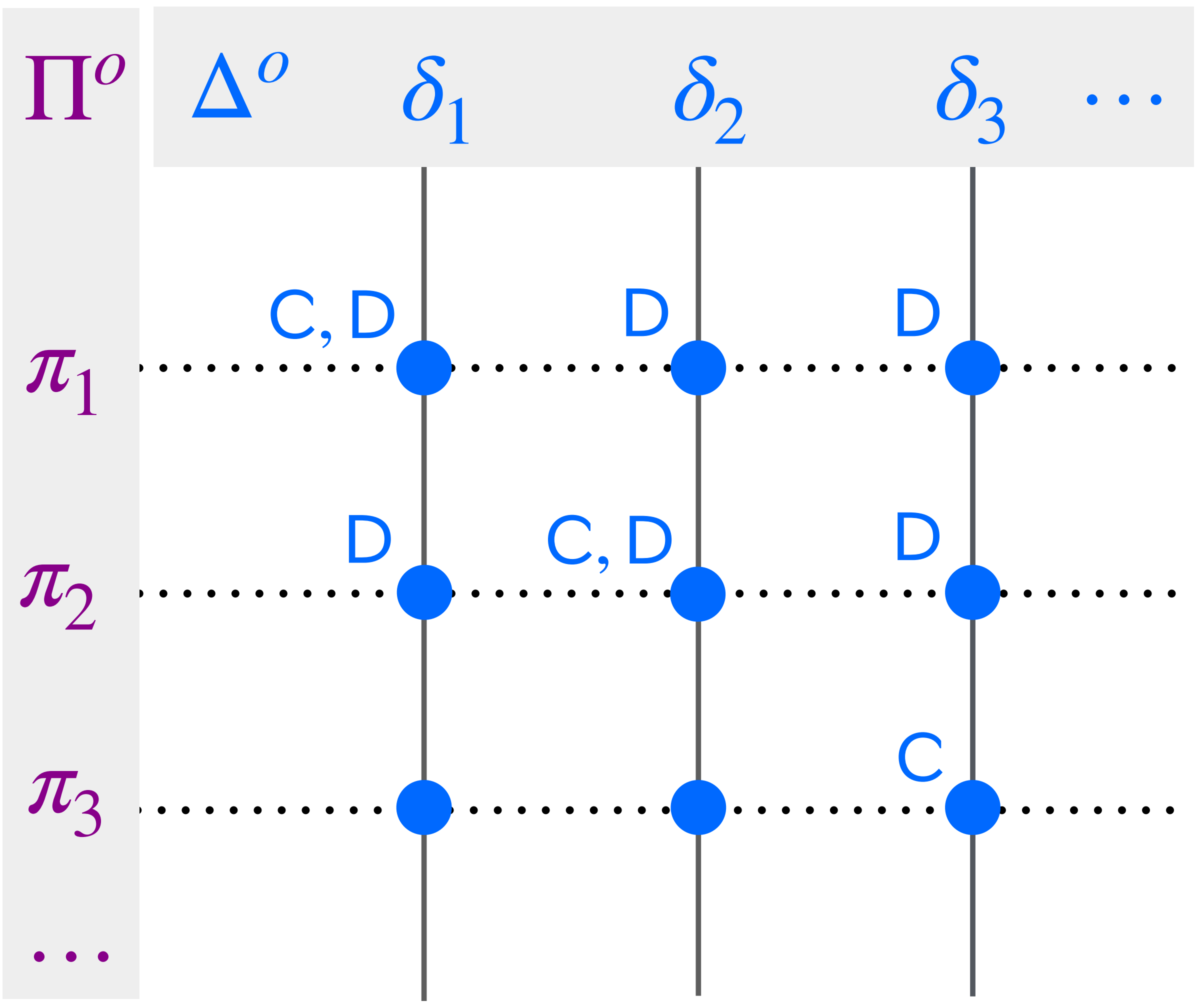

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

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- Witness standpoint non-emptiness
- Witness $\Diamond_s C$ membership

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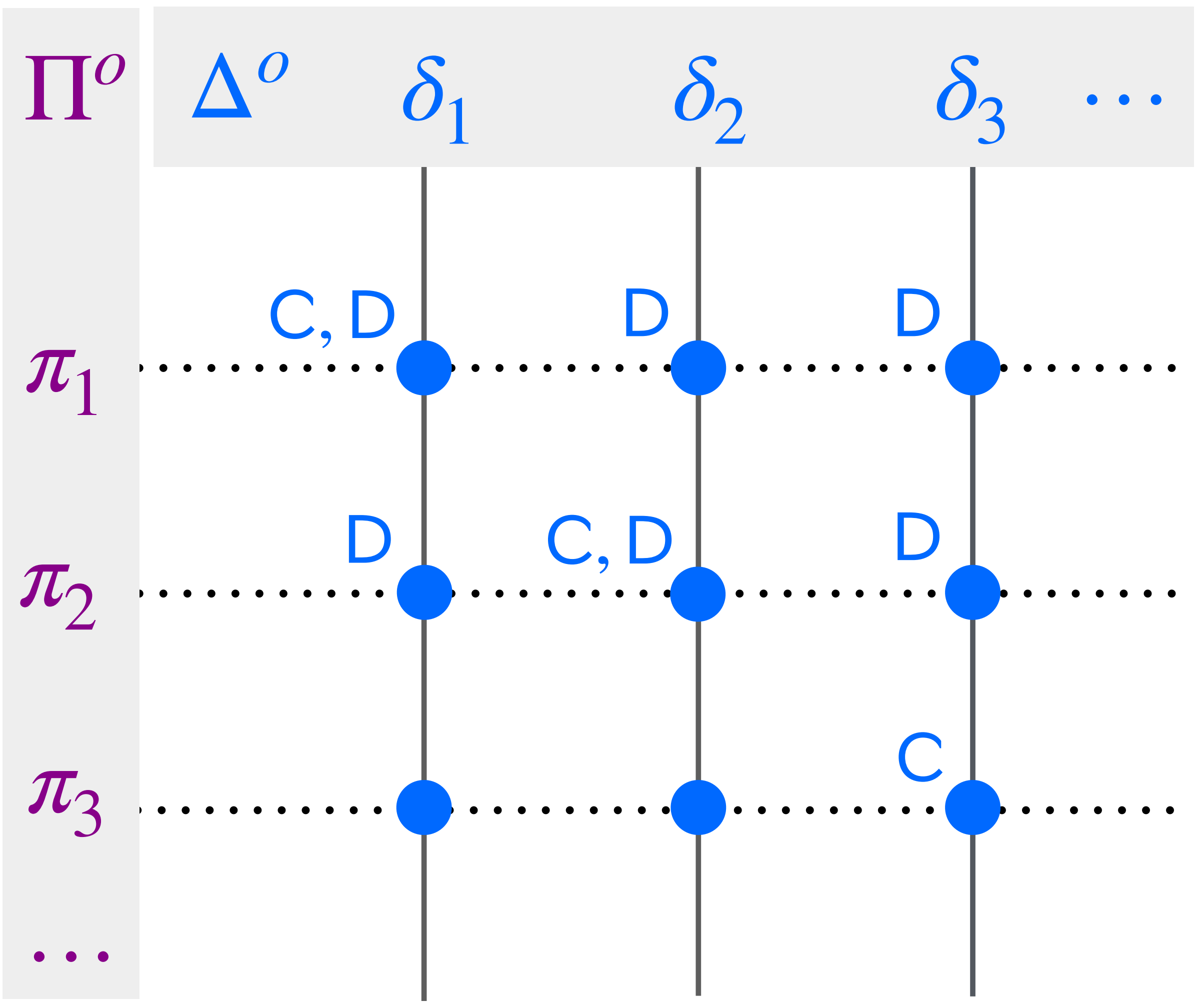

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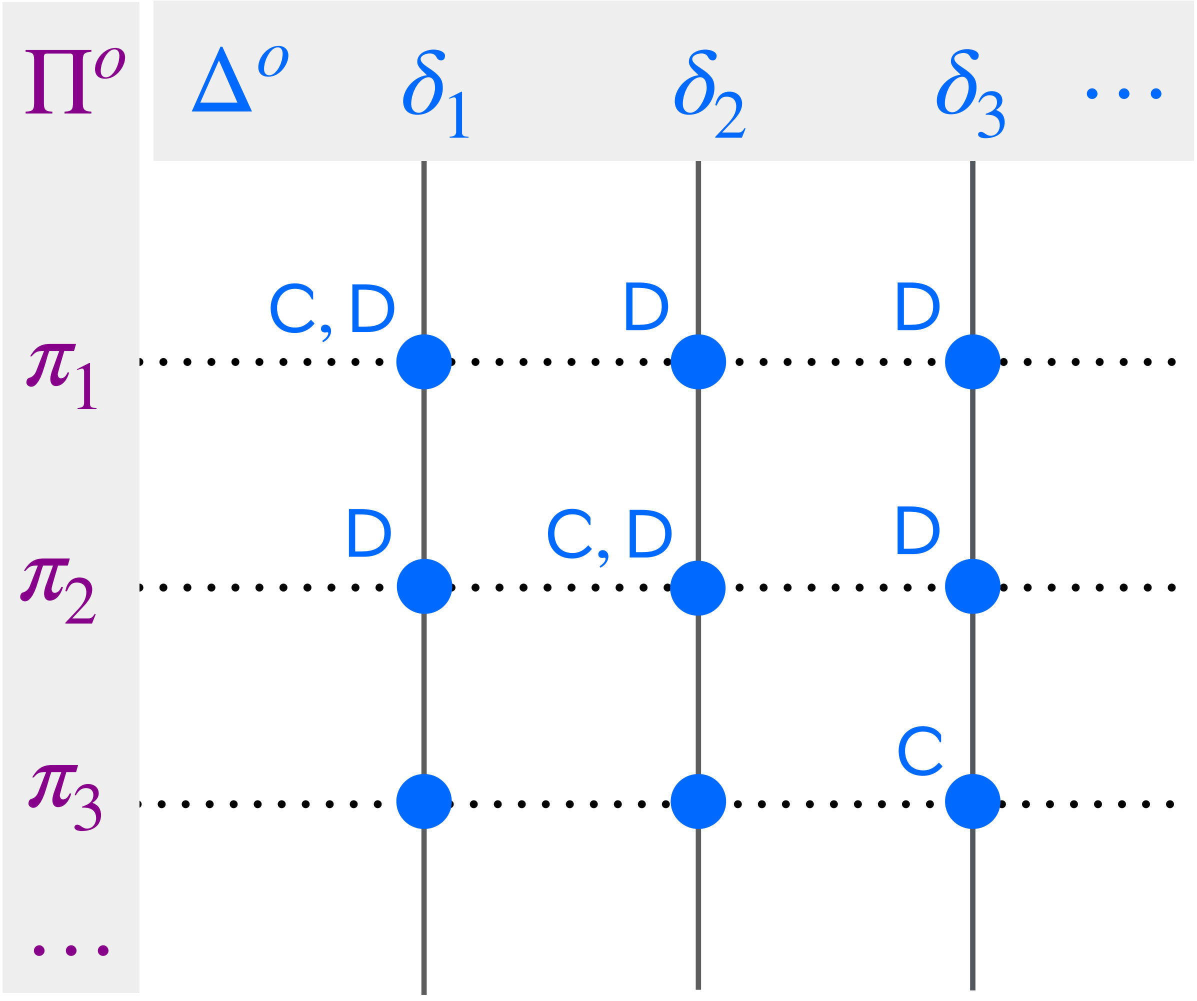

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

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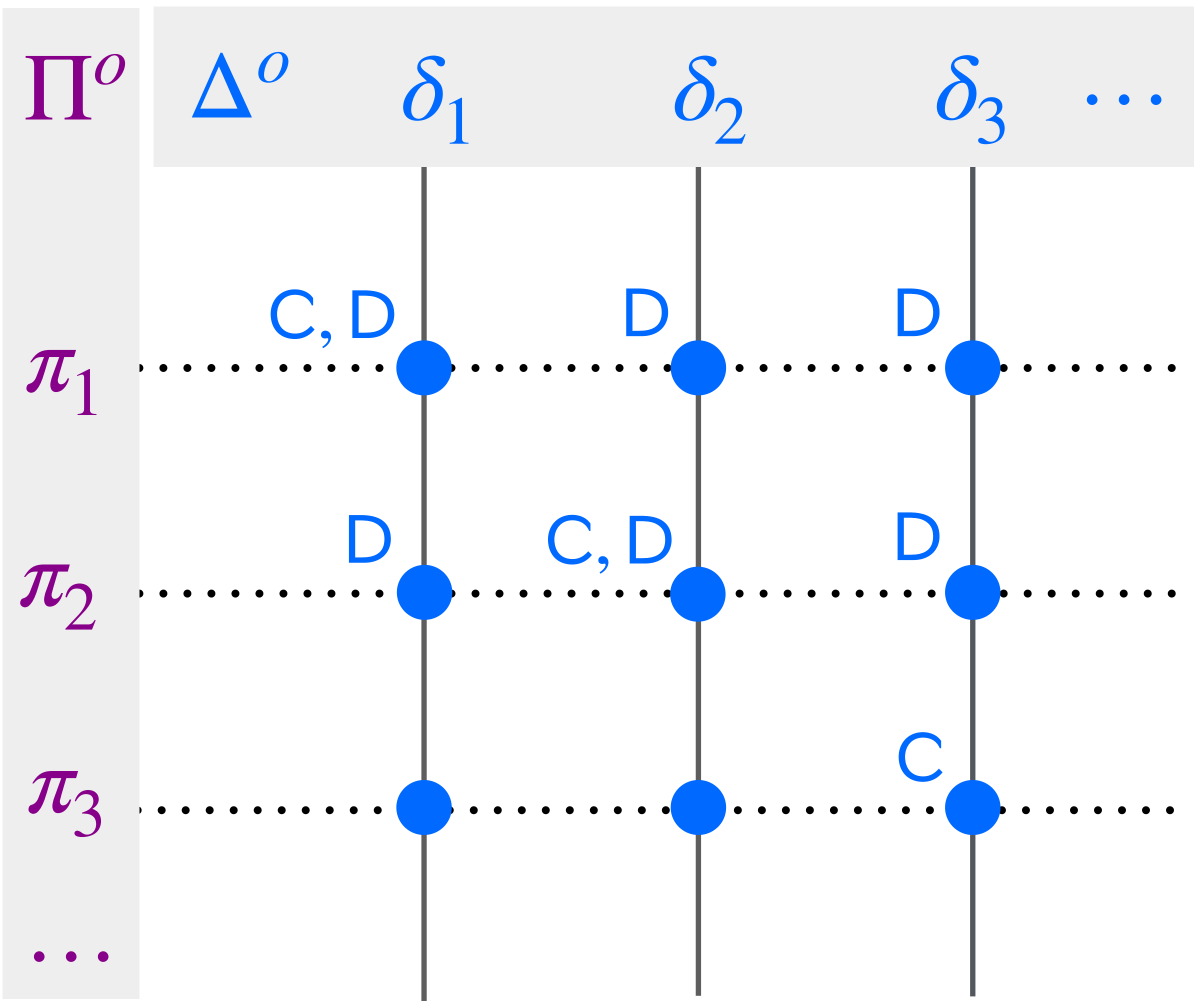

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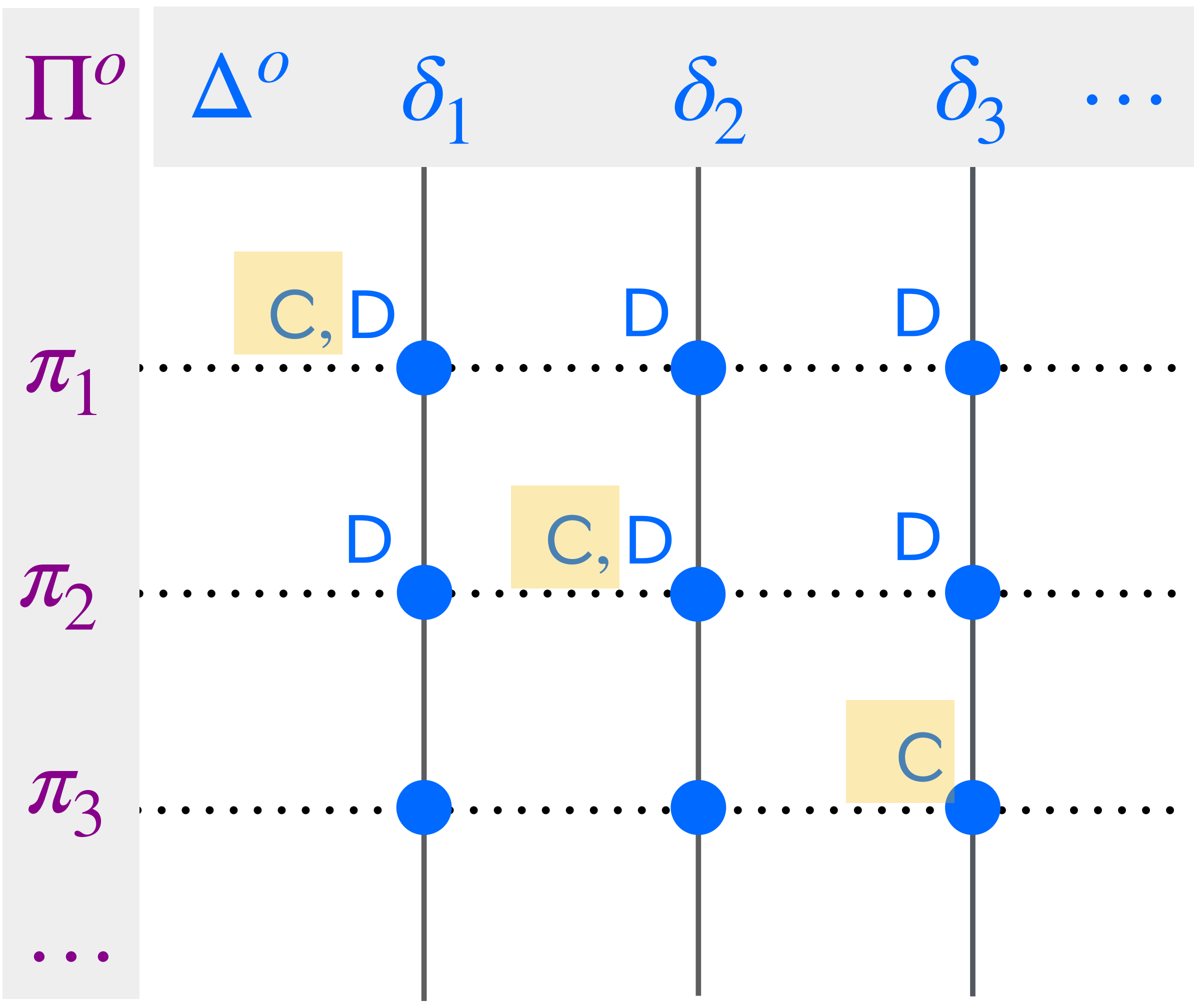
- Witness $\lozenge_s C$ membership
  - How do we pick witnesses?

$$\mathscr{K} = \{ \square_\star (\top \sqsubseteq D), \; \square_\star (\top \sqsubseteq \lozenge_\star C) \}$$



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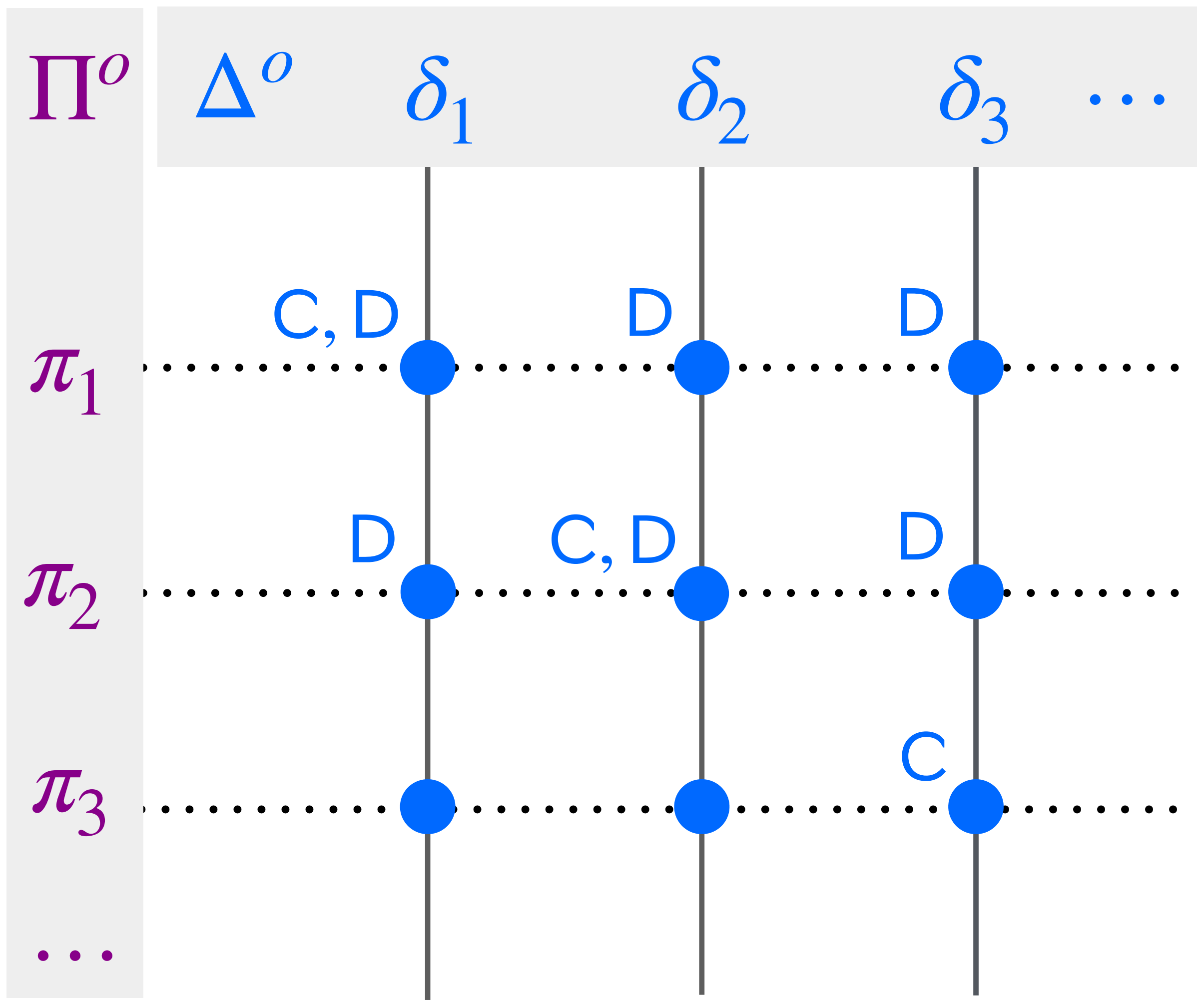

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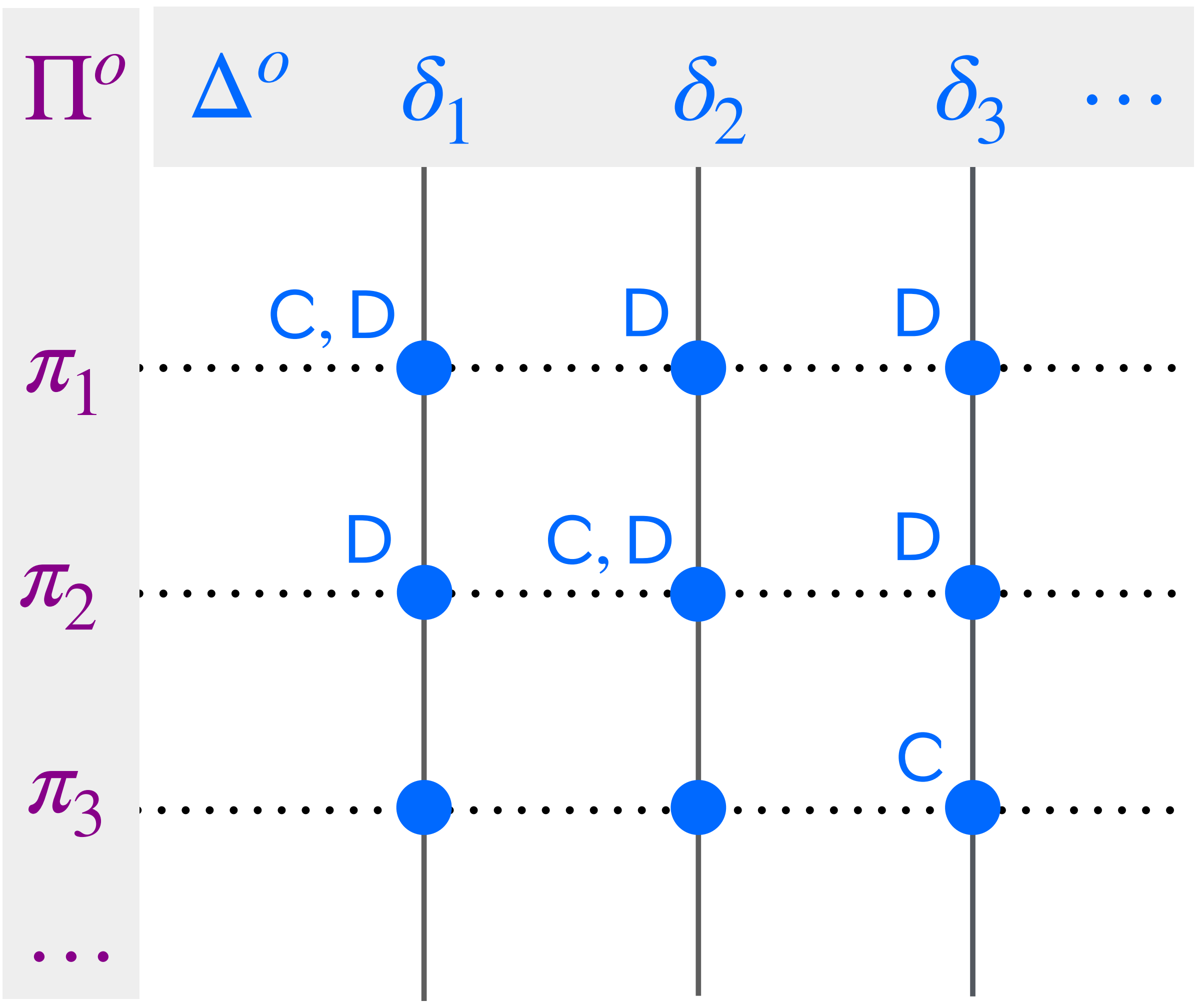

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For witnessing $\Diamond_{\mathrm{s}} C$ membership:

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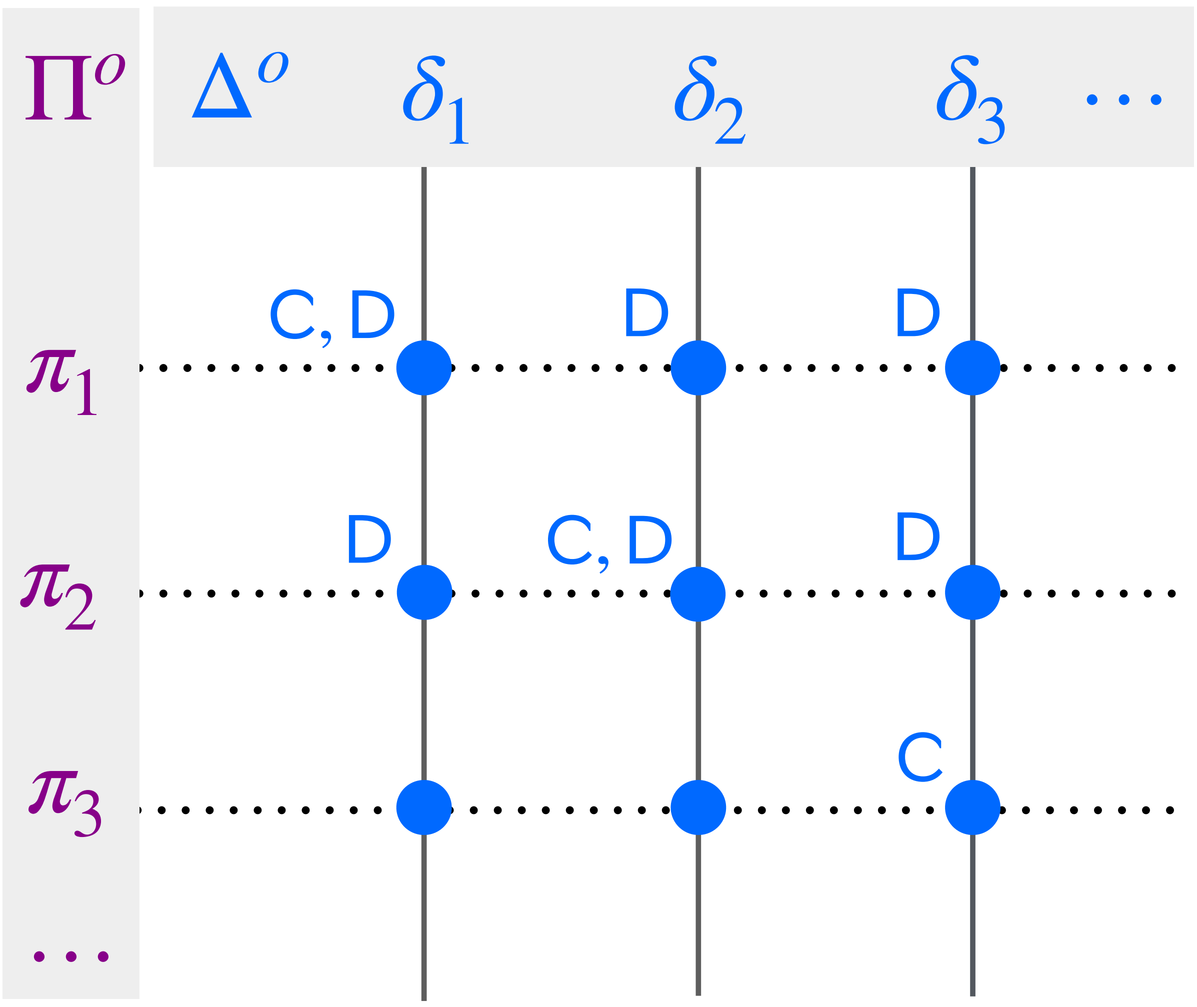

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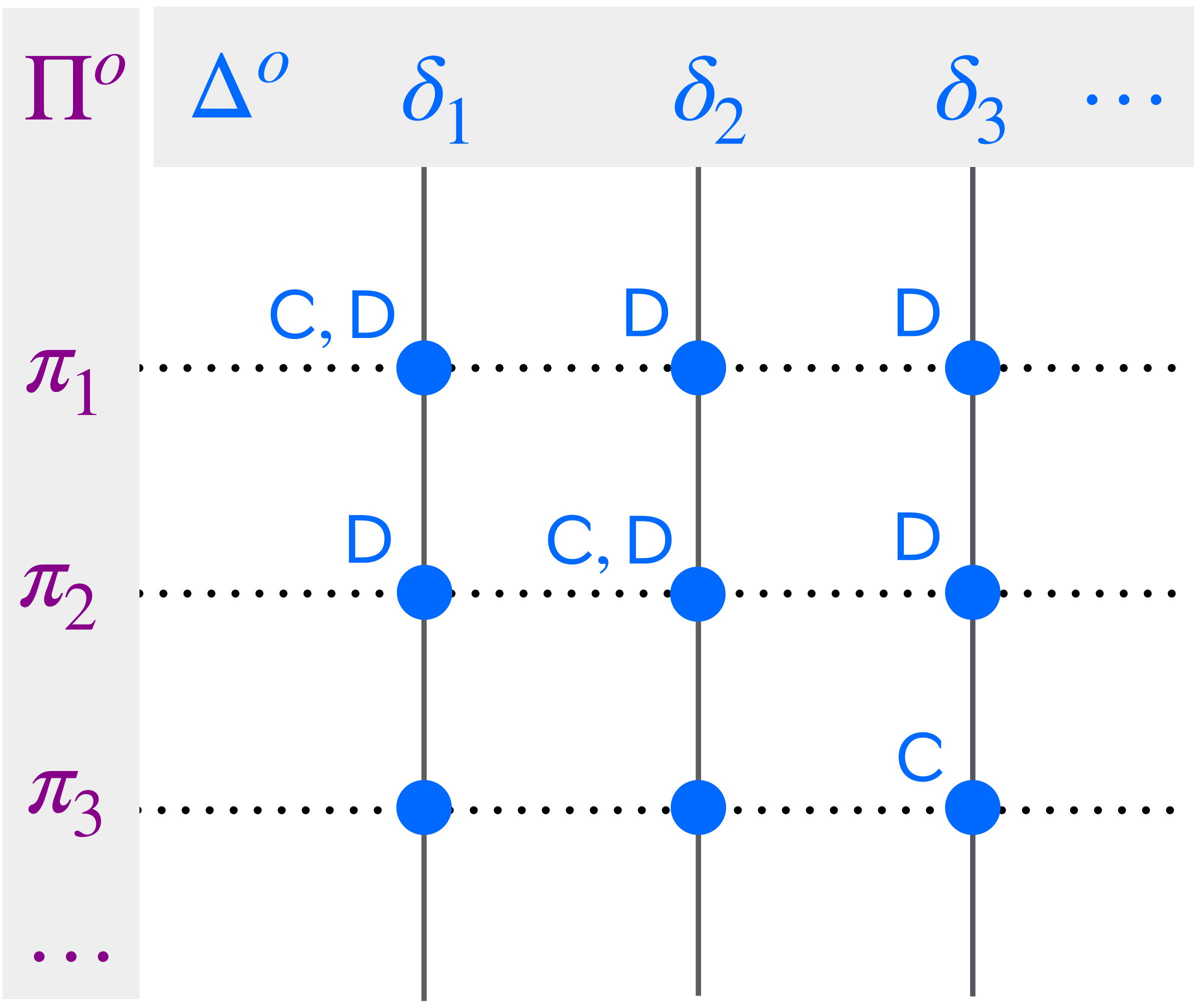

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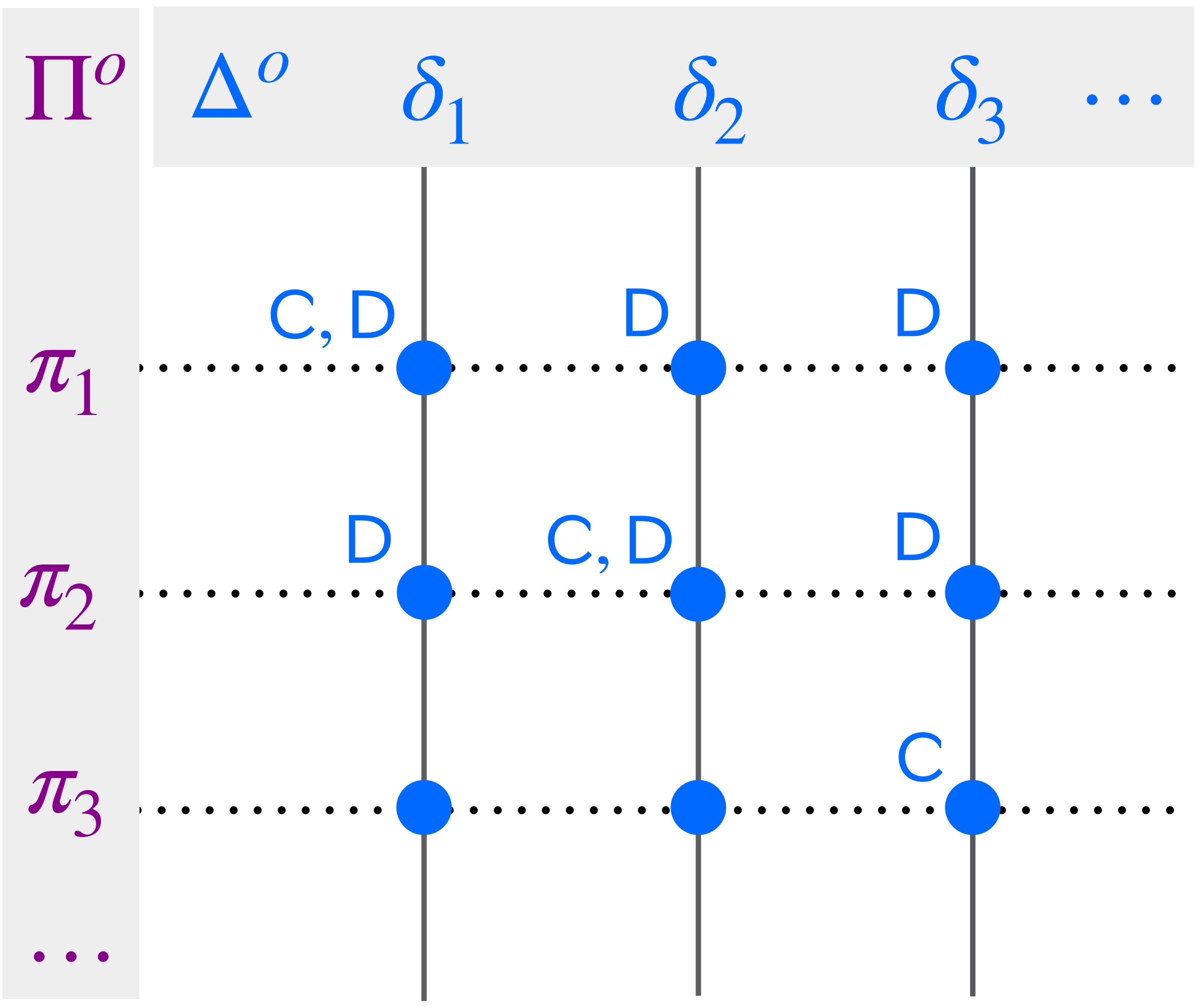

✤ **in SHIQ, the disjoint union of two or more models will be a model again**

✤ **many precisifications can co-exist inside one precisification!**

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

Tidy models

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- π_{s} for every standpoint s
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- $\pi^{0}_{\mathrm{s},C}$, $\pi^{1}_{\mathrm{s},C}$ for each $\Diamond_{\mathrm{s}}C$

For witnessing $\Diamond_{\mathrm{s}}C$ membership:

- A witness for each **named individual** a
- For **unnamed individuals**:

➡ Squeeze infinite copies of all s precisifications

➡ Arrange them so for each δ, either $\pi^{0}_{\mathrm{s},C}$ or $\pi^{1}_{\mathrm{s},C}$ deals with the $\Diamond_{\mathrm{s}}C$ membership

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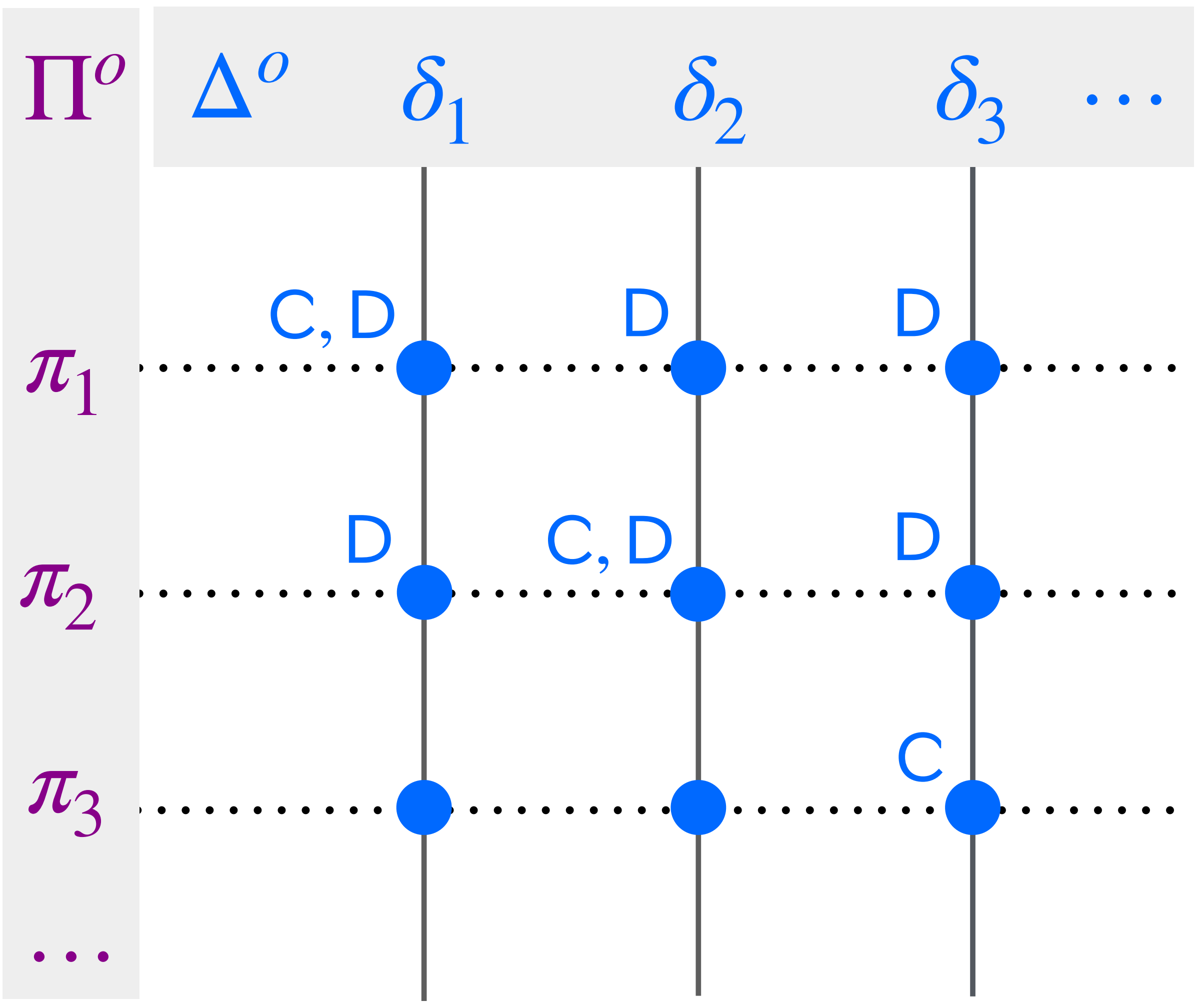

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$$
\begin{aligned}
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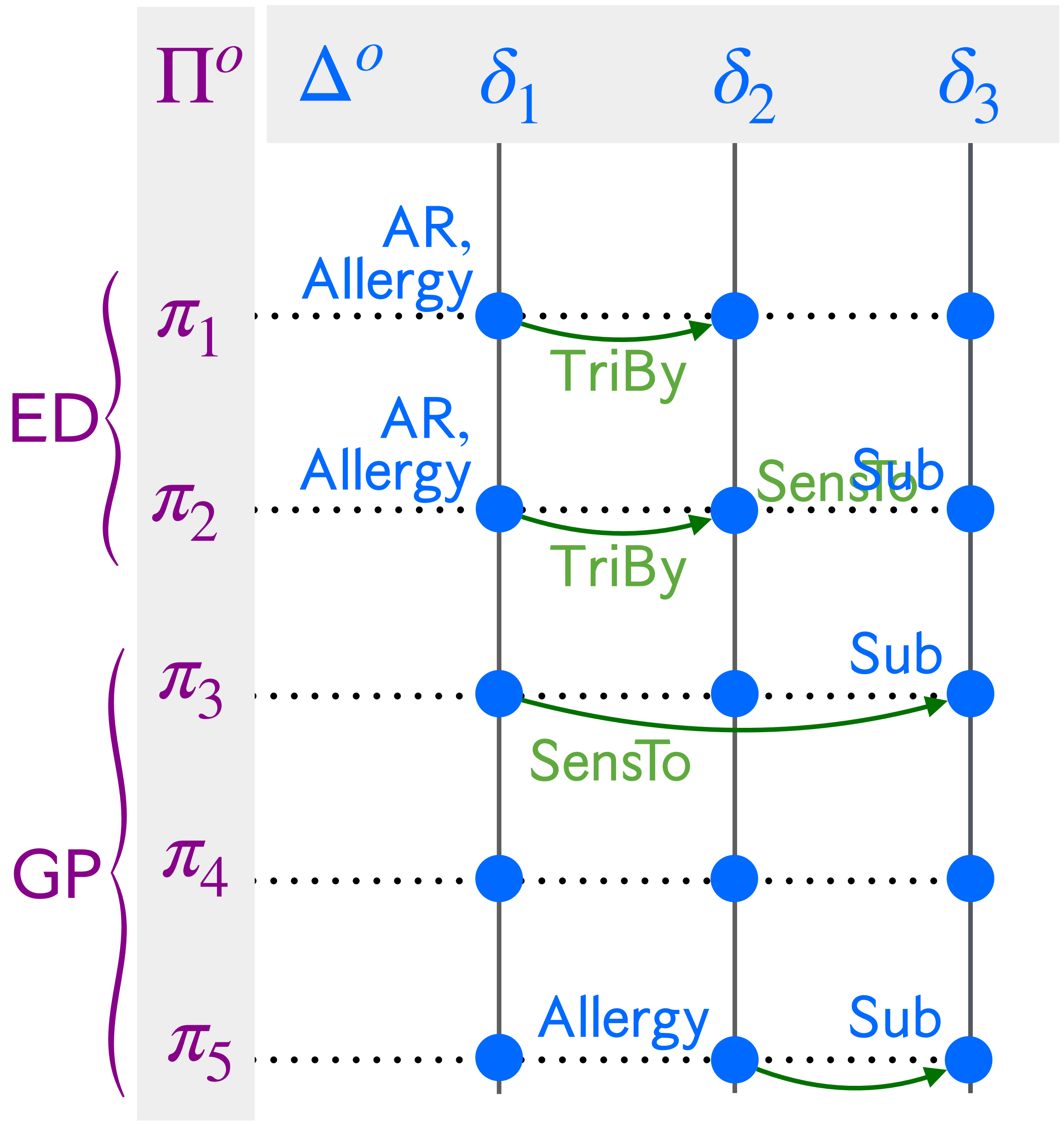
\end{aligned}
$$



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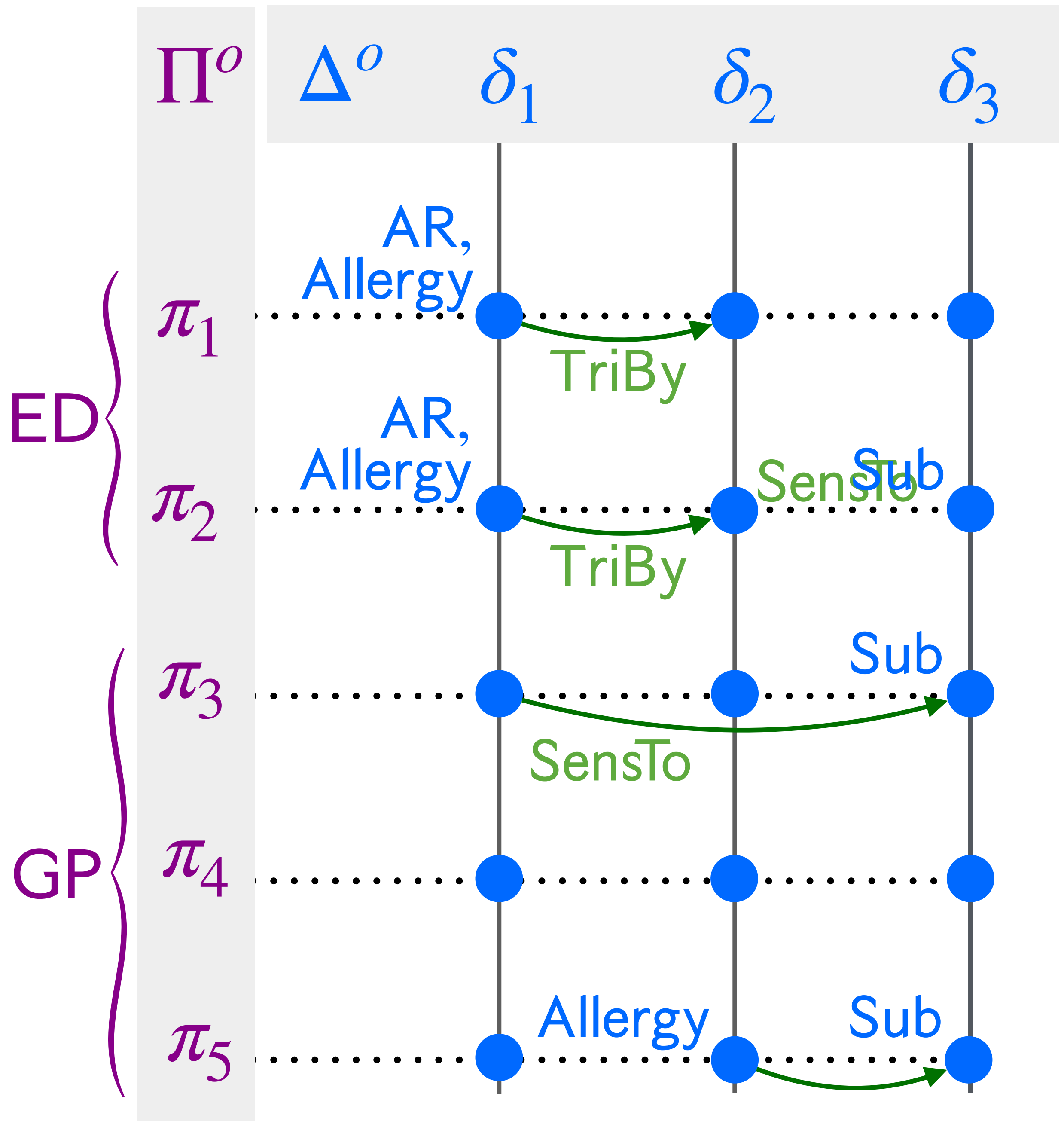

Δ

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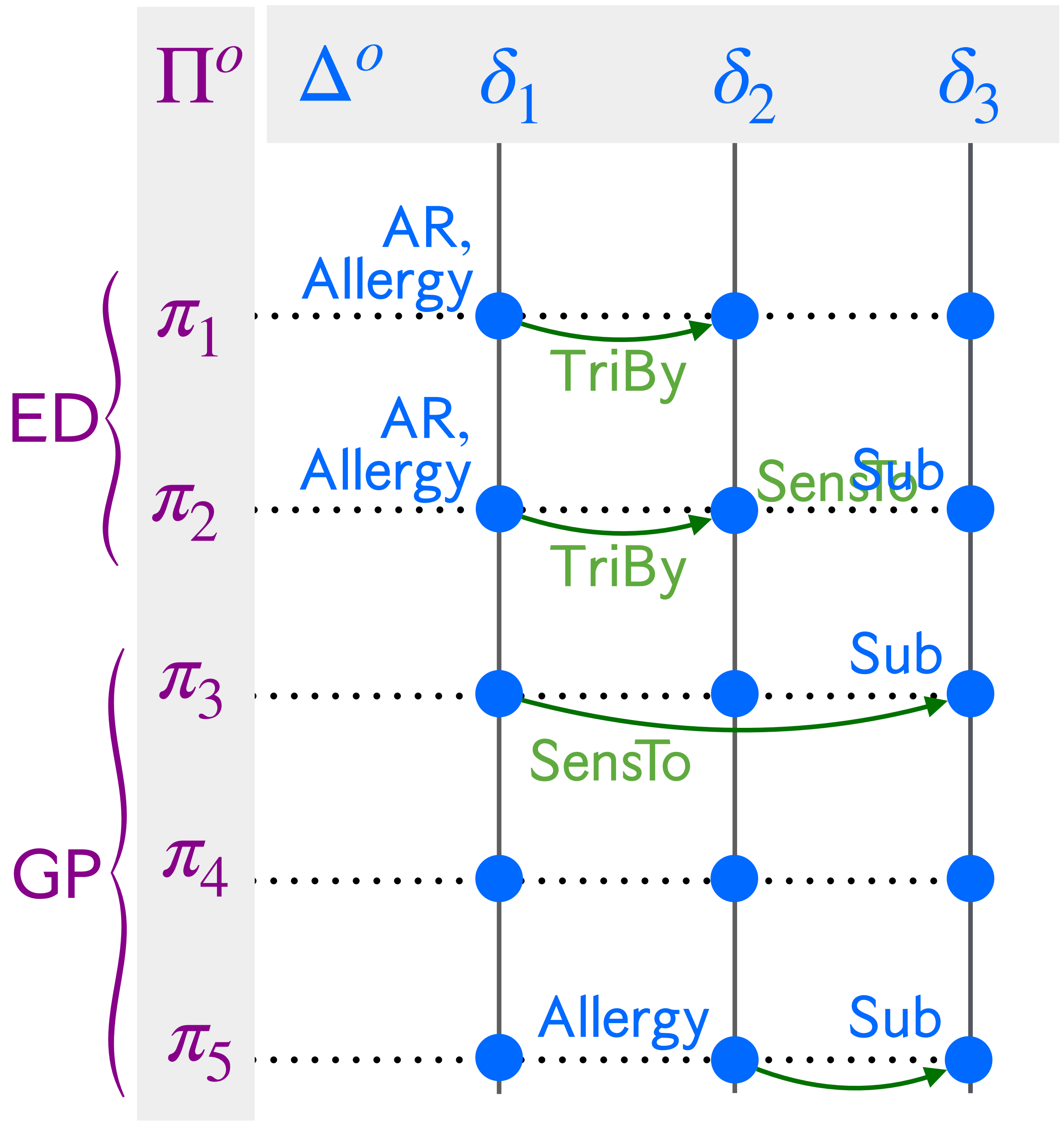

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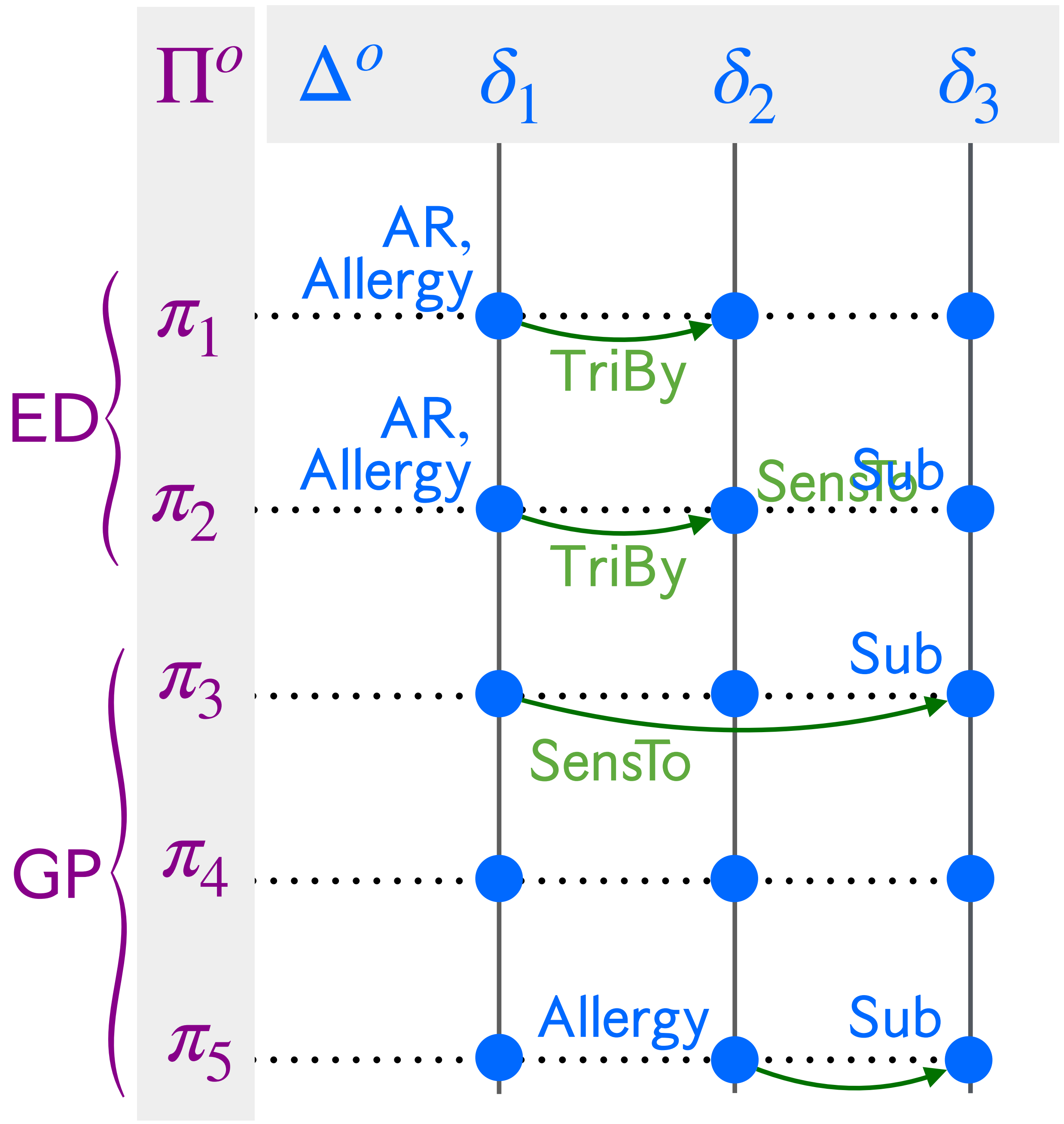

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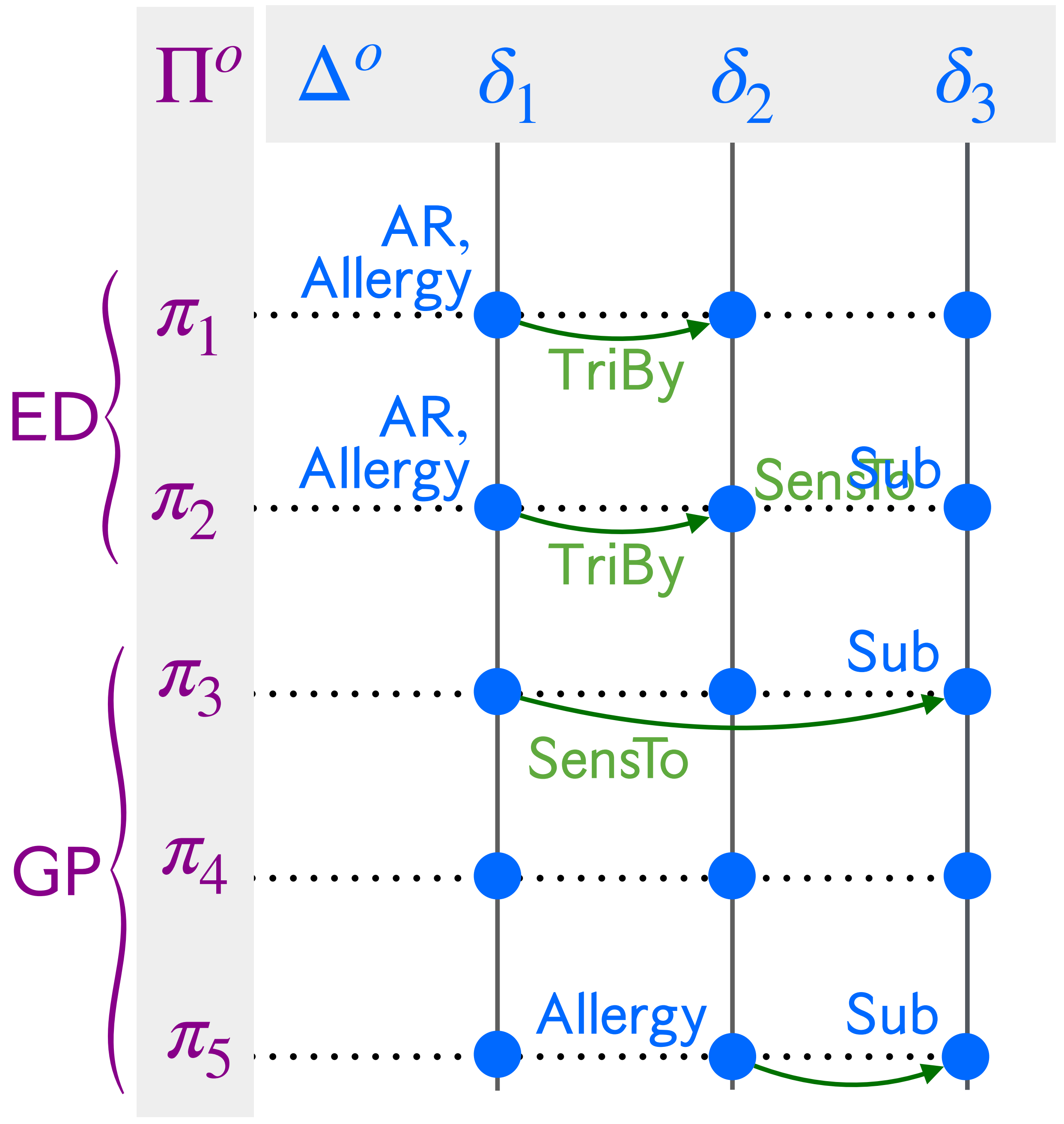

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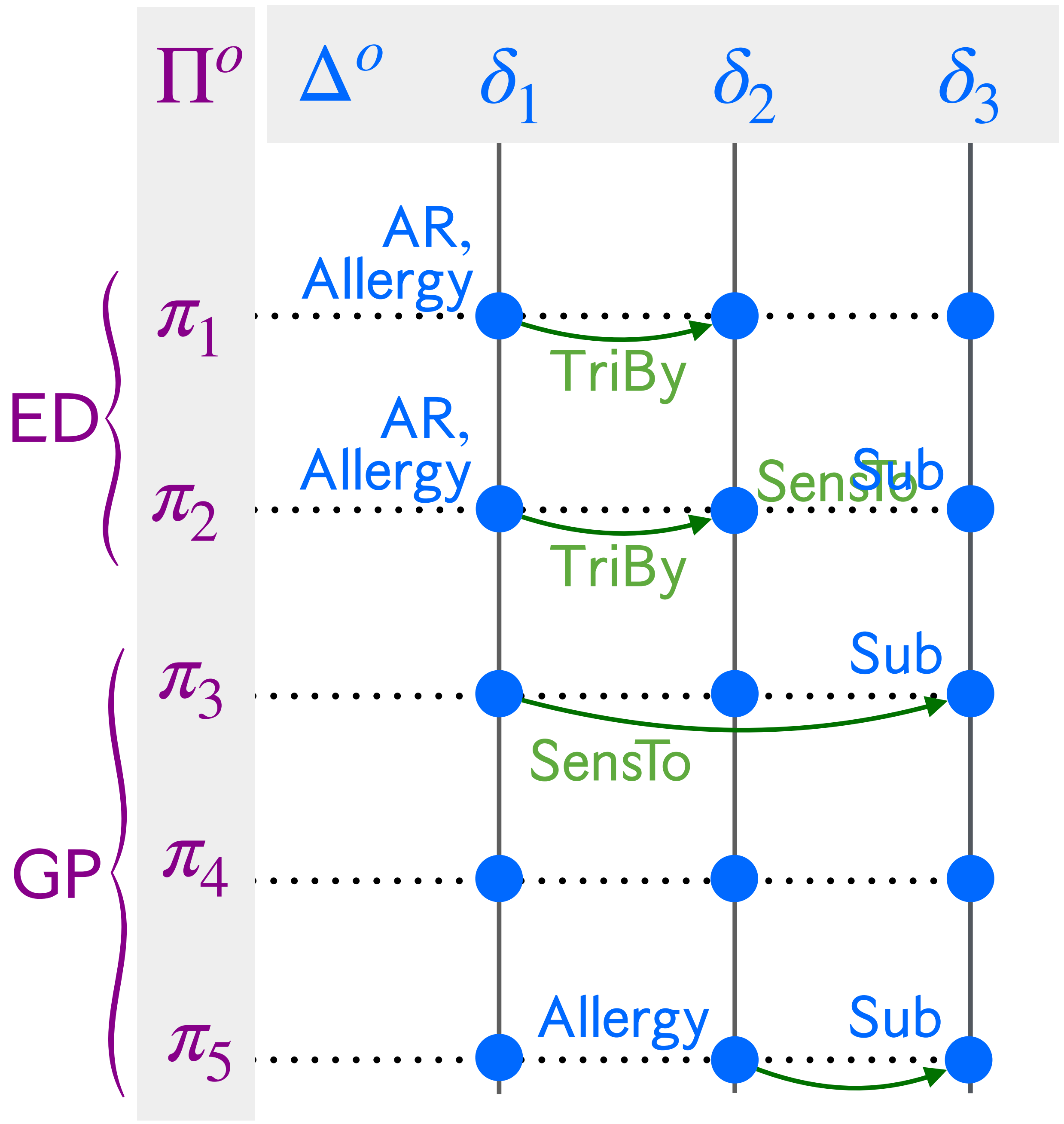

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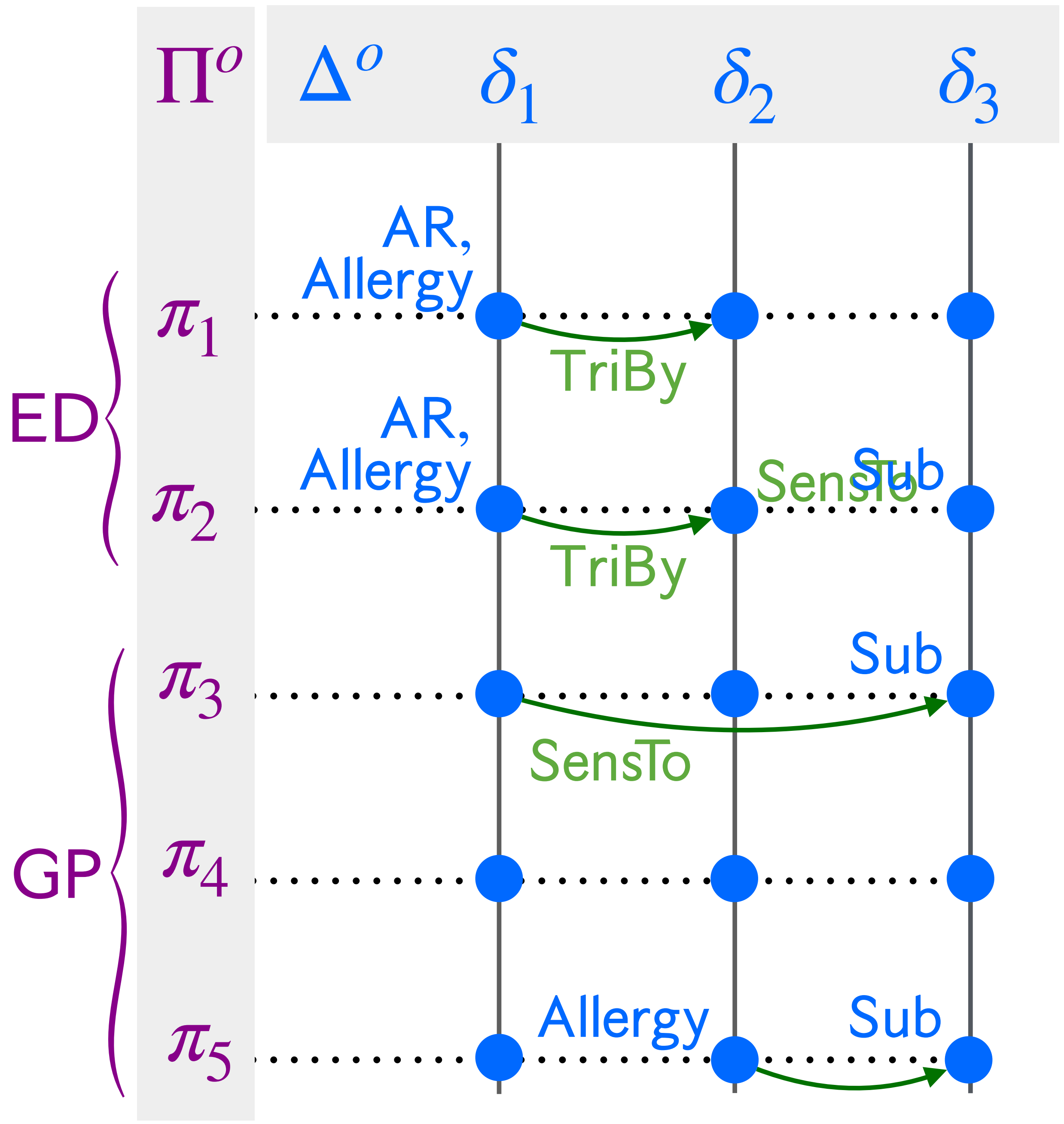

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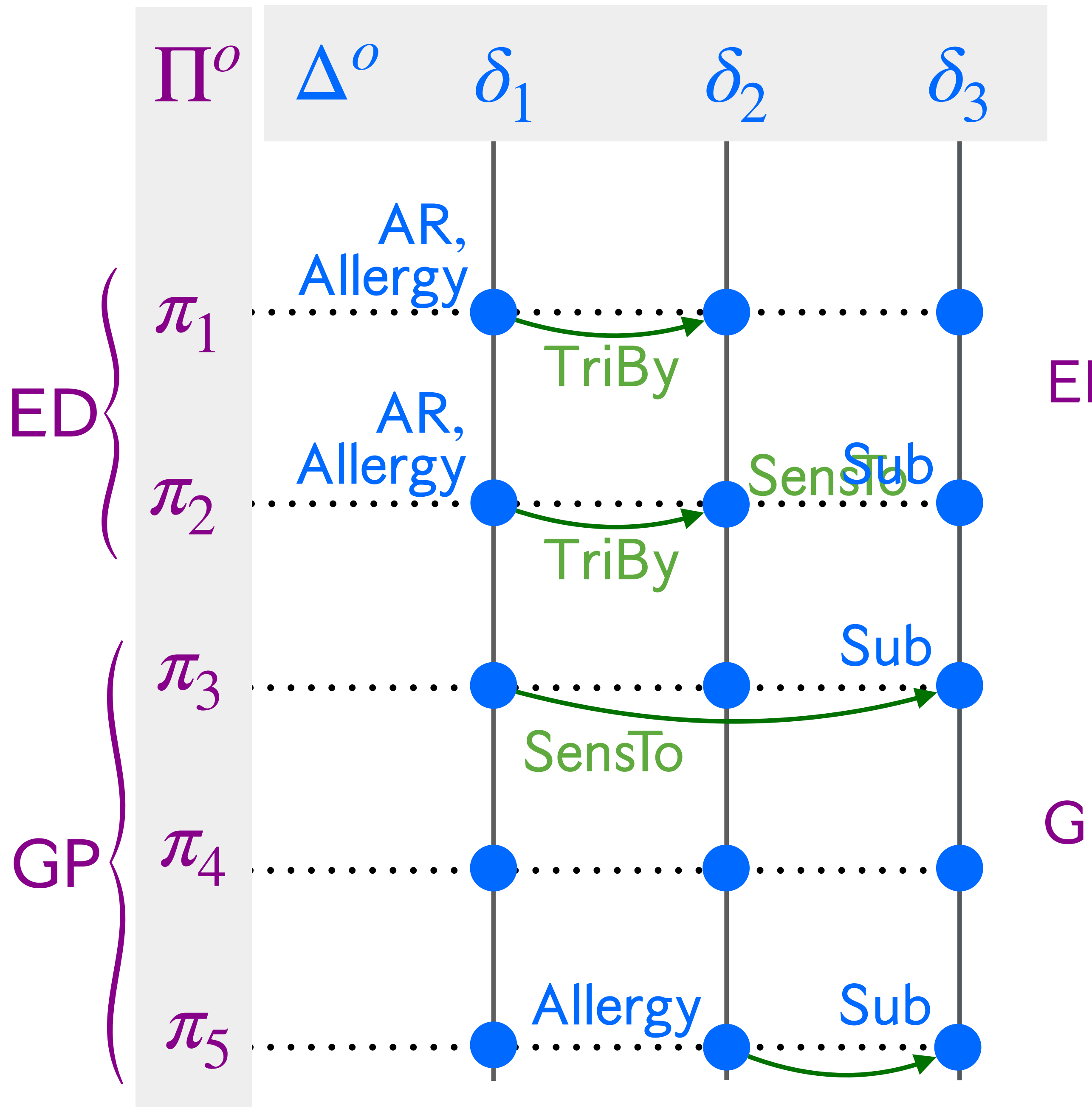

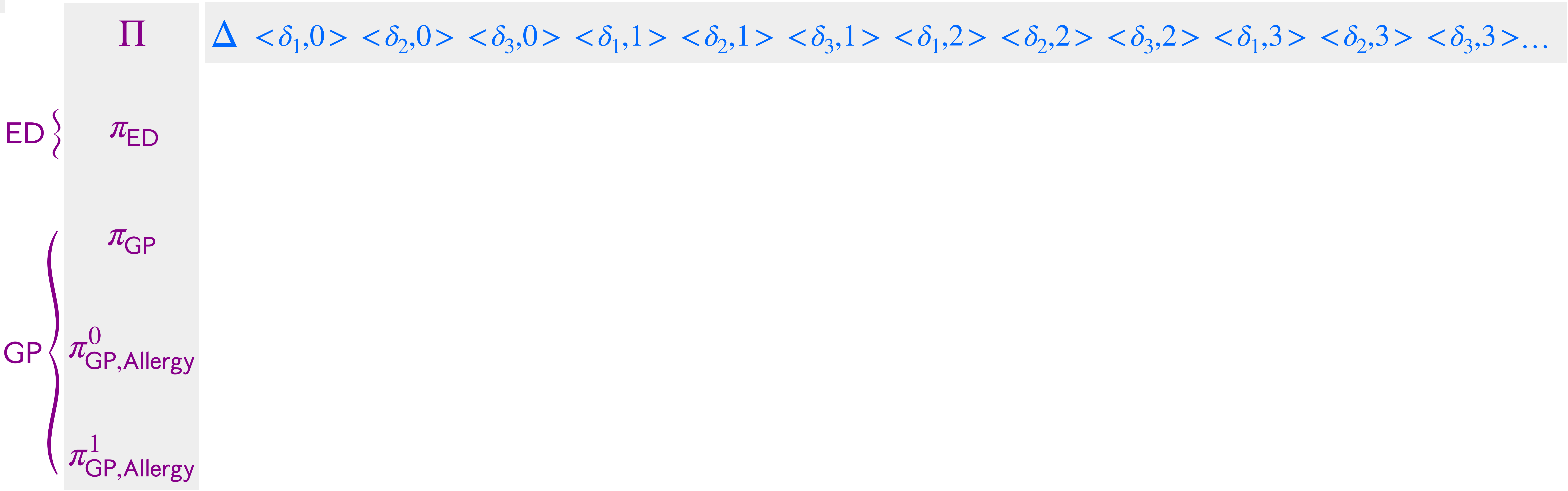

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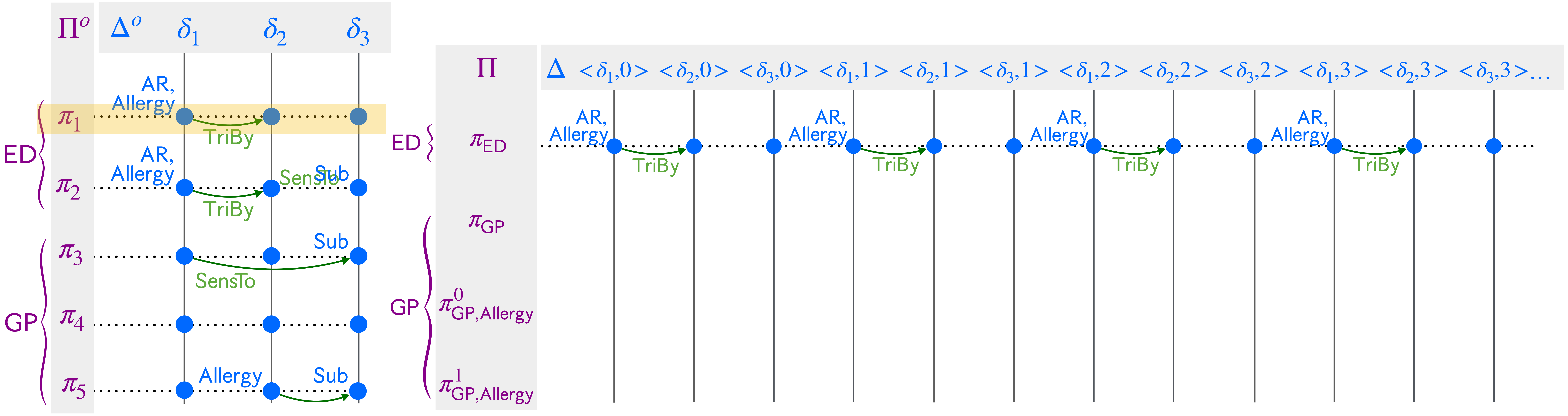
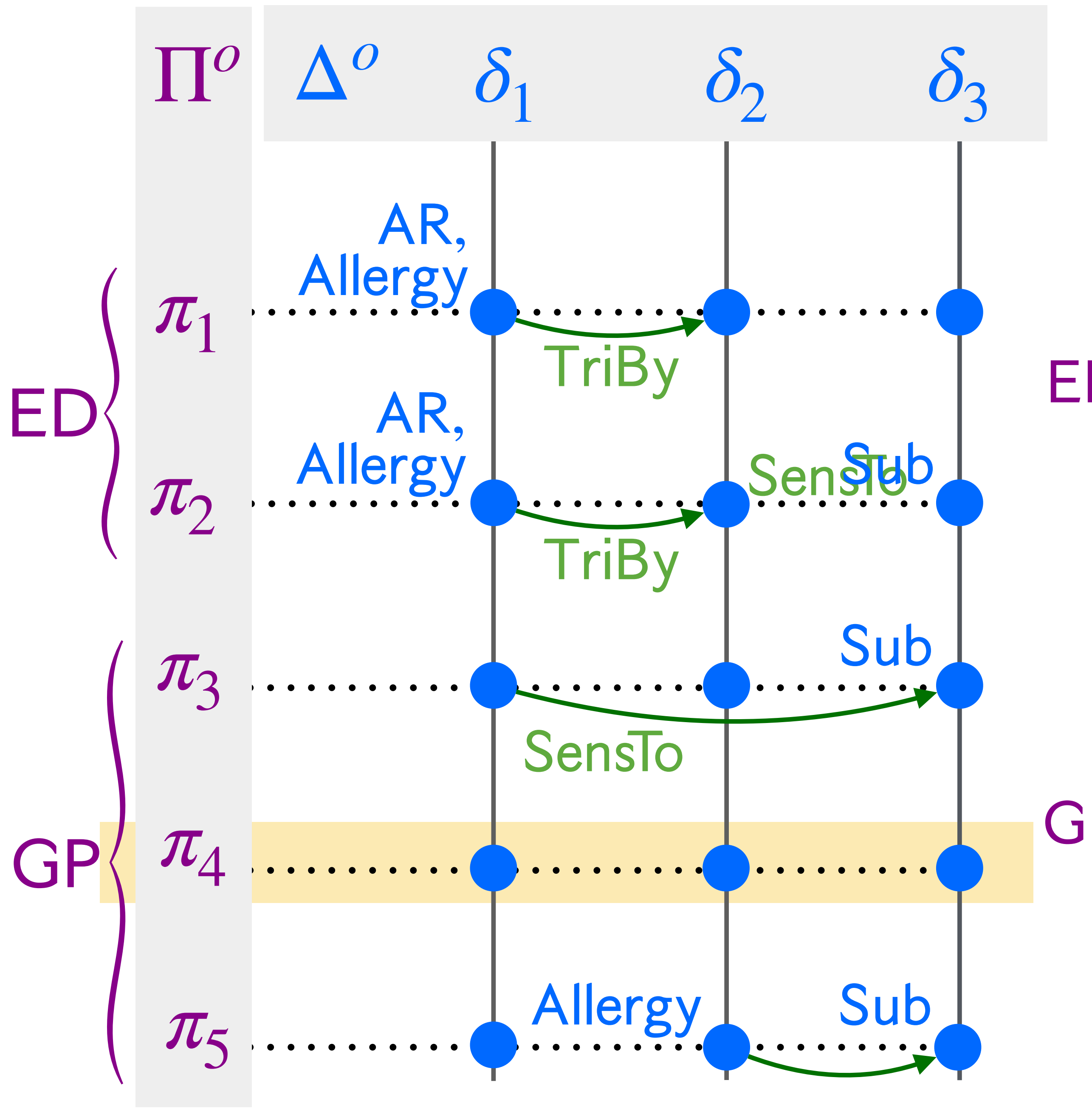
\end{aligned}
$$

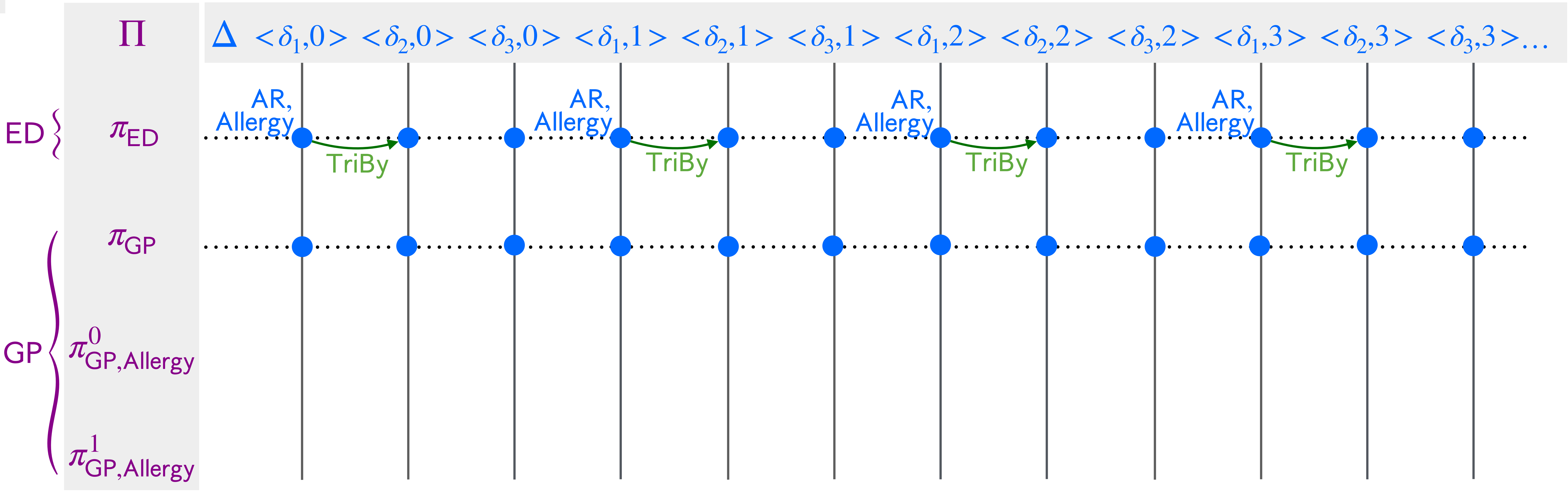


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathcal{K} = \{\ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq\ =1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\ \}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	$\dots$
π_{ED}	δ_1	$\pi_1, 0$	$\pi_1, 1$	$\pi_1, 2$	$\pi_1, 3$	
	δ_2	$\pi_1, 0$	$\pi_1, 1$	$\pi_1, 2$	$\pi_1, 3$	
	δ_3	$\pi_1, 0$	$\pi_1, 1$	$\pi_1, 2$	$\pi_1, 3$	
π_{GP}	δ_1	$\pi_4, 0$	$\pi_4, 1$	$\pi_4, 2$	$\pi_4, 3$	
	δ_2	$\pi_4, 0$	$\pi_4, 1$	$\pi_4, 2$	$\pi_4, 3$	
	δ_3	$\pi_4, 0$	$\pi_4, 1$	$\pi_4, 2$	$\pi_4, 3$	

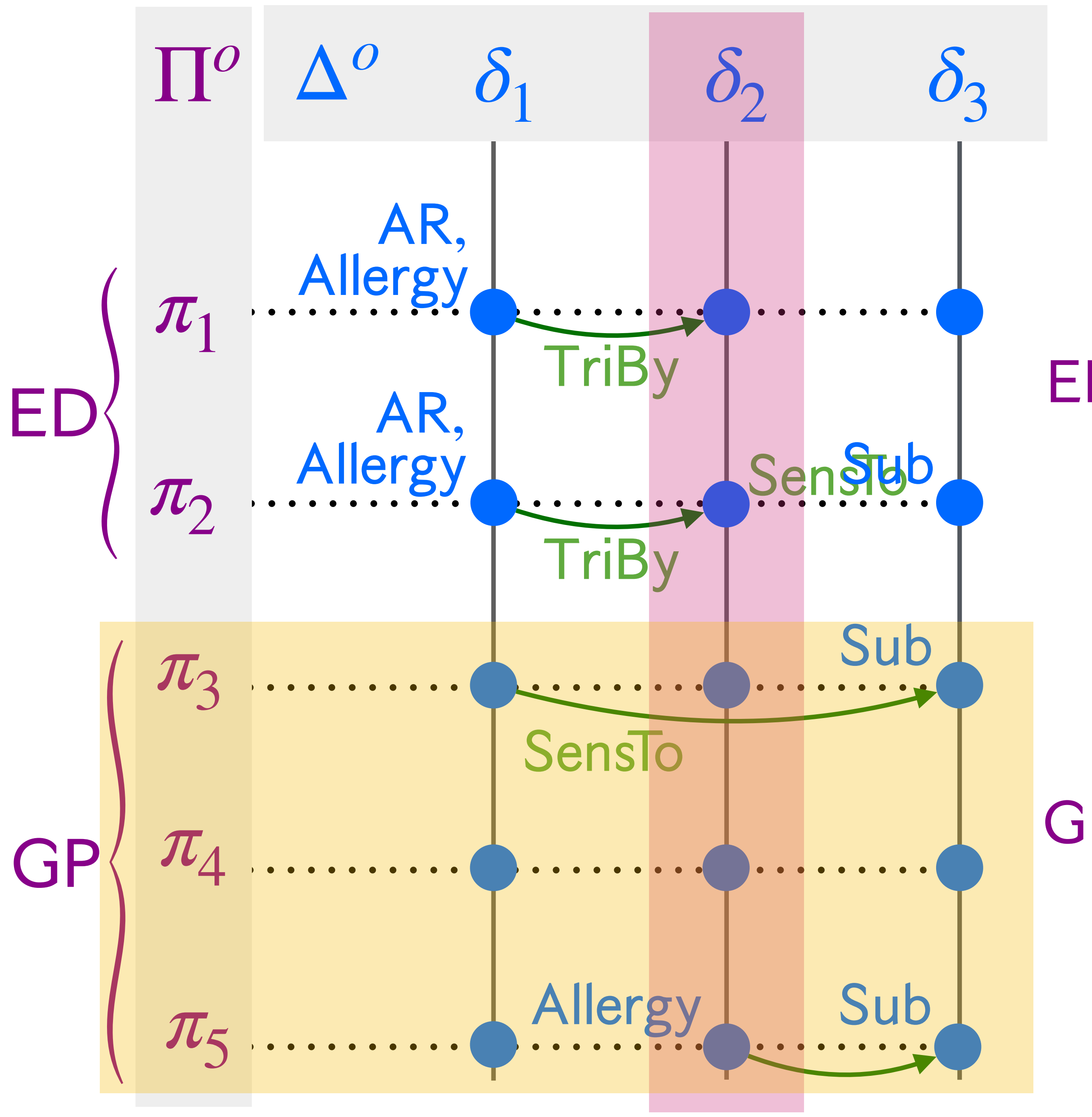


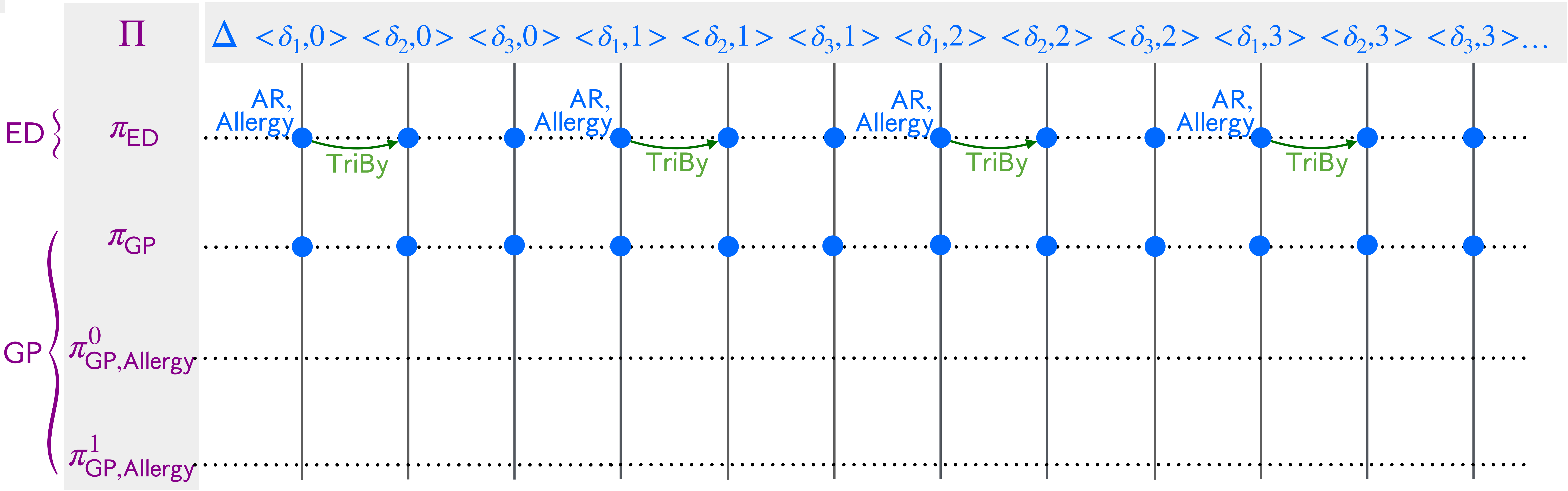


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{\, & \square_{\mathrm{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathrm{GP}}\,(\mathsf{Allergy} \sqsubseteq\, =1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathrm{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathrm{GP}}\mathsf{Allergy})\,\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	$\dots$
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

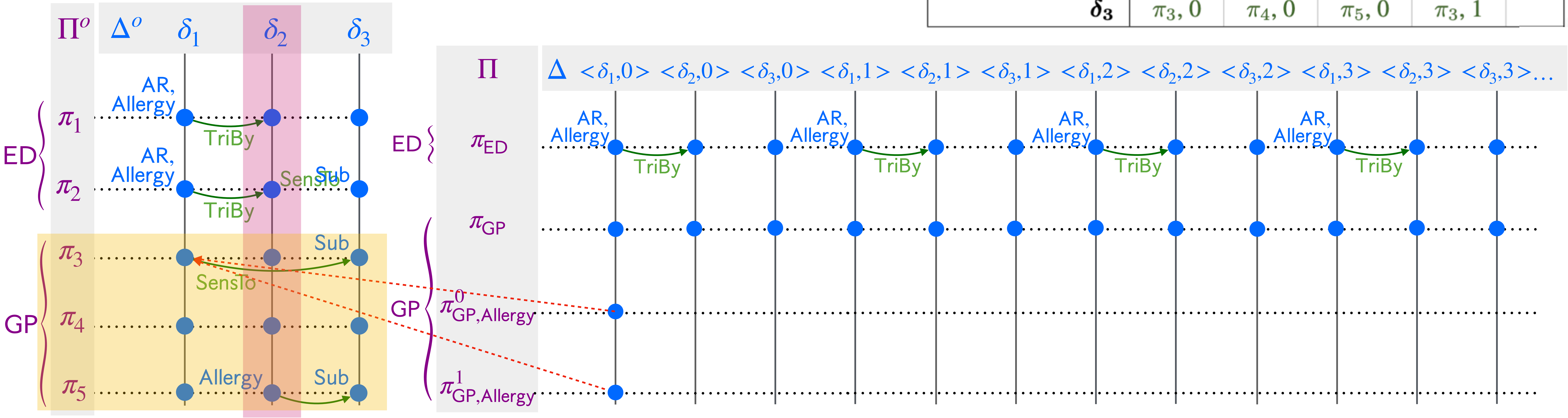




Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathcal{K} = \{\, &\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ &\Box_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq {=}\,1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ &\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\,\}\end{aligned}$$

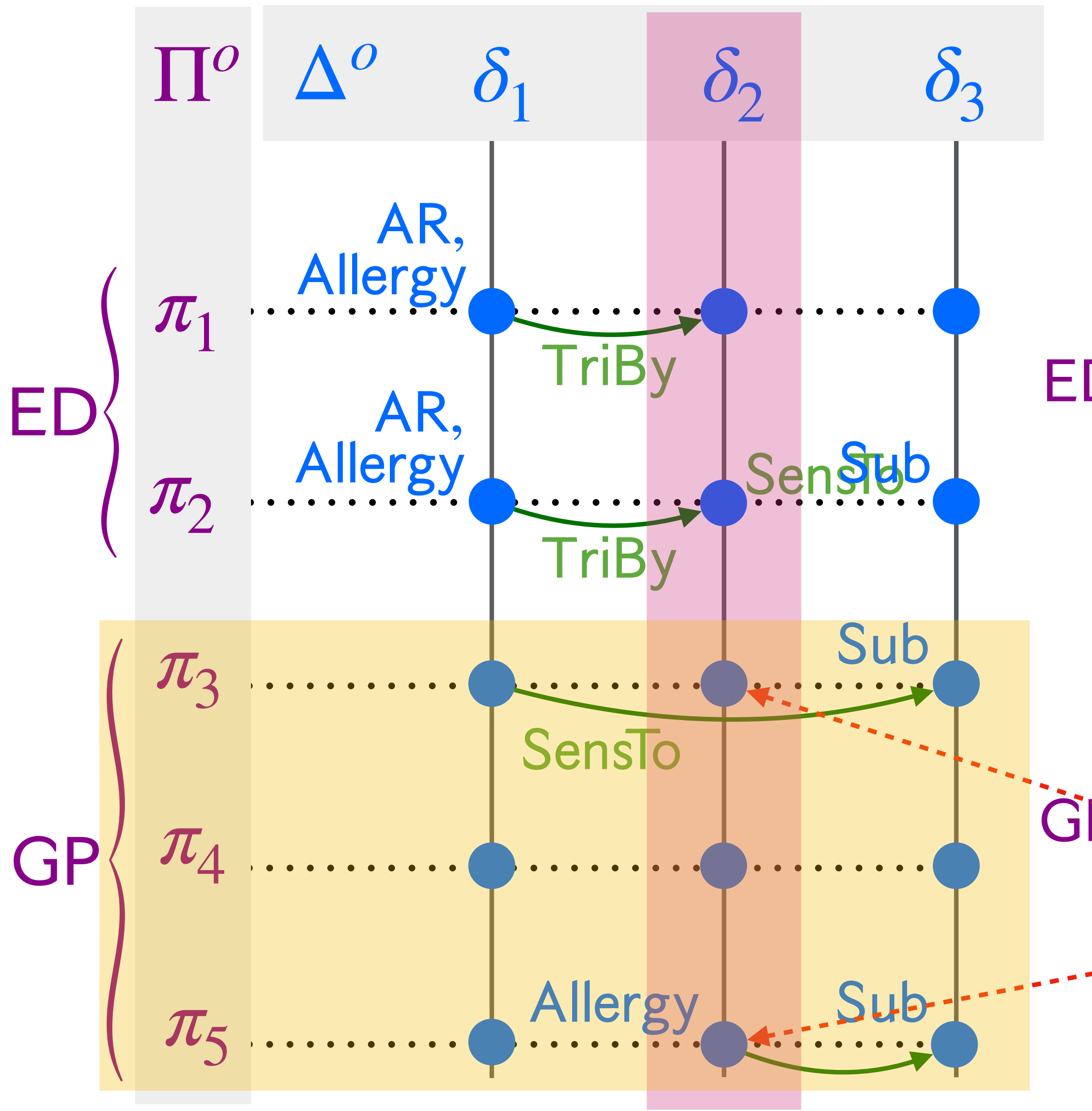
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\texttt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\texttt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

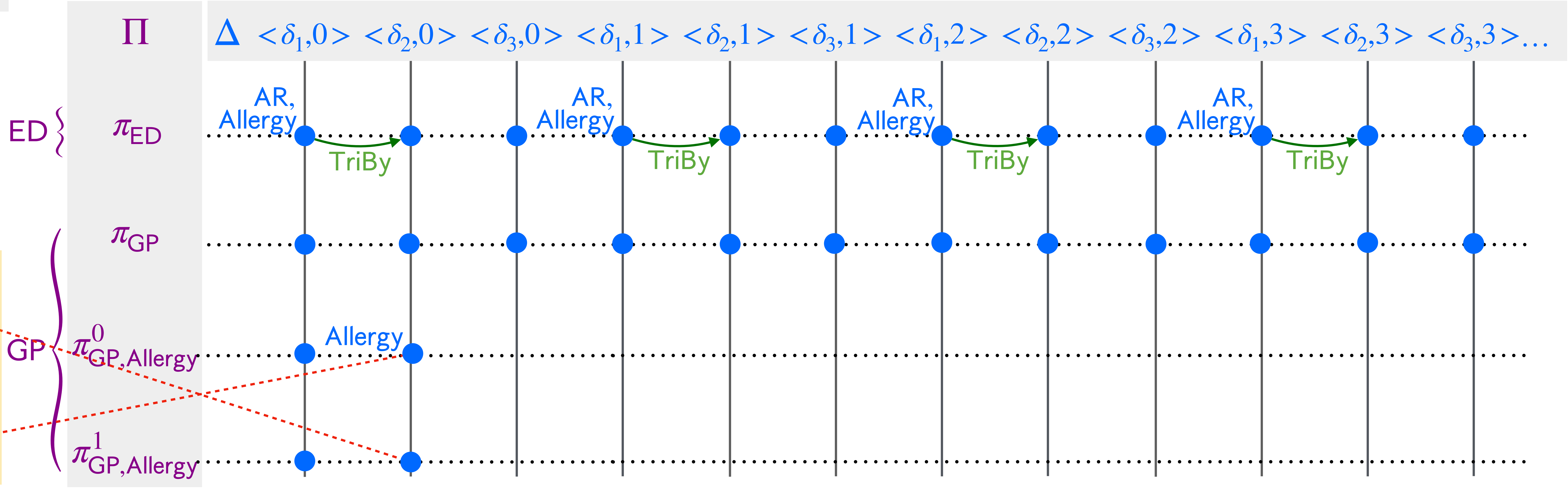


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq {=}\,1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$: $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

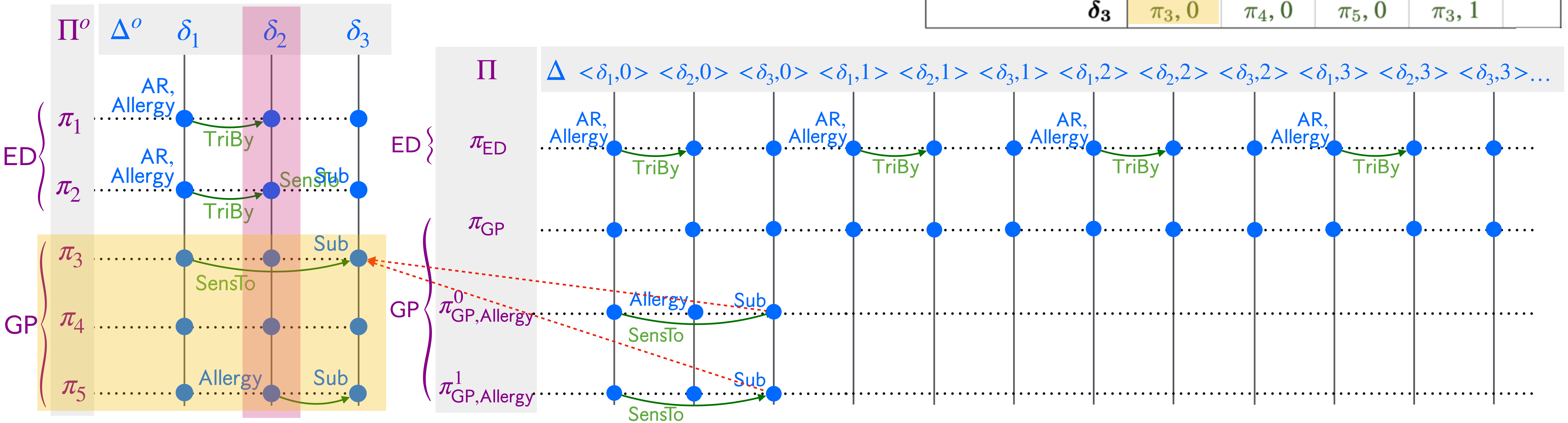




Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathcal{K} = \{\ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq {=}1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\lozenge_{\mathsf{GP}}\mathsf{Allergy})\ \}\end{aligned}$$

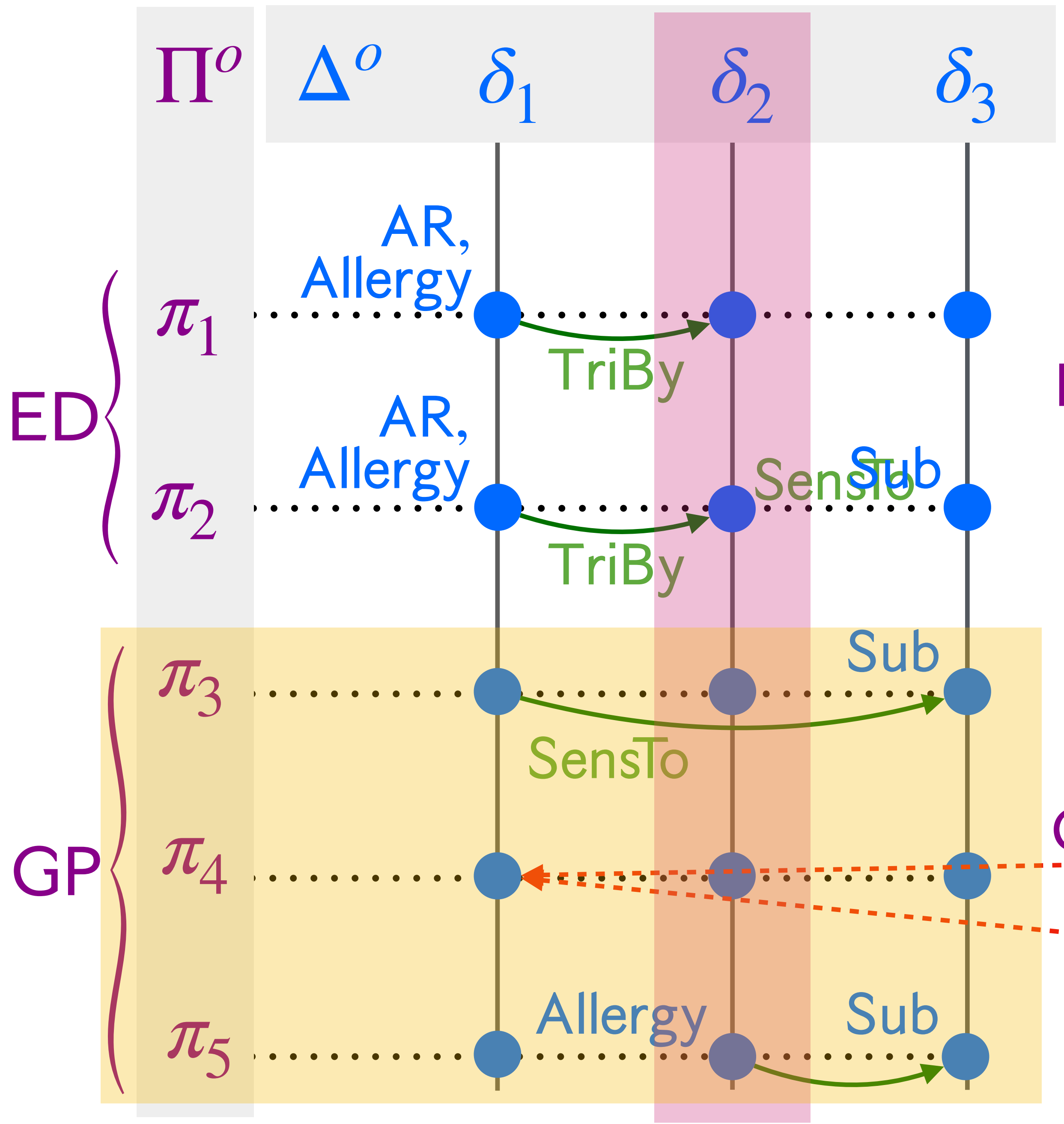
$\mathbf{\Pi}$	$\mathbf{\Delta}$	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

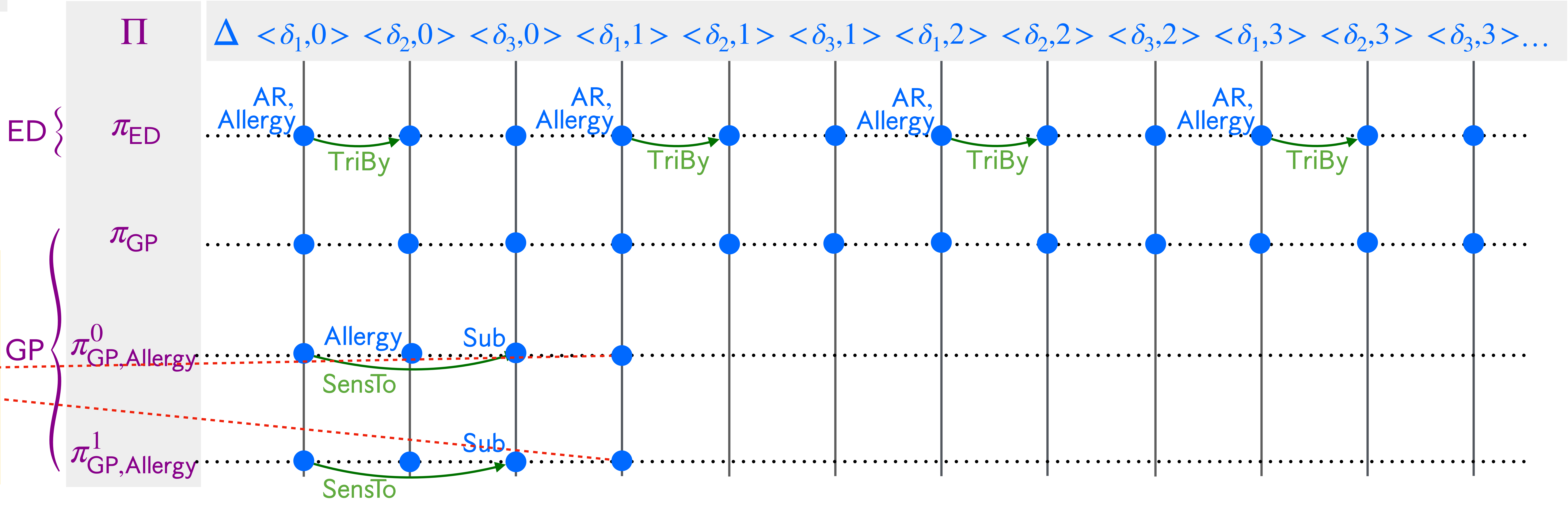


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{\, & \square_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathsf{GP}}\,(\mathsf{Allergy} \sqsubseteq\, =1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\,\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

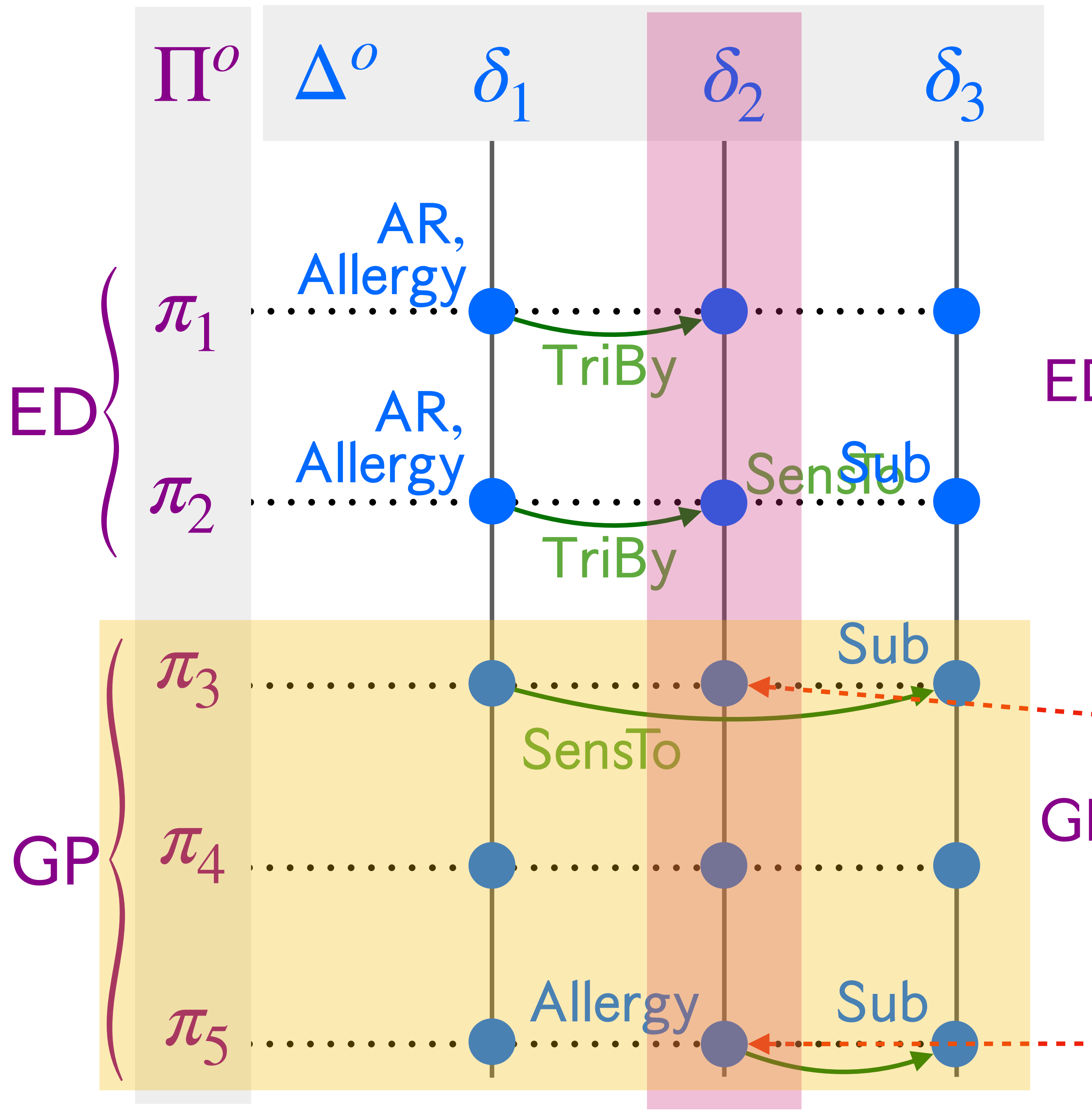


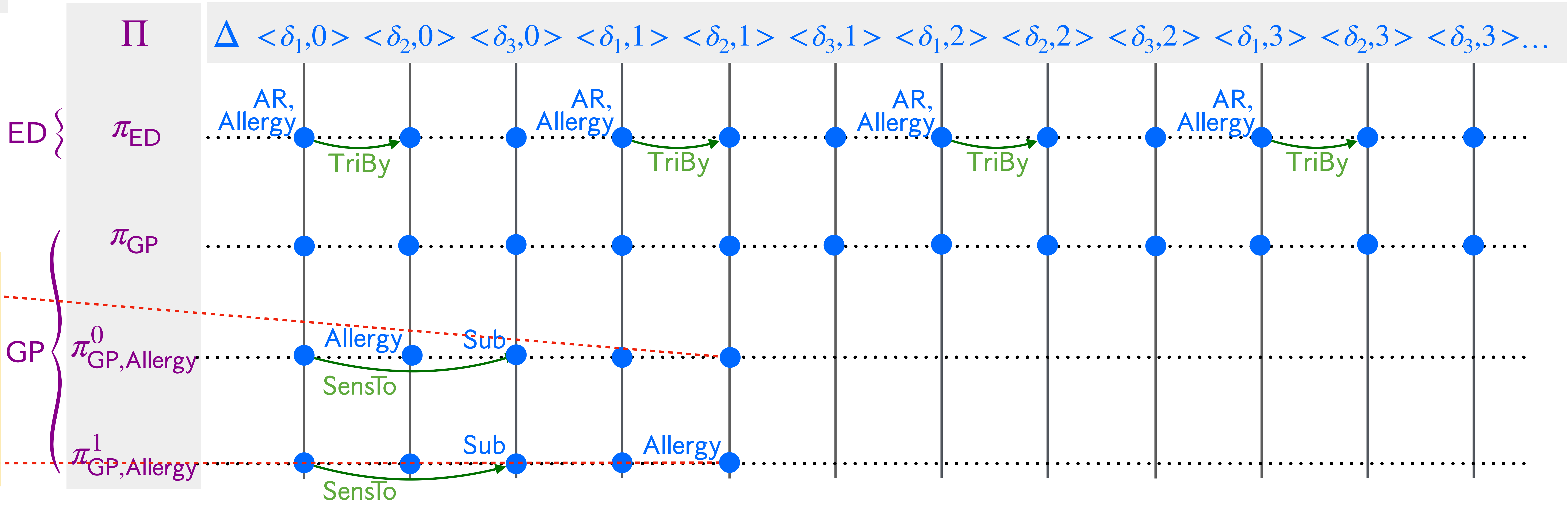


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathscr{K} = \{\, \Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq\ = 1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
$$\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists \mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\,\}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

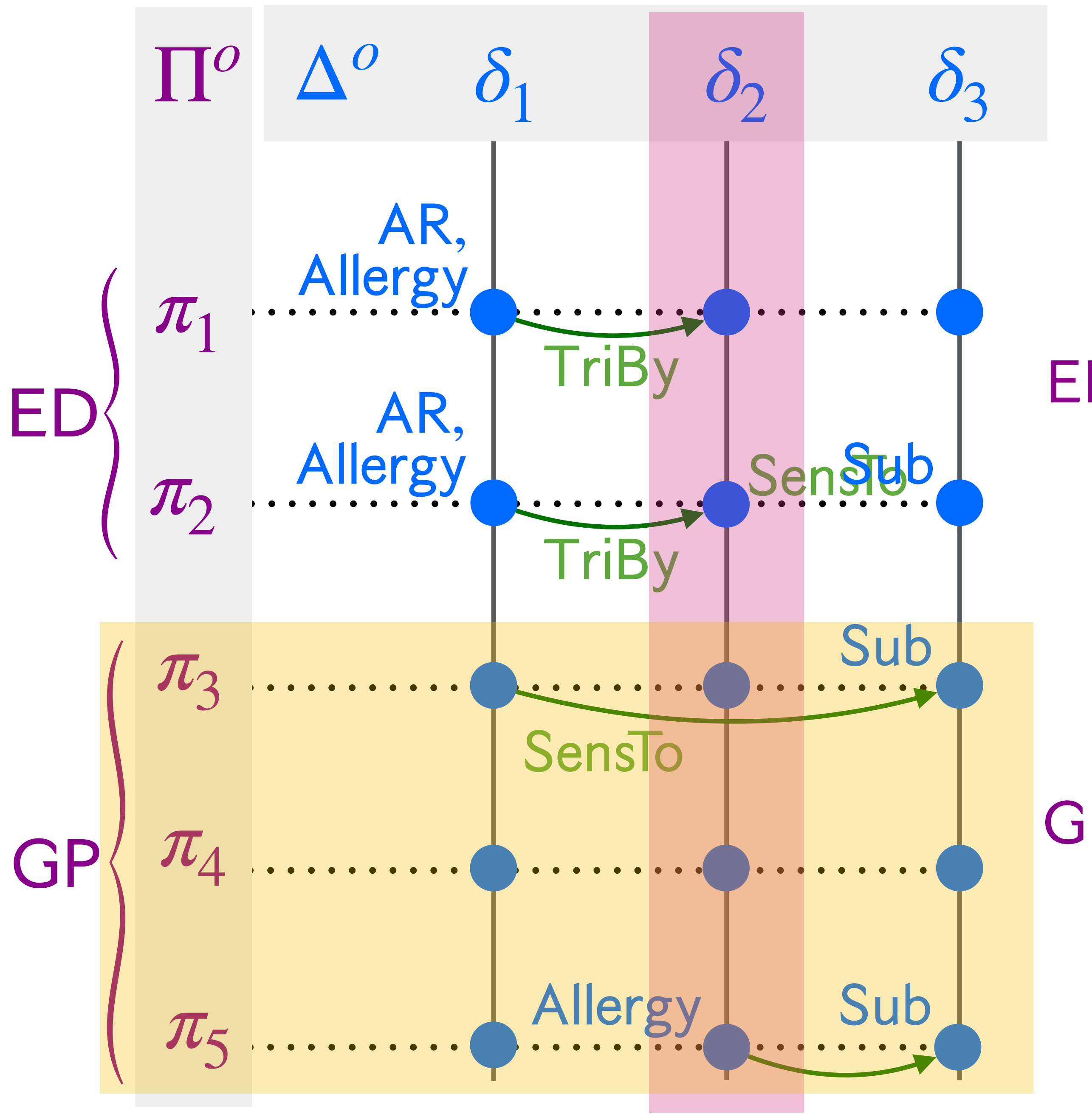


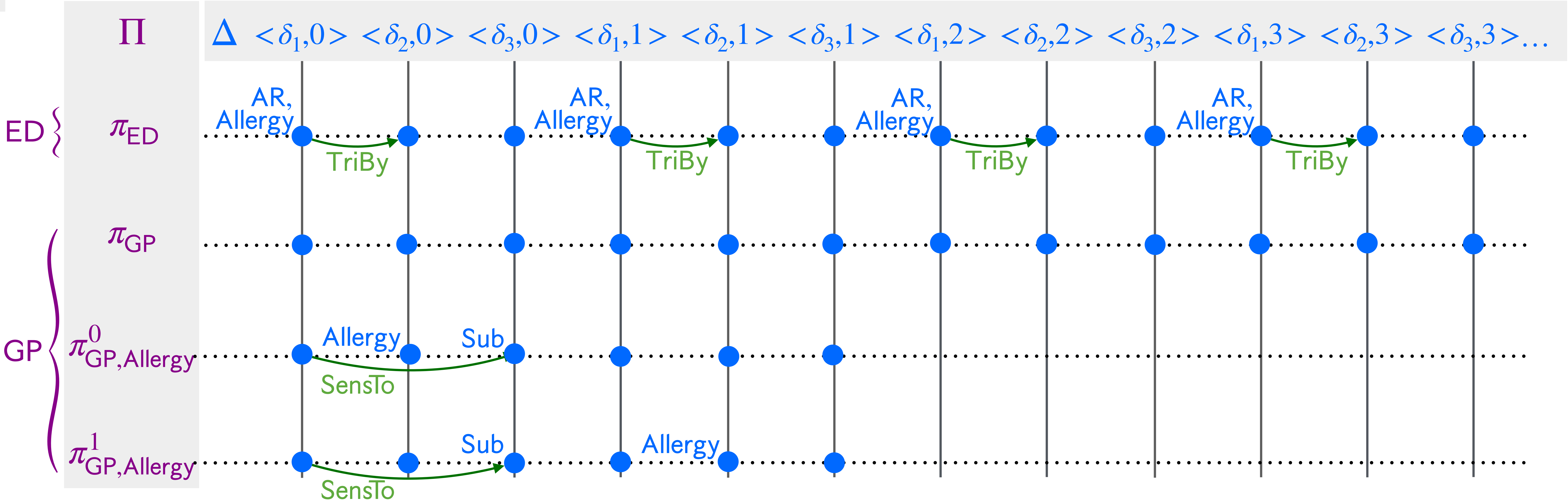


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathcal{K} = \{\ &\square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ &\square_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq {=}1\mathsf{SensitivityTo}.\mathsf{Substance}),\\ &\square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}.\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	





Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

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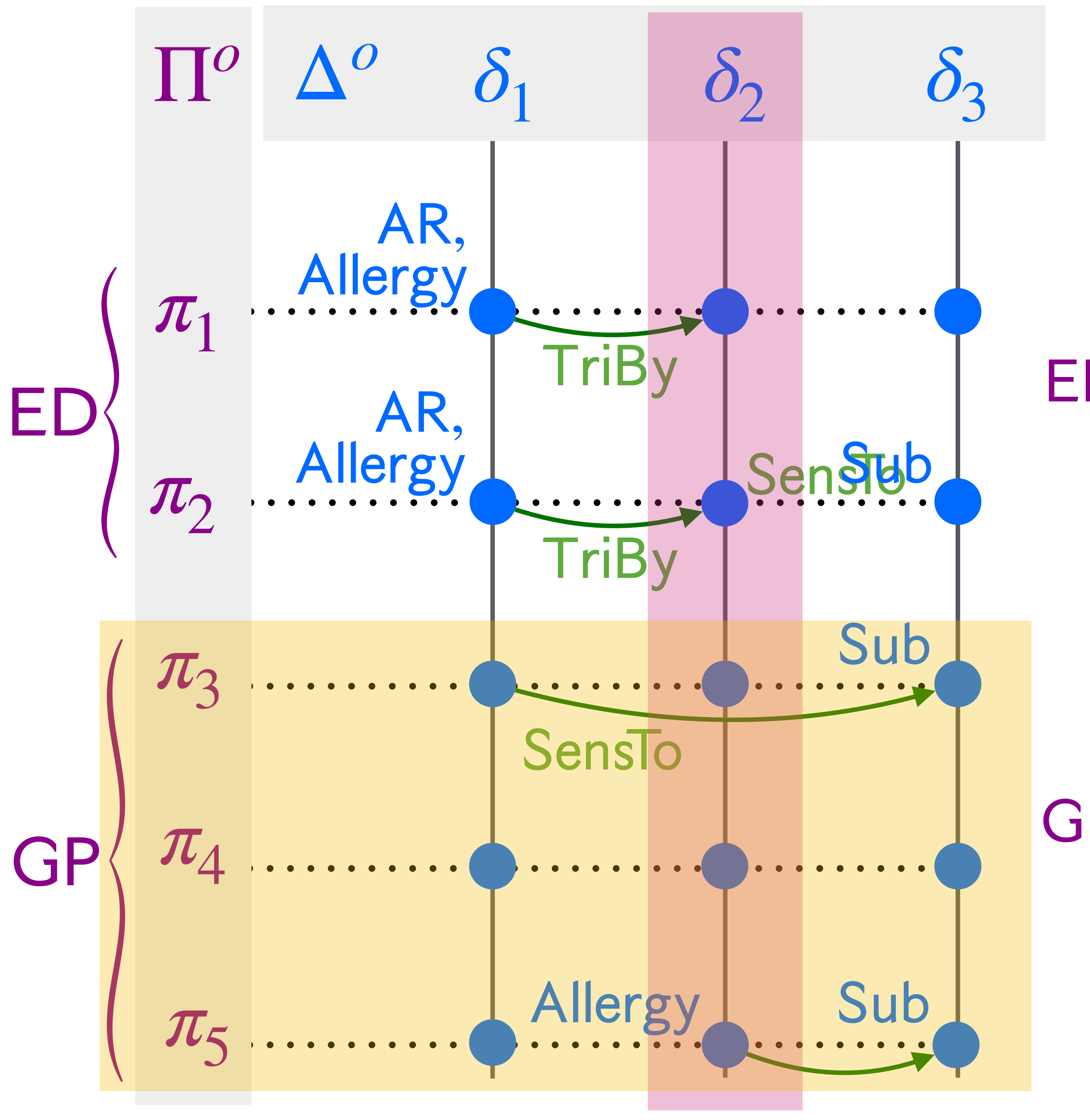
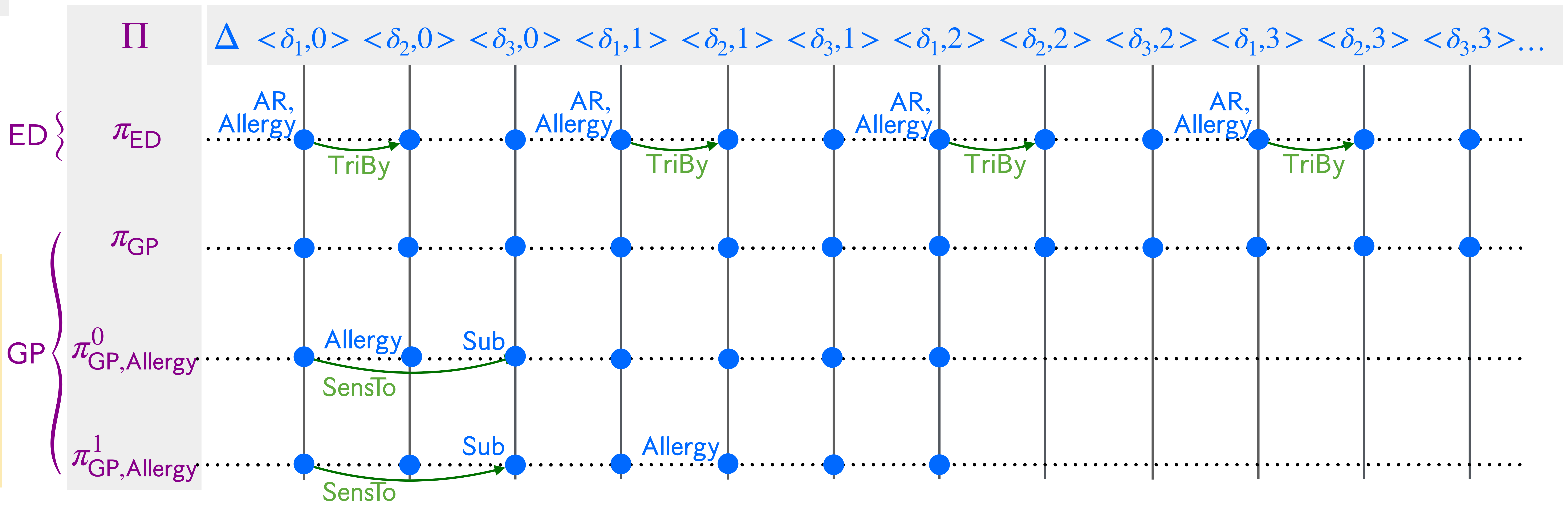
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

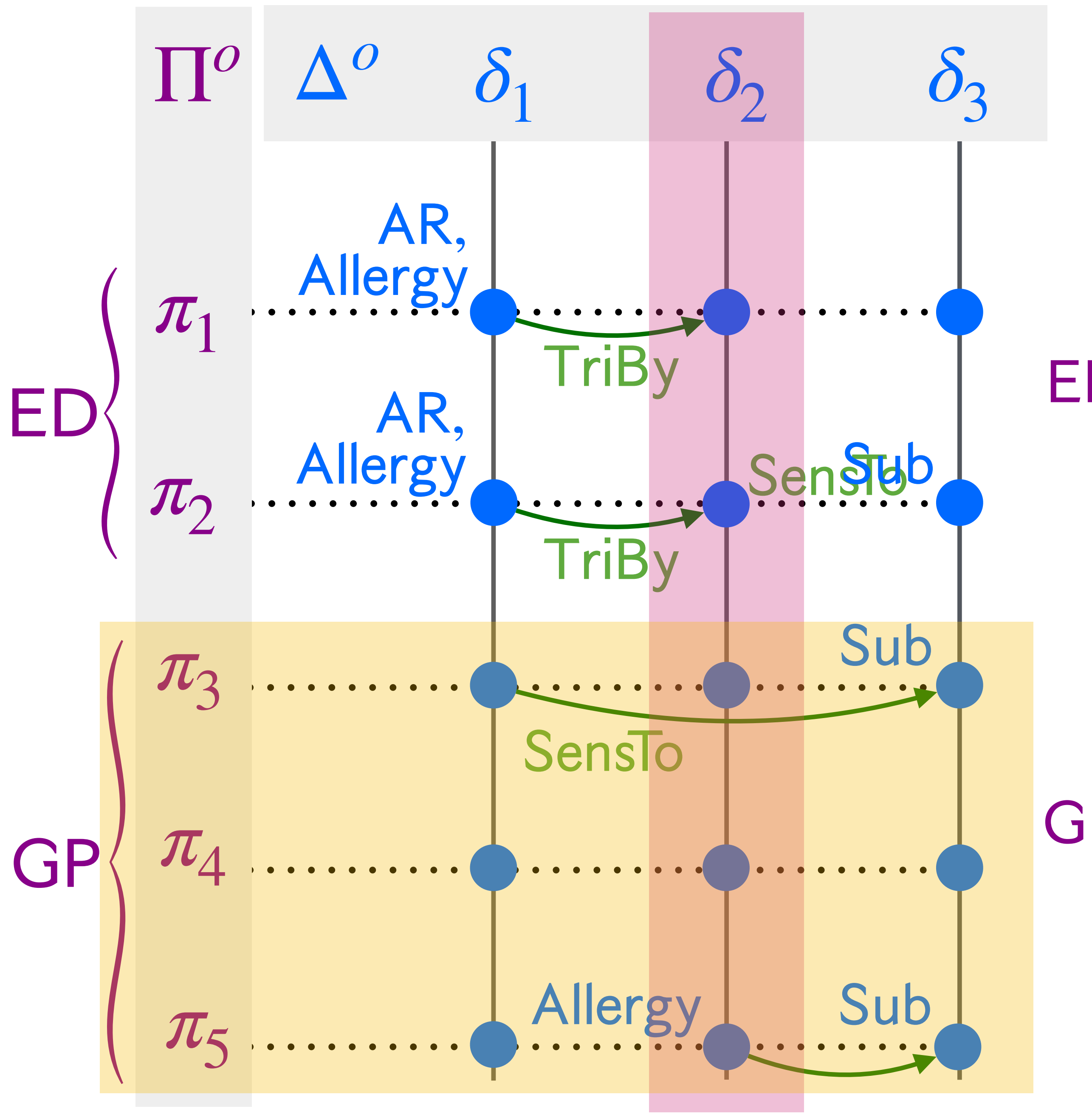


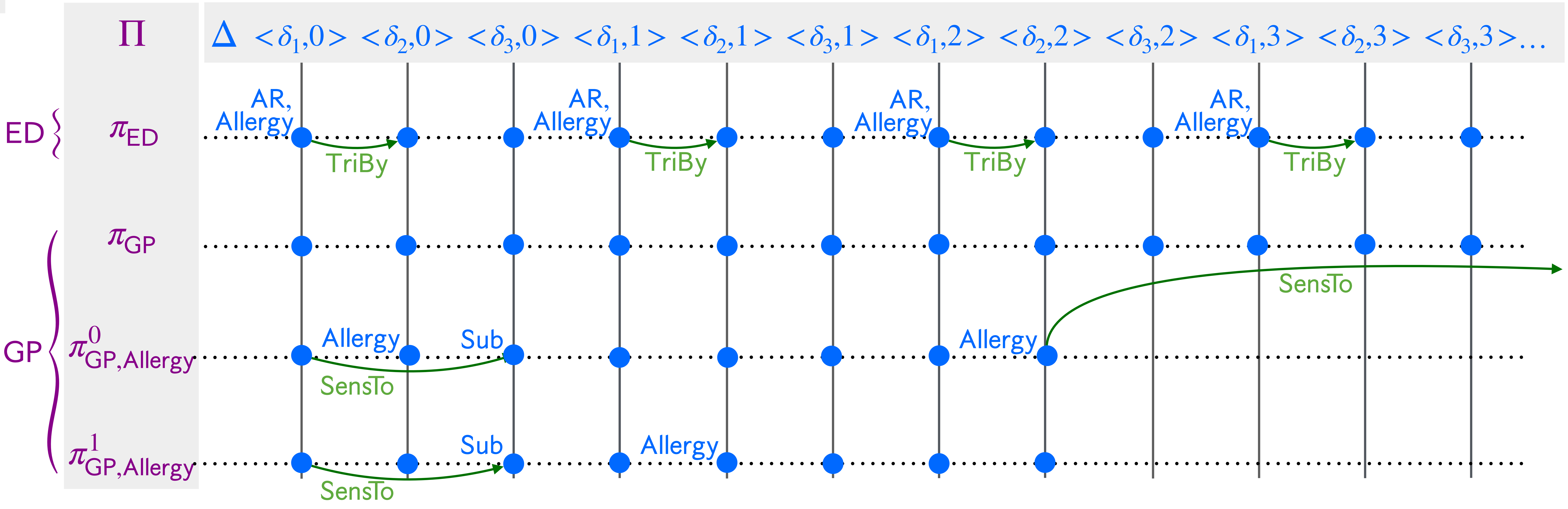


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{\ &\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ &\Box_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq\ = 1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ &\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$: $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\texttt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\texttt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	





Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

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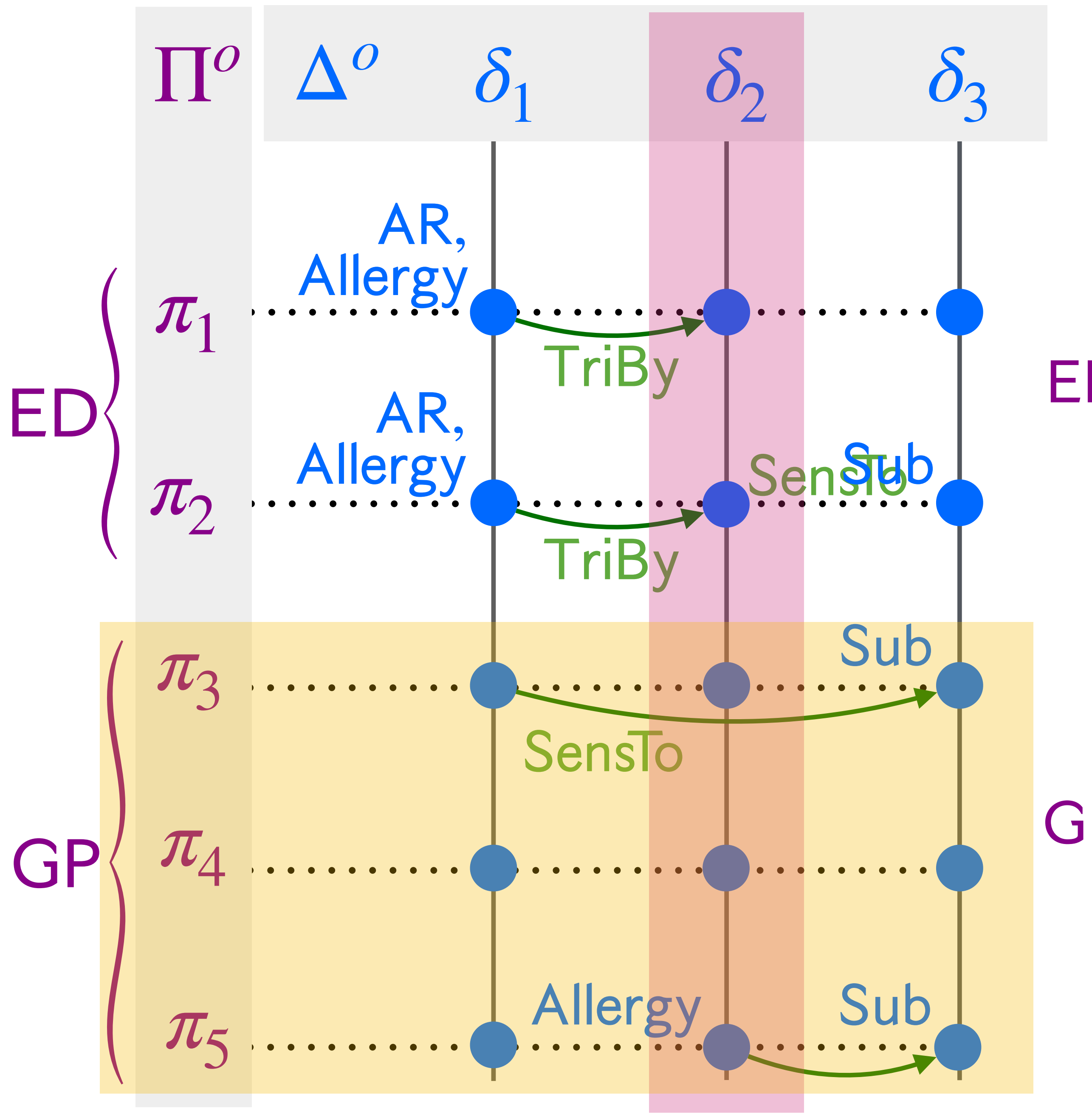
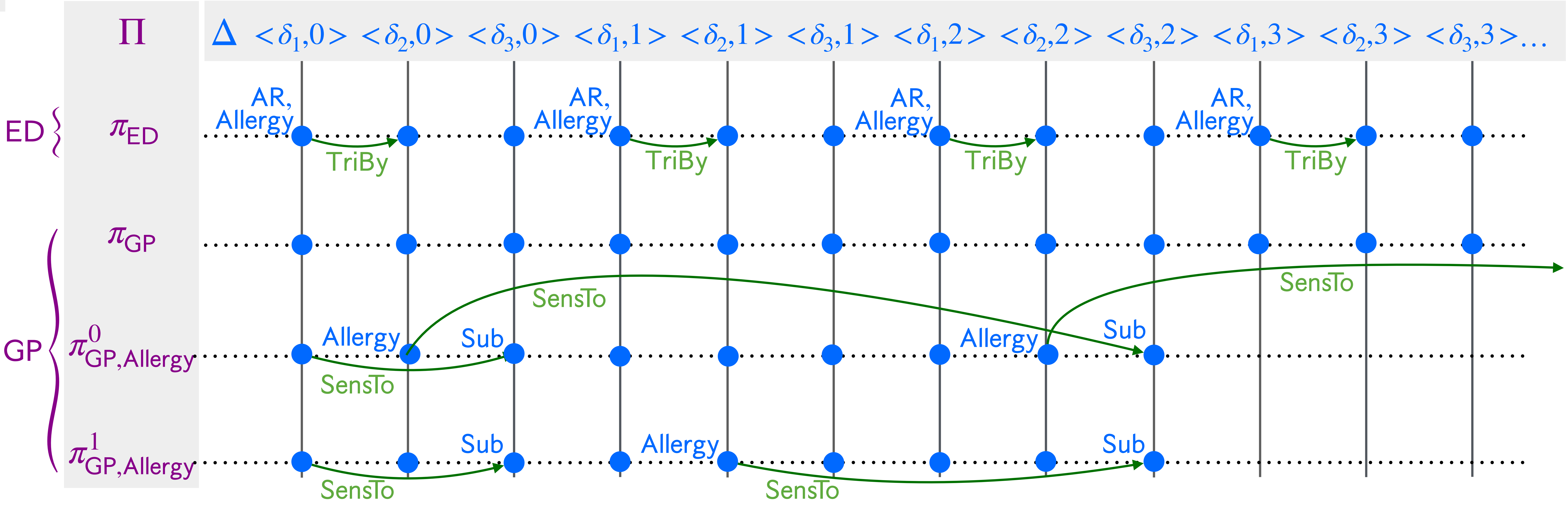
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

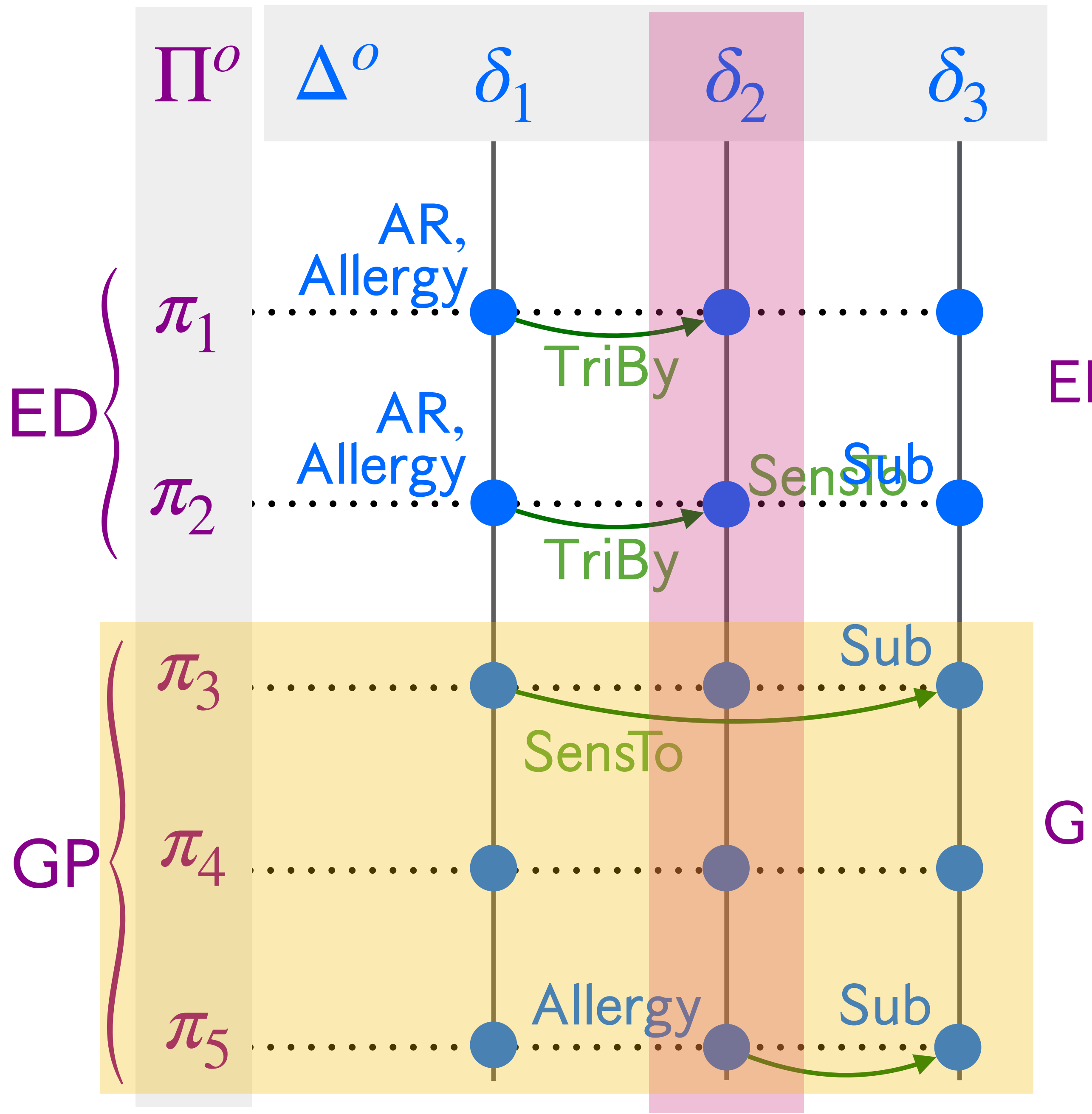


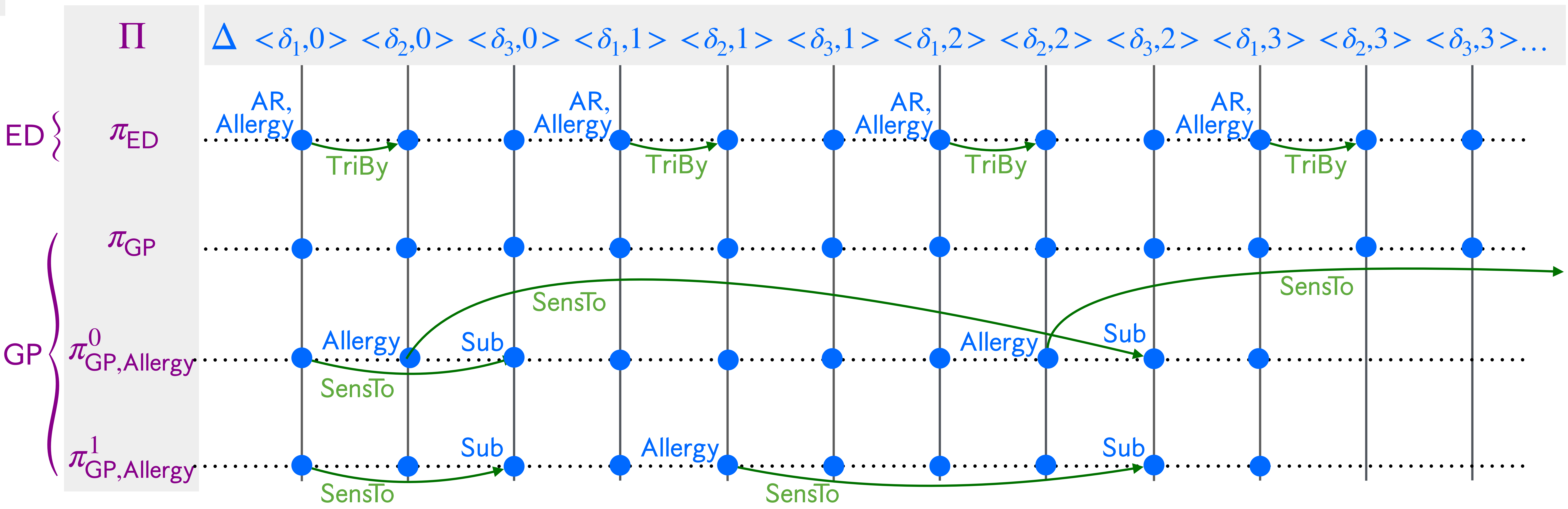


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{\, & \square_{\mathrm{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathrm{GP}}\,(\mathsf{Allergy} \sqsubseteq\, =1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathrm{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathrm{GP}}\mathsf{Allergy})\,\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	…
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

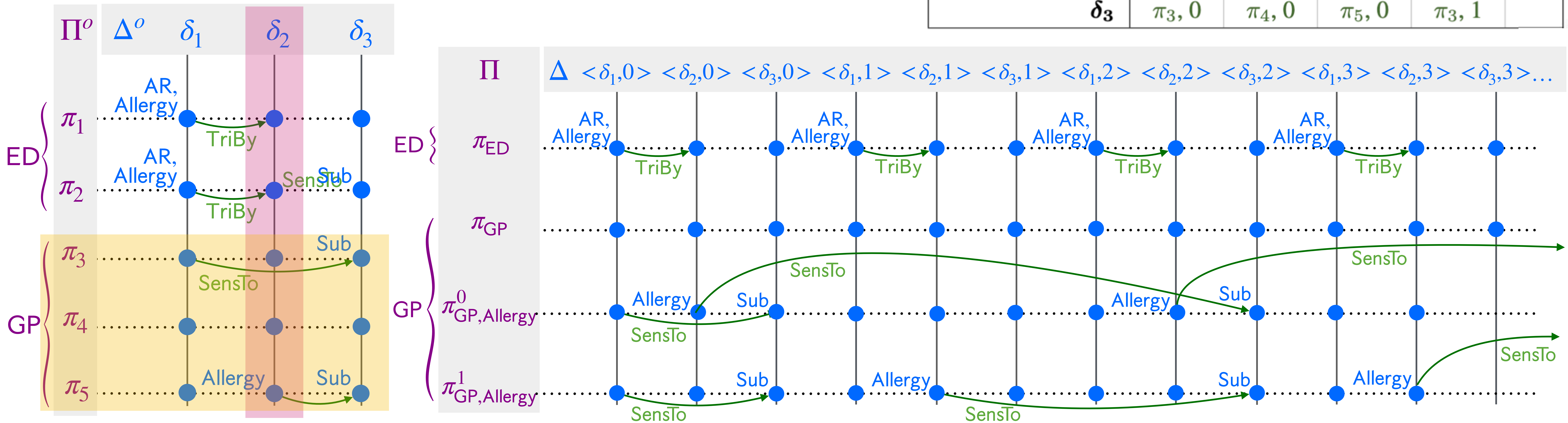




Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathcal{K} = \{\, \Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq\, =1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
$$\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\,\}$$

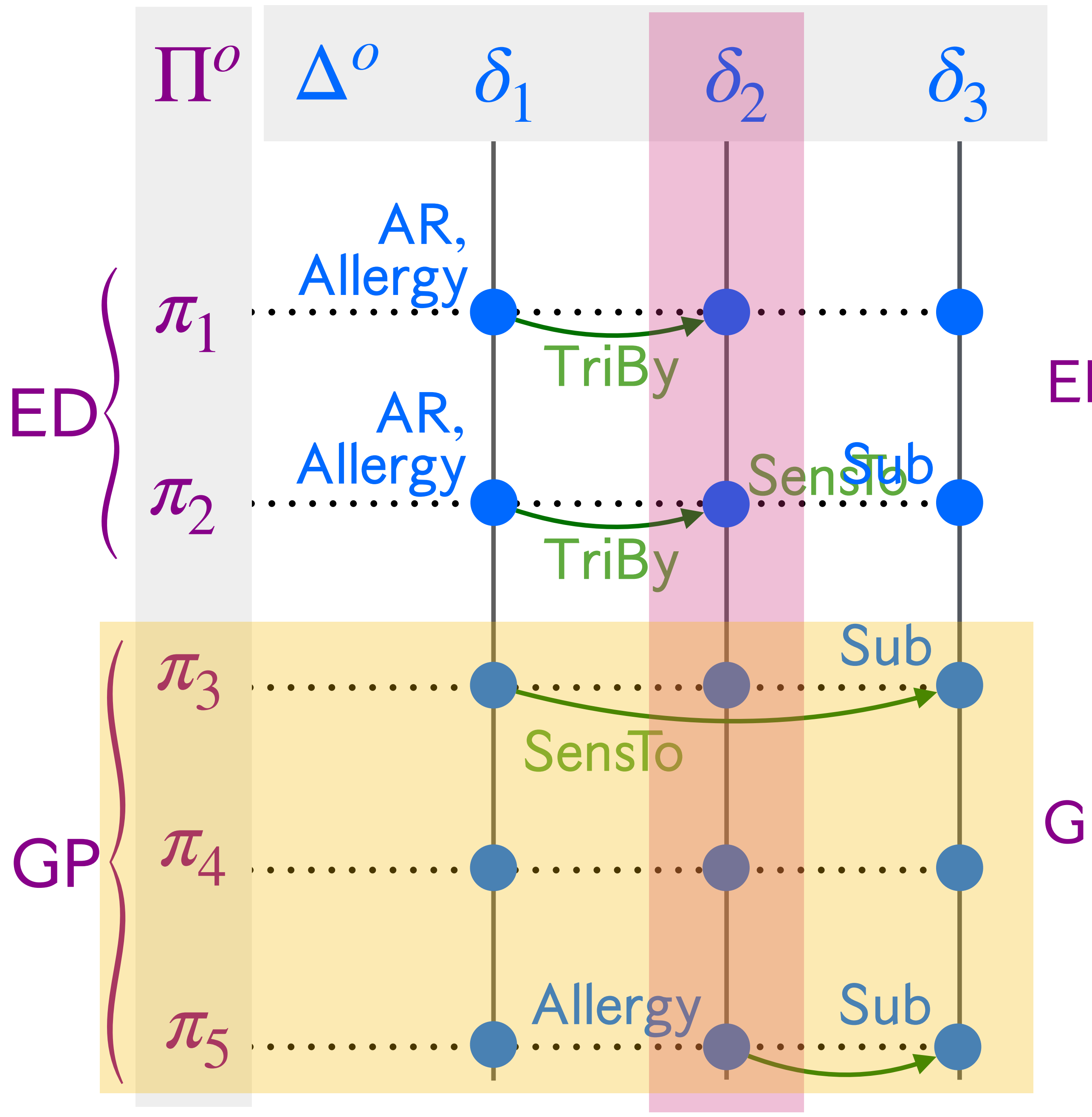
		$f(\pi,\delta,k), g(\pi,\delta,k)$				
Π	Δ	$k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

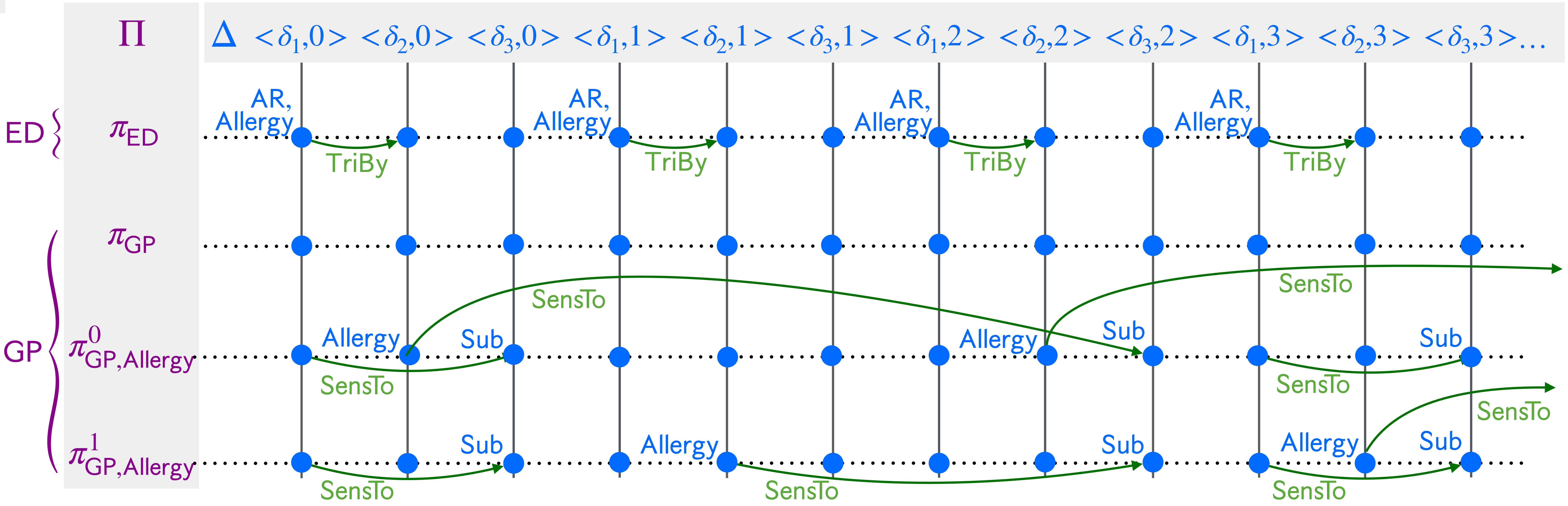


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq {=}1\mathsf{SensitivityTo}.\mathsf{Substance}),\\ & \square_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}.\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$: $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

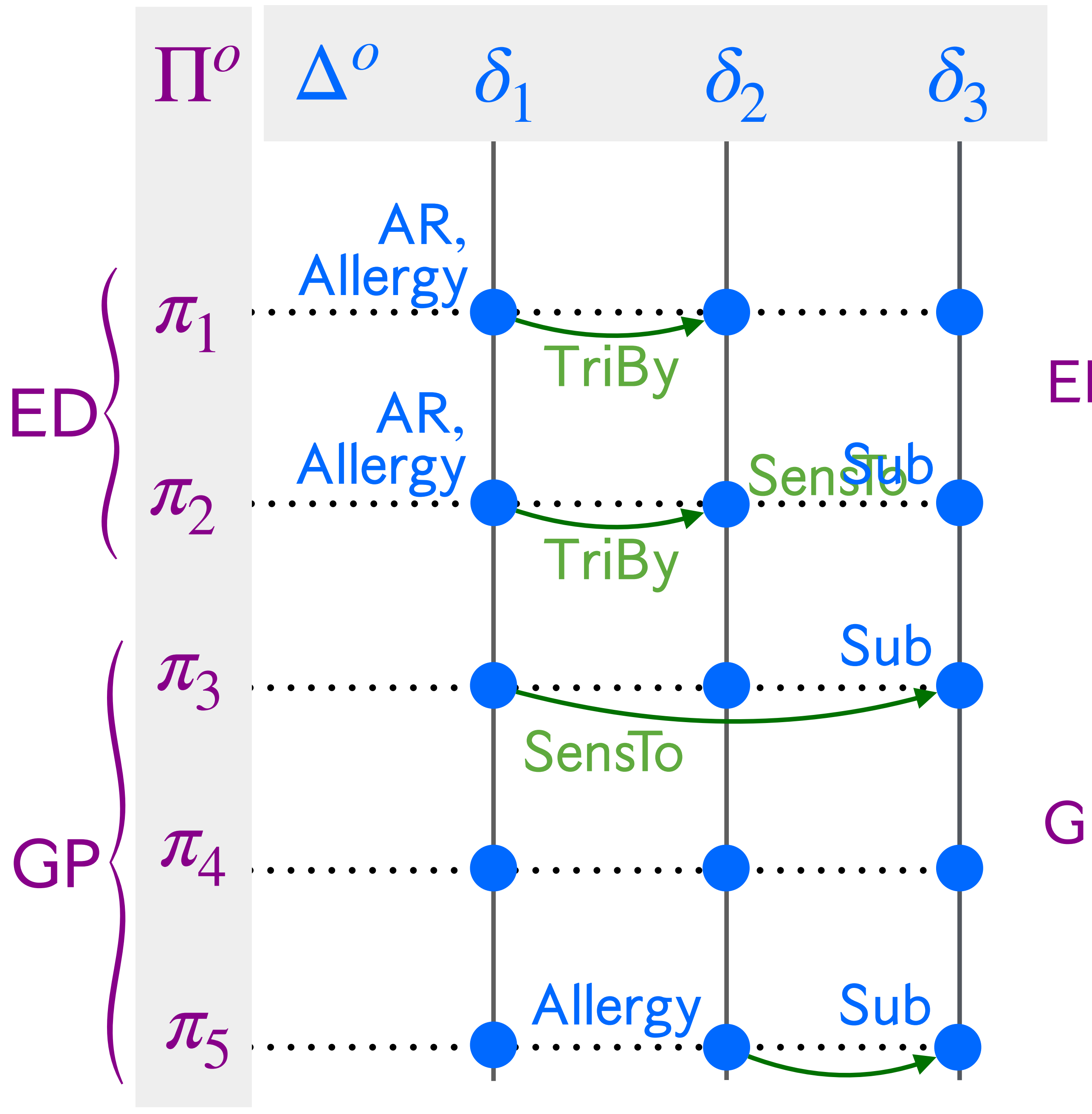


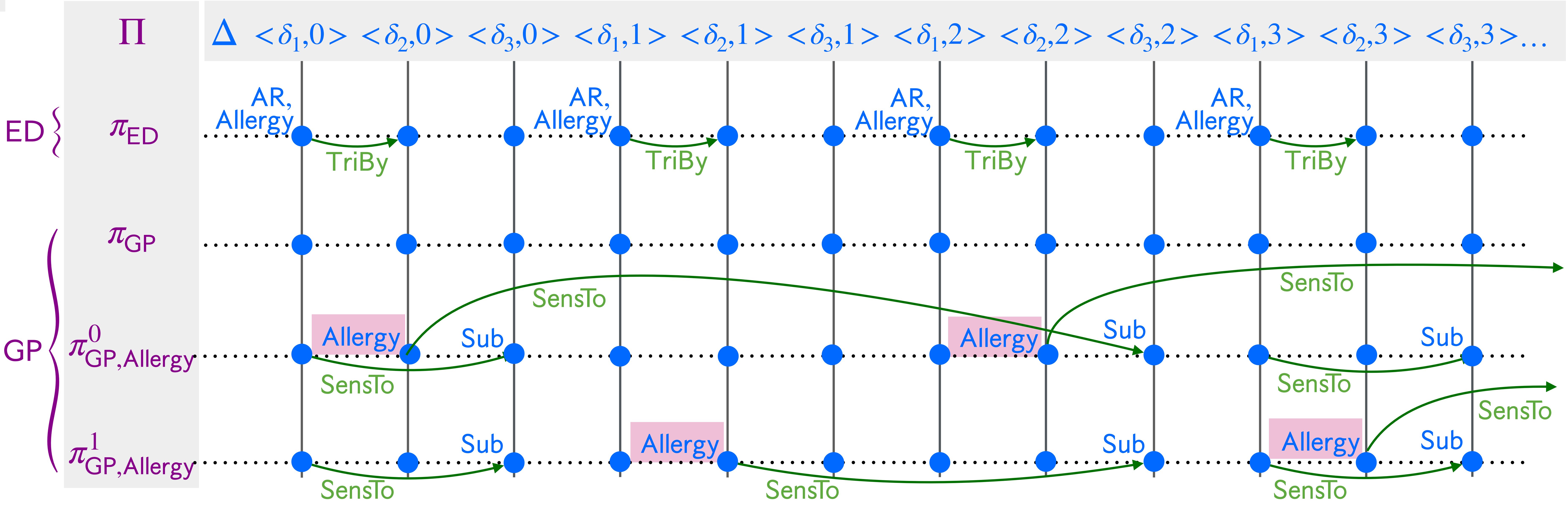


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\begin{aligned}\mathscr{K} = \{\, & \square_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),\\ & \square_{\mathsf{GP}}\,(\mathsf{Allergy} \sqsubseteq\, =1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),\\ & \square_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\,\}\end{aligned}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathtt{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	





Future Research Goals and Challenges

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Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

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Objectives:

Future Research Goals and Challenges

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Objectives:

(O1) **Decidability and complexity**

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

(O1) **Decidability and complexity**

(O2) **Efficient algorithms and implementations**

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

(O1) **Decidability and complexity**

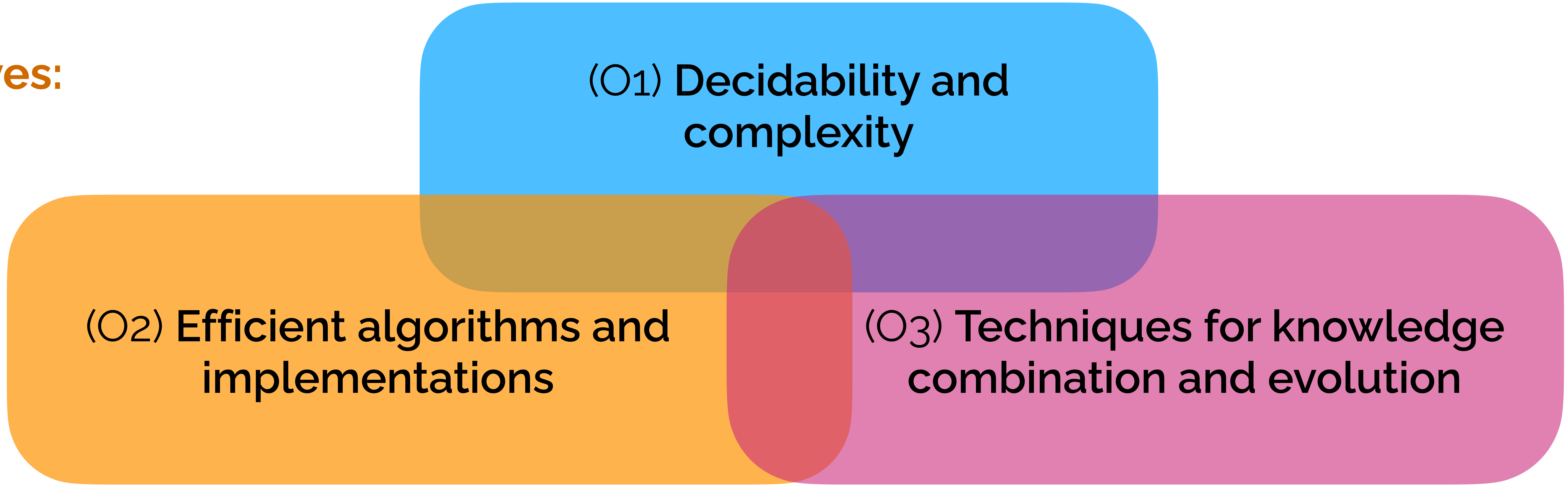
(O2) **Efficient algorithms and implementations**

(O3) **Techniques for knowledge combination and evolution**

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

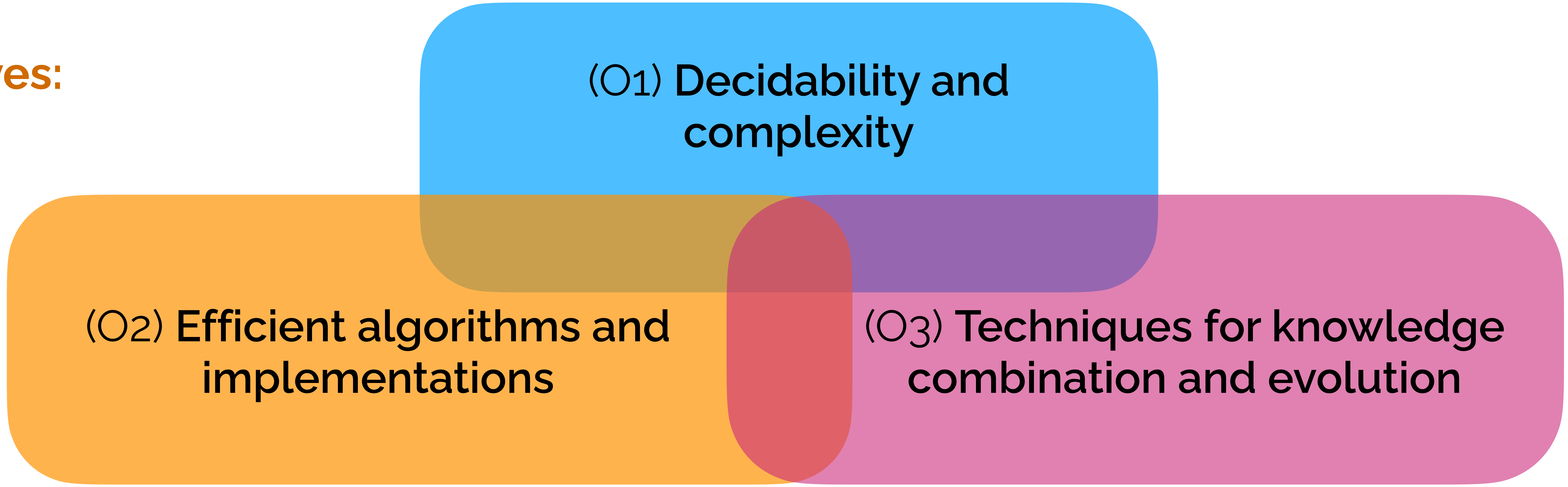


Challenge: Expressivity — Efficiency trade-off

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:



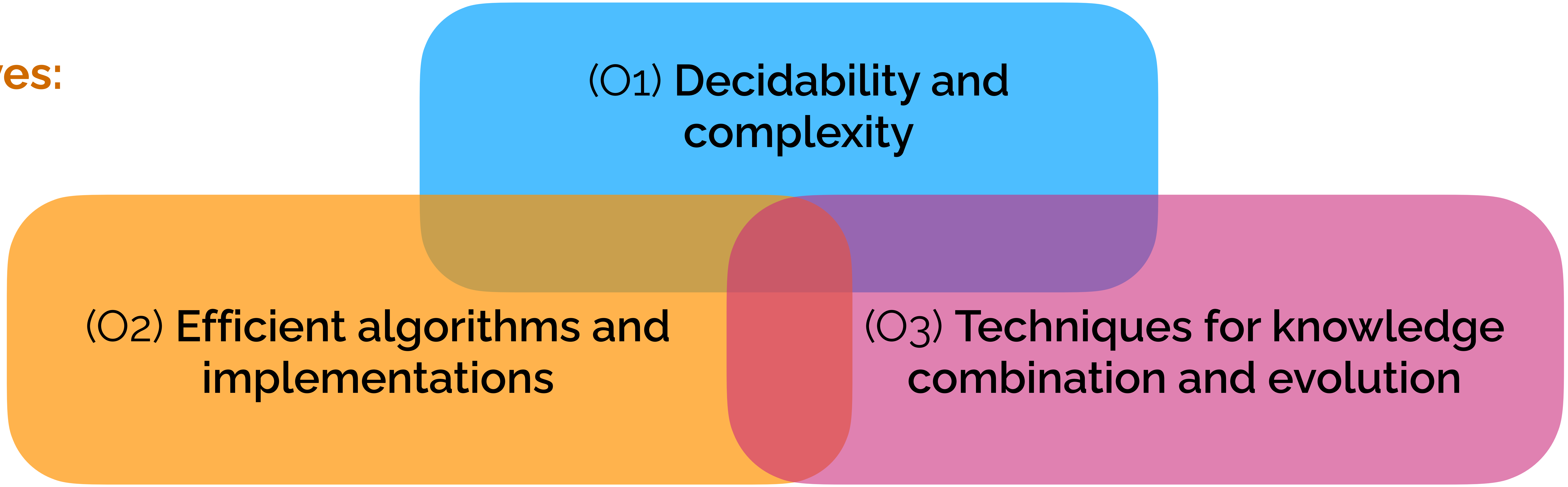
Challenge: Expressivity — Efficiency trade-off

* Knowledge available is <u>highly diverse</u>

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:



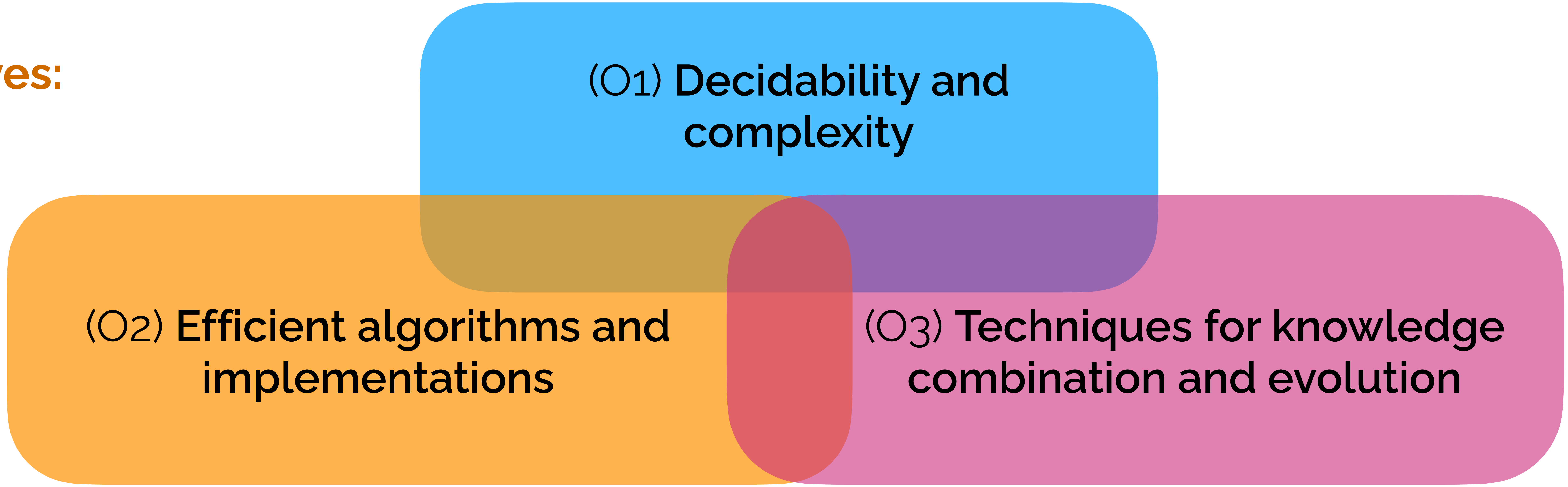
Challenge: Expressivity — Efficiency trade-off

* Knowledge available is <u>highly diverse</u>
* Multi-perspective frameworks give rise to <u>complex reasoning tasks</u>

Future Research Goals and Challenges

Goal: **towards a viable framework for reasoning with heterogeneous knowledge communities**

Objectives:



Challenge: Expressivity — Efficiency trade-off

- Knowledge available is <u>highly diverse</u>
- Multi-perspective frameworks give rise to <u>complex reasoning tasks</u>
- The Semantic Web contains extremely <u>large knowledge sources</u>