An Introduction to Reasoning with Existential Rules

David Carral
LIRMM, Inria, University of Montpellier, CNRS

November 17, 2021

About to the Seminar

Program

1. Morning Session from 9:30 to 12:00 (more general/introductory):
   - "Introduction to ∃-Rules" by D. Carral
   - "Reasoning with Guarded ∃-Rules" by M. Thomazo
   - Coffee Break (outside of the seminar room)
   - "Compressing Rule-based Reasoning" by J. Urbani

2. Lunch Break from 12:00 to 13:30 in Building 5 (did you register?)

3. Afternoon Session from 13:30 to 17:00 (more specific/technical):
   - "Capturing Homomorphism-Closed Decidable Queries with ∃-Rules" by S. Rudolph (winner of the Ray Reiter Best Paper Prize 2021)
   - "Answering Counting Queries over Lightweight Ontologies" by M. Thomazo
   - Coffee Break (outside of the seminar room)
   - "Derivation Graphs, Greediness, and Bounded-treewidth in the Context of ∃Rules" by T. Lyon
   - "∃-rules: Intersecting FO-Rewritability and Core Termination" by P. Ostropolski-Nalewaja

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Remark

Complete schedule available at:
`https://www-sop.inria.fr/members/David.Carral/events/2021-montpellier-existential-rules-seminar.html`

I will also upload the slides by the end of the week!

Google “David Carral Sophia Inria” to find my personal webpage, then look under the “Events” tab.

Acknowledgements

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Funding provided by:

- The LIRMM through the “Action Transverse Logique”
- The GraphIK team
- The Chair of Computational Logic at TU Dresden (that is, Sebastian’s research group)

Intro to Existential Rules

Goals of this Presentation

What this talk is about:

- General intro to $\exists$-rules
- Provide context for the other talks
- Research goals in this field

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Remark: Contact me for more info!

- Email: `david.carral@inria.fr`
- Office 3.129 at Building 5

Syntax: Existential Rules

Definition: Existential Rules

An *(existential) rule* is a first-order logic formula $\forall \vec{x}, \vec{z}.\beta[\vec{x}, \vec{z}] \rightarrow \exists \vec{y}.\eta[\vec{x}, \vec{y}]$ where β and η are conjunctions of atoms without function symbols.

In a nutshell: $\exists$-rules = Datalog rules + $\exists$-quantifiers in the head

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Example: Existential Rules

$$\forall x.\mathit{PhDStudent}(x) \rightarrow \exists y.\mathit{MainSupervisor}(x, y) \wedge \mathit{Professor}(y)$$
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Remark: Database Theory

Diff. notation: tuple/equality generating dependencies and constraints.

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Remark

Can we decide BCQ entailment?

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Determine if $\mathcal{K}$ entails γ: determine if infinitely many models (which may be infinite!) satisfy γ

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Proof Intuition: universal models are the *logically weakest* models

Example

Consider the KB $\mathcal{K} = \langle \{R(x,y) \rightarrow \exists z.R(y,z)\}, \{R(a,b)\} \rangle$; the BCQs

$$q = \exists x,y,z.R(x,y) \wedge R(y,z) \wedge R(z,x) \text{ and } q' = \exists x,y,z.R(x,y) \wedge R(y,z);$$

and the following models of $\mathcal{K}$:

$$\mathcal{M} = \{R(a,b), R(b,u), R(u,v), R(v,b)\}$$
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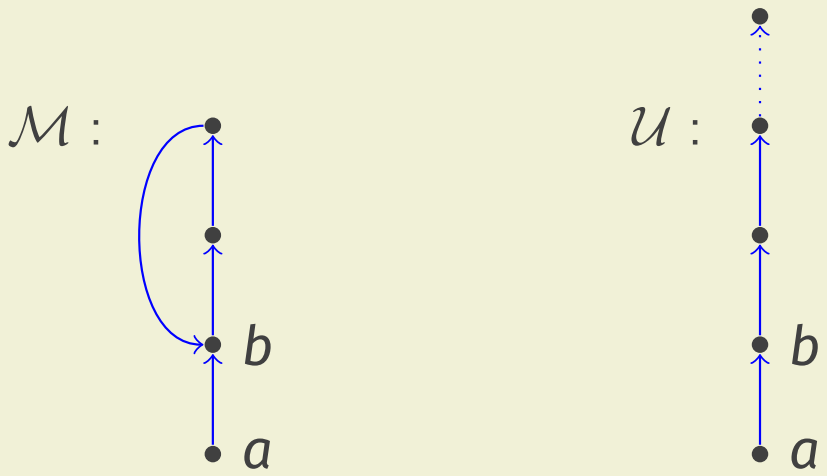
Graphical representation:



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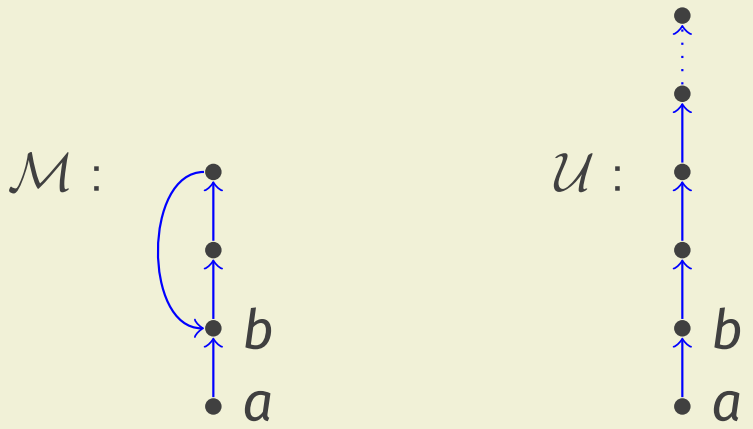

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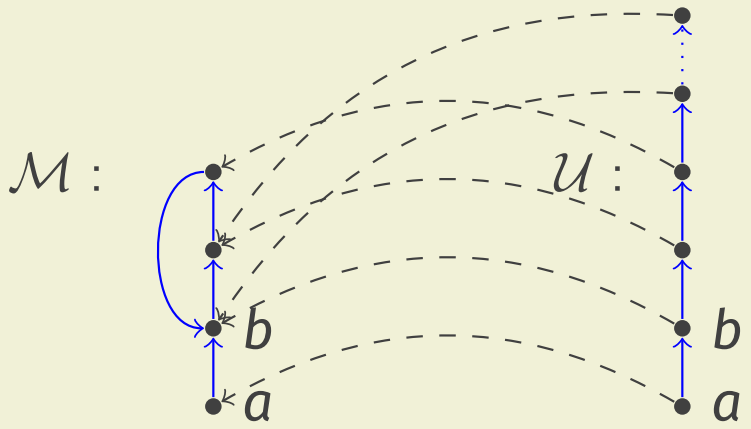

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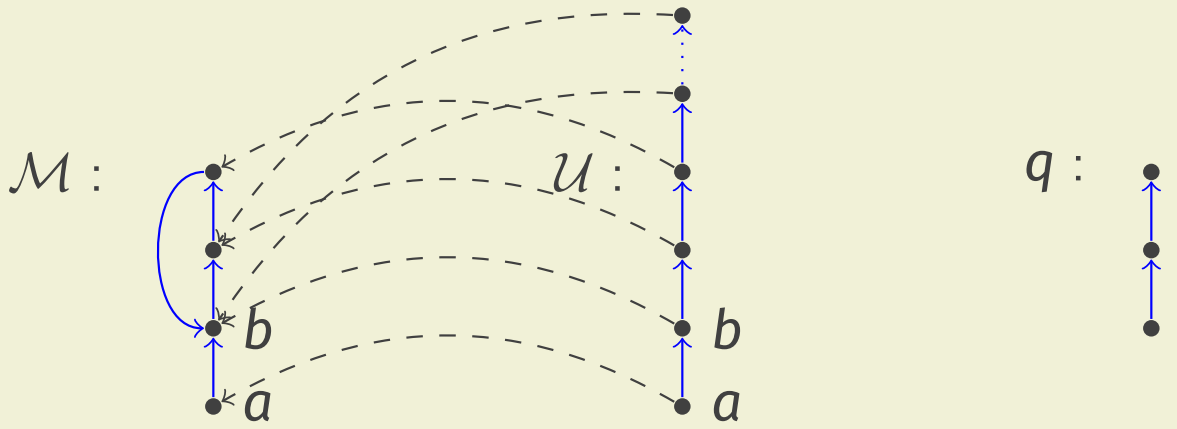

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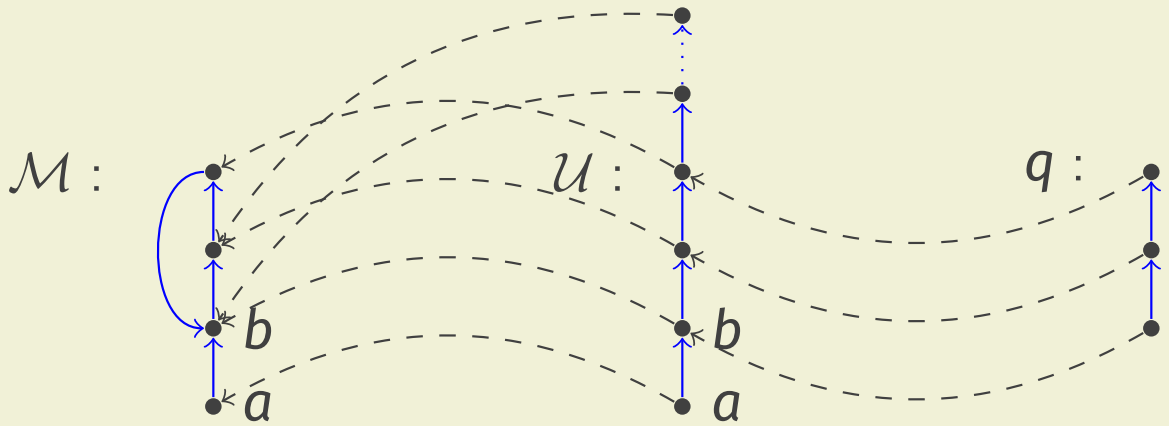

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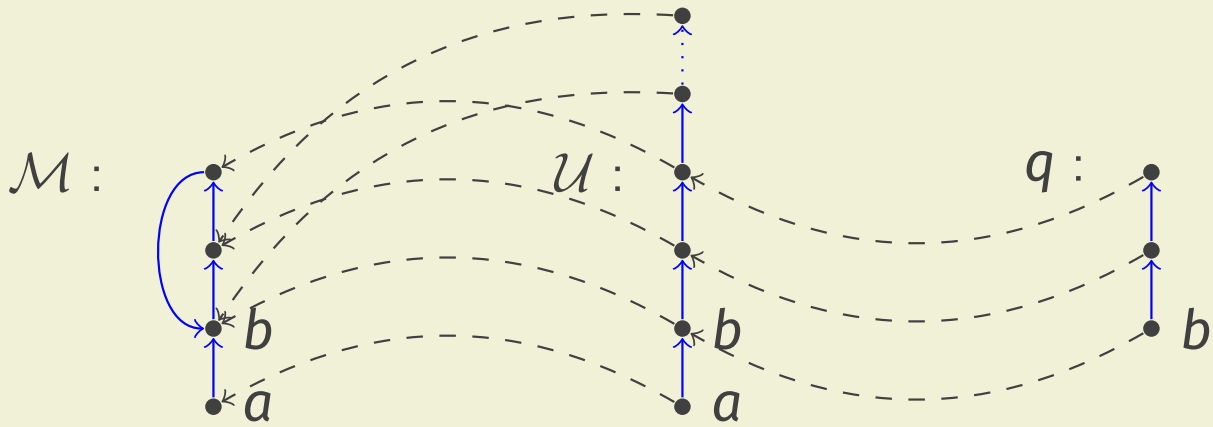

Theorem

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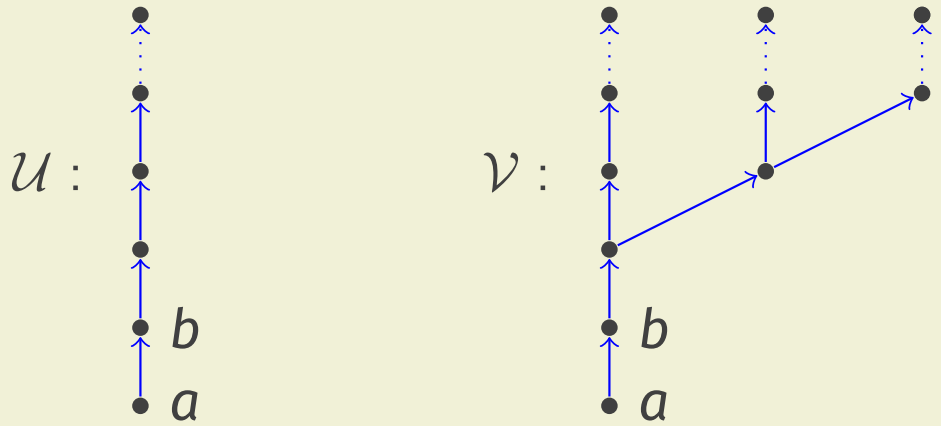

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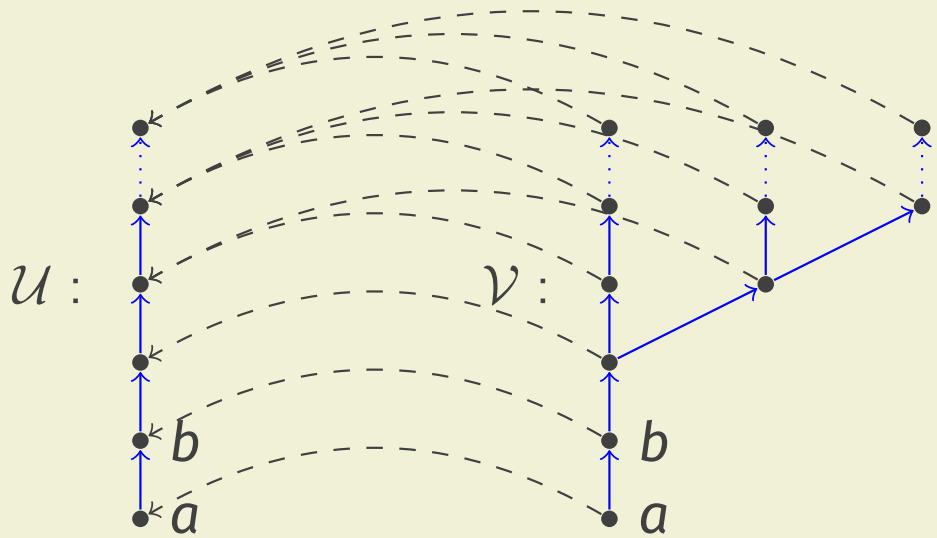

Universal Models: Skolem Chase

$\text{Features}(x, y) \longrightarrow \text{Actor}(y)$

$\text{ActsIn}(x, y) \longrightarrow \text{Features}(y, x)$

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Director(spielberg)

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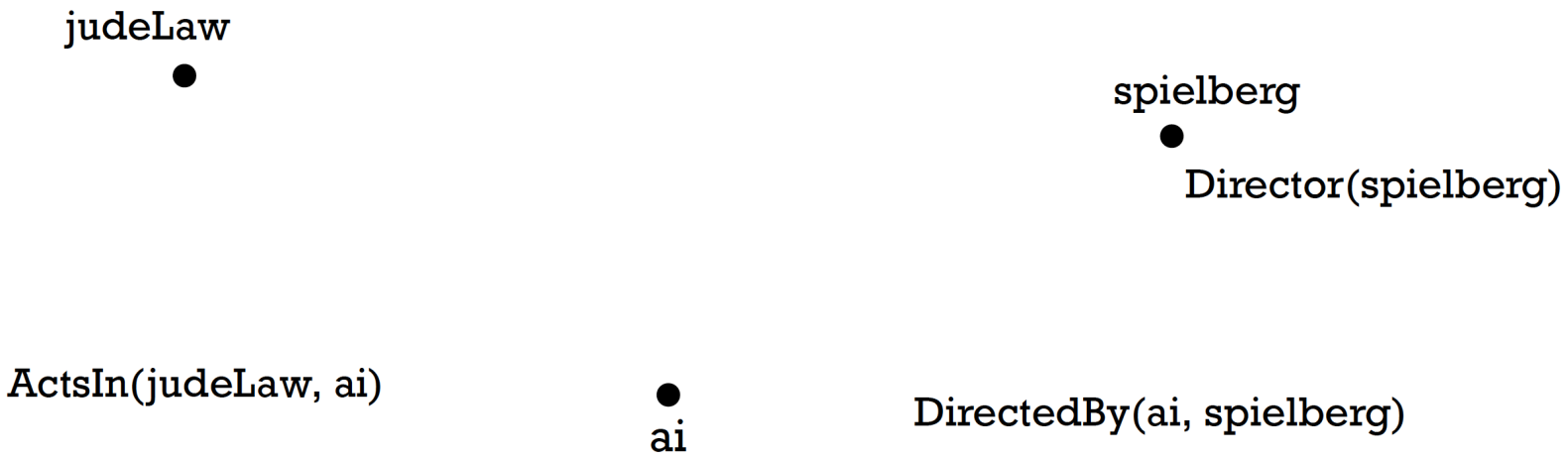

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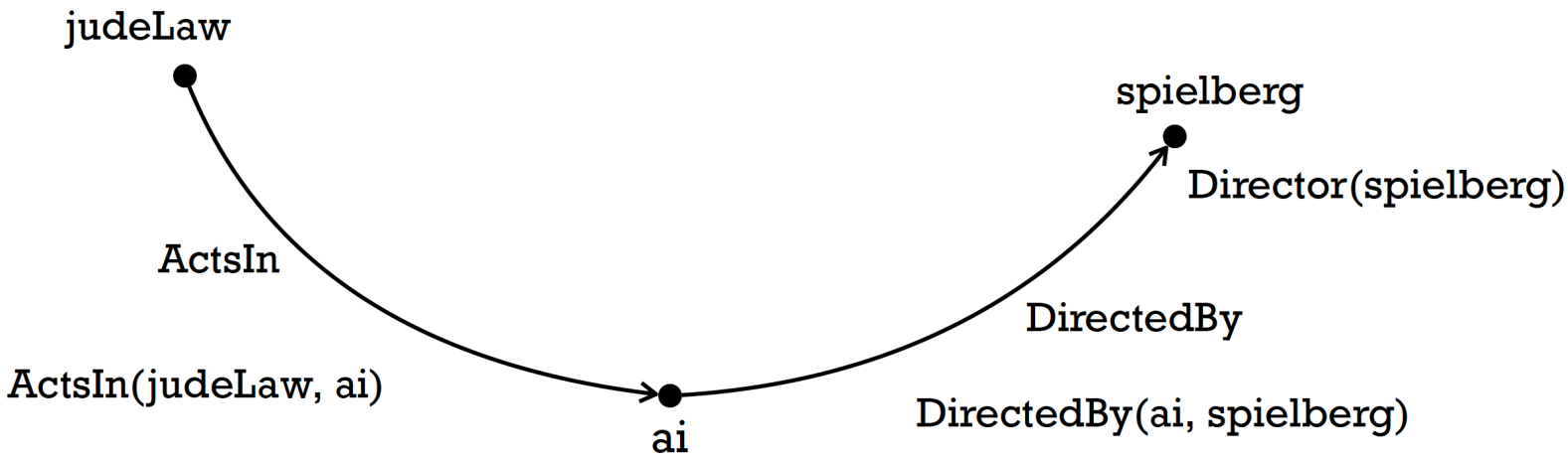

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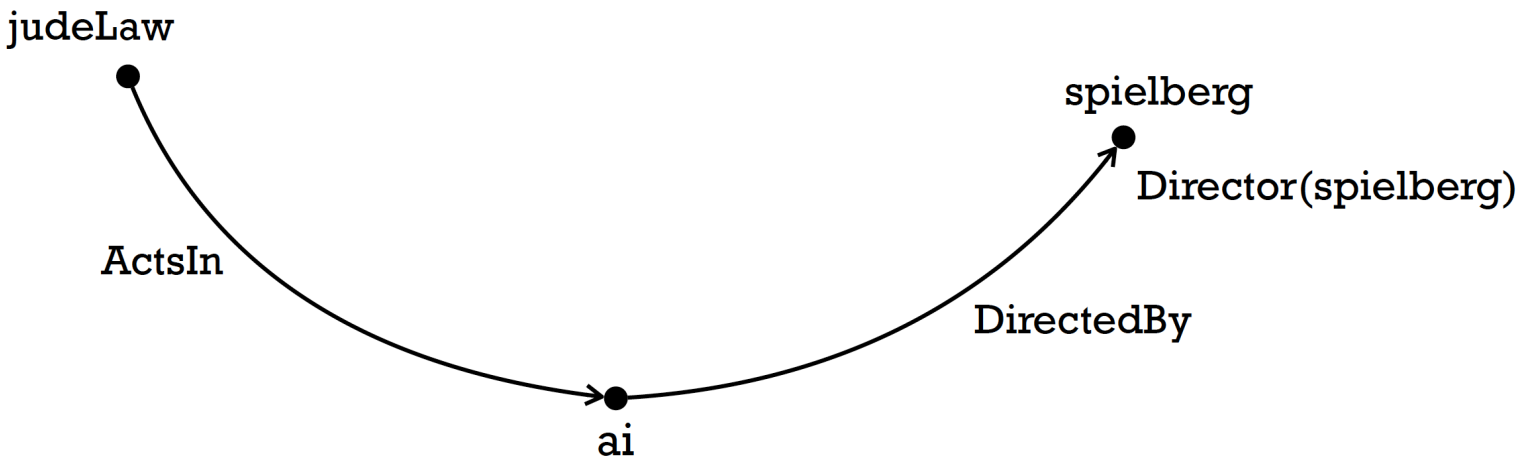

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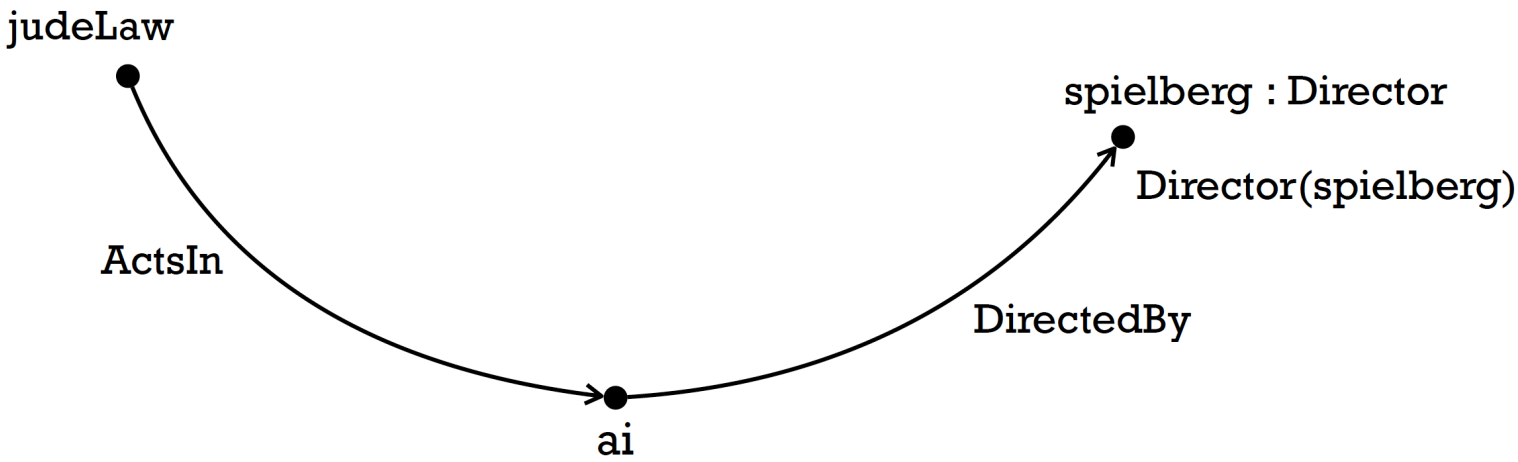

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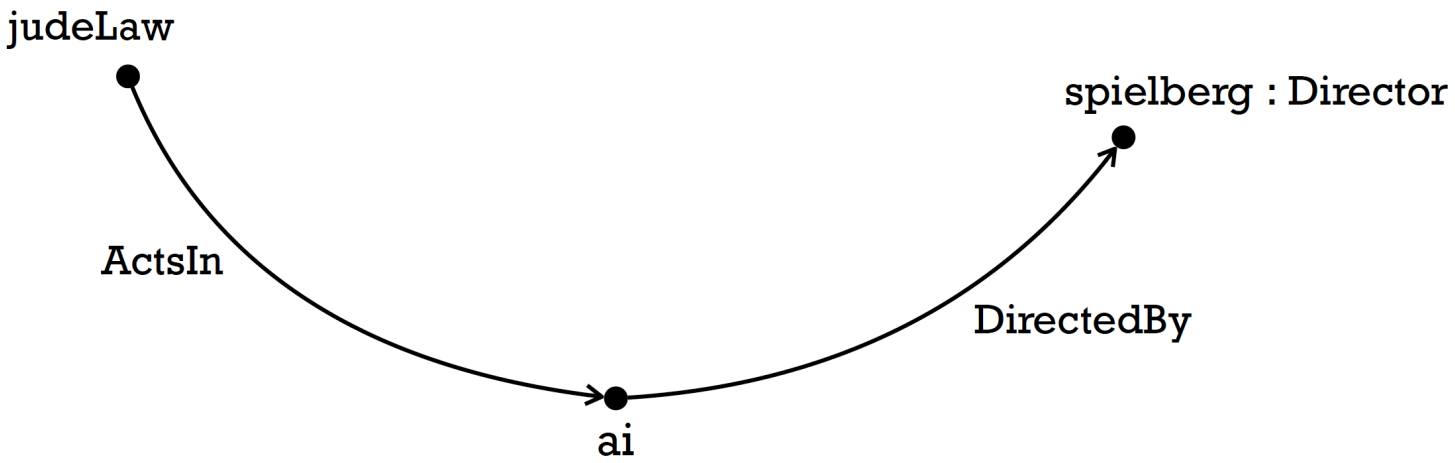

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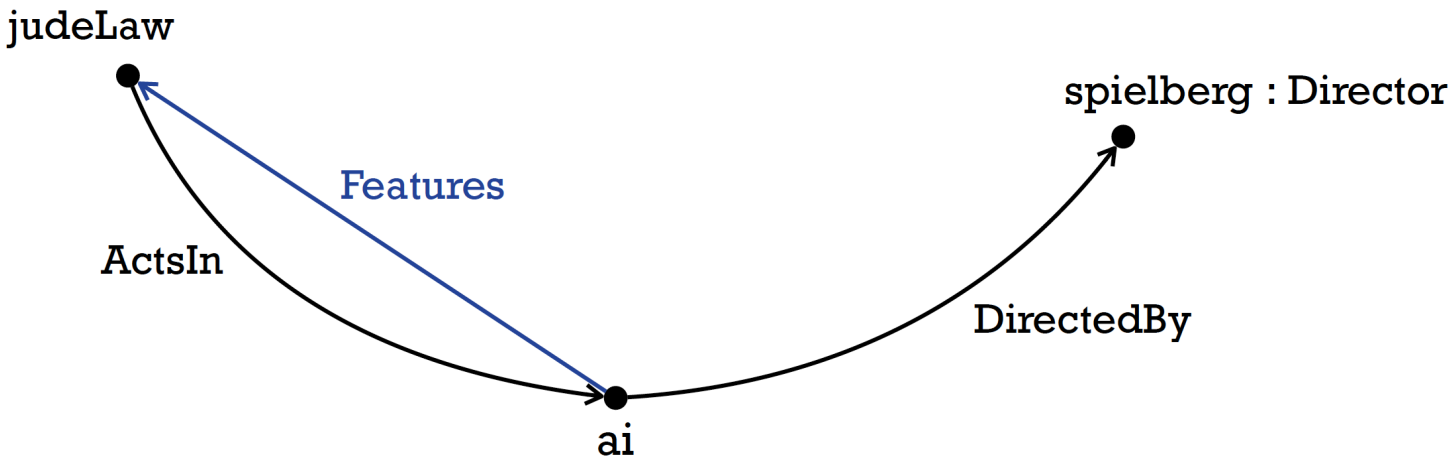

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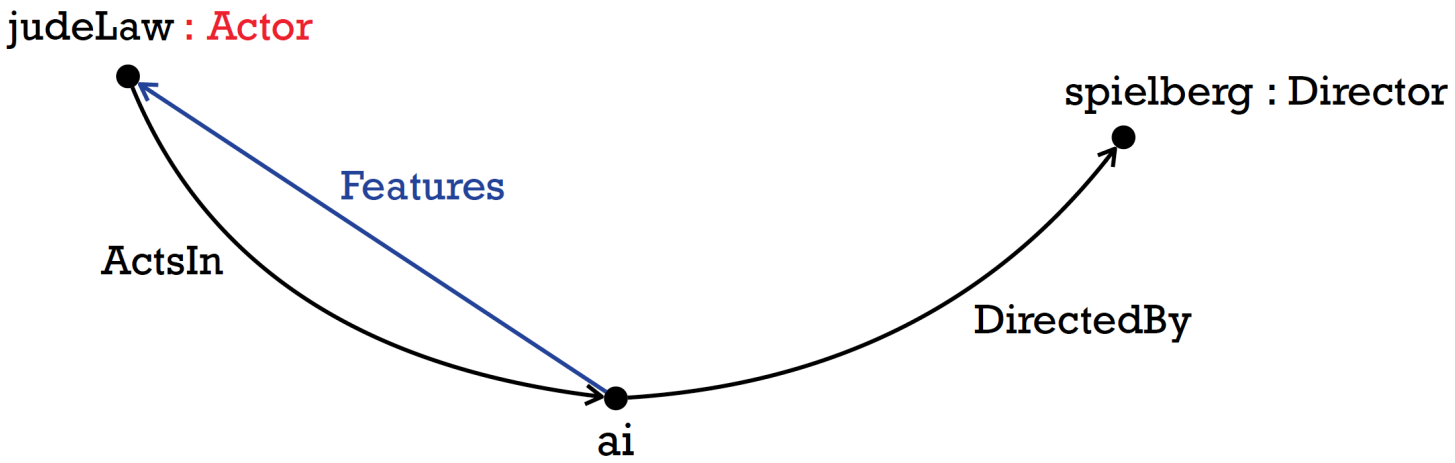

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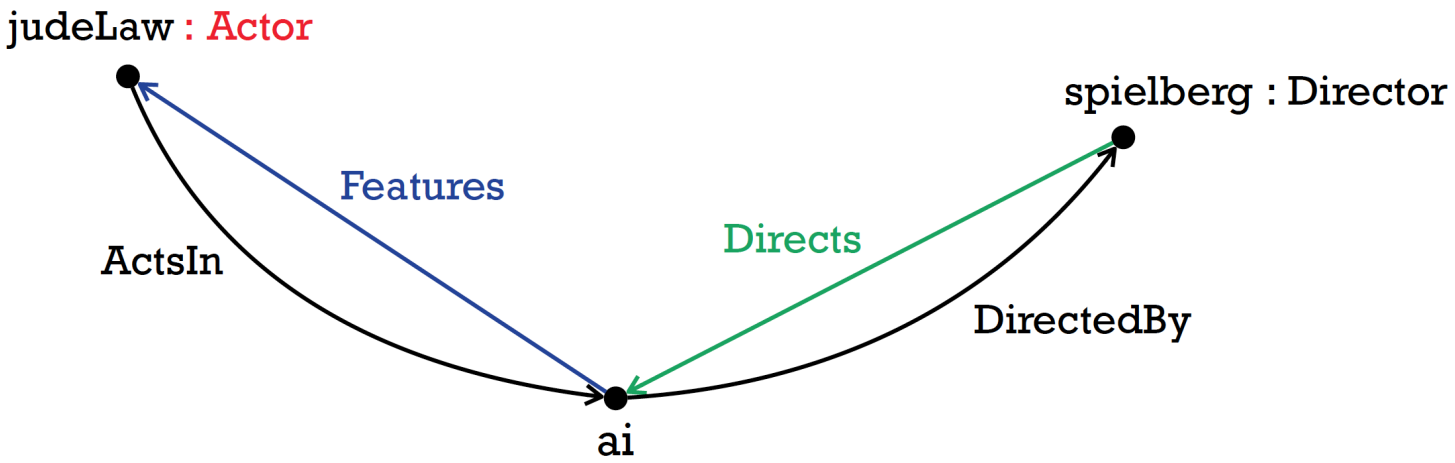

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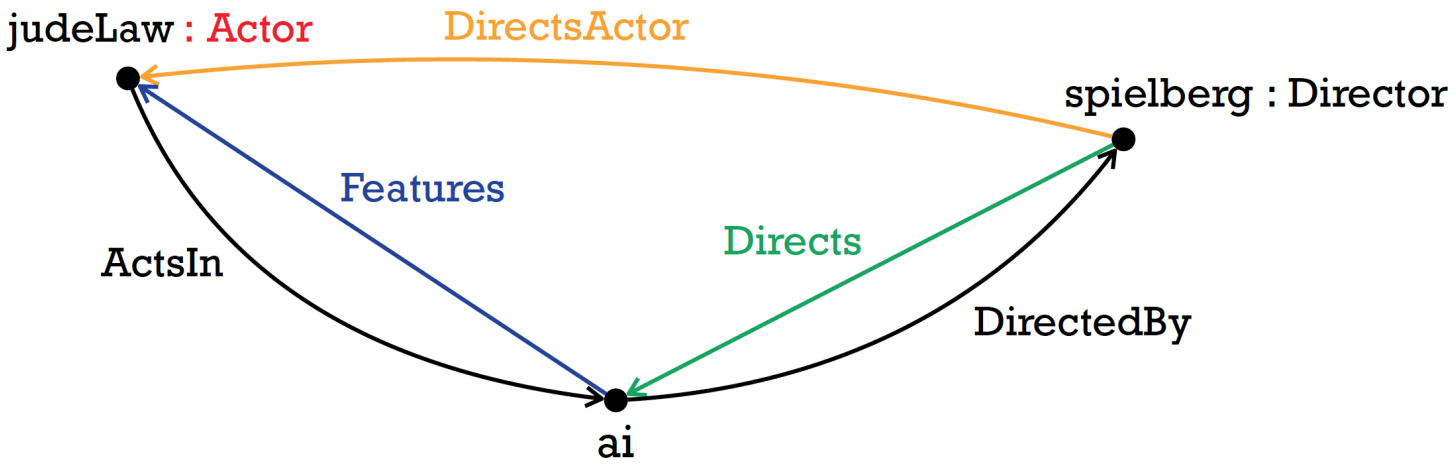

Universal Models: Skolem Chase

$\text{Bicycle}(x) \longrightarrow \exists v \,.\, \text{HasPart}(x, v) \wedge \text{Wheel}(v)$

$\text{Wheel}(x) \longrightarrow \exists w \,.\, \text{IsPartOf}(x, w) \wedge \text{Bicycle}(w)$

$\text{HasPart}(x, y) \longrightarrow \text{IsPartOf}(y, x)$

$\text{IsPartOf}(x, y) \longrightarrow \text{HasPart}(y, x)$

b : Bicycle

Universal Models: Skolem Chase

$\text{Bicycle}(x) \longrightarrow \text{HasPart}(x, f_v(x)) \wedge \text{Wheel}(f_v(x))$

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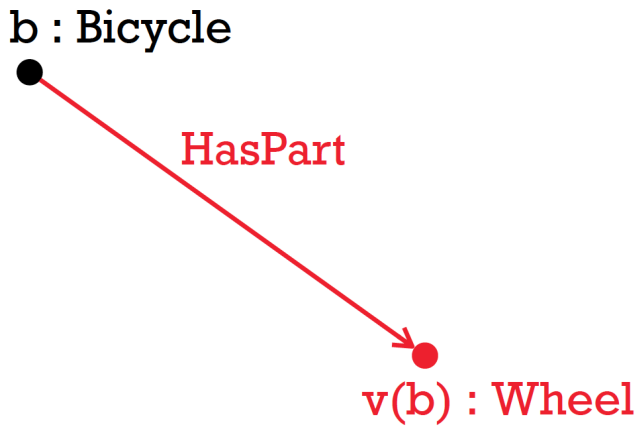

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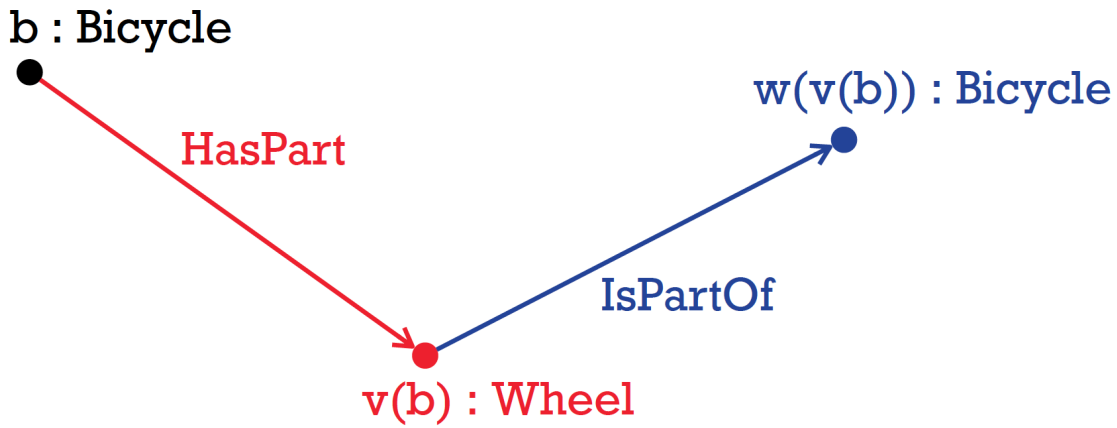

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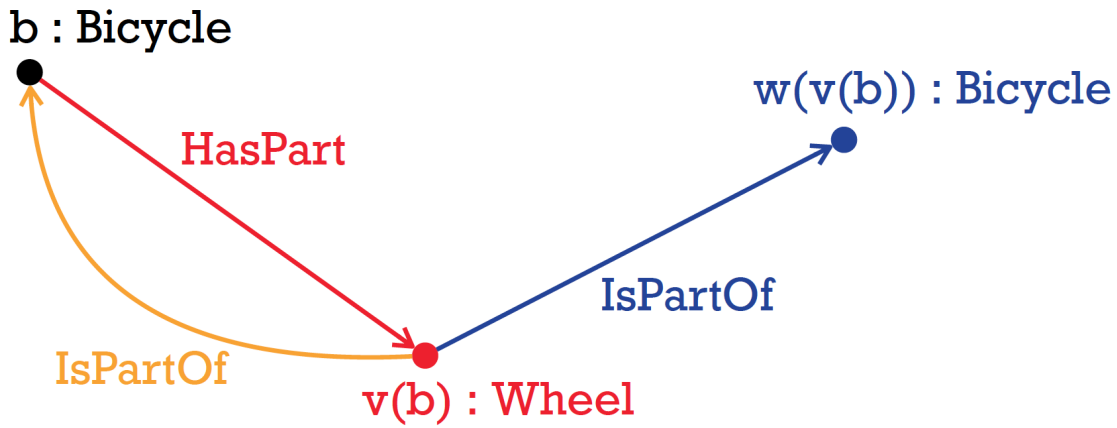

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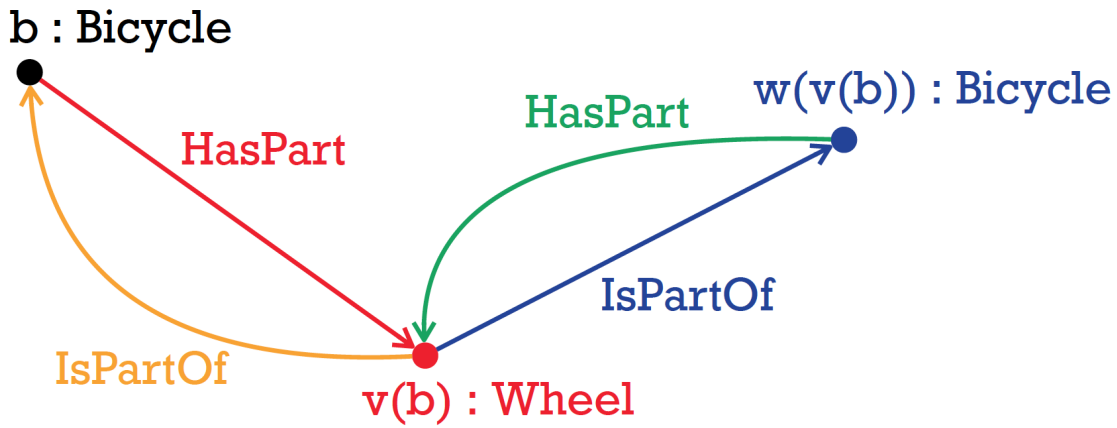

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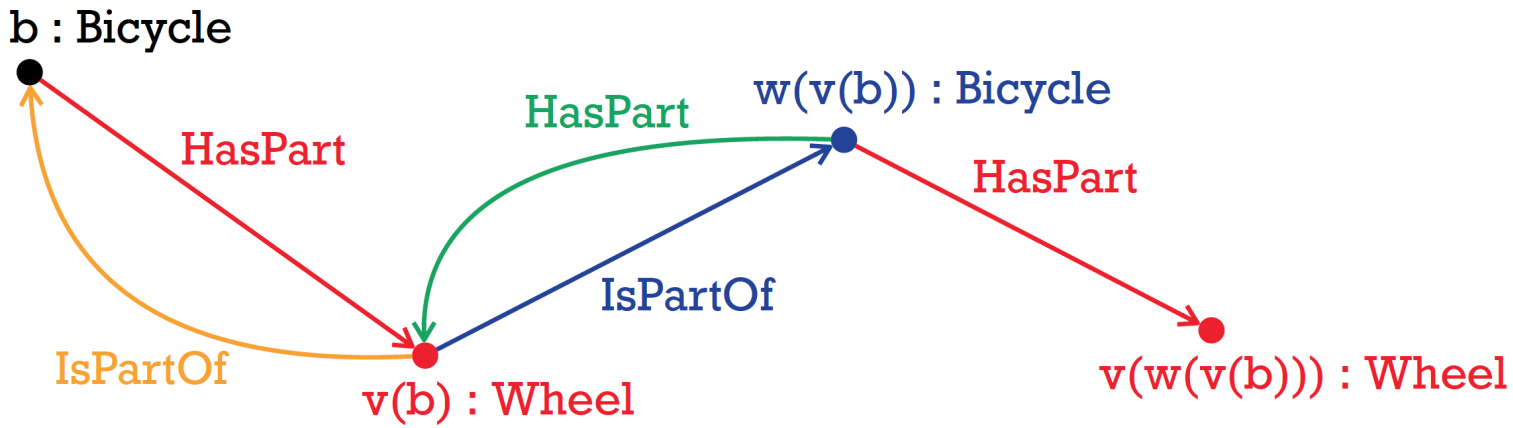

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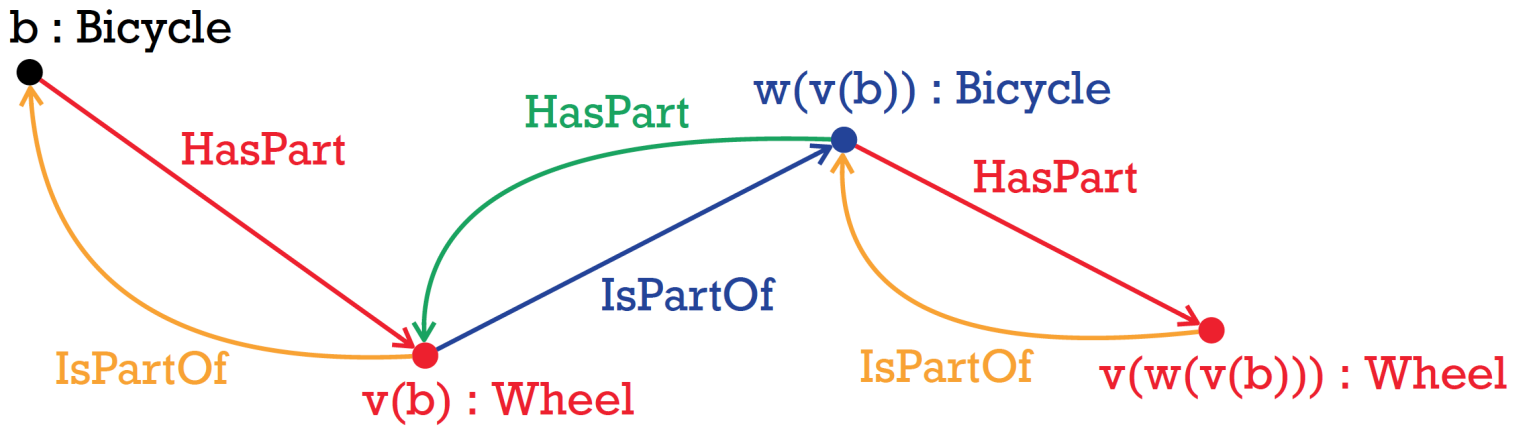

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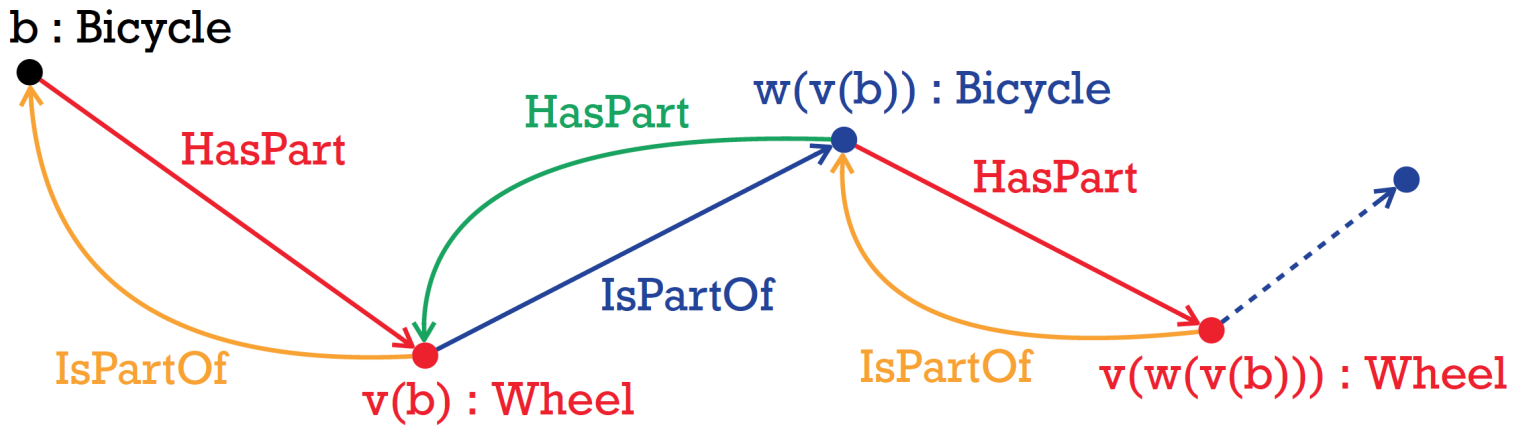

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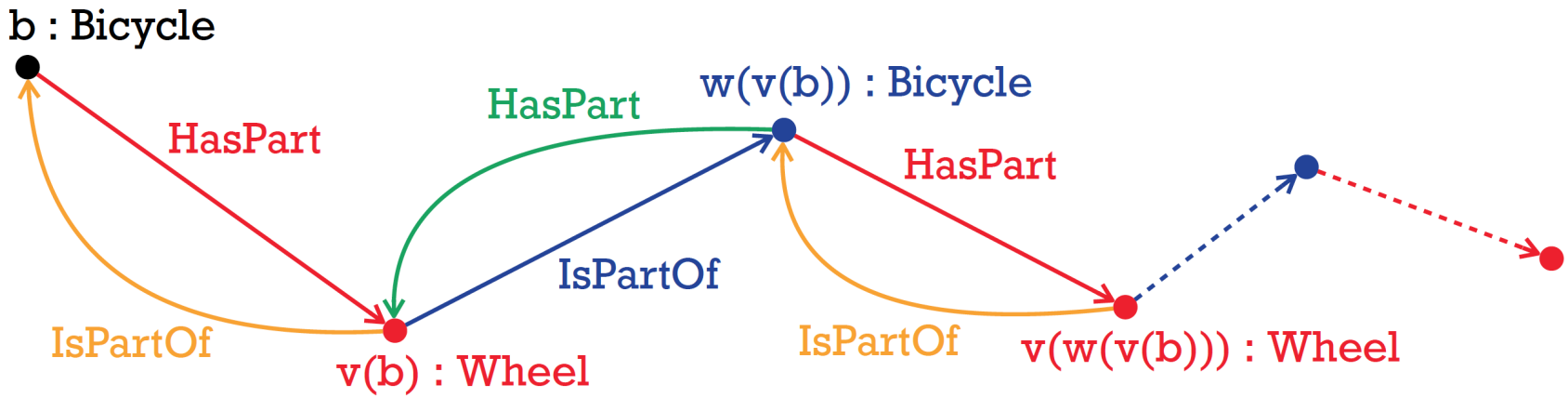

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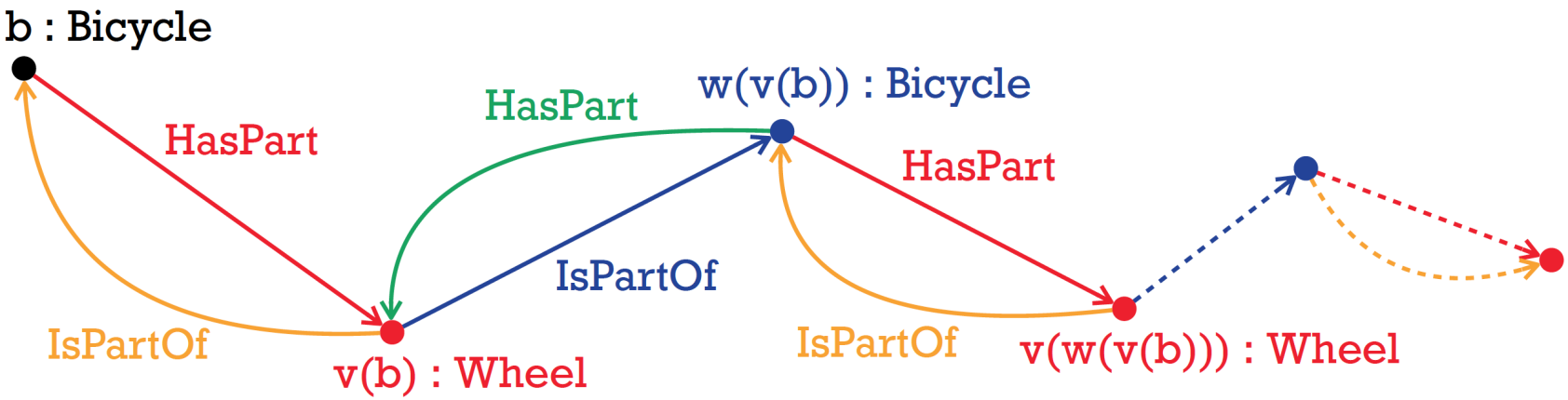

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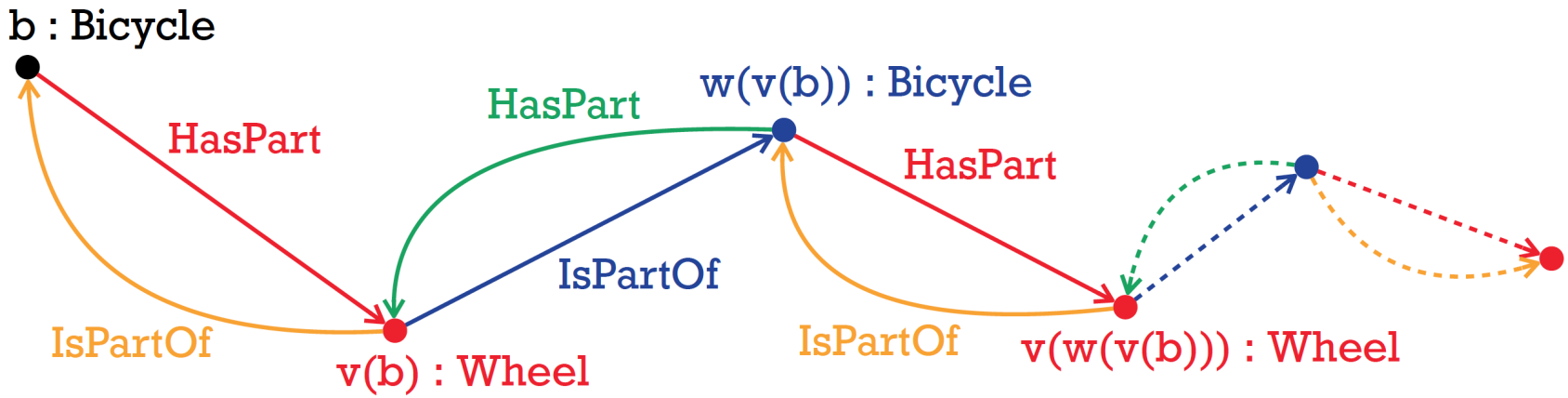

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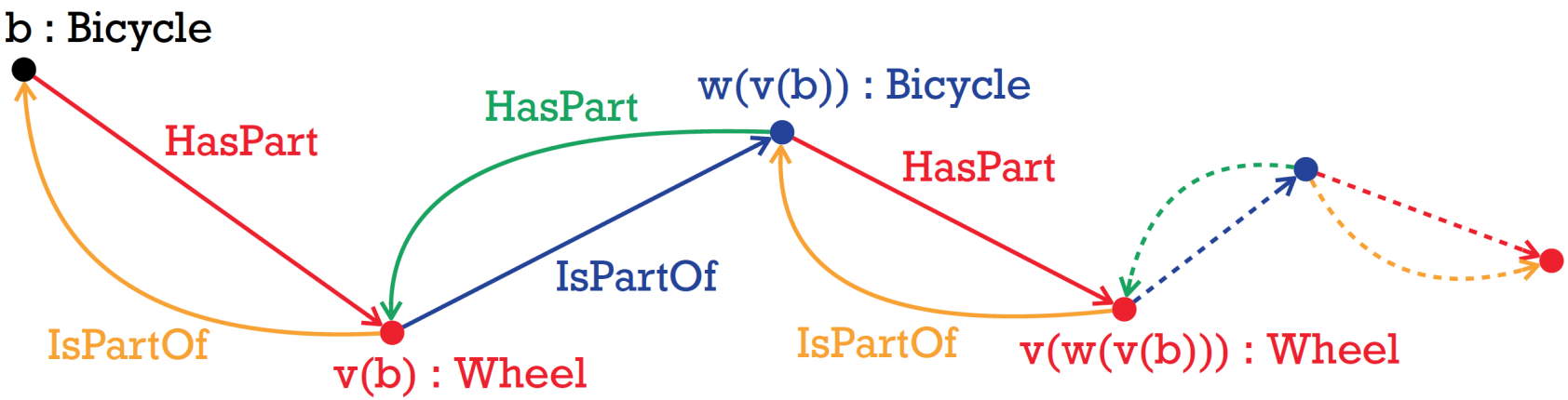

Seminar Talk

Jacopo will discuss an implementation of the chase algorithm that uses compression to improve performance.

Also, see [Mar09] for a formal definition!

Is BCQ Entailment Decidable?

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Reduction from:

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Definition

A *context-free grammar (CFG)* G consists of a start non-terminal S and a set of production rules of the form

$$A \to B_1 \cdot \ldots \cdot B_n \qquad A \to \epsilon$$

where A is a non-terminal and each B_i is a terminal or a non-terminal. Moreover, there is some production rule of the form $S \to B_1 \cdot \ldots \cdot B_n$ in G.

Wlog assumptions: all CFGs are defined over the binary alphabet $\{0, 1\}$; non-terminals do not reoccur across different grammars

BCQ Entailment is Undecidable (1)

How to encode a CFG with an existential rule KB:

Definition

For a CFG G, let $\mathcal{K}_G = \langle \mathcal{R}_G, \{V(c)\} \rangle$ wher $\mathcal{R}_G$ is the rule set that contains:

$$V(x) \to \exists y.R_0(x,y) \wedge V(y)$$
$$V(x) \to \exists y.R_1(x,y) \wedge V(y)$$
$$R_{B_1}(x_0,x_1) \wedge \ldots \wedge R_{B_n}(x_{n-1},x_n) \to R_A(x_0,x_n) \quad \text{for all } A \to B_1 \cdot \ldots \cdot B_n \in G$$
$$\to R_A(x,x) \quad \text{for all } A \to \epsilon \in G$$

In the above, each R_s is a fresh binary predicate unique for the non-terminal/terminal symbol s.

BCQ Entailment is Undecidable (2)

Example

Consider the CFG G with start symbol S (left) and the rule set $\mathcal{R}_G$ (right):

$$S \to \mathrm{o}S\mathrm{1} \qquad V(x) \to \exists y.R_{\mathrm{o}}(x,y) \wedge V(y) \qquad V(x) \to \exists y.R_{\mathrm{1}}(x,z) \wedge V(z)$$

$$S \to \epsilon \qquad R_{\mathrm{o}}(x,y) \wedge R_S(y,z) \wedge R_{\mathrm{1}}(z,w) \to R_S(x,w)$$

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Note that $\mathcal{L}(G) = \mathrm{o}^n\mathrm{1}^n$.

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Output of the Skolem chase on input $\mathcal{K}_G = \langle \mathcal{R}_G, \{V(c)\} \rangle$:

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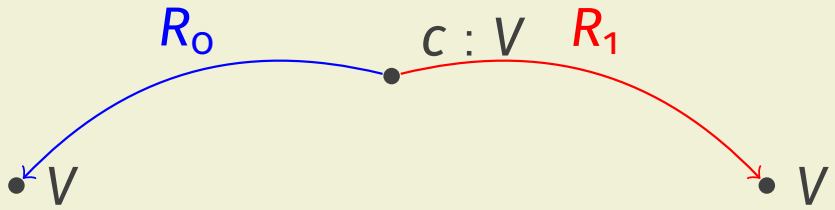

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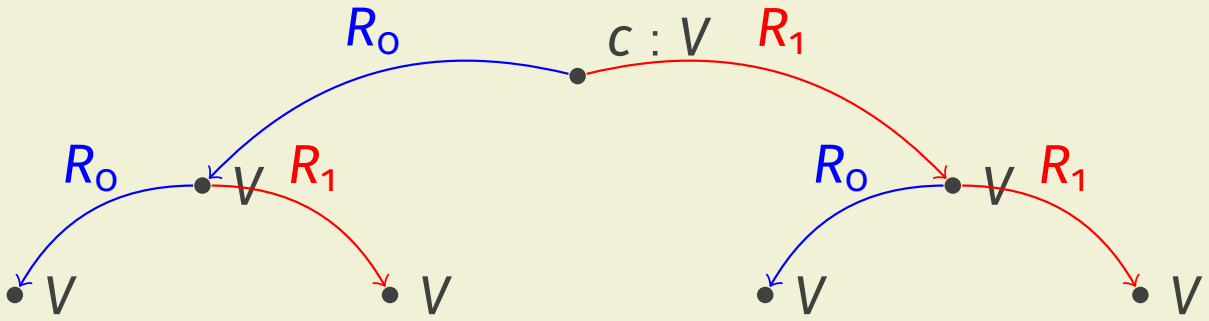

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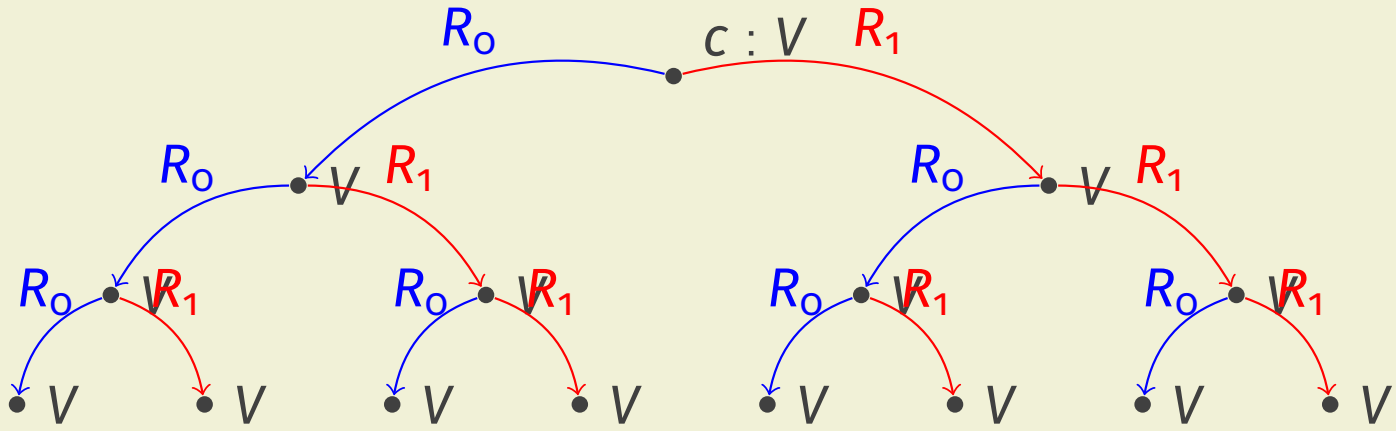

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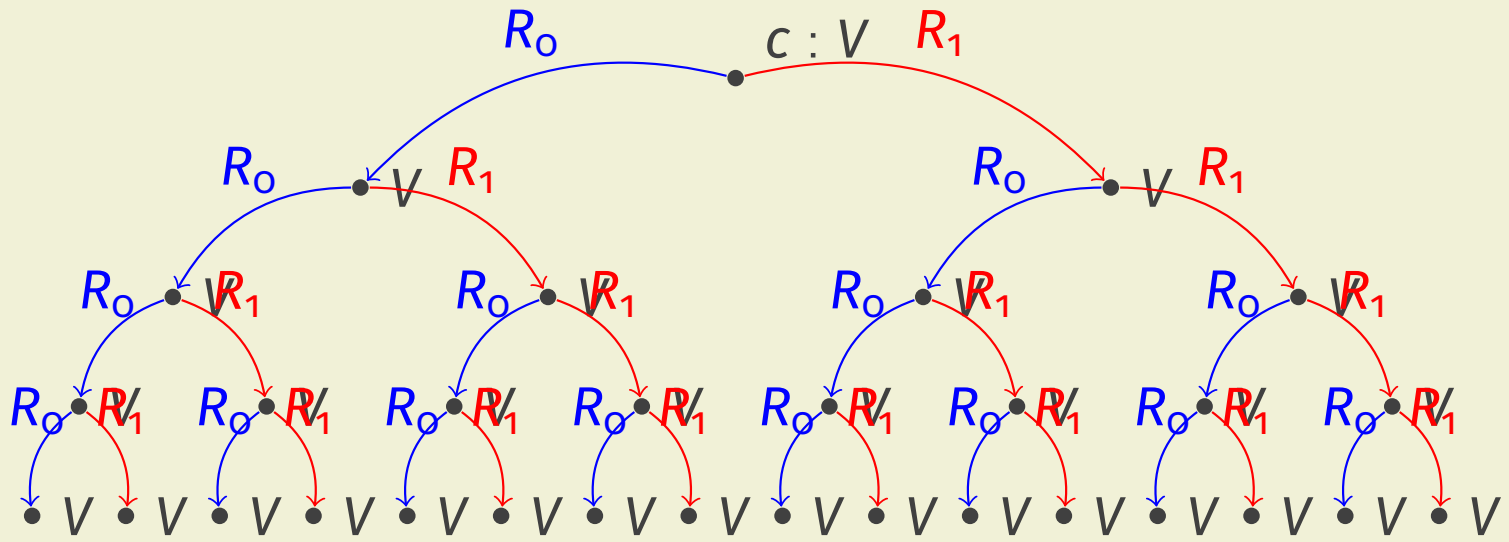

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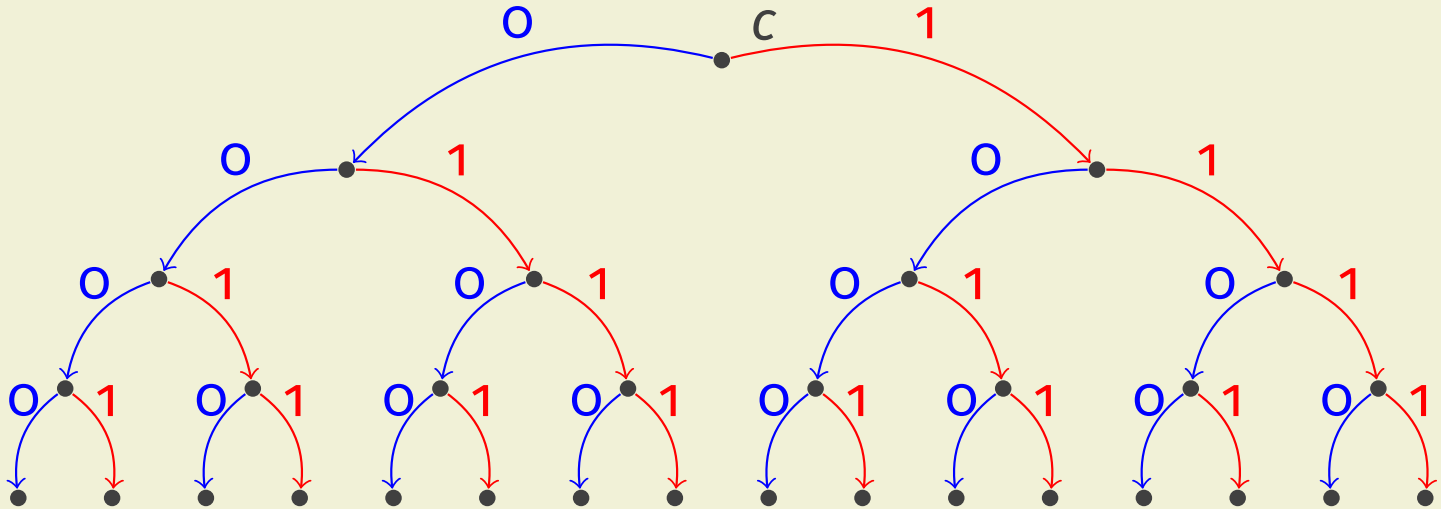

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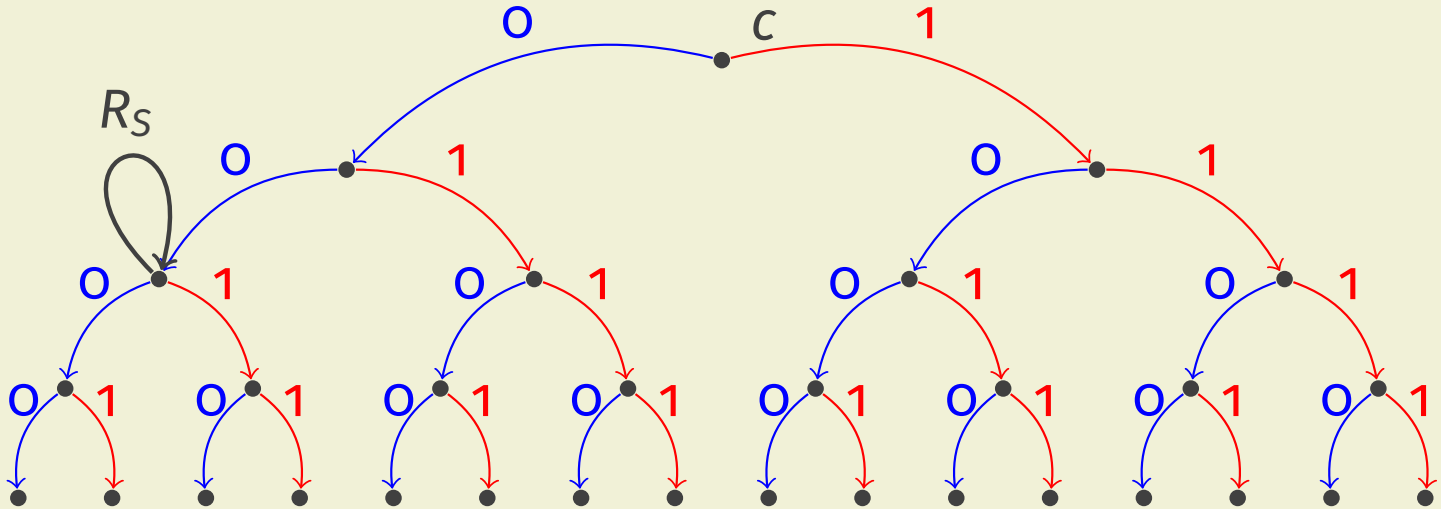

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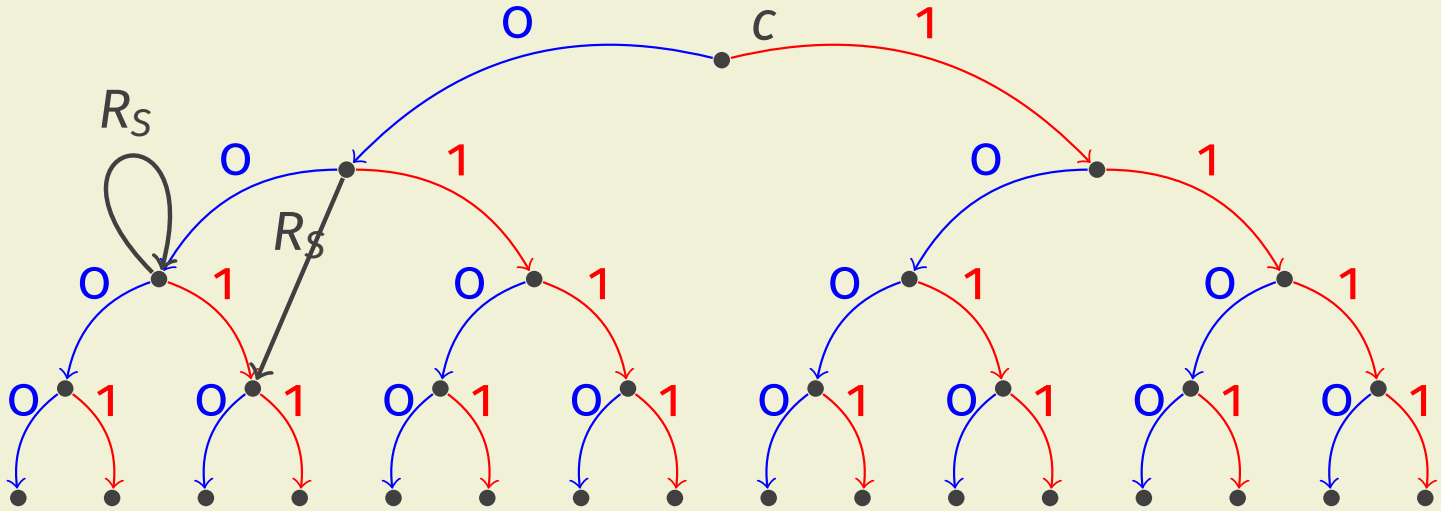

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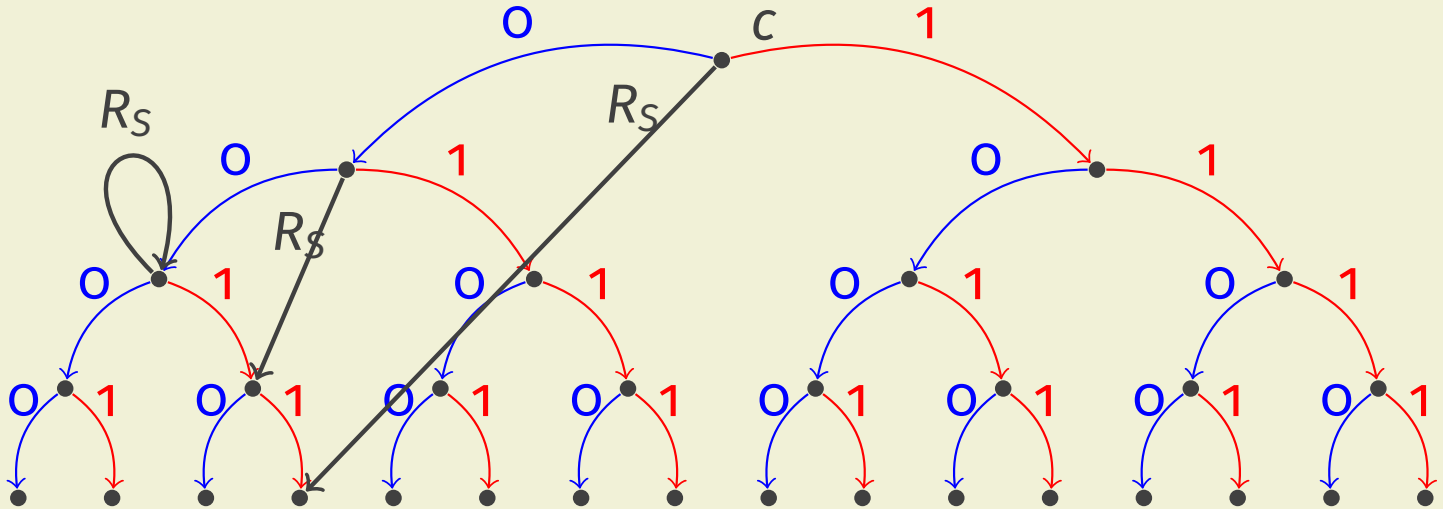

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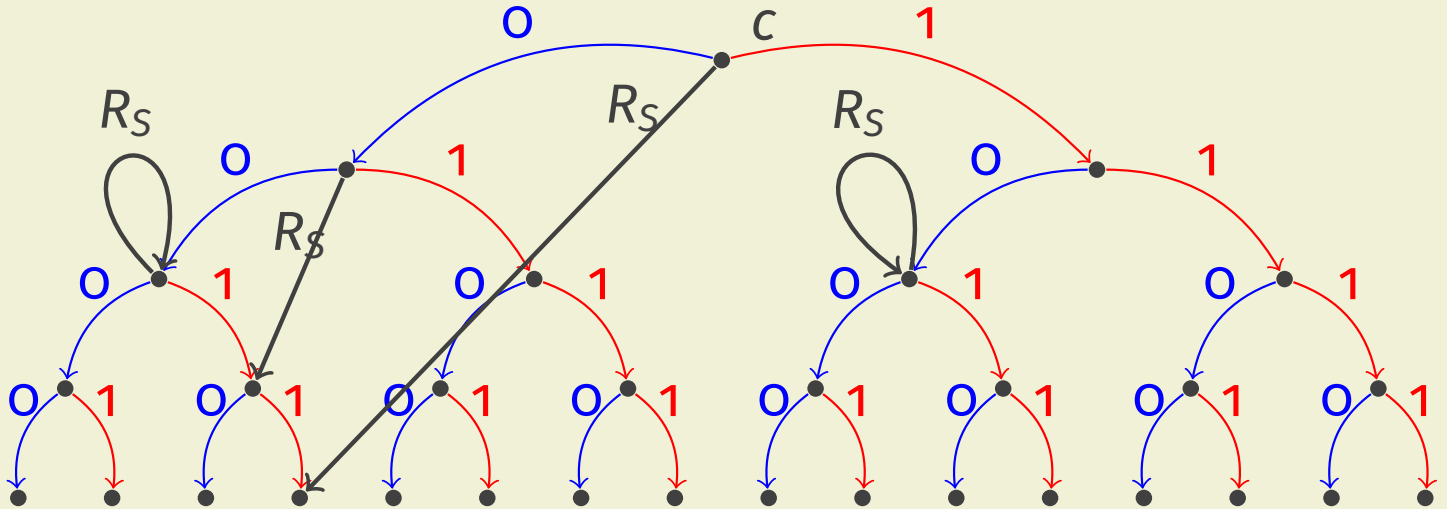

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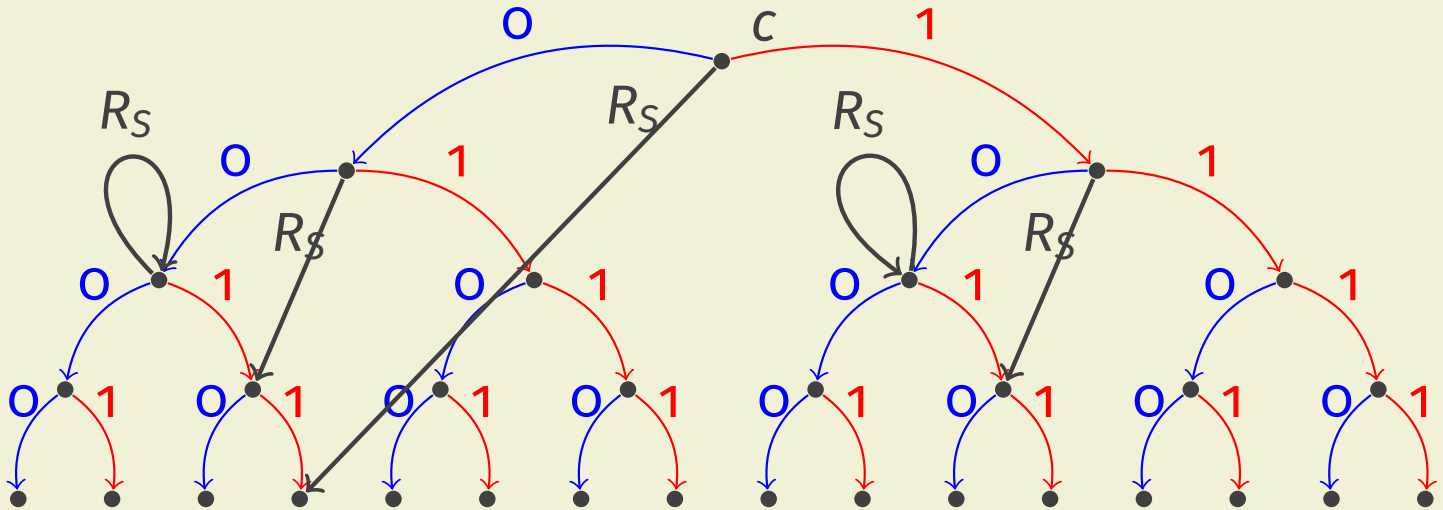

BCQ Entailment is Undecidable (3)

Definition: Reduction

Consider some CFGs G_1 and G_2 with start symbols S_1 and S_2, respectively. Then, let $\mathcal{K}_{G_1}^{G_2}$ be the KB:

$$\langle \mathcal{R}_{G_1} \cup \mathcal{R}_{G_2} \cup \{R_{S_1}(x,y) \wedge R_{S_2}(x,y) \rightarrow Int\}, \{V(c)\} \rangle$$

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$$\langle \mathcal{R}_{G_1} \cup \mathcal{R}_{G_2} \cup \{R_{S_1}(x,y) \wedge R_{S_2}(x,y) \rightarrow Int\}, \{V(c)\} \rangle$$

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Given some CFGs G_1 and G_2,

$$\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset \iff \mathcal{K}_{G_1}^{G_2} \not\models Int$$

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See [BV81] for an alternative proof of the above result.

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Remark: Universal Termination

Note that we consider a check that verifies if the chase of $\mathcal{R}$ terminates with respect to any fact set.

Retrieving Decidability (in Some Cases) (2)

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2. Compute the Skolem chase of $\mathcal{K}$ and check if its result entails γ

Remark

Weak acyclicity [FKMP05] is a sufficient condition that guarantees universal Skolem chase termination.

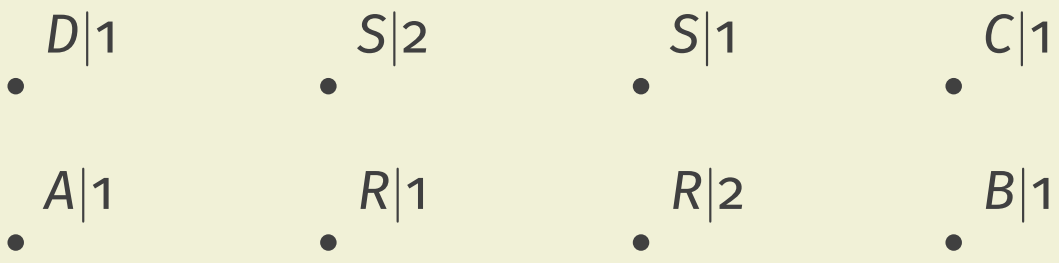
Weak Acyclicity (1)

Example

Consider the rule set $\mathcal{R}$

$$A(x) \to \exists y.R(x,y) \wedge B(y) \qquad B(x) \to C(x)$$
$$C(x) \to \exists y.S(x,y) \wedge D(y) \qquad D(x) \to A(x)$$

and its weakly acyclic graph



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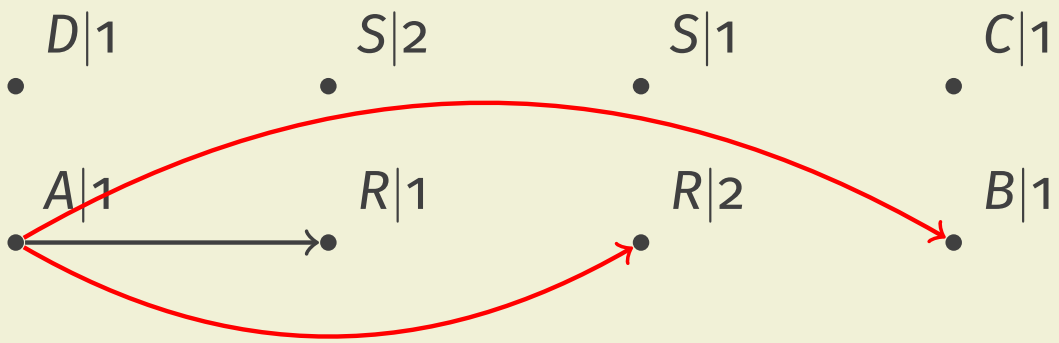

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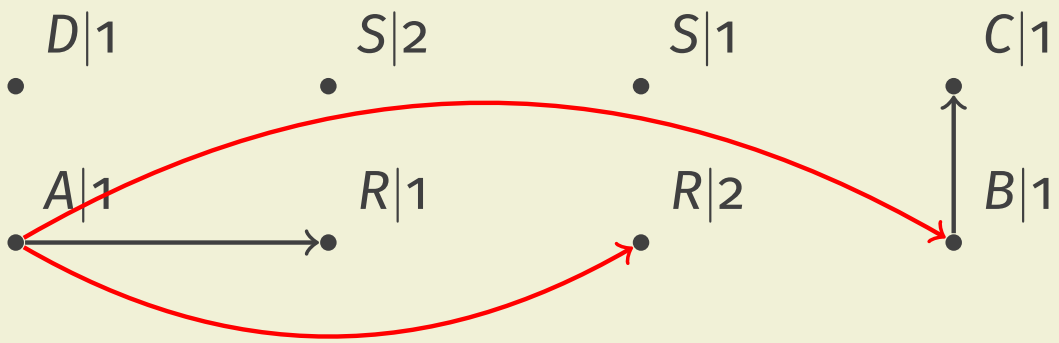

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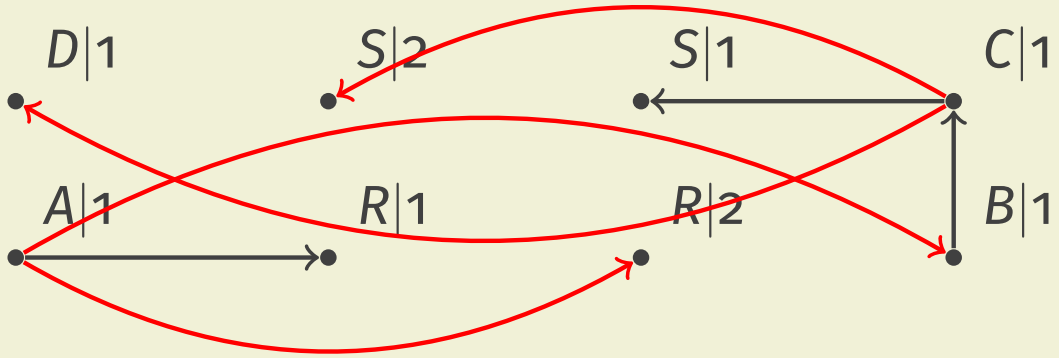

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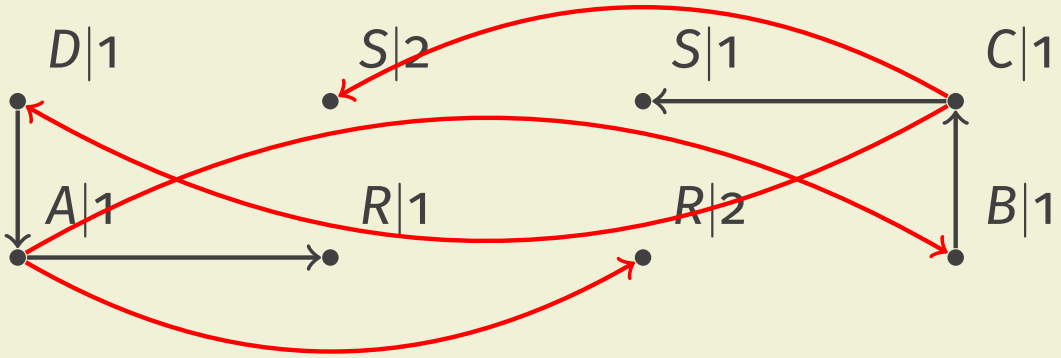

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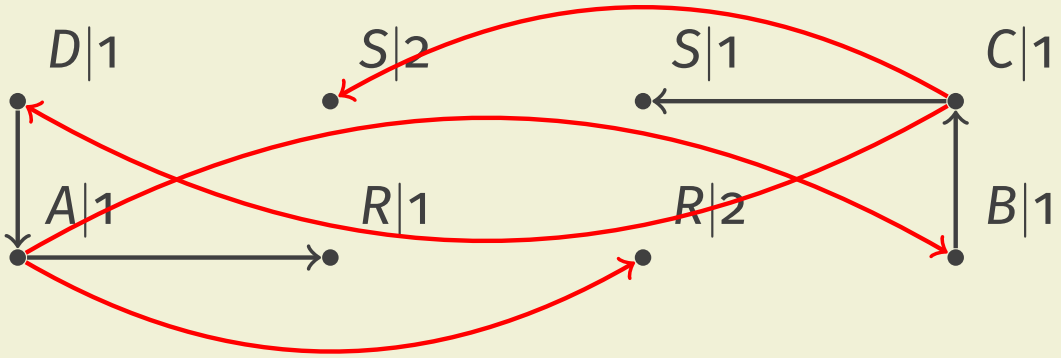

Fails the test: true negative!

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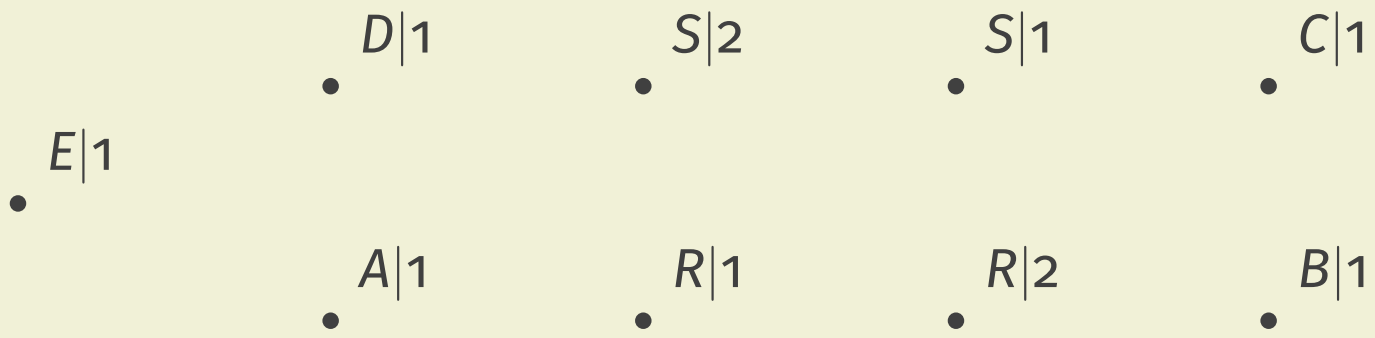

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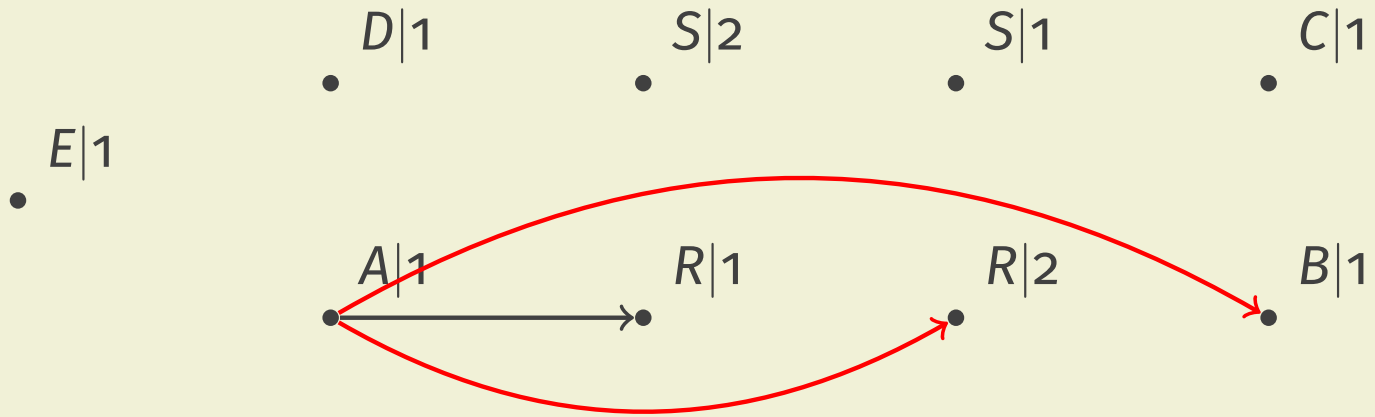

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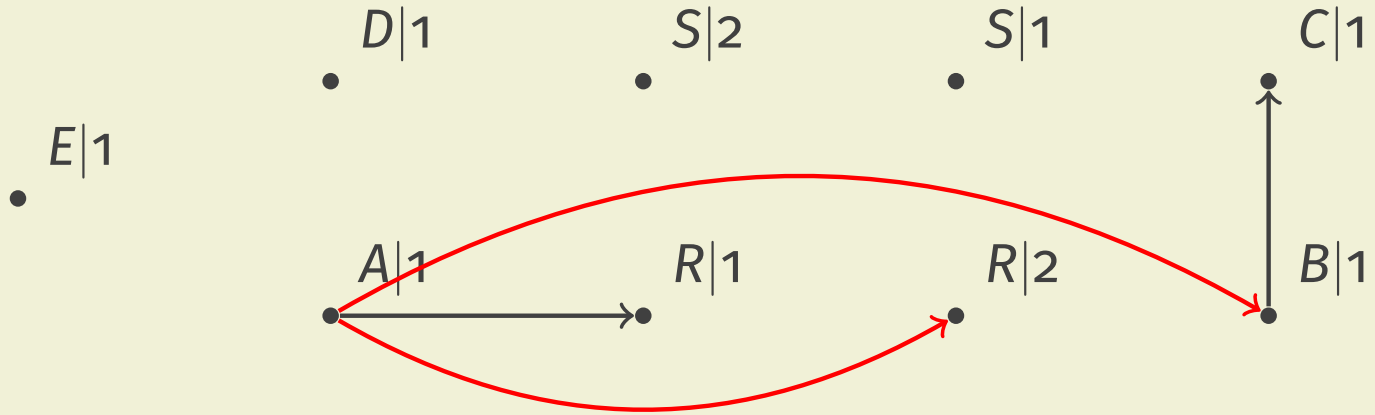

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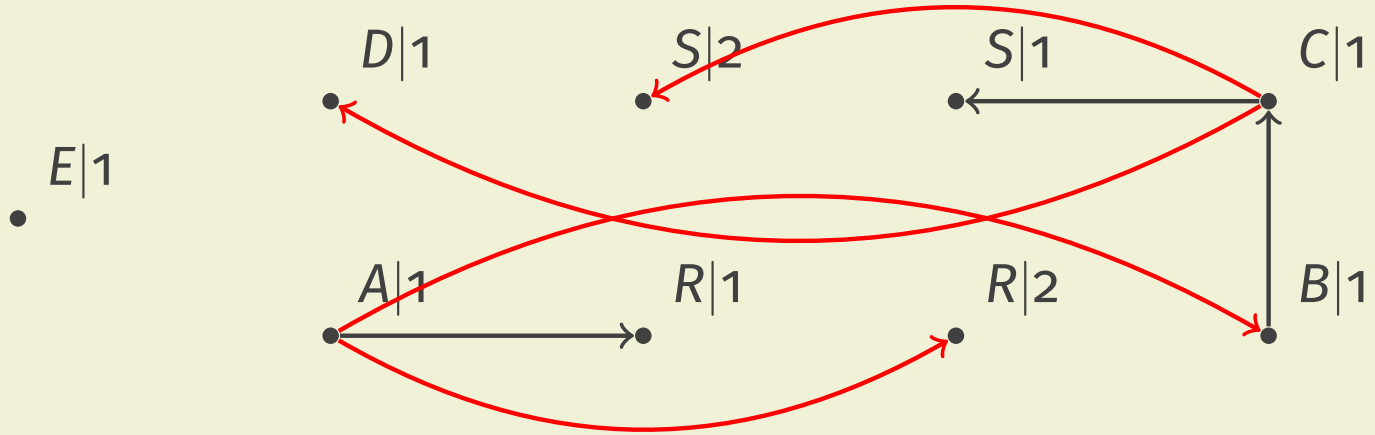

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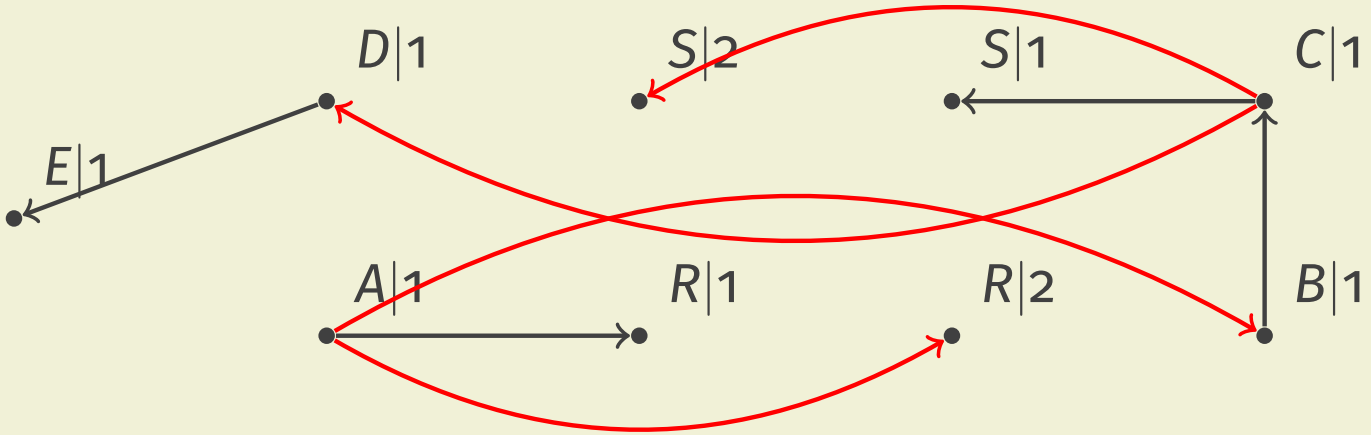

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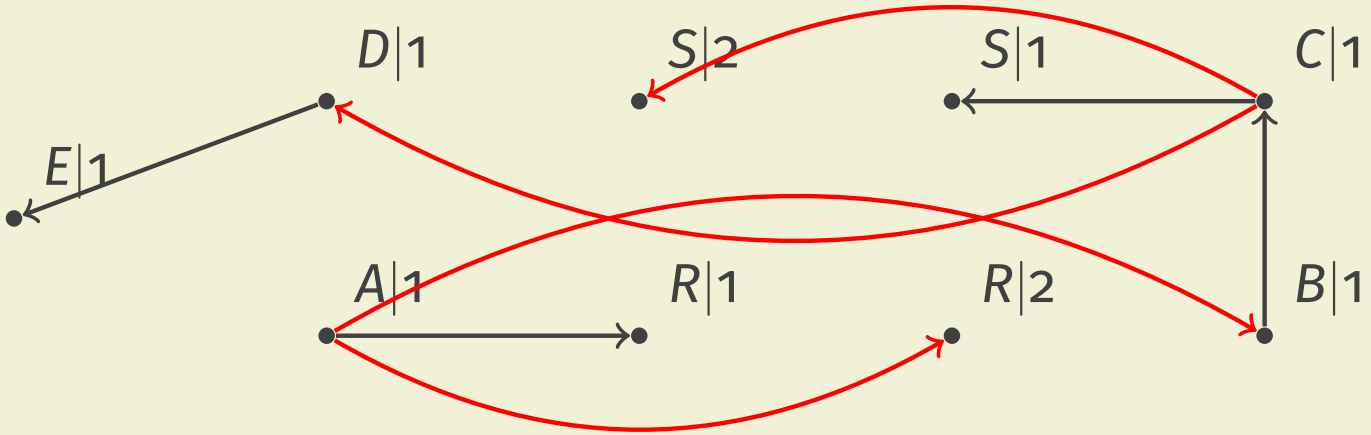

Passes the test: (true) positive!

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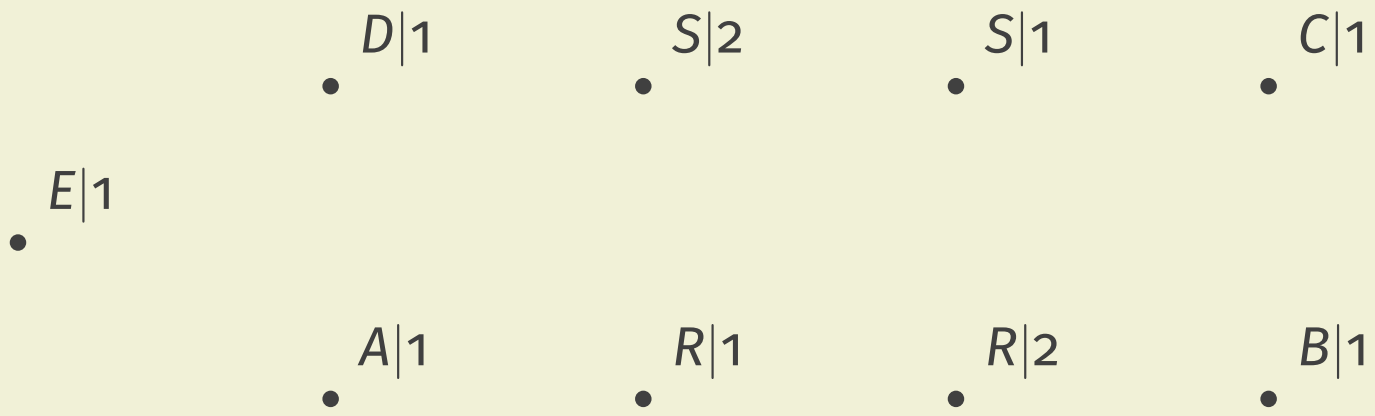

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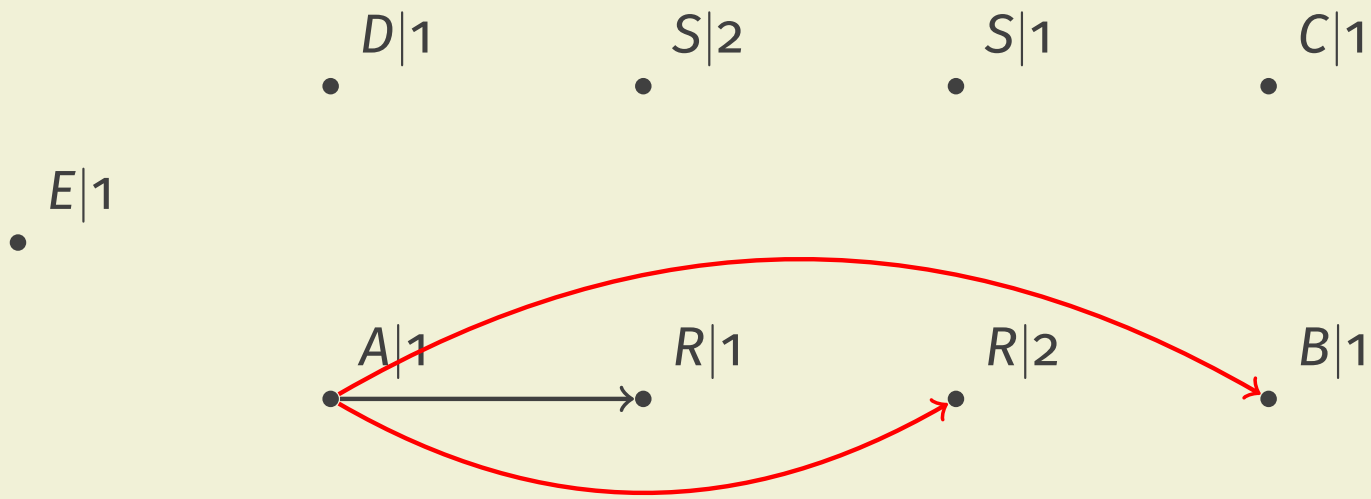

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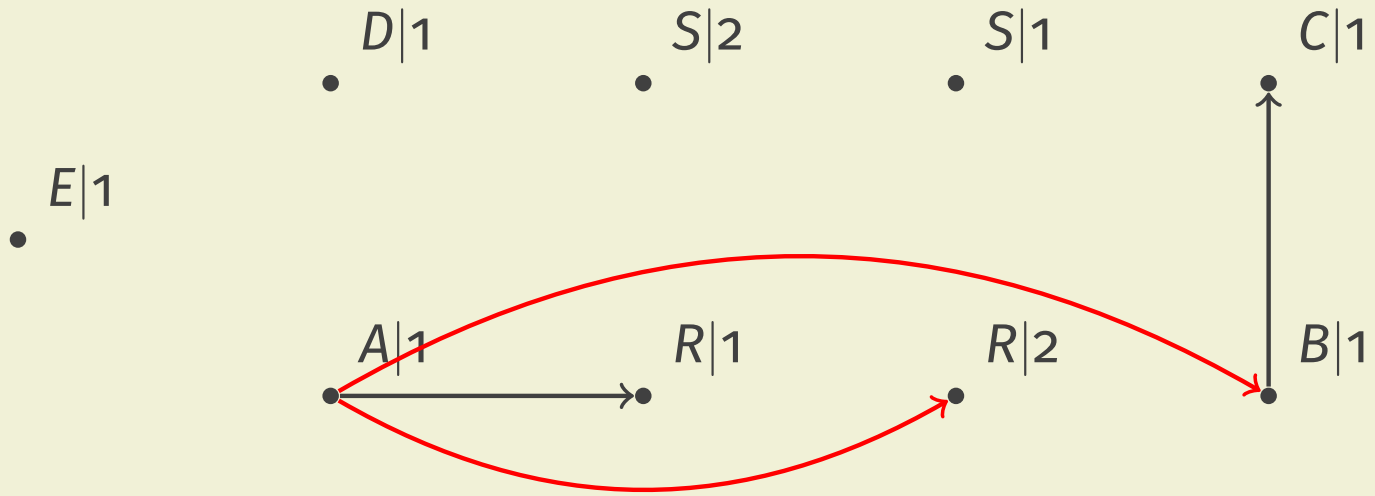

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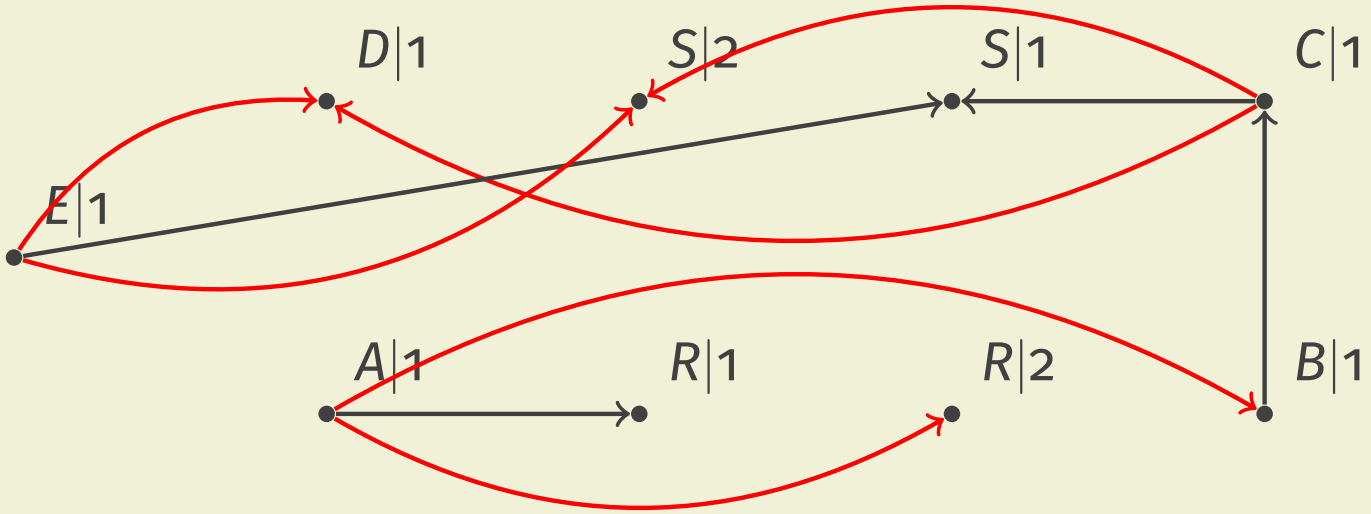

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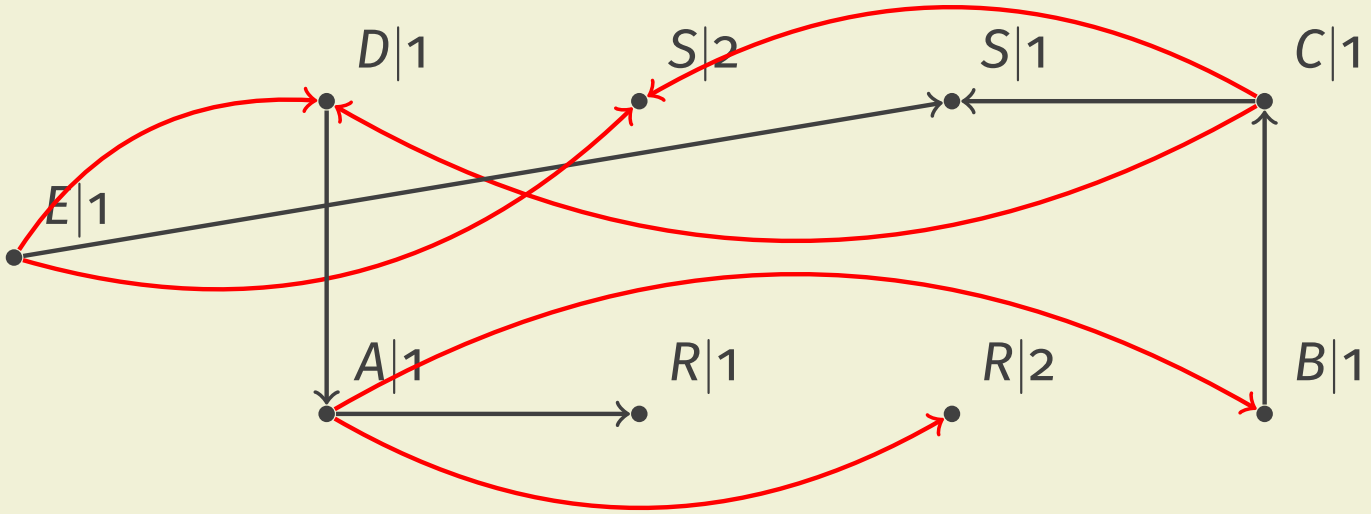

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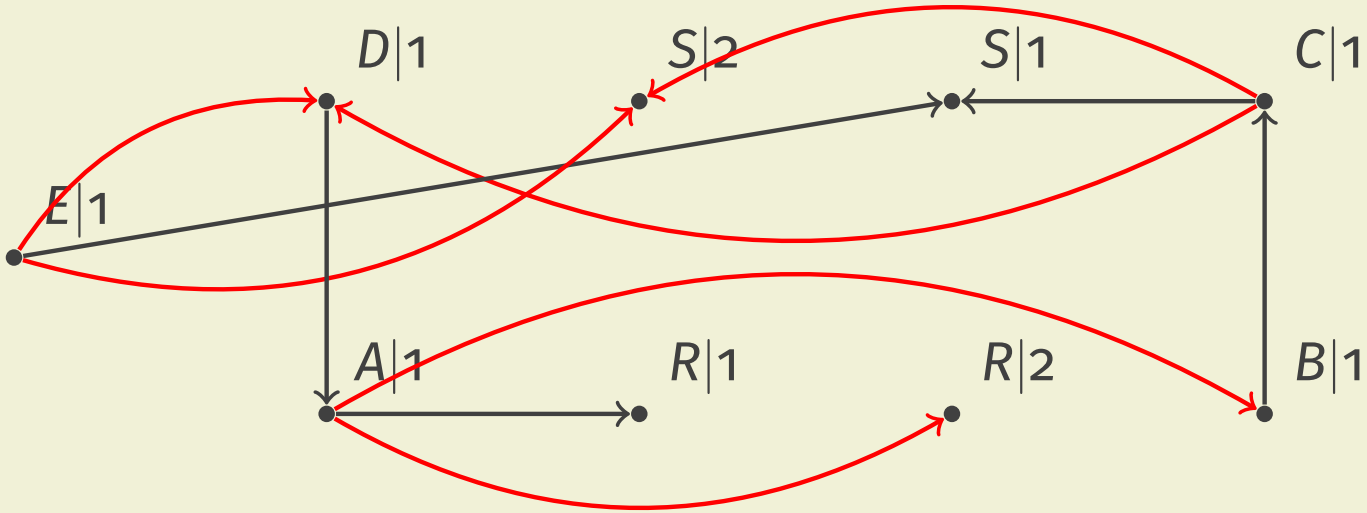

Fails the test: false negative!

A Recipe for Decidability

To decide if a KB $\mathcal{K} = \langle \mathcal{R}, \mathcal{F} \rangle$ entails a query γ:

1. *Decidability property:* the Skolem chase terminates on $\mathcal{R}$
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Seminar Talk: Michaël Thomazo (morning)

1. *Decidability property:* bounded tree-width model property
2. *Decidable language:* guarded existential rules
3. *Algorithm:* compute a finite rep. of a (possibly) infinite model

Research Goals

The recipe:

1. Choose a property that guarantees decidability
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- Study the expressivity of decidable classes of rule sets (Sebastian's talk).

Thanks for your attention!

References I

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References II

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