Ontology-Based Query Answering under Guarded Rules

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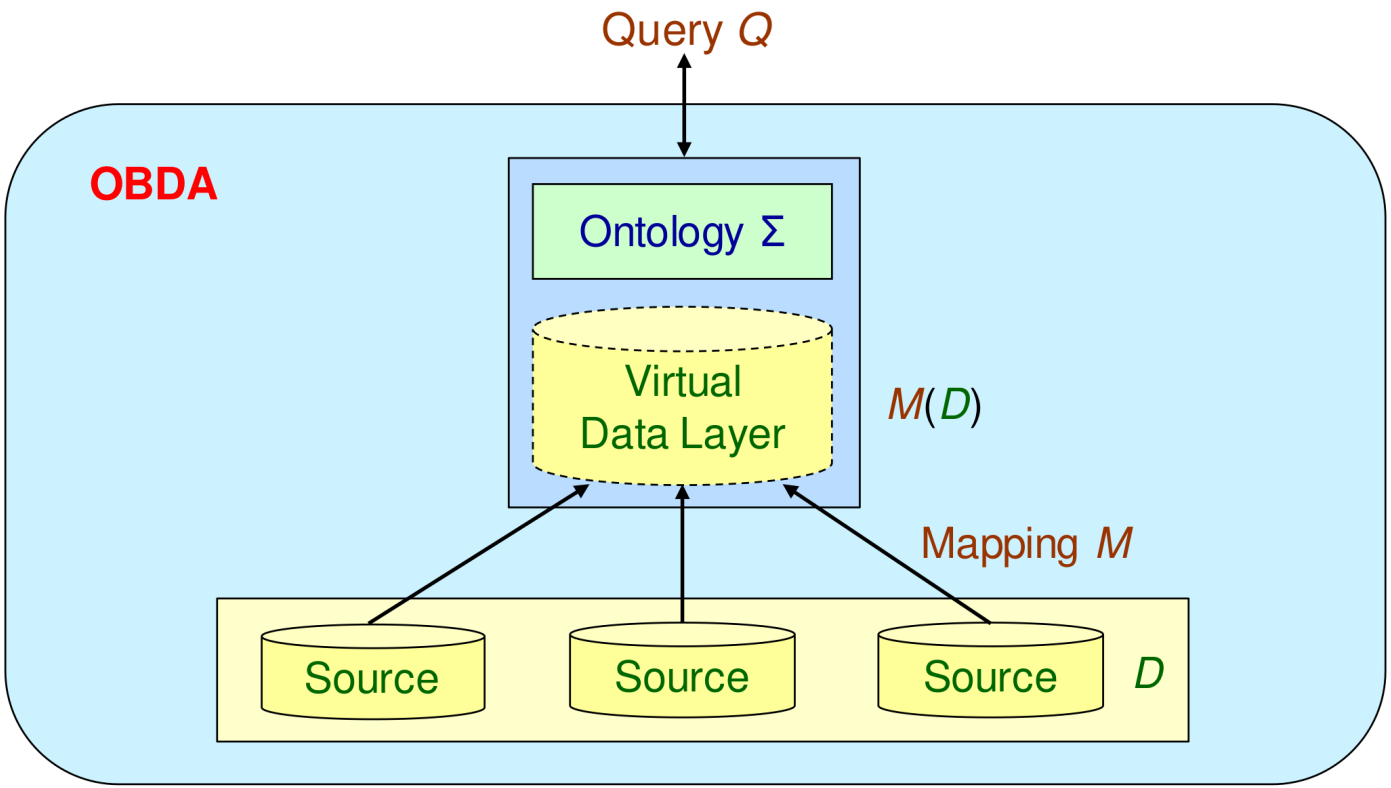
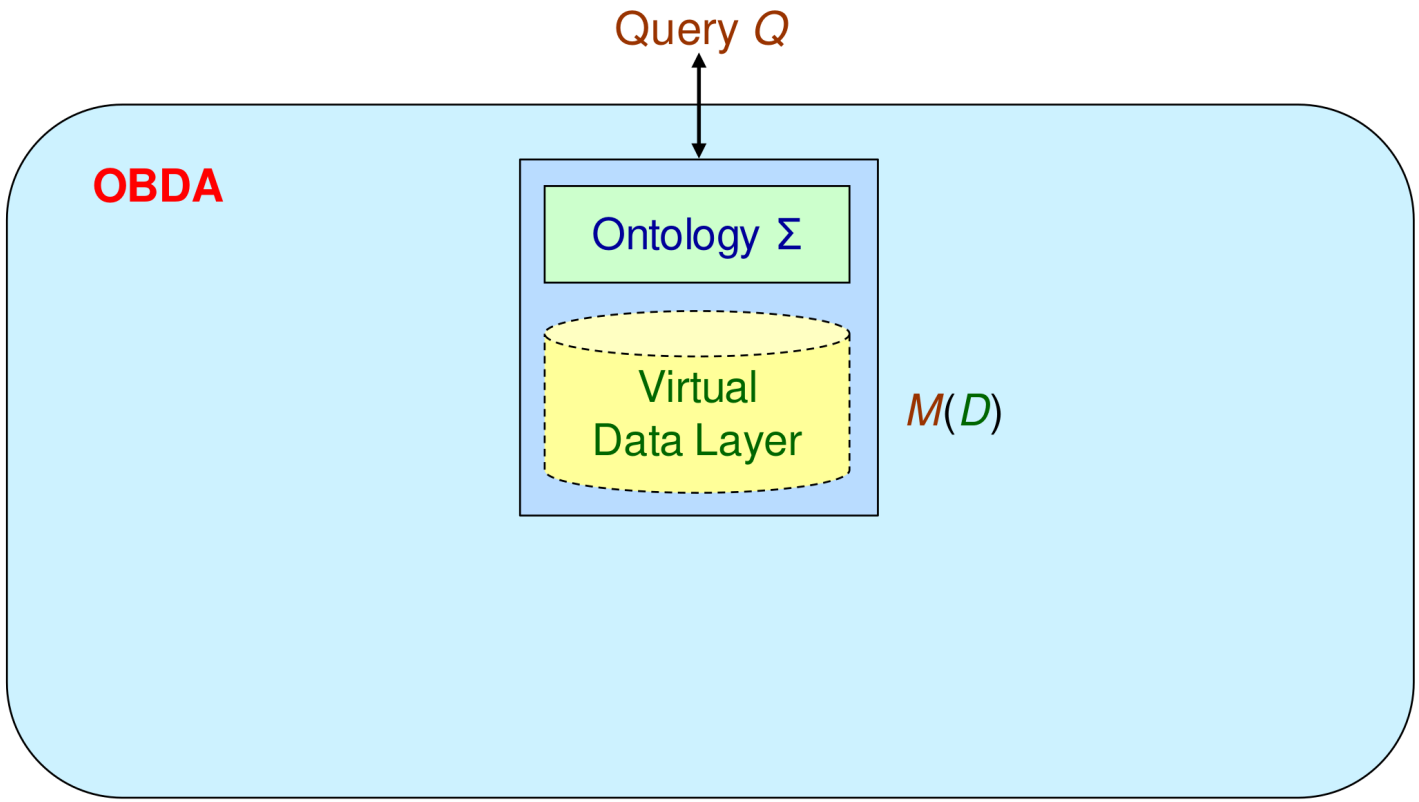
Journée de la logique, Montpellier

Ontology-Based Data Access



Ontology-Based Query Answering



Ontology Language

Existential rules

$$\forall \vec{x}\ \forall \vec{y}\ (\ \mathrm{Body}[\vec{x}, \vec{y}] \rightarrow \exists \vec{z}\ \mathrm{Head}[\vec{x}, \vec{z}]\ )$$

$\vec{y}$ is called the frontier of the rule.

Our Problem

Given a set of ground atoms D, a set of existential rules $\mathcal{R}$ and a Boolean conjunctive query q, does it hold that

$$D, \mathcal{R} \models q?$$

Complexity

- Data complexity: is calculated by considering only the database as part of the input, while the ontology and the query are fixed

- Combined complexity: is calculated by considering, apart from the database, also the ontology and the query as part of the input

- Data complexity vs. Combined complexity
  - Data complexity tends to be a more meaningful measure - ontologies and queries tend to be small; databases tend to be large
  - Nevertheless, the combined complexity is a relevant measure - identifies the real source of complexity

Description Logics and their Translation to Existential Rules

DL-Lite: Popular family of DLs - at the basis of the OWL 2 QL profile of OWL

DL-Lite Axioms	First-order Representation
$A \sqsubseteq B$	$\forall X\ (A(X) \rightarrow B(X))$
$A \sqsubseteq \exists R$	$\forall X\ (A(X) \rightarrow \exists Y\ R(X,Y))$
$\exists R \sqsubseteq A$	$\forall X \forall Y\ (R(X,Y) \rightarrow A(X))$
$\exists R \sqsubseteq \exists P$	$\forall X \forall Y\ (R(X,Y) \rightarrow \exists Z\ P(X,Z))$
$A \sqsubseteq \exists R.B$	$\forall X\ (A(X) \rightarrow \exists Y\ (R(X,Y) \wedge B(Y)))$
$R \sqsubseteq P$	$\forall X \forall Y\ (R(X,Y) \rightarrow P(X,Y))$
$A \sqsubseteq \neg B$	$\forall X\ (A(X) \wedge B(X) \rightarrow \bot)$

DL-Lite form the core of the OWL 2 QL Semantic Web profile.

$\mathcal{EL}$ as Existential Rules

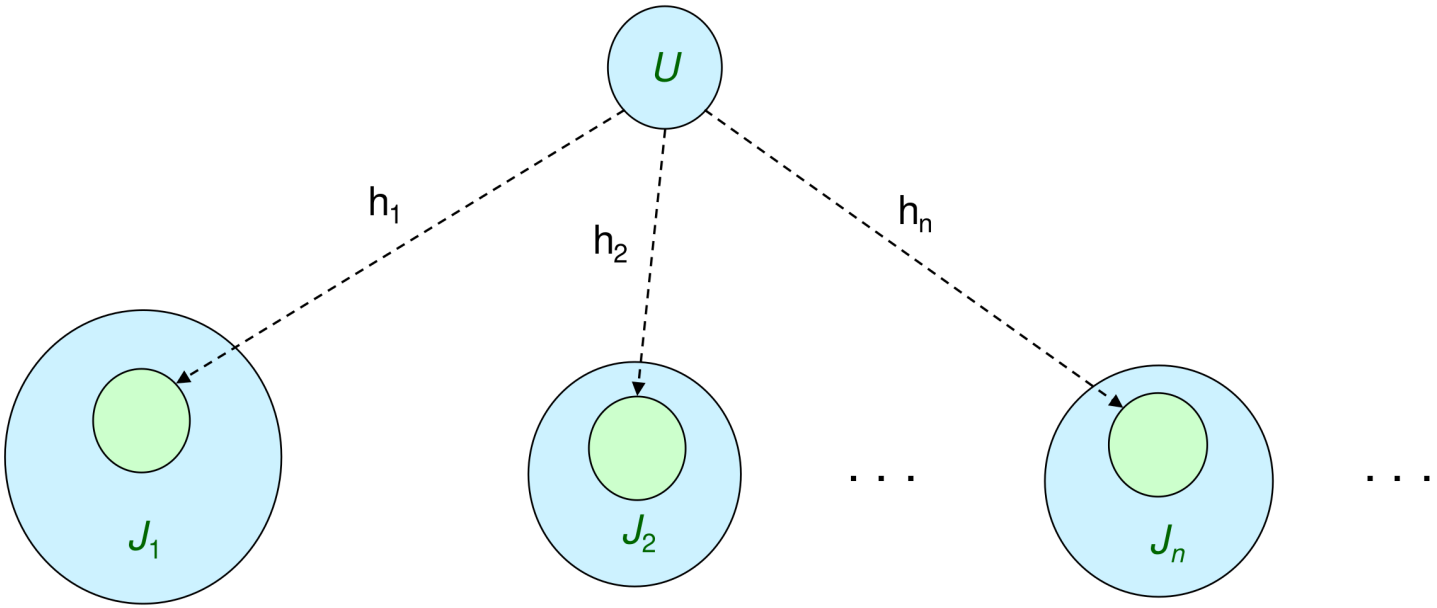
EL: Popular DL for biological applications - at the basis of OWL 2 EL profile

EL Axioms	First-order Representation
$A \sqsubseteq B$	$\forall X\ (A(X) \rightarrow B(X))$
$A \sqcap B \sqsubseteq C$	$\forall X\ (A(X) \wedge B(X) \rightarrow C(X))$
$A \sqsubseteq \exists R.B$	$\forall X\ (A(X) \rightarrow \exists Y\ (R(X,Y) \wedge B(Y)))$
$\exists R.B \sqsubseteq A$	$\forall X \forall Y\ (R(X,Y) \wedge B(Y) \rightarrow A(X))$

Outline of the Talk

1. Limitations of terminating chase and first-order rewritability
2. Guarded Rules: Definition and Properties of the Chase
3. An Algorithm for Guarded Rules
4. Other Approaches

Universal Models



An instance U is a universal model of $D \wedge \Sigma$ if the following holds:

1. U is a model of $D \wedge \Sigma$
2. $\forall J \in$ models$(D \wedge \Sigma)$, there exists a homomorphism h_J such that $h_J(U) \subseteq J$

(Non)-Finiteness of the Chase

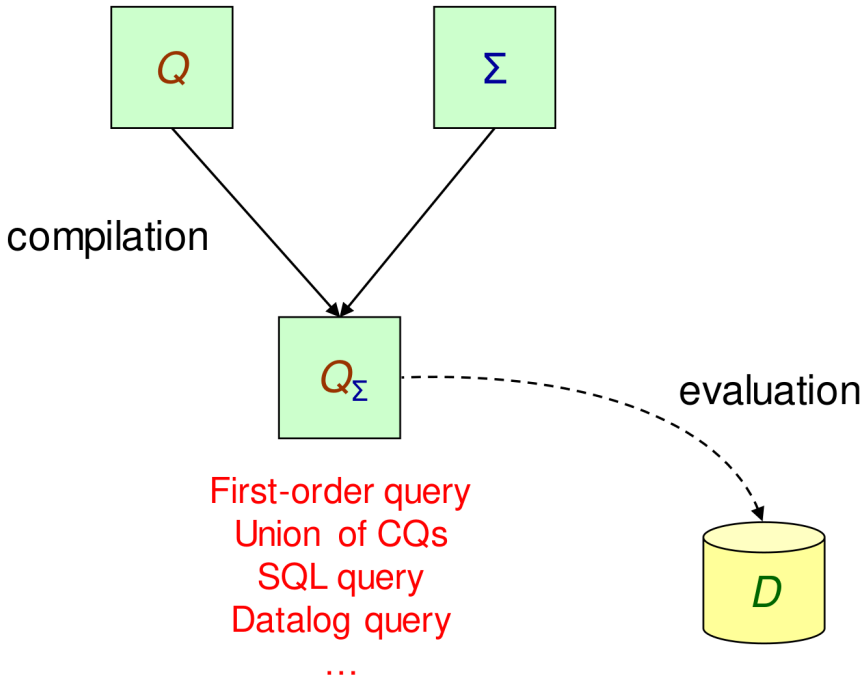
Σ

$$\forall X\,(Person(X) \rightarrow \exists Y\,(hasParent(X,Y) \wedge Person(Y)))$$

$$chase(D,\Sigma) = D \cup \{hasParent(Alice, z_1), Person(z_1),$$
$$hasParent(z_1, z_2), Person(z_2),$$
$$hasParent(z_2, z_3), Person(z_3), \ldots$$

1. **Existential quantification**
2. **Recursive definitions**

Existence of a First-Order Rewriting



$\forall D : D \wedge \Sigma \models Q \quad \Leftrightarrow \quad D \models Q_\Sigma$

evaluated and optimized by

exploiting existing technology

Ensures small data complexity: in $AC_0 \subseteq$ LogSpace

An "Easy" Example

Consider the following ruleset:

$$p(x) \to \exists y\ r(x, y) \wedge p(y)$$
$$p(x) \wedge r(x, y) \to p(y)$$

None of the mentionned approaches are applicable:

- chase does not terminate because of the first rule;
- first-order rewriting does not exist because of the second rule.

Guarded Rules

Guard

A *guard* in a rule is an atom of its body that contains all the body variables.

Guarded Rule

A rule is guarded is it admits a guard. A ruleset is guarded if all its rules are guarded.

Guarded Rules

Guard

A *guard* in a rule is an atom of its body that contains all the body variables.

Guarded Rule

A rule is guarded is it admits a guard. A ruleset is guarded if all its rules are guarded.

The following rule is guarded:

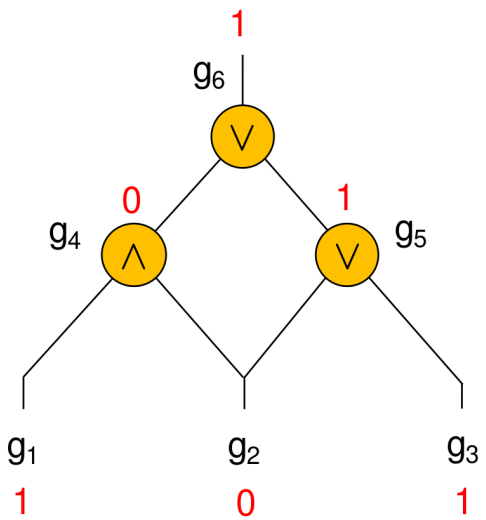
$$p(x) \rightarrow \exists y\ r(x, y) \wedge p(y)$$

The following rule is not guarded:

$$r(\mathbf{x}, y) \wedge r(y, \mathbf{z}) \rightarrow r(x, z)$$

Data Complexity of Guarded Rules

We can show PTime hardness in data complexity of the reasoning with guarded rules.



Does the circuit evaluate to *true*?

encoding of the circuit as a database D

$T(g_1)$ $T(g_3)$

$AND(g_4,g_1,g_2)$ $OR(g_5,g_2,g_3)$ $OR(g_6,g_4,g_5)$

evaluation of the circuit via a *fixed* set Σ

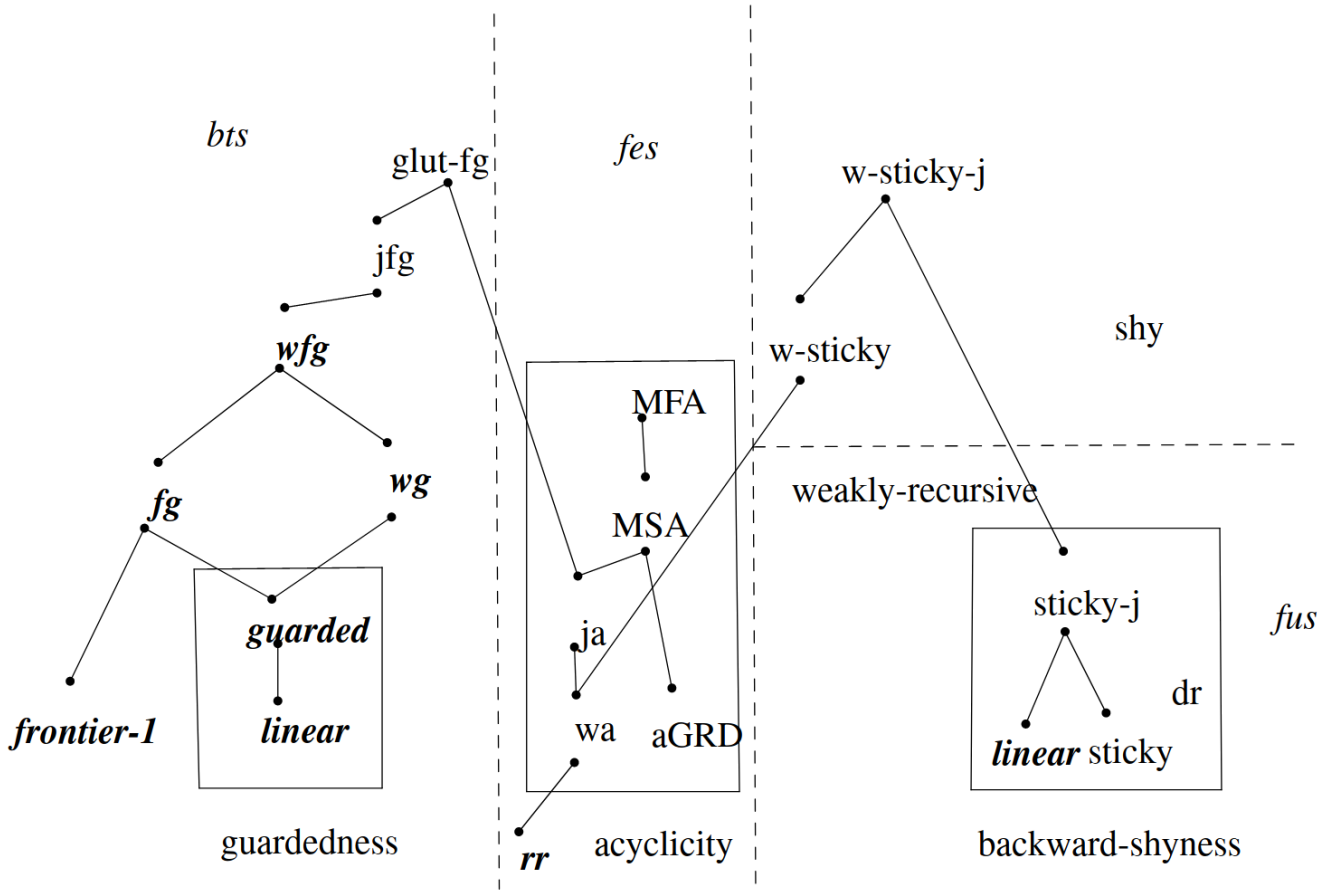
$\forall X \forall Y \forall Z\ (T(X) \wedge OR(Z,X,Y) \rightarrow T(Z))$

$\forall X \forall Y \forall Z\ (T(Y) \wedge OR(Z,X,Y) \rightarrow T(Z))$

$\forall X \forall Y \forall Z\ (T(X) \wedge T(Y) \wedge AND(Z,X,Y) \rightarrow T(Z))$

Circuit evaluates to *true* iff $D \wedge \Sigma \models T(g_6)$

Relationships between Guarded Rules and Other Classes



Guarded Rules and $\mathcal{EL}$

EL: Popular DL for biological applications - at the basis of OWL 2 EL profile

EL Axioms	First-order Representation
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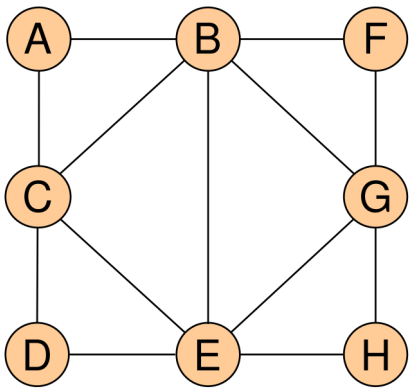
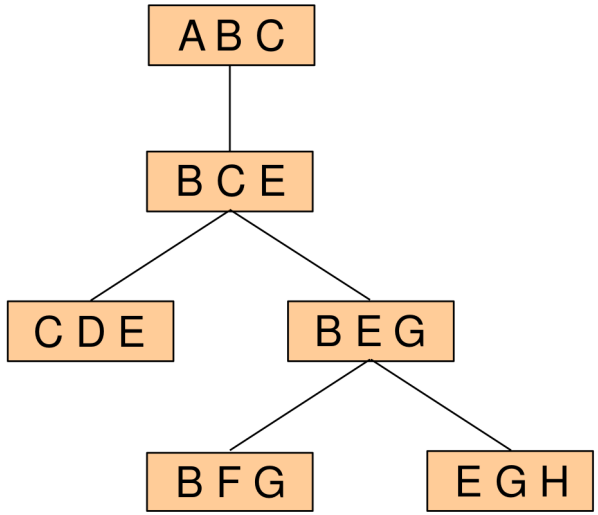
Tree Decomposition

Tree decomposition - a mapping of a graph into a tree

Graph $G = (\mathbf{V},\mathbf{E})$



Tree decomposition $T = (\mathbf{V'},\mathbf{E'})$ of G



1. For each v $\in$ **V**, there exists u $\in$ **V'** such that v $\in$ u
2. For each (v,w) $\in$ **E,** there exists u $\in$ **V'** such that {v,w} $\subseteq$ u
3. For each v $\in$ **V**, {u | v $\in$ u} induces a connected subtree

Treewidth and Consequences

Example

- $w(T) = \mathsf{max}_{B \in T}(\mathsf{size}(B)) - 1$
- $tw(G) = \mathsf{min}_{T \text{ for } G}(w(T))$

Small Treewidth Makes (Some) Problems Easy

Many problems that are hard on general graphs become easy on graphs of bounded treewidth.

Gaifman Graph

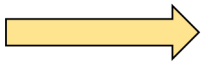
A.k.a Primal Graph

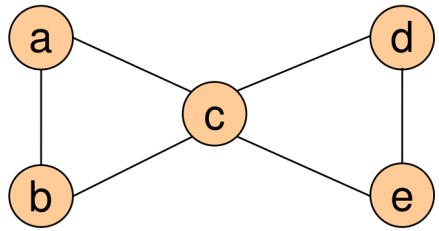
- An instance J can be represented as a graph $\mathcal{G}_J$ - Gaifman graph

R(a,b,c)

S(c,d)

T(c,d,e)





- The treewidth of J, denoted tw(J), is defined as tw($\mathcal{G}_J$)
- Thus, we can talk about the treewidth of the chase

Guardedness and Tree Decomposition

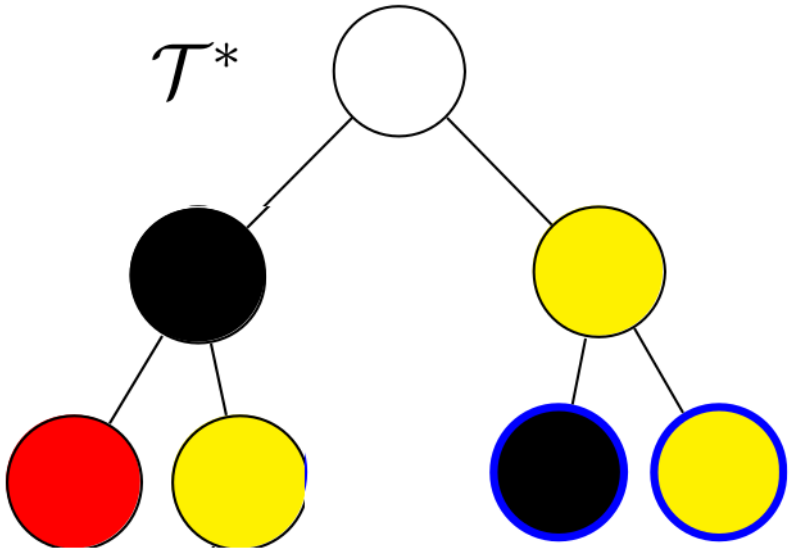
Property

For any guarded rule set, for any database, there exists b such that, for any k, $\mathsf{chase}_k(D, \mathcal{R})$ has treewidth at most b.

Key idea: maintain a tree decomposition at each step of the chase.

Blocking Technique: Finite Representation of the Chase

Finite representation of the chase:



Creating this Finite Representation is not Trivial

- defining what it means that two bags are equivalent;
- actually computing this equivalence relation.

On an easy example:

- $R_1 : p(x) \rightarrow \exists y\ r(x, y)$
- $R_2 : r(x, y) \rightarrow \exists z\ s(y, z)$
- $R_3 : q(x) \wedge r(x, y) \rightarrow q(y)$
- $R_4 : q(x) \wedge s(x, y) \wedge r(z, x) \wedge h(x)$

and consider $D = \{q(a_1), r(a_1, b_1), r(b_1, c_1), p(c_1), r(a_2, b_2), r(b_2, c_2), p(c_2)\}$

Ultimate Similarity

$$\frac{}{\mathbb{P} \rightsquigarrow \pi_{|\sigma(fr(R))}.\mathbb{P}_{\text{init}}(\mathbb{B}(R,\sigma))}\ \text{Prop. 5}$$

$$\frac{\mathbb{P}_1 \rightsquigarrow \lambda.\mathbb{P}_2}{\mathbb{P}_1 \rightsquigarrow \lambda.\mathit{Join}_{\text{l}}(\mathbb{P}_1,\lambda,\mathbb{P}_2)}\ \text{Prop. 6} \qquad \frac{\mathbb{P}_1 \rightsquigarrow \lambda.\mathbb{P}_2}{\mathbb{P}_1 \rightsquigarrow \mathit{Join}_{\text{u}}(\mathbb{P}_1,\lambda,\mathbb{P}_2)}\ \text{Prop. 7}$$

$$\frac{\mathbb{P}_1 \rightsquigarrow \mathbb{P}_2 \quad \mathbb{P}_2 \rightsquigarrow \mathbb{P}_3}{\mathbb{P}_1 \rightsquigarrow \mathbb{P}_3}\ \text{Prop. 8}$$

$$\frac{\mathbb{P}_1 \rightsquigarrow \mathbb{P}_2 \quad \mathbb{P}_1 \rightsquigarrow \lambda.\mathbb{P}_3}{\mathbb{P}_2 \rightsquigarrow \lambda.\mathbb{P}_3}\ \text{Prop. 9} \qquad \frac{\mathbb{P}_1 \rightsquigarrow \lambda.\mathbb{P}_2 \quad \mathbb{P}_2 \rightsquigarrow \mathbb{P}_3}{\mathbb{P}_1 \rightsquigarrow \lambda.\mathbb{P}_3}\ \text{Prop. 10}$$

Figure 5: Deduction calculus for the pattern saturation rules. For the first inference rule, $\mathbb{P}$ is any pattern, $R \in \mathcal{R} \cup \{\rightarrow I\}$, π is a homomorphism from $\mathit{fr}(R)$ to $\mathit{terms}(\mathit{support}(\mathbb{P}))$ such that $(\mathit{body}(R),\pi) \in \mathbb{P}$, and σ is the fusion of $\mathit{fr}(R)$ induced by π.

Other Approaches for Guarded Rules

- chase does not terminate, but one can decide whether a guarded ruleset ensures universal chase termination;
- query rewriting may not always exist, but one can decide whether a query is first-order rewritable;
- another interesting approach: the combined approach.

The Combined Approach

- An approach proposed in the context of description logics
- Several promising results - applied on (extensions of) EL, and members of the DL-Lite family

$D = \{P(a), S_1(a), P(b), S_2(b)\}$

$\Sigma = \{\forall X\,(P(X) \rightarrow \exists Y\,(R(X,Y) \wedge P(Y)))\}$

$Q = \exists X \exists Y \exists Z\,(R(X,Y) \wedge R(Z,Y) \wedge S_1(X) \wedge S_2(Z))$

auxiliary constant for satisfying $\exists$-variables

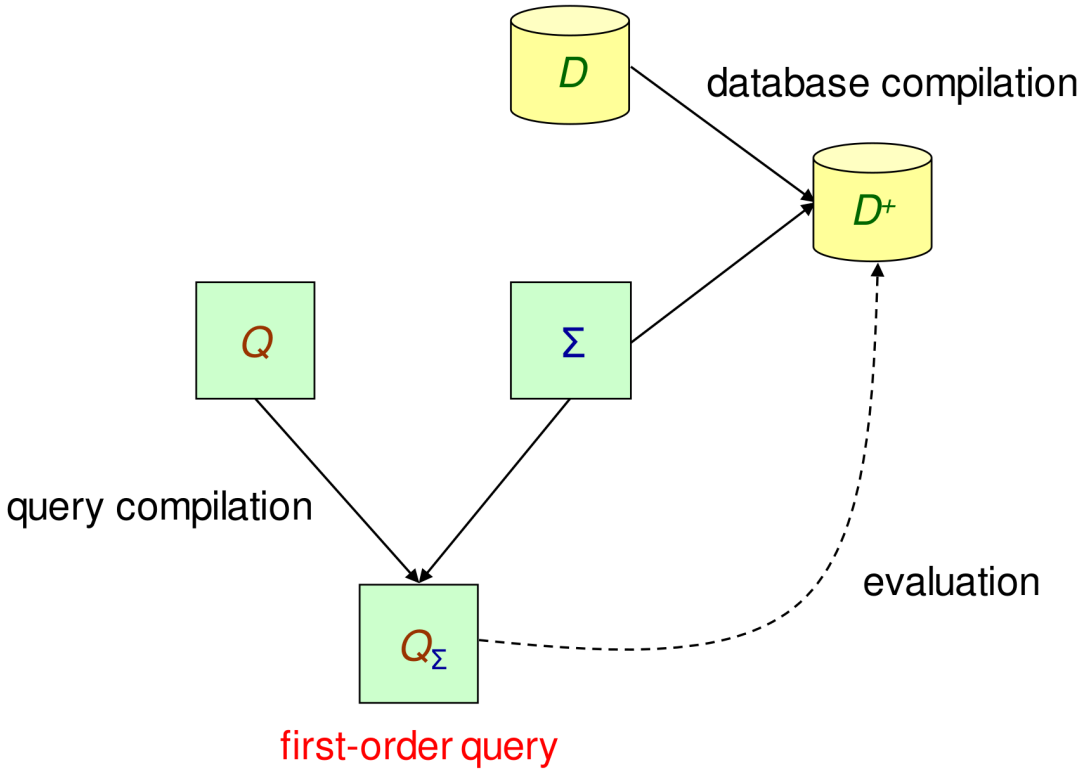
Step 1: Saturate the database, without inventing new nulls

$$D^+ = \{P(a), S_1(a), P(b), S_2(b)\} \cup \{Ex(c)\} \cup \{R(a,c), R(b,c), P(c), R(c,c)\}$$

Step 2: Eliminate unsound answers by rewrting the query into a FO-query

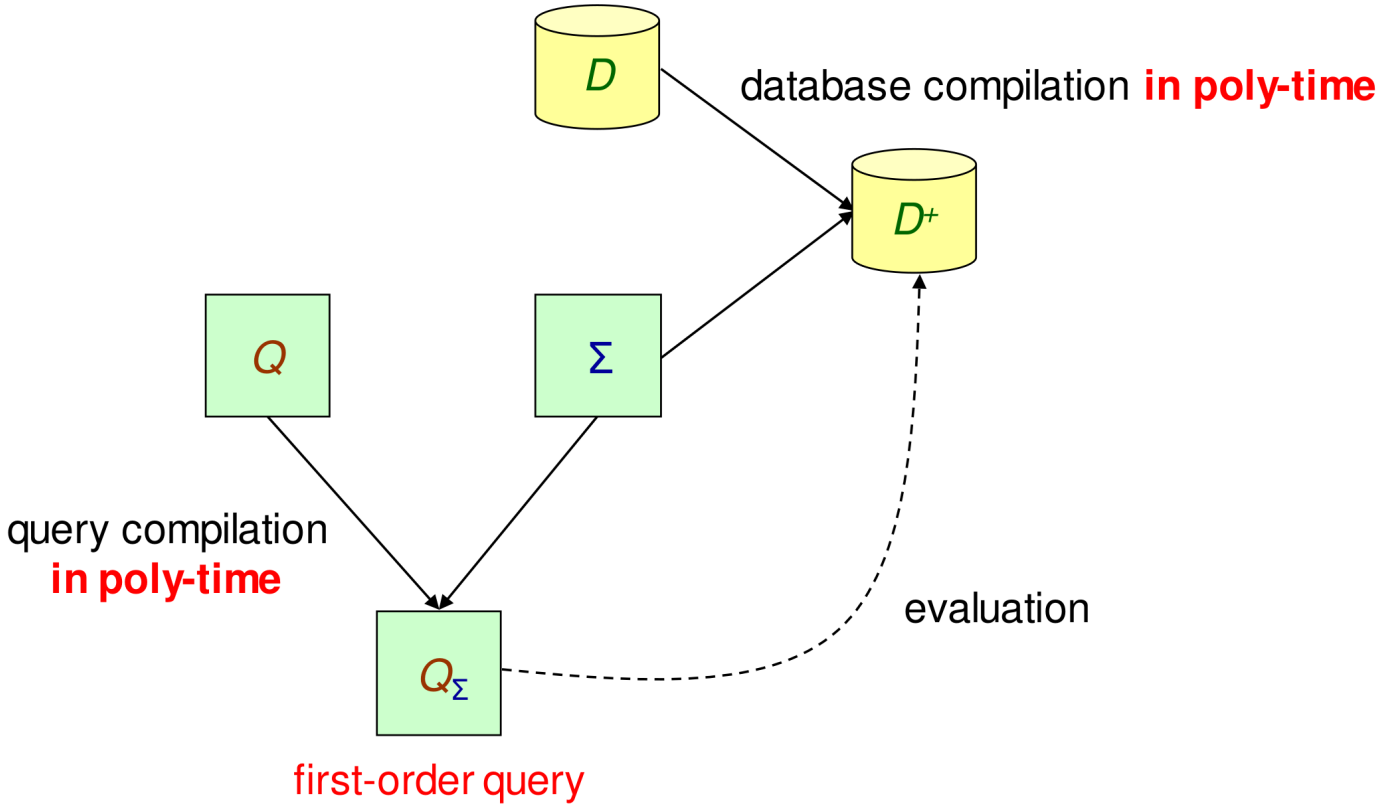
$$Q_{FO} = \exists X \exists Y \exists Z\,((R(X,Y) \wedge R(Z,Y) \wedge S_1(X) \wedge S_2(Z)) \wedge (Ex(Y) \rightarrow X = Z))$$

The Combined Approach



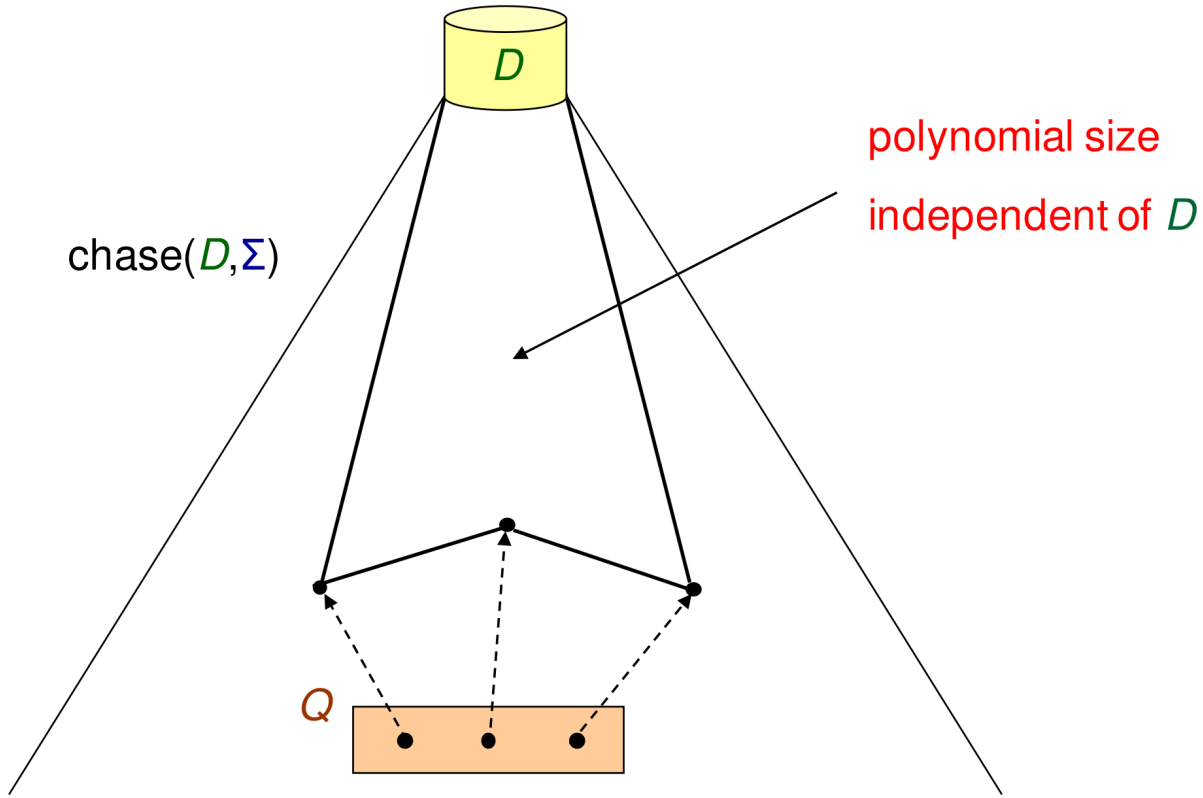
$$\forall D : D \wedge \Sigma \models Q \quad \Leftrightarrow \quad D^{+} \models Q_{\Sigma}$$

Polynomially Combined First-Order Rewritability



$$\forall D : D \wedge \Sigma \models Q \quad \Leftrightarrow \quad D^{+} \models Q_{\Sigma}$$

The Polynomial Witness Property



chase(D,Σ) $\models Q$ $\Rightarrow$ the query admits a small witness

The Polynomial Witness Property

Theorem: The PWP implies that BCQ-Answering is polynomially combined FO-rewritable

Proof (hint):

- We simulate the polynomially sized witness via a polynomially sized first-order query (query compilation)
- Notice that the number of nulls that appear in the witness depends on the query, and thus can not be explicitly added in the database
- We simulate these nulls via tuples of 0s and 1s - the constants 0 and 1 are explicitly added in the database (database compilation)

Applicability

- applicable for a subset of guarded rules, namely *linear* rules
- further conditions required for guarded rules.
- of practical interest for $\mathcal{EL}$, less so for linear rules.

Conclusion

- Guarded rules ensure a well-behaved chase: bounded treewidth
- Working with tree decompositions allow to finitely represent the chase
- Not seen today, but guardedness ensure good properties beyond existential rules

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