

CGL 0 - Introduction

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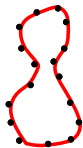
MPRI 2020-2021

Toy Example

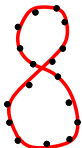
What is this shape?



1



2



3

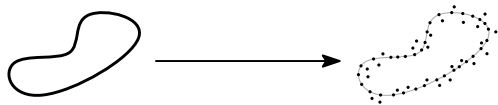


- A A squashed circle?
- B A figure eight?
- C A bunch of points?

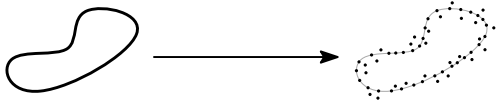
Topological Inference Problem



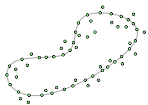
Topological Inference Problem



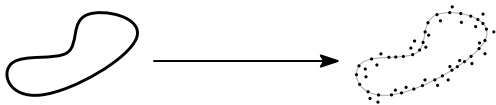
Topological Inference Problem



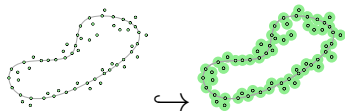
scale: \cdot



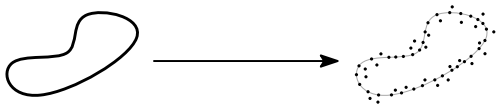
Topological Inference Problem



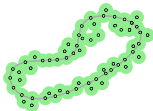
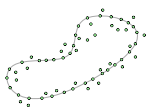
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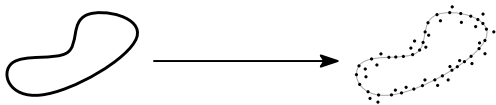
Topological Inference Problem



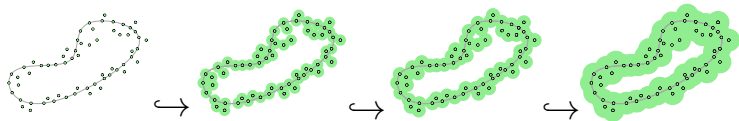
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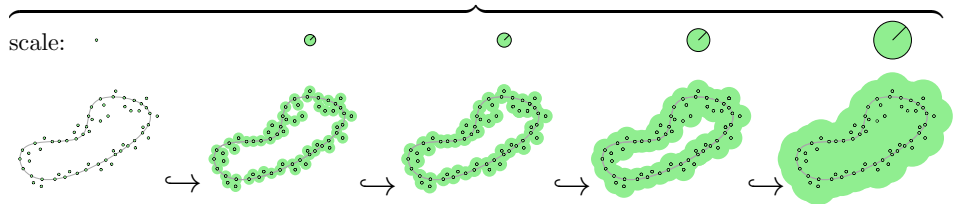
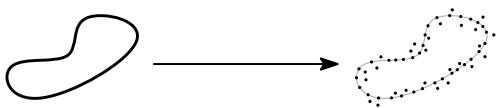
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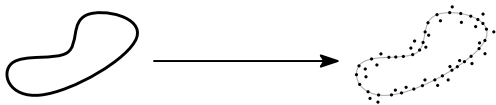
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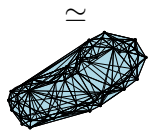
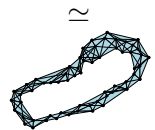
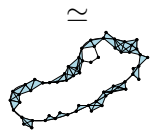
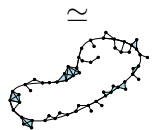
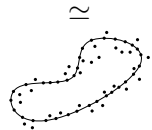
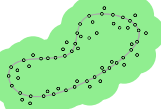
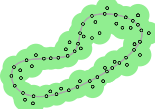
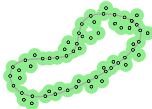
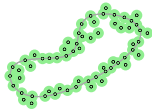
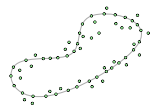
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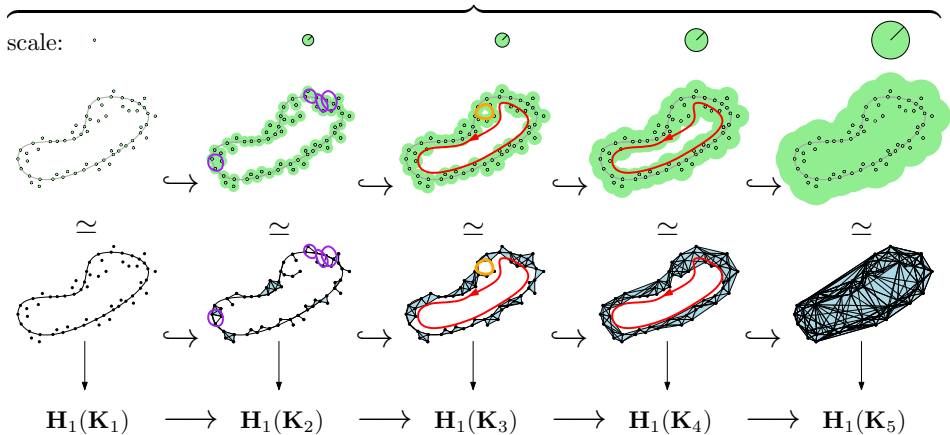
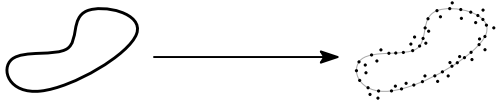
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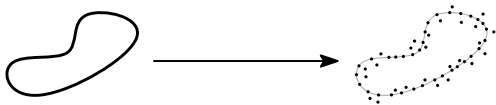
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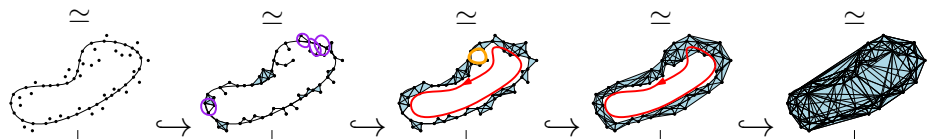
Topological Inference Problem



Topological Inference Problem



scale: scale:

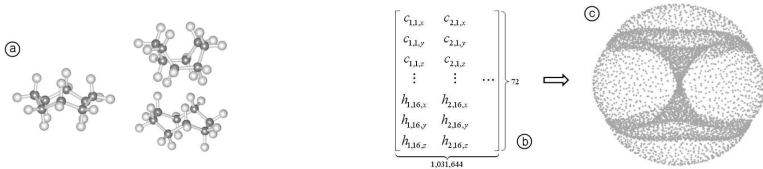


$H_1(K_1) \longrightarrow H_1(K_2) \longrightarrow H_1(K_3) \longrightarrow H_1(K_4) \longrightarrow H_1(K_5)$

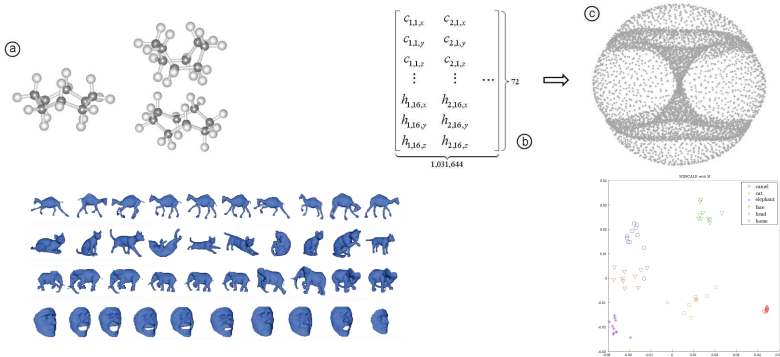
barcode



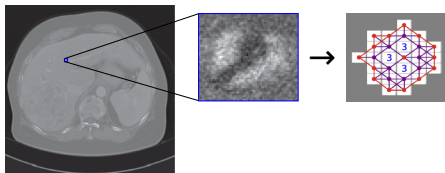
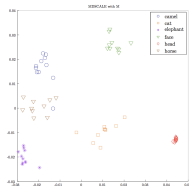
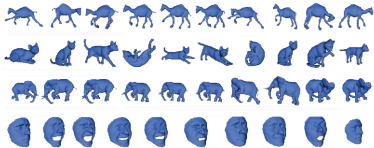
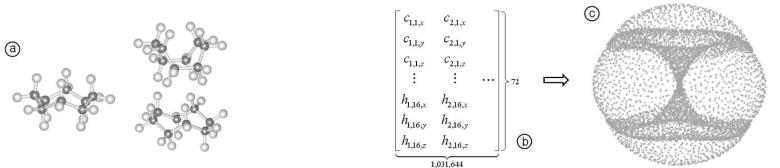
Some Applications



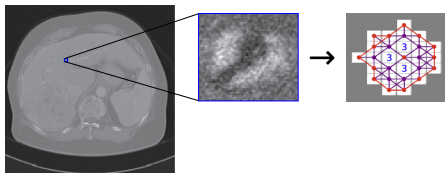
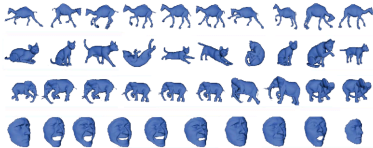
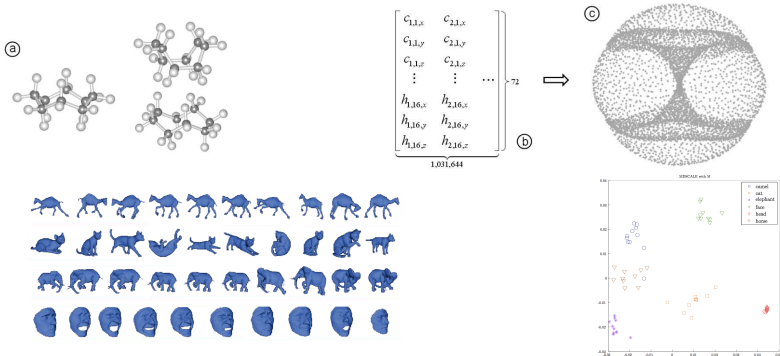
Some Applications



Some Applications



Some Applications

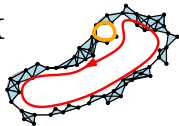


Usually: **Thousands** of points in **very high dimensional** spaces, with complicated metrics.

Overview

vector spaces, of dimension related to:

simplicial
complex \mathbf{K}



↖ $\mathbf{H}_0(\mathbf{K})$

num. connected comp.

▶ $\mathbf{H}_1(\mathbf{K})$

num. of “holes”

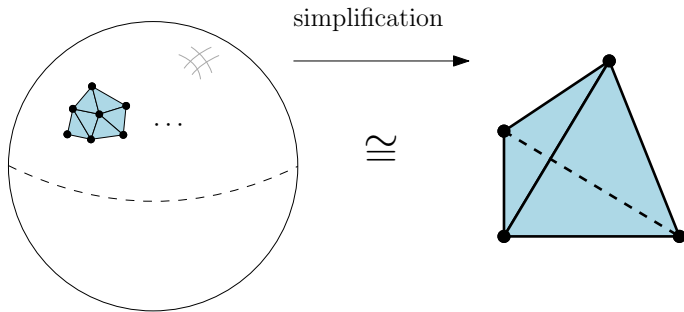
↙ $\mathbf{H}_2(\mathbf{K})$

num. of “voids”

...

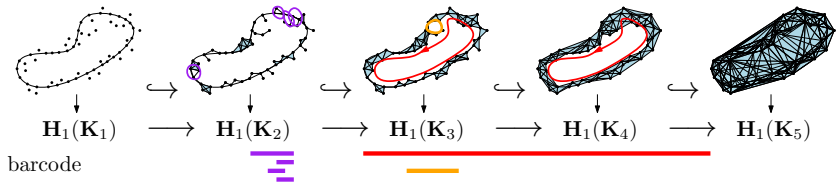
[07/10] **Homology theory**: formalize algebraically the “notion of shape” of a simplicial complex.

Overview



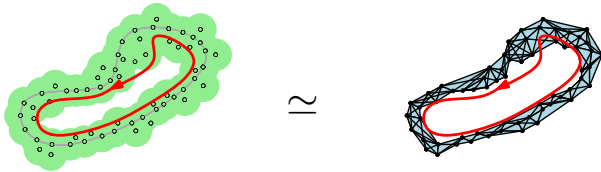
[21/10] **Discrete Morse theory**: define combinatorial transformations to simplify a shape while preserving its topology \longrightarrow accelerate computation.

Overview



[28/10] **Persistent homology**: study the idea of persistence of the topology \rightarrow multiscale approximation of the topology of a point cloud.

Overview



[04/11] **Stability and topological inference:** ensure that the reconstruction *point cloud to simplicial complex* approximates correctly the topology of the underlying shape.