0 - Introduction

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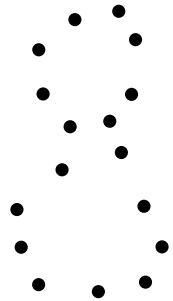
Toy Example

What is this shape?



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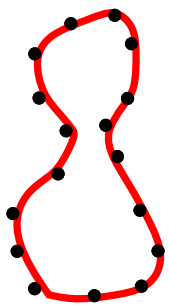

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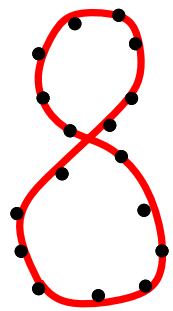

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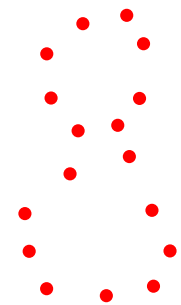
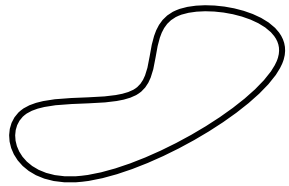


A A squished circle?

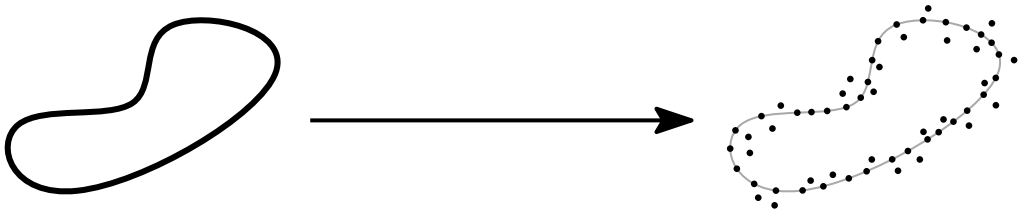
B A figure eight?

C A bunch of points?

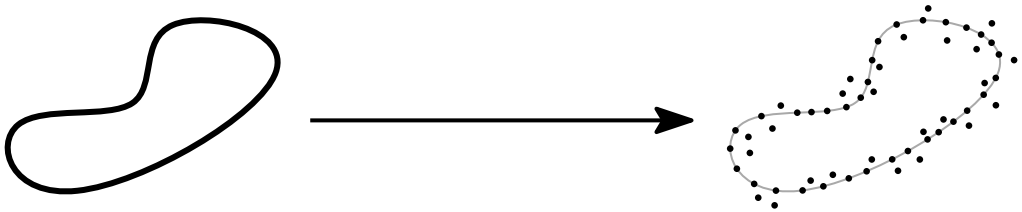
Topological Inference Problem



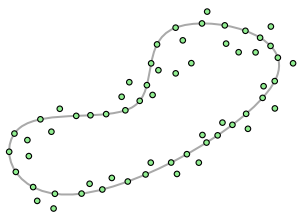
Topological Inference Problem



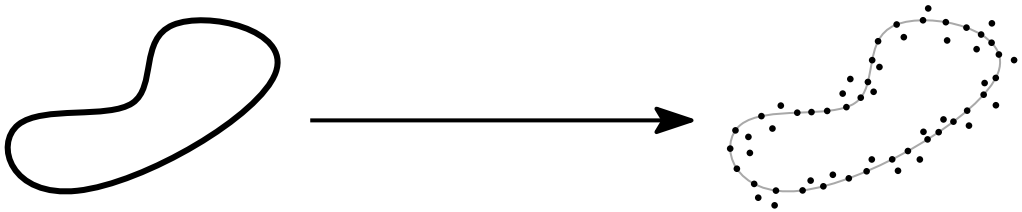
Topological Inference Problem



scale:



Topological Inference Problem



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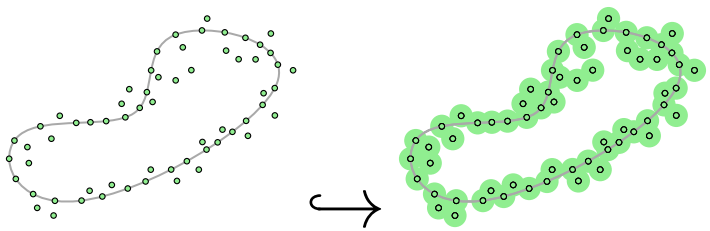

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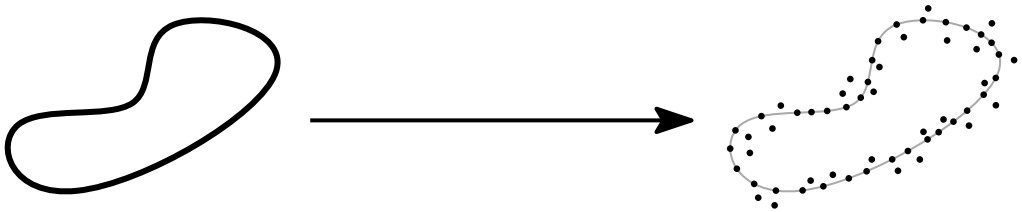
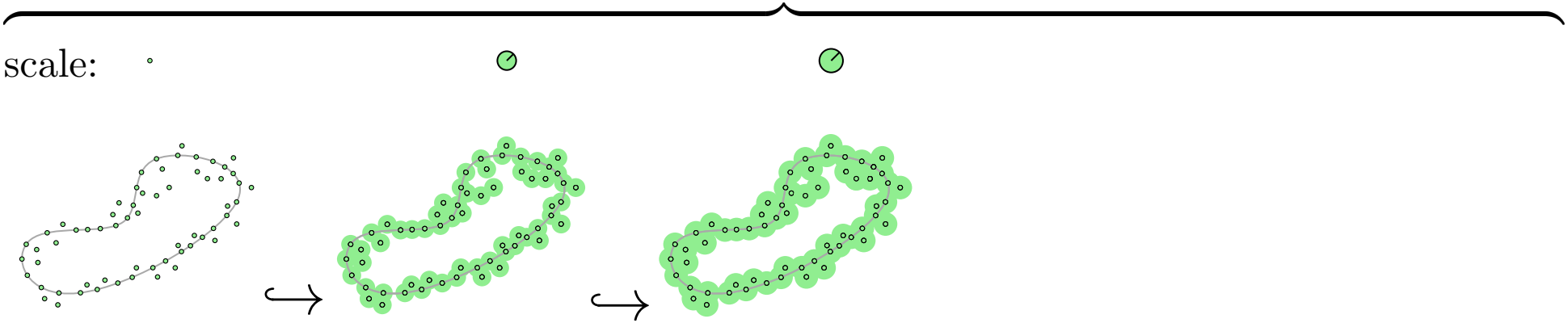
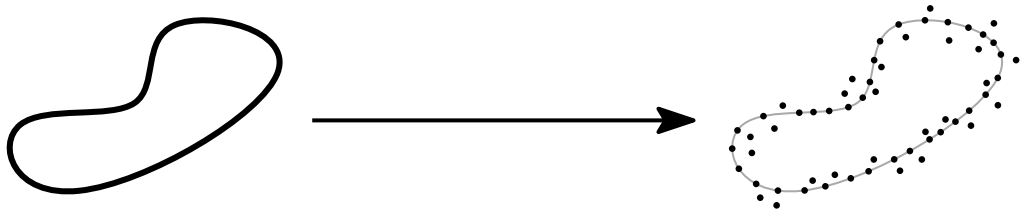
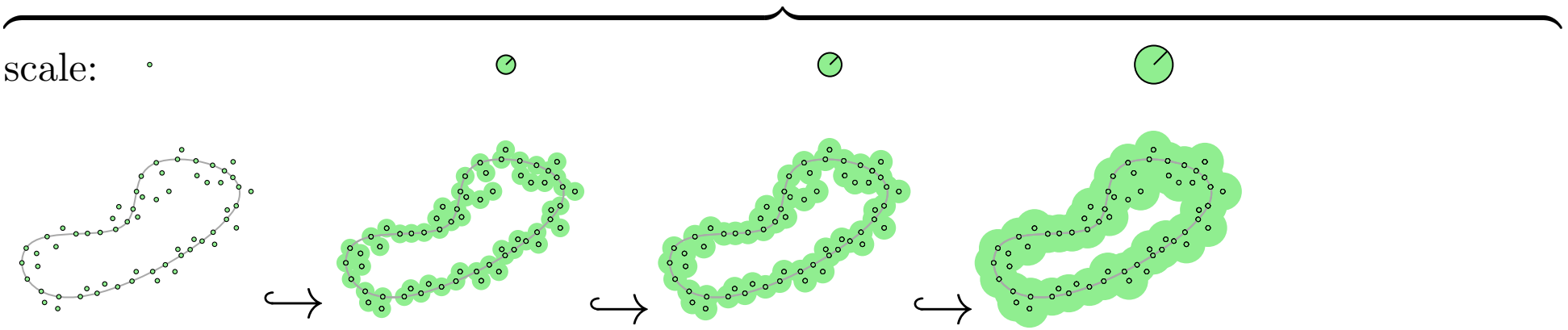




Topological Inference Problem





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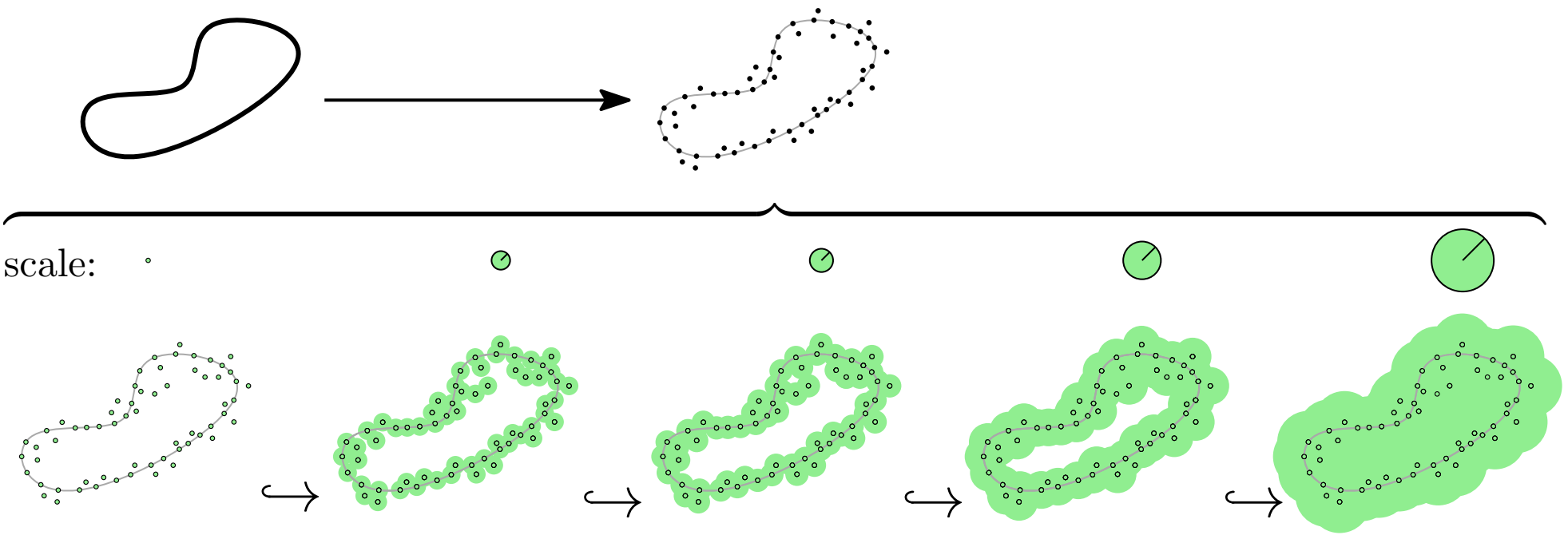

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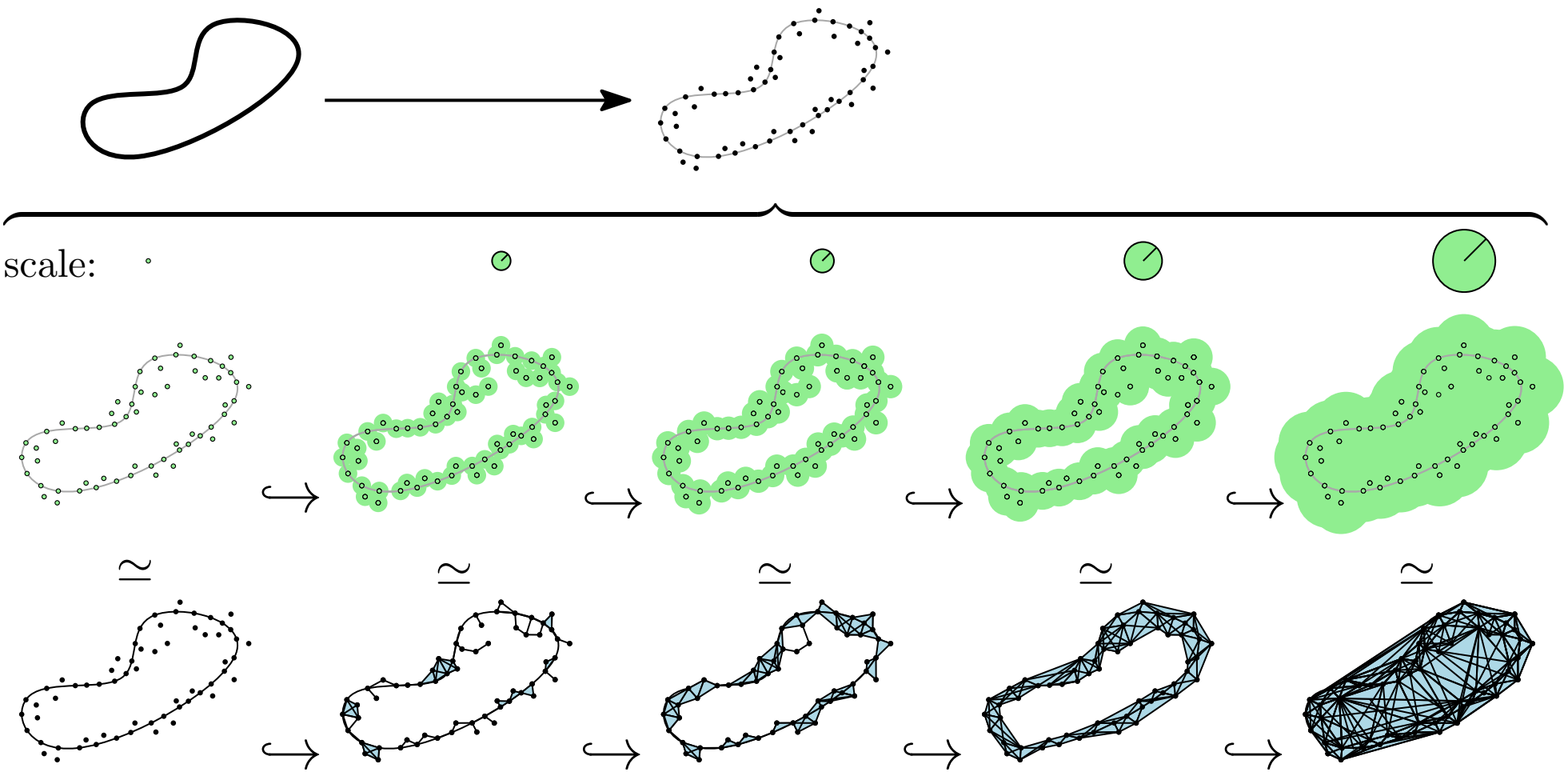

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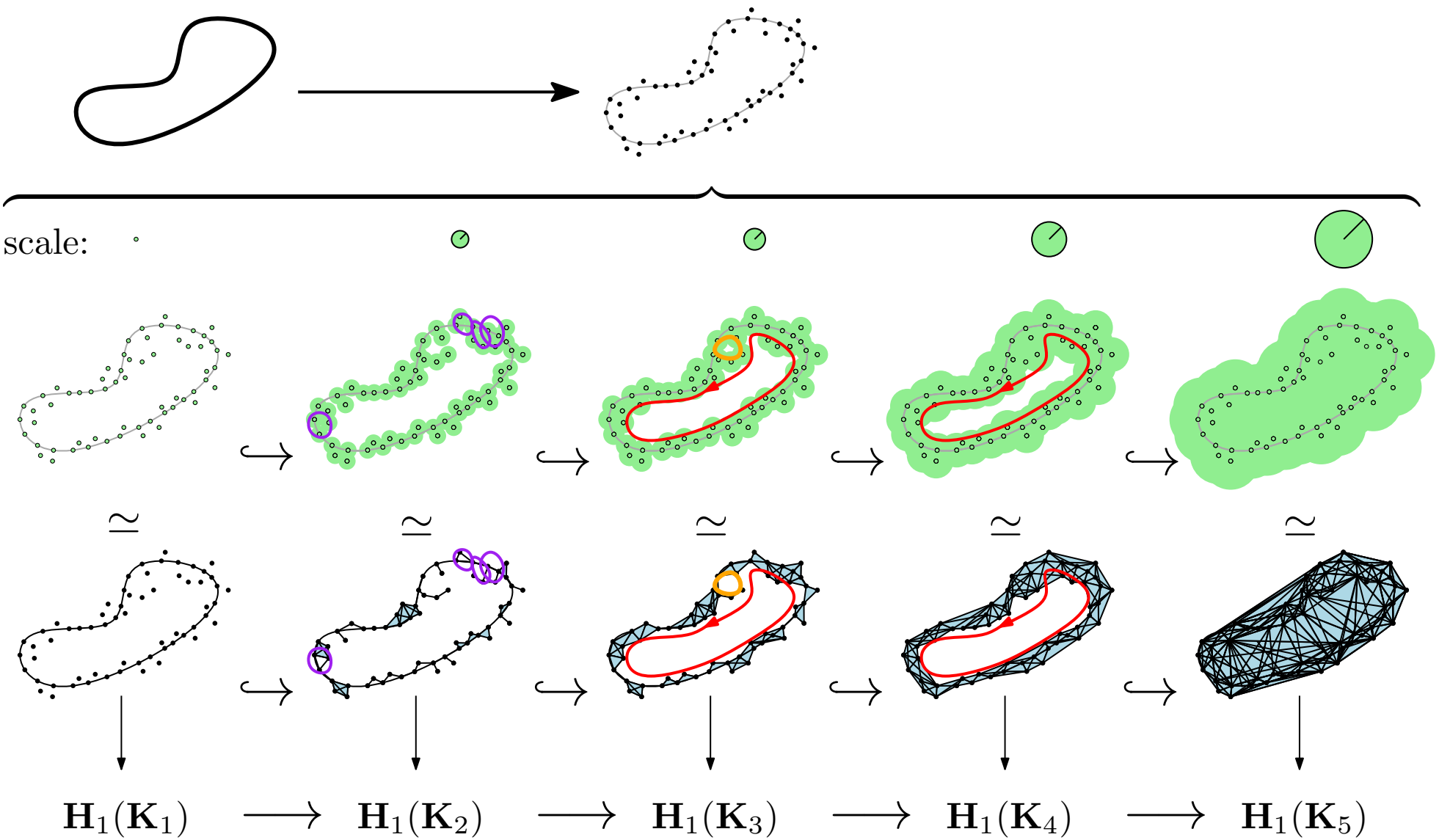
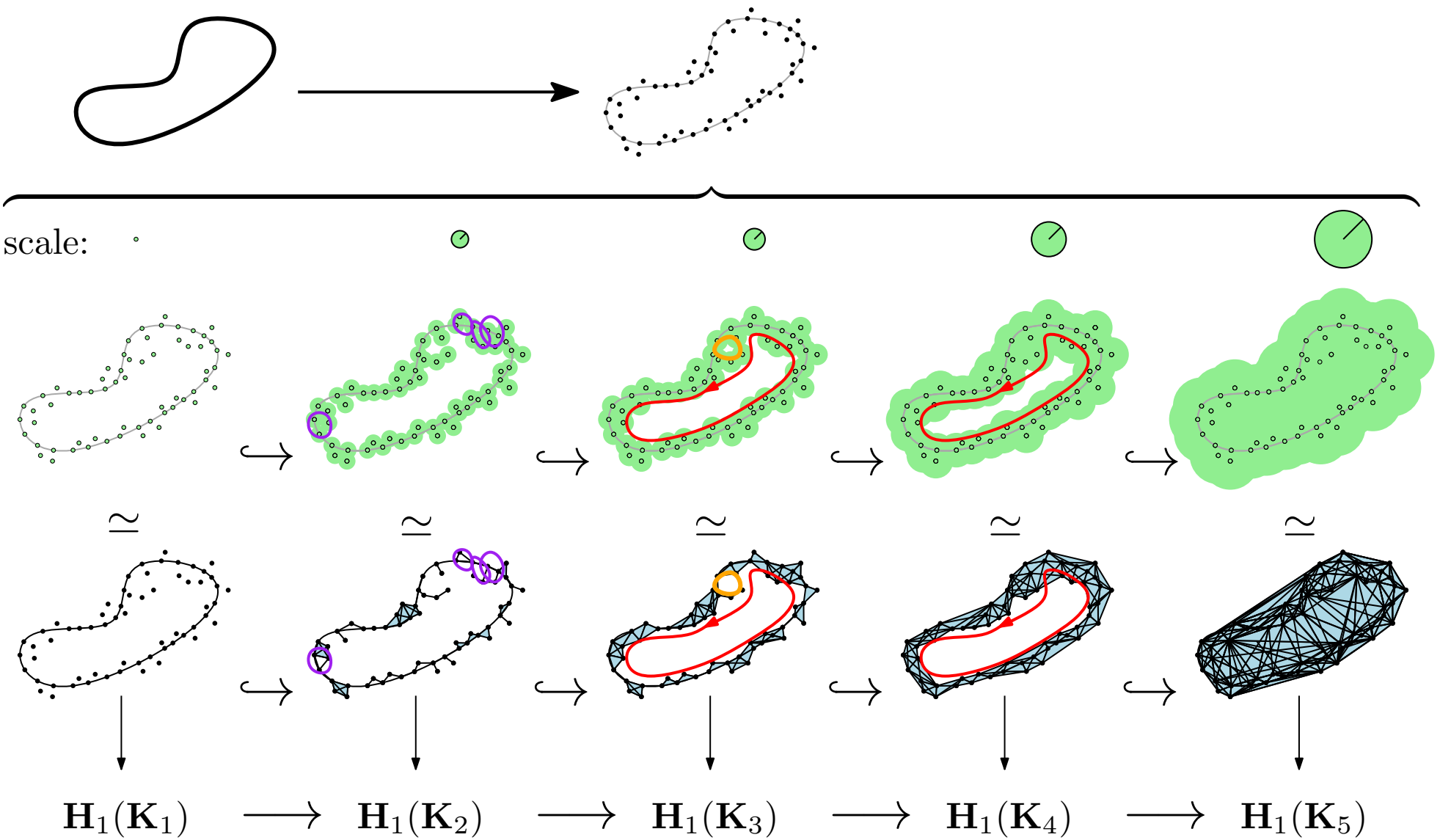
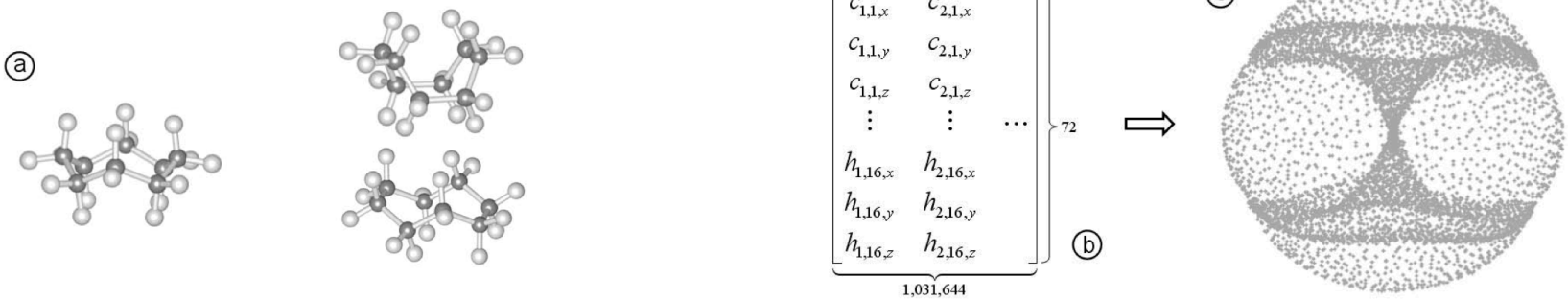


Topological Inference Problem



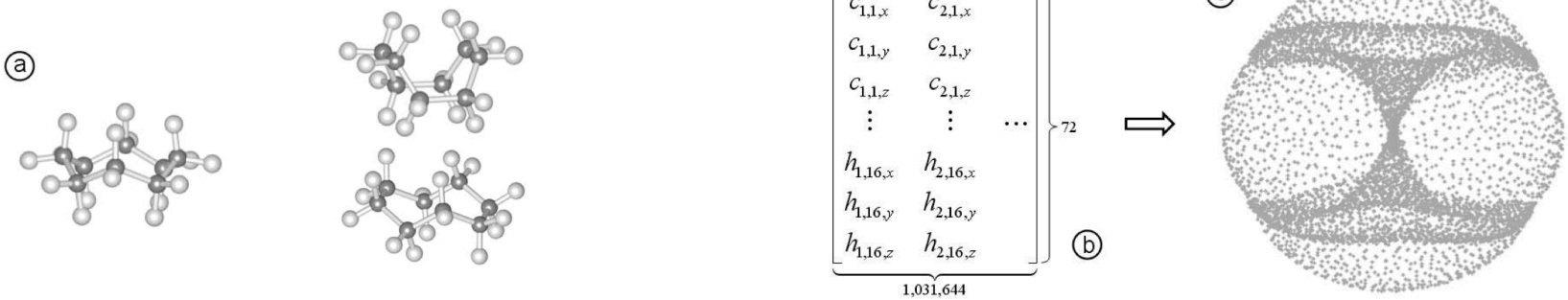
Some Applications

[Martin et al. ’10]

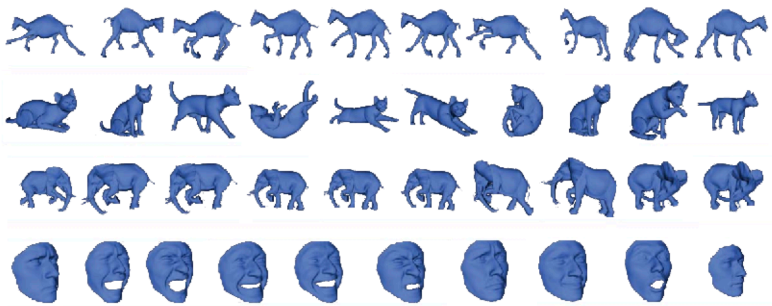


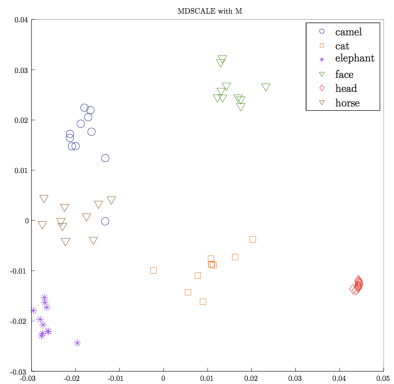
Some Applications

[Martin et al. ’10]



[Chazal et al. ’09]





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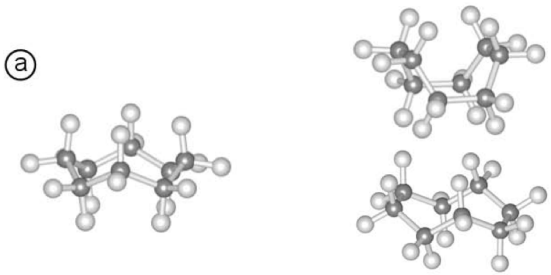

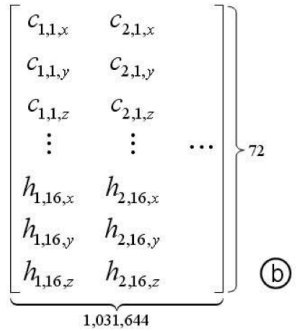

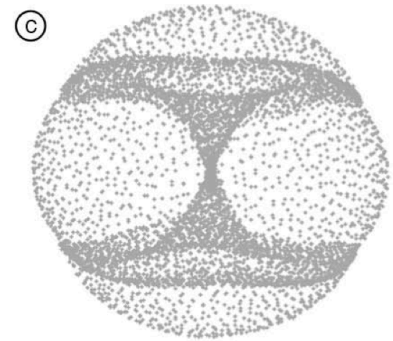

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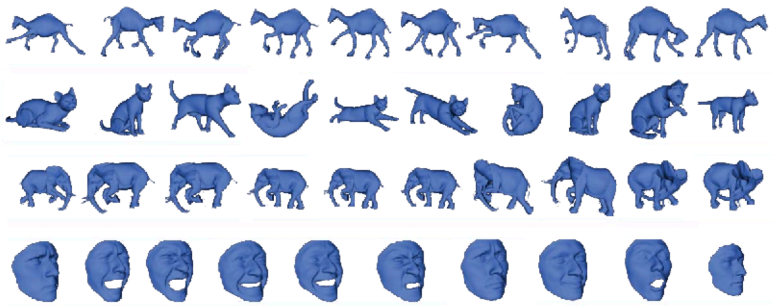
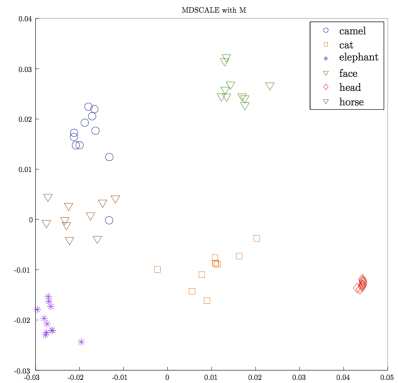
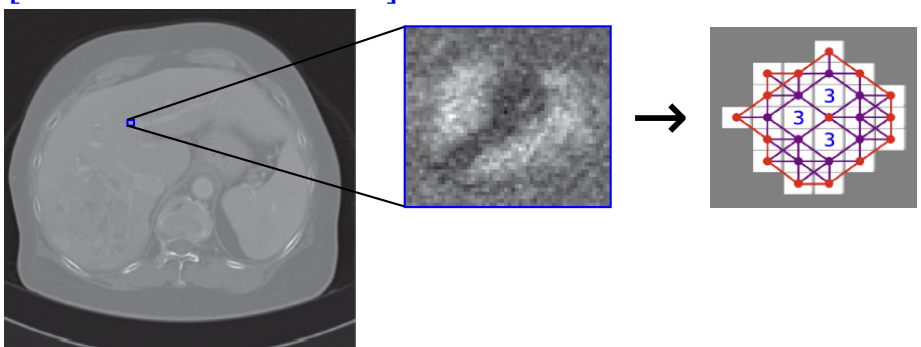


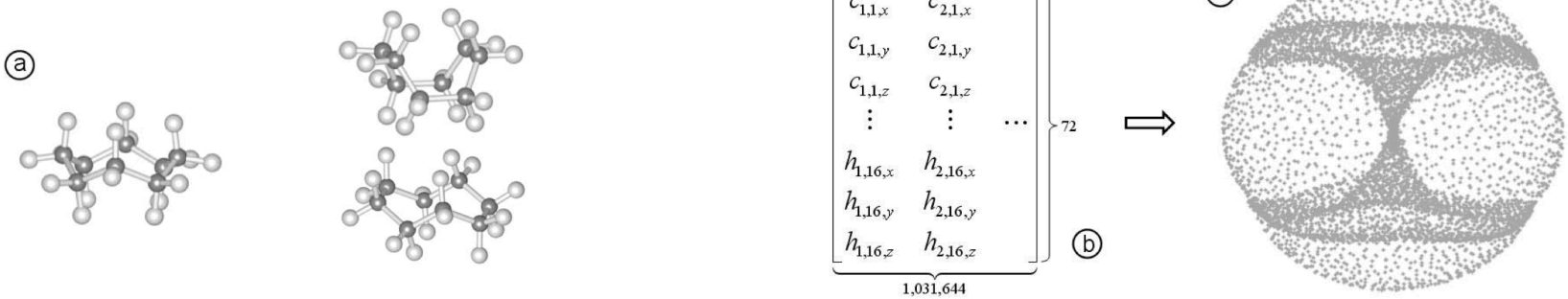


[Adcock et al. ’14]



Some Applications

[Martin et al. '10]



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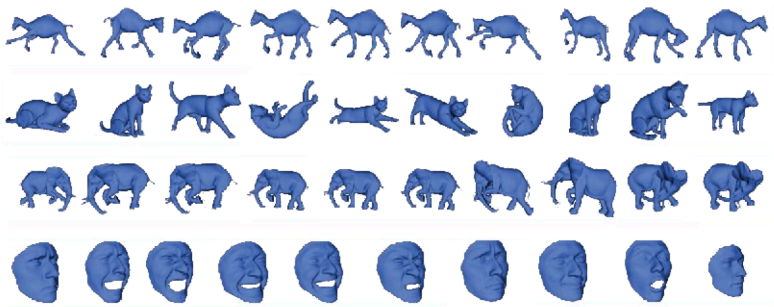

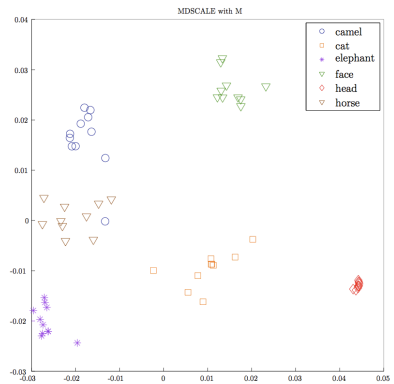

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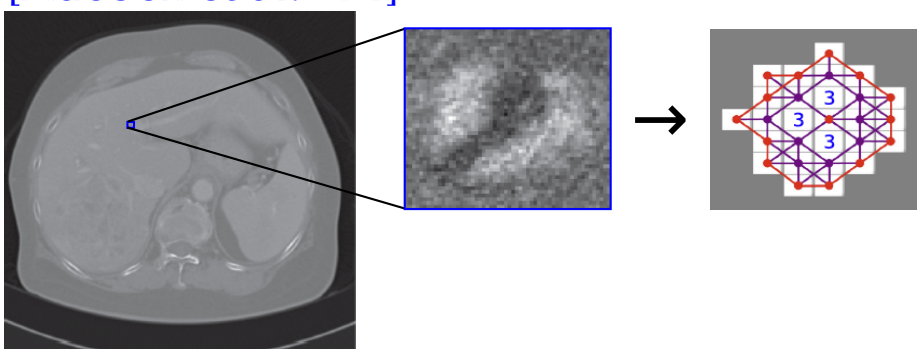
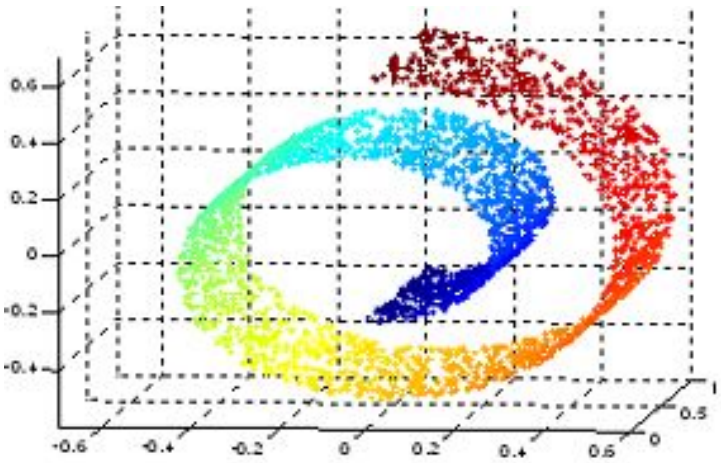


Usually: Thousands of points in very high dimensional spaces, with complicated metrics.

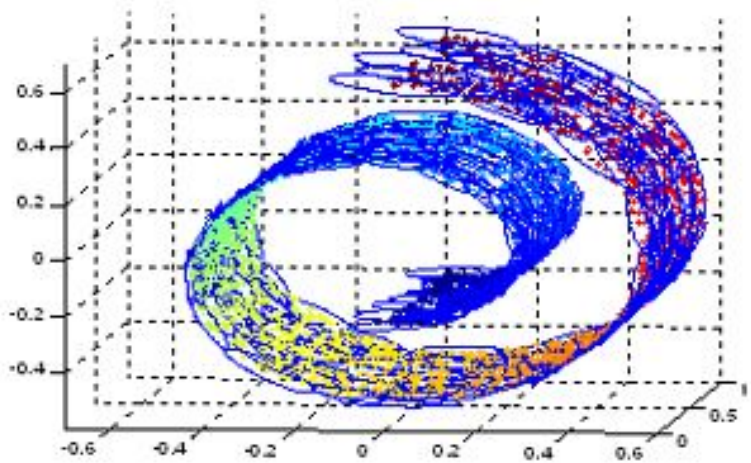
Why Topology?

Intuitively: Topology = Set of points + local proximity

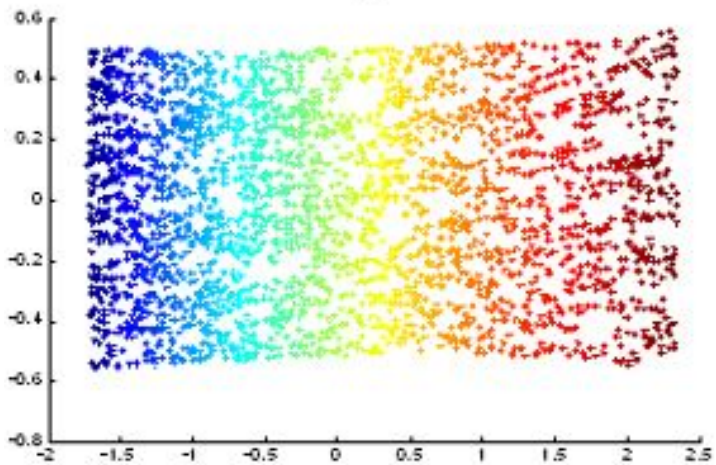
$\longrightarrow$ Stability w.r.t. deformation: change of coordinates, noise, etc.



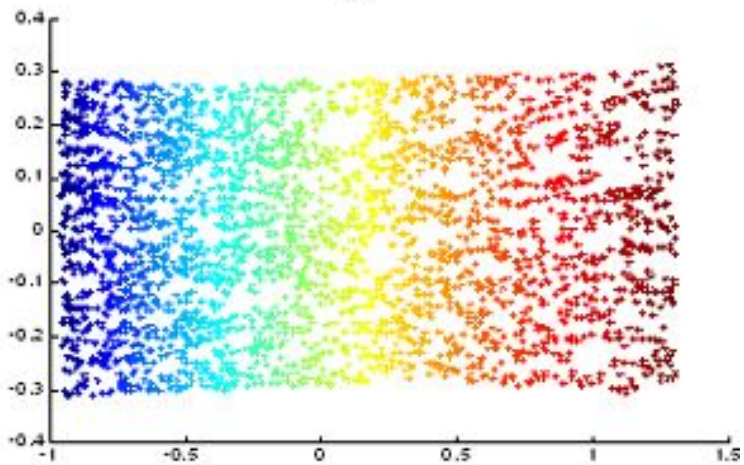
(a)



(b)



(c)

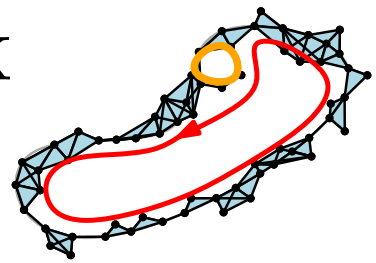


(d) [Tenenbaum et al '00]

Overview

vector spaces, of dimension related to:

simplicial complex $\mathbf{K}$



$\mathbf{H}_0(\mathbf{K})$ num. connected comp.

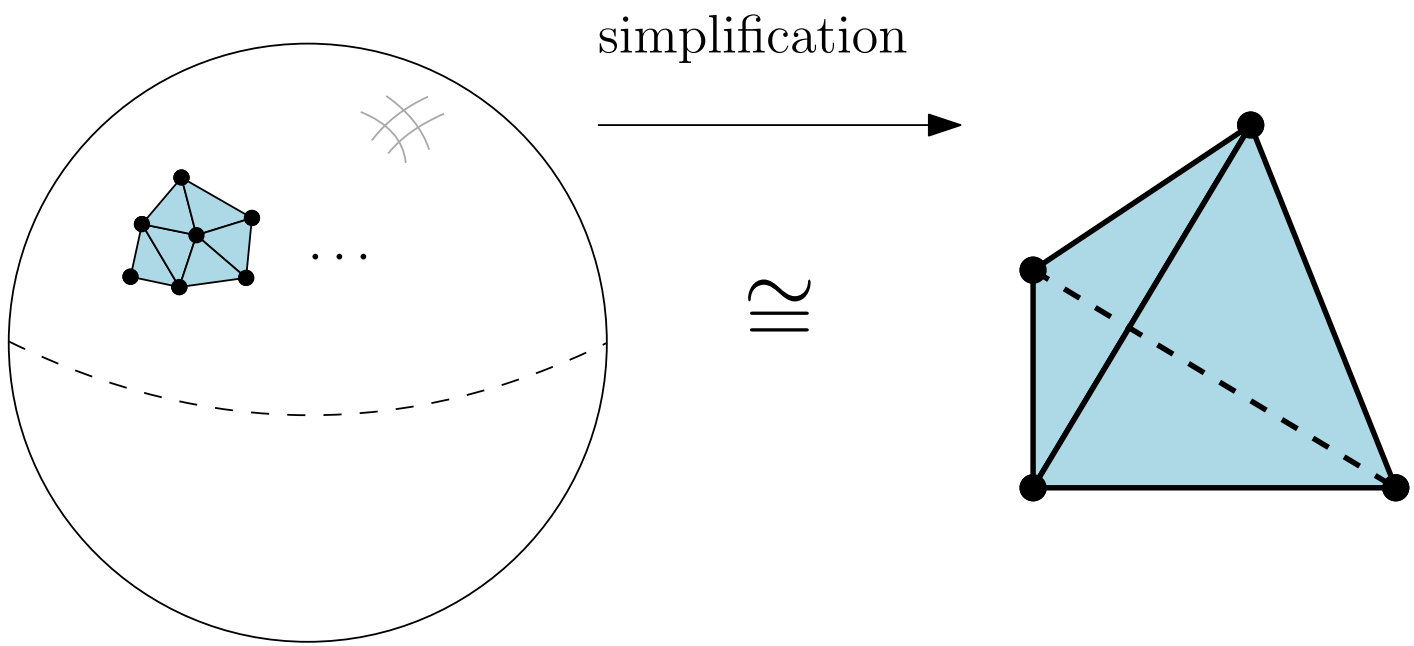
$\mathbf{H}_1(\mathbf{K})$ num. of "holes"

$\mathbf{H}_2(\mathbf{K})$ num. of "voids"

$\ldots$

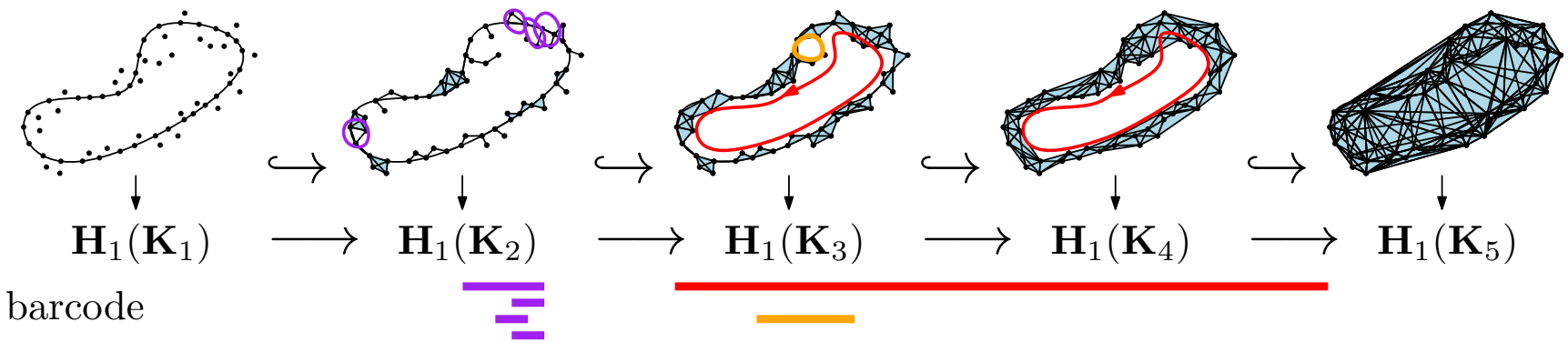
[Mon 16/01] Homology theory: formalize algebraically the "notion of shape" of a simplicial complex.

Overview



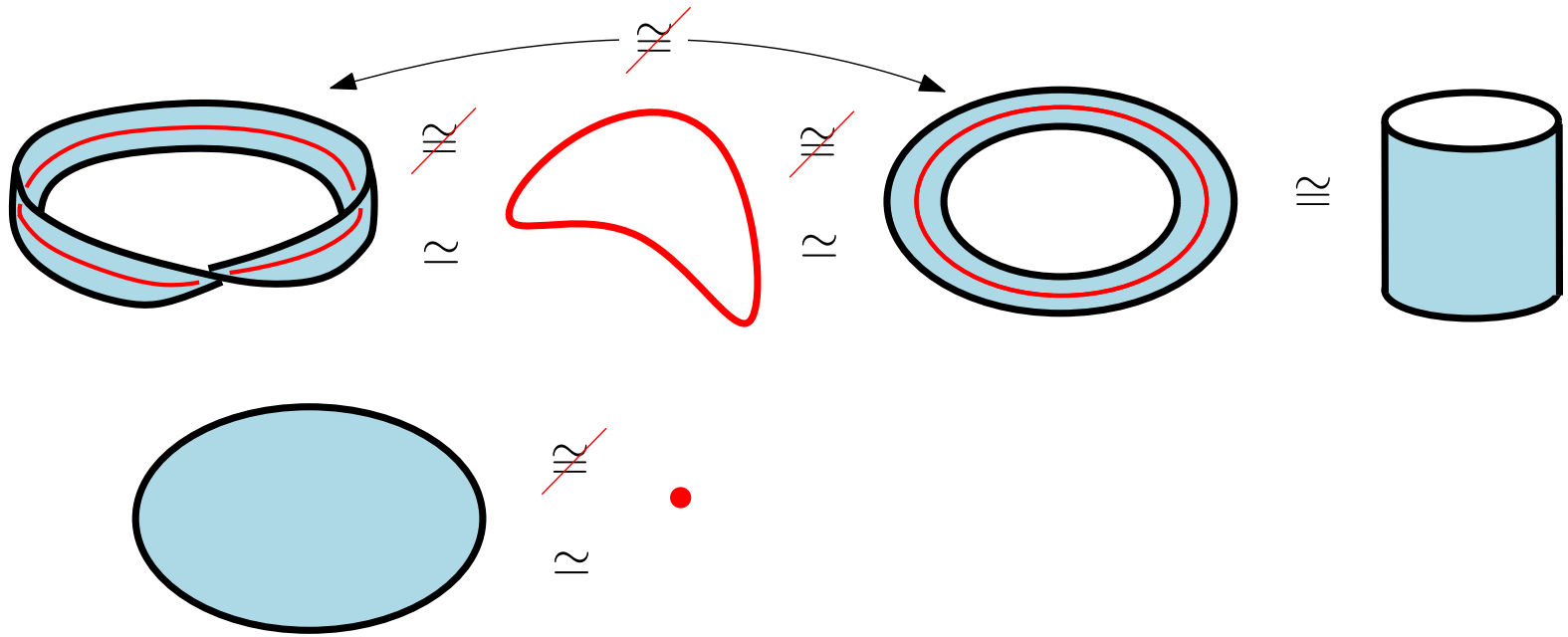
[Wed 18/01] Discrete Morse theory: define combinatorial transformations to simplify a shape while preserving its topology $\longrightarrow$ accelerate computation.

Overview



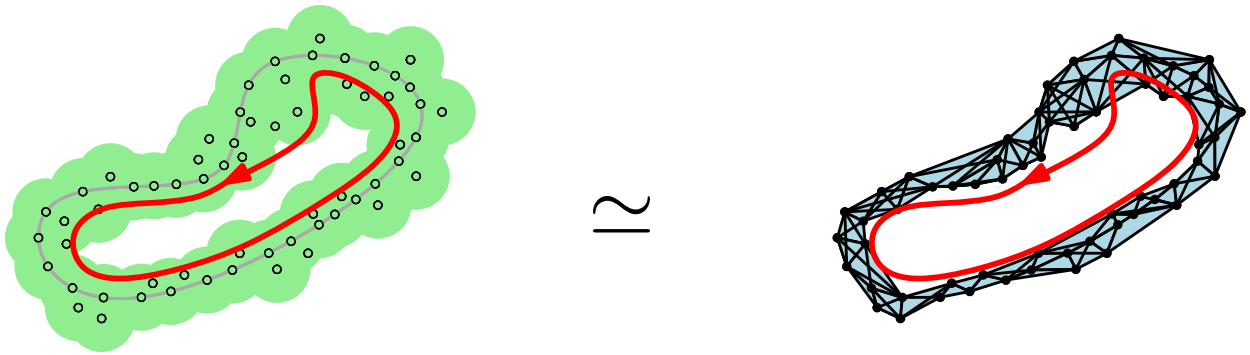
[Fri 20/01] Persistent homology: study the idea of persistence of the topology $\longrightarrow$ multi-scale approximation of the topology of a point cloud.

Overview



[Mon 23/01] General topology: general concepts of topology in order to connect the combinatorial world of simplicial complexes to continuous shapes.

Overview



[Wed 25/01] Stability and topological inference: ensure that the reconstruction *point cloud* to *simplicial complex* approximates correctly the topology of the underlying shape.