

Stack Machine

Advice

- Write the **status of the stack** as a **comment**, after each instruction
- **Example :**

```
push A      ; | A
push B      ; | A B
bcl: cmp     ; stack unchanged and
             ; test of A-B
beqz done   ; stack unchanged and
             ; branch if A == B
bltz else   ; stack unchanged and
             ; branch if A < B
swap        ; | B A
```

gcd

Algorithm: Euclid (by subtraction)

// in: A and B: positive integers

// out: gcd(A,B): positive integer

while $A \neq B$ do

if $A > B$ then

$A \leftarrow A - B$

else

$B \leftarrow B - A$

endif

done

gcd $\leftarrow A$

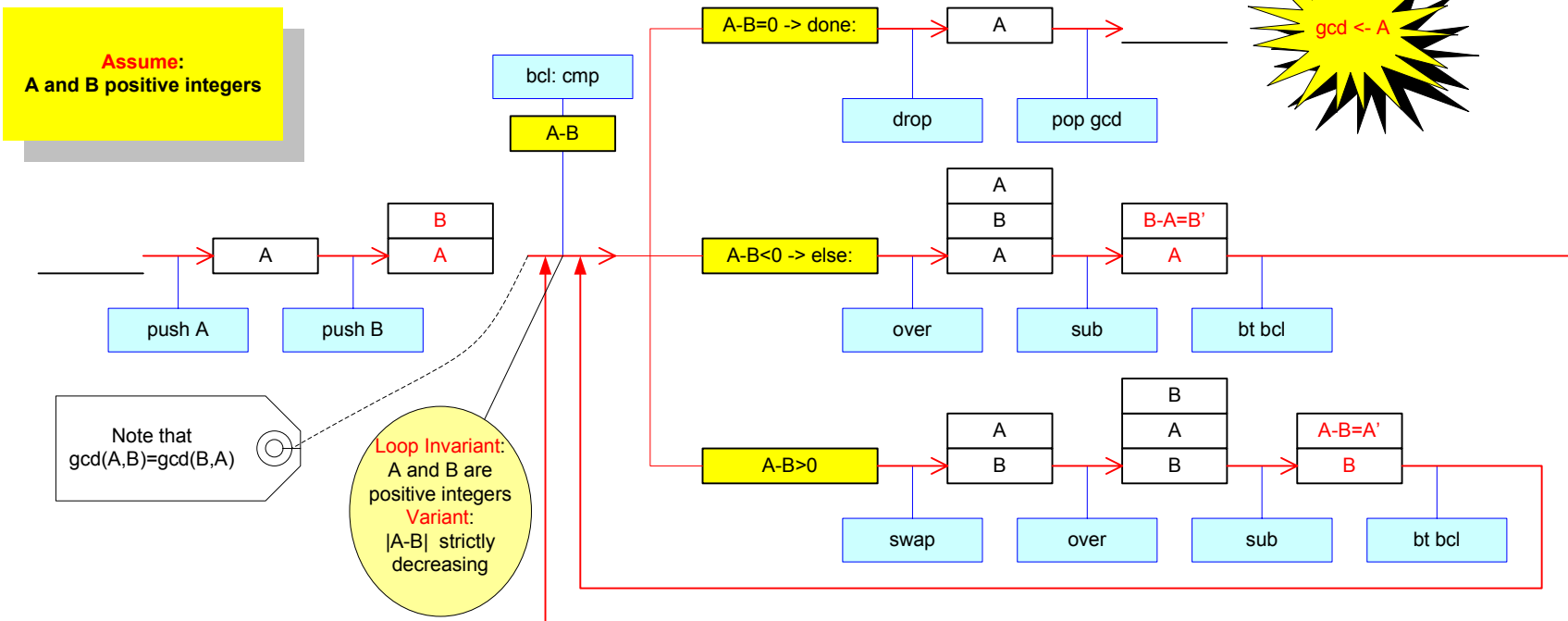
Loop Invariant:
A and B positive
integers

Variant:
 $|A-B|$ strictly
decreasing

gcd: stack implementation

IN: A, B

OUT: gcd



Sum of the first N odd numbers

Definition:

$$S = \sum_{k=0}^{N-1} (2*k + 1)$$

Algorithm

```
// in: N: positive integer
// out: S: positive integer
odd:=1:integer;    S = 0; // init
do N times
    S = S + odd;
    odd = odd + 2
done
```

Simple addition
Thanks to the
use of a local
variable

Advice

- The less memory reference, the better
- Use `dup`, `over` to save temporary values in the stack
- Use `swap`, `roll` to re-order values in the stack
- Use `drop`, `pop` to discard useless values
- Find out orderings that are loop invariants

Stack implementation

```
; initializations
```

```
pushi    1
```

```
pushi    0
```

```
push     N
```

```
; main loop
```

```
bcl: pushi    1
```

```
sub
```

```
bltz     done
```

```
roll
```

```
over
```

```
add
```

```
roll
```

```
; | odd
```

```
; | odd S
```

```
; | odd S N
```

This structure
should be a
loop invariant

```
; | odd S N 1
```

```
; | odd S  $N-1=N'$ 
```

```
; branch if  $N' < 0$ 
```

```
; |  $N'$  odd S
```

```
; |  $N'$  odd S odd
```

```
; |  $N'$  odd  $S+odd=S'$ 
```

```
; |  $S'$   $N'$  odd
```

New value
of N

New value of
S

Stack implementation

pushi 2	; S' N' odd 2	New value of odd
add	; S' N' odd+2=odd'	
roll	; odd' S' N'	
bt bcl	;	
; after the loop		Structure Loop invariant
done: drop	; odd S	
pop S	; odd	
	; and S memory set	
drop	;	