Post-Doc proposition
Entanglement of Quantum states via moments and tensor analysis

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1 Subject

Entanglement is a physical phenomenon of particle systems that reveals a fundamental difference between quantum and classical mechanics. It can be used as an important additional resource in quantum information processing, for various tasks such as quantum computation, quantum cryptography, quantum communication, etc. A fundamental question in quantum information is to detect entanglement or, equivalently, the separability of quantum states.

Given a bipartite quantum state ρ (that is, a positive semidefinite matrix acting on the tensor product $C^n \otimes C^n$), the task is to decide whether ρ can be written as a conic combination of product states of the form $xx^* \otimes yy^*$ for some $x, y \in C^n$, a well-known NP-hard problem.

Different hierarchies of tractable semidefinite relaxations have been studied to address this question. Most notably, Doherty, Parrilo, Spedaglieri [DPS04] developed complete hierarchies based on state extensions. Approximate representations by separable states of quantum states with Bosonic symmetry (in fact, of their partial traces) from [CKMR07] have been recently used to develop other hierarchies for the closely related optimization problem on the sphere, with a focus on their convergence rates [FF20], [LJ23], [BL24]. The convergence rate analysis of such hierarchies is indeed currently witnessing many developments (e.g. [BM22], [LS22], [Slo22]).

The separability problem can also be seen as a tensor decomposition problem, for which extension techniques have also been investigated [BCMT10]. Equivalently, it can be stated as a Generalized Moment Problem, which benefits from extensive studies (see e.g., [GM25])

Determining the separable rank, that is, the smallest number of rank one states entering the decomposition of a separable state is closely connected to the notion of rank of a tensor and to the rank of Hankel operators, via flat extension properties [LM09]. Recent works (see e.g.[GLS22]) have been investigating this separable rank.

The aim of the project is to investigate new and efficient characterizations of separable states by combining different expertises and advances in the domains of tensor decomposition, moment sequence optimization, and measure representation. We target the construction of new hierarchies of moment relaxations, inspired by tensor decomposition problems [GM25], which are more efficient and well adapted to the characterization of separable quantum states with low separable rank. Establishing stronger performance guarantees for more semialgebraic sets going beyond the sphere is an open problem that we propose to tackle in this postdoc project. Our objective is to push and exploit the link to quantum de Finetti type results. Another objective is to expand the applicability of spectral bounds to polynomial optimization on more sets, beyond the sphere. Developing further complexity analysis for these problems is also another interesting challenge. Direct applications of these approaches to quantum information are foreseen, opening up new avenues at the crossroads of tensor analysis, moment optimisation and quantum information.

References

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We will be interacting with Marc-Olivier Renou (Junior Professor Chair at Inria Saclay in the team PhiQus) for his expertise in Quantum Information and theoretical physics.