Carbon sequestration policies in leaky reservoirs: Sufficient conditions for optimality and Economic interpretations

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Outline

Progress

Motivation: Carbon capture

It is well known that there still exist huge reserves of fossil carbon energy sources, accessible at low cost, such as coal. Without the greenhouse problem, this low cost would allow the current development of our energy-based society for a while (Fouquet, 2008).

However, the use of these resources generates CO_2 and other greenhouse-effect gases in the atmosphere.
The renewable energy sources with low pollution (wind, sun, biomass, ...) are still much more costly.
The capture of pollutants is a possible alternative, insofar it can be done at a reasonable cost.

Capture technologies

There exist several types of carbon capture:

biological	carbon pits, forests, oceans $\Longrightarrow$ not mature, difficult to model: out of the scope of this paper
mechanical	storage in underground sites, depleted mines/oil/gas reservoirs

Some papers consider the problem of carbon emission by capturing and storing the CO_2 away. (Moreaux *et al.*), "Optimal sequestration policy with ceiling on the stock of carbon in the atmosphere".

- sequestration must be implemented once pollution ceiling is reached
- price path for the energy are continuous and monotonous

Leaks in storage

Carbon stored in reservoirs may escape!

- either accidentally and brutally (industrial accident, combustion, lake Nyos-type degassing...)
  $\Longrightarrow$ risk management
- either slowly but constantly

In the latter case, is it relevant to capture CO_2 which is going to be released eventually in the atmosphere?

Leaks in storage. Empirical results

A first investigation has been given by Ha-Duong and Keith (2003)

- using an integral assessment numerical model (DIAM) to explore the role of discount rate and leakage when the discount rate is 4% they find that a leakage rate of 0.1% is nearly the same as prefect storage while a leakage rate of 0.5% renders storage unattractive.

Van der Zwaan et Gerlagh (2008, 2009).

- using carbon sequestration and storage policies with leaky reservoirs does not permit to escape a big switch to renewable non polluting resource if a pollution ceiling of 450 ppmv has to be enforced.

Main questions

Is it relevant to capture CO_2 which is going to be released eventually in the atmosphere?
To what extent does the presence of leaks change optimal paths?

- simultaneity/sequentiality of phases w.r.t. capture, use of clean energy
- partial capture situations
- monotonicity of consumption, pollution paths

The present presentation is devoted to the theoretical analysis of this question.

Main results

There are changes indeed!

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Technical:

- Optimal control model with 3 state variables, 3 controls, 2 state constraints, 3 controls constraint: 32 distinct configurations (9 really useful)
- Endogenous viability constraint
- Discontinuities in adjoint variables

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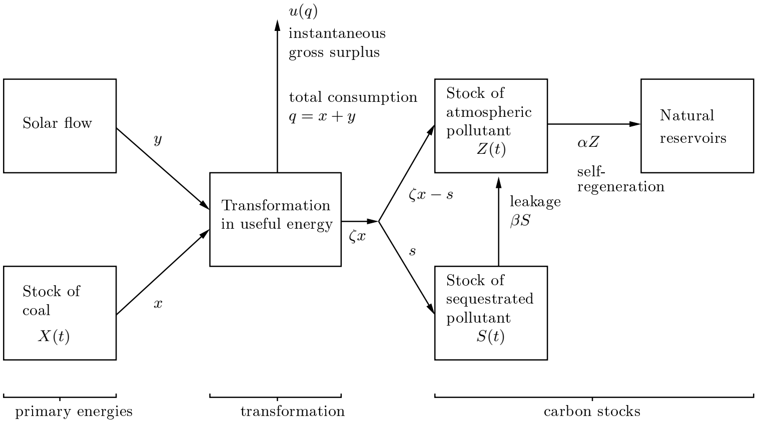
Economics:

- Optimal paths staying in the frontier can go inside the admissible domain to come back later to the frontier; several ceiling phases, "M"-shaped curves
- Optimal energy price can be discontinuous and non monotonous
- Simultaneous consumption of clean/dirty energies
- Capture when the ceiling is not reached

Progress

The Physical Model

Flows of energy and pollution in our model



The dynamics

Energy consumption, carbon emission, assimilation and sequestration :

- x units of polluting energy generates ζx units of CO_2
- quantity s of emission can be sequestered in a stock S,
- sequestered stock leaks at rate β
- rest of emission $\zeta x - s$ goes in the atmospheric stock Z,
- atmospheric carbon is assimilated at rate α

Basic controlled dynamics

$$\left\{ \begin{array}{rcl} \dot{X} & = & -x \\ \dot{S} & = & -\beta S + s \\ \dot{Z} & = & -\alpha Z + \beta S + \zeta x - s \end{array} \right. \tag{1}$$

Economic parameters

Optimization involves the following parameters and functions:

- ρ discount factor
- x nonrenewable resource consumption rate (dirty energy)
- y renewable resource consumption rate (clean energy)
- $u(q)$ gross instantaneous surplus produced by the consumption rate $q = x + y$ of useful energy
- c_x constant unitary extraction cost of polluting energy
- c_y constant unitary extraction cost of clean energy
- c_s constant unitary capture cost
- $\overline{Z}$ maximal allowed atmospheric stock of carbon

The social planner problem

The social planner faces the optimization problem:

$$\max_{s,x,y} \int_0^\infty \left[u(x(t)+y(t)) - c_s s(t) - c_x x(t) - c_y y(t)\right] e^{-\rho t} \mathrm{d}t$$

given the controlled dynamics (1) and the constraints on state variables and controls: for all t,

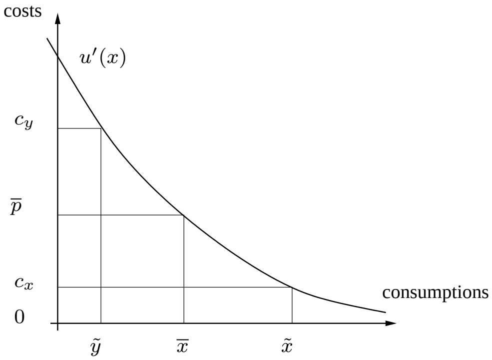
$$\begin{array}{rcl} X(t) & \geq & 0 \\ y(t) & \geq & 0 \\ Z(t) & \leq & \overline{Z} \\ \zeta x(t) \geq s(t) & \geq & 0\,. \end{array}$$

Typical assumptions

Maximal consumption of coal when this threshold is attained:

$$\overline{x} = \frac{\alpha \overline{Z}}{\zeta}$$

The typical assumptions on the shape of functions and relative values of costs are summarized in the diagram:



Progress

State dynamics absent any control

When there is no consumption of the polluting resource, the state evolves as:

$$\left\{ \begin{array}{rcl} \dot{Z} & = & -\alpha Z + \beta S \\ \dot{S} & = & -\beta S \ . \end{array} \right.$$

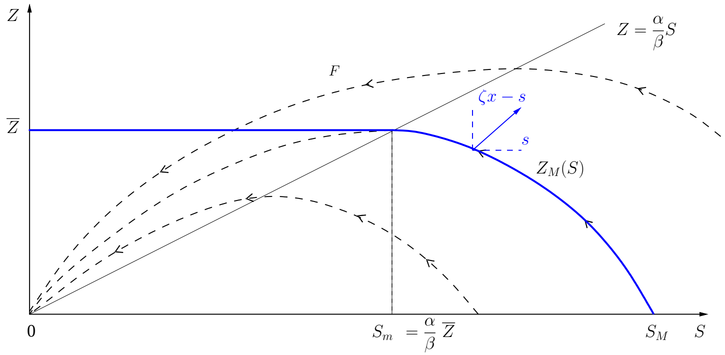
Integration yields:

$$\begin{array}{rcl} Z(t) & = & Z^0 e^{-\alpha(t-t^0)} - S^0 \dfrac{\beta}{\alpha - \beta} \left(e^{-\alpha(t-t^0)} - e^{-\beta(t-t^0)} \right) \\ S(t) & = & S^0 e^{-\beta(t-t^0)} \ . \end{array}$$

The trajectories are curves in the domain (S, Z):

$$Z = Z(S) \ = \ Z^0 \left(\frac{S}{S^0} \right)^{\alpha/\beta} - \frac{\beta}{\alpha - \beta} \left(S^0 \left(\frac{S}{S^0} \right)^{\alpha/\beta} - S \right) \ .$$

Viability Domain: Not all trajectories respect the maximal value $\overline{Z}$



Control vector $(s, \zeta x - s)$ points outwards

- $S_m := \alpha \overline{Z} / \beta$: maximal possible value of the sequestrated stock, when the atmosphere is saturated
- S_M: maximal feasible sequestrated stock

Progress

Lagrange multipliers

For the original problem:

$$\begin{array}{rrcl}
(\nu_X) & X(t) & \geq & 0 \\
(\nu_Z) & \overline{Z} & \geq & Z(t) \\
(\gamma_y) & y(t) & \geq & 0 \\
(\gamma_{sx}) & \zeta x(t) & \geq & s(t) \\
(\gamma_s) & s(t) & \geq & 0\,.
\end{array}$$

Lagrange multipliers

For the problem with explicit viability constraint:

$$
\begin{array}{rrcl}
(\nu_X) & X(t) & \geq & 0 \\
(\nu_Z) & \widetilde{Z}(S(t)) & \geq & Z(t) \\
(\gamma_y) & y(t) & \geq & 0 \\
(\gamma_{sx}) & \zeta x(t) & \geq & s(t) \\
(\gamma_s) & s(t) & \geq & 0
\end{array}
$$

where

$$
\widetilde{Z}(S) = \begin{cases} \overline{Z}, & 0 \leq S \leq S_m \\ Z_M(S), & S_m \leq S \leq S_M. \end{cases}
$$

First-Order Conditions

The first order conditions are then the following. First, optimality of the control yields:

$$\begin{aligned}0 &= -c_s - \lambda_Z + \lambda_S + \gamma_s - \gamma_{sx} \\ 0 &= u'(x+y) - c_x - \lambda_X + \zeta\lambda_Z + \zeta\gamma_{sx} \\ 0 &= u'(x+y) - c_y + \gamma_y \ .\end{aligned}$$

Dynamics of the costate variables are

$$\begin{aligned}\dot{\lambda}_X &= \rho\lambda_X - \nu_X \\ \dot{\lambda}_Z &= (\rho+\alpha)\lambda_Z - \nu_Z \\ \dot{\lambda}_S &= (\rho+\beta)\lambda_S - \beta\lambda_Z \ .\end{aligned}$$

Transversality conditions:

$$\lim_{t\to\infty}\{e^{-\rho t}\lambda_X X, e^{-\rho t}\lambda_Z Z, e^{-\rho t}\lambda_S S\} = 0 \ .$$

Solution Strategy

We adopt the following strategy:

- Depending on what constraints on states and control are bound, this defines "phases" characterized by specific consumption/capture functions command x, y, s and specific dynamics for state variables S, Z, X, and co-state variables λ_X, λ_S, λ_Z.
- Optimal trajectories are obtained by chaining such phases; depending on the parameters, phase configurations may be feasible or not.

Many configurations turn out to be feasible...

$\Longrightarrow$ classification complete when $X = +\infty$

$\Longrightarrow$ some characterizations for $X < +\infty$

(not in this presentation)

Theoretical tools

Mangasarian's suff. cond.

Theorem (Seierstad and Sydsæter (1977), Theorems 6 and 10)

Suppose $(x^*(t), u^*(t))$ is an admissible state/control pair. Suppose further that there exist functions $\gamma(t) = (\gamma_1(t), \ldots)$ and $\lambda(t) = (\lambda_1(t), \ldots)$, where $\lambda(t)$ is continuous and $\dot{\lambda}(t)$ and $\gamma(t)$ are piecewise continuous, such that the FOC are satisfied. Suppose H is concave in x, u and differentiable at (x^*, u^*) for all t. Then $(x^*(t), u^*(t))$ is catching-up optimal for problem.

$$\max_{u(\cdot)} \int_0^\infty f_0(x(t), u(t), t)dt$$

under constraints $\dot{x} = f(x, u, t)$ and $g_j(x, u, t) \geq 0, j = 1, \ldots, s$, provided that the g_j are quasi-concave in x, u and differentiable at x^*, u^*.

Theoretical tools (ctd.)

But sometimes, continuity of $\lambda(\cdot)$ cannot be obtained! It is allowed that $\lambda(t)$ is piecewise continuous, and $\exists \beta_k \geq 0$ s.t.:

$$\lambda_i(t_1^+) - \lambda_i(t_1^-) \geq \sum_k \beta_k \frac{\partial g_k}{\partial x_i}(x^*(t_1^-), u^*(t_1^*), t_1^-)$$

Theorem (Seierstad and Sydsæter (1999), Theorem 11)

Suppose $(x^*(t), u^*(t))$ is an admissible state/control pair, that there exist vector functions $\gamma(t)$ and $\lambda(t)$, where $\lambda(t)$ is piecewise continuous as above and $\dot{\lambda}(t)$ and $\gamma(t)$ are piecewise continuous, such that the FOC are satisfied. Suppose H is concave in x, u. Then $(x^*(t), u^*(t))$ is catching-up optimal for the problem under constraints $g_j(x, u, t) \geq 0$, $j = 1, \ldots, s$, provided that the g_j are quasi-concave in x, u and C^2, and f and f_0 are C^1.

Bad luck: the function $\widetilde{Z}$ is not C^2, and f_0 not always C^1.

Theoretical tools (ctd)

Not so bad luck: for a given value of parameters,

- either costate variables are continuous on every optimal trajectory
- or no optimal trajectory touches $Z = Z_M(S)$, except one.

$\Longrightarrow$ one of the two theorems covers the situation.

Progress

Optimal Capture

Optimal capture obeys a sort of "bang-bang" principle.

Lemma

Consider a piece of optimal trajectory located in the interior of the domain, such that $x(t) > 0$. Then for every time instant t, either $s(t) = 0$, or $s(t) = \zeta x(t)$.

Consider the function, issued from first-order conditions:

$$\gamma(t) \;:=\; -c_s - \lambda_Z(t) + \lambda_S(t) \;=\; \gamma_{sx}(t) - \gamma_s(t) \;.$$

Its sign determines the capture, when $x(t) > 0$:

- $\gamma(t) > 0 \implies \gamma_{sx} > 0,\ \gamma_s = 0$: $s = \zeta x$
- $\gamma(t) < 0 \implies \gamma_s > 0,\ \gamma_{sx} = 0$: $s = 0$
- $\gamma(t) = 0 \implies \gamma_s = 0,\ \gamma_{sx} = 0$: $s \in (0, x)$, only if $Z = \overline{Z}$

Type of energy consumption

Consumption of non-renewable resource ($x > 0$) and renewable resource ($y > 0$) is exclusive in the interior.

Lemma

Consider a piece of optimal trajectory located in the interior of the domain. Then either $x(t) > 0$ or $y(t) > 0$ but not both.

The case of abundant resources

From now on: $X = +\infty$

$\implies \lambda_X \equiv 0$

States or phases that can be terminal

Taking into account constraints and transversality conditions, only three situations may occur when $t \to \infty$. It depends on the following critical values for the unitary capture cost c_s:

$$\hat{c}_s := \frac{\rho}{\rho + \beta} \, \frac{\overline{p} - c_x}{\zeta} \; .$$

- Phase P: $s = y = 0$, $Z = \overline{Z}$, $S \to 0$; only if $c_s > \hat{c}_s$
- Phase Q: $y = 0$, $Z = \overline{Z}$, S constant; only if $c_s = \hat{c}_s$
- Phase S: $y = 0$, $x = \overline{x}$, $s = \zeta\overline{x}$, $Z = \overline{Z}$, $S = S_m$ constant; only if $c_s < \hat{c}_s$.

A trajectory perturbation argument

Reference: $Z(t) = \overline{Z}$, $S(t) = S_m$, $x(t) = \overline{x}$, $s(t) = \zeta\overline{x}$.
Modification:
1) On $[0, \Delta t]$, consumption is $x(t) = \overline{x} - \Delta x$ (constant) and capture $s(t) = \beta S(t) - \zeta\Delta x$ so that $Z(t) = \overline{Z}$ still holds.
Difference in profit between trajectories is

$$D_1 = (\overline{p} - c_x - \zeta c_s)\Delta x \Delta t + o(\Delta x)\Delta t + o(\Delta t).$$

2) On $[\Delta t, \infty)$, capture is restored to the nominal level $\zeta\overline{x}$, and consumption is such that $Z = \overline{Z}$. The difference is:

$$D_2 = \int_{\Delta t}^{\infty} e^{-\rho t}[u(\overline{x}) - u(\overline{x} + \beta(S_m - S)/\zeta) + c_x\beta(S_m - S)/\zeta]\mathrm{d}t$$

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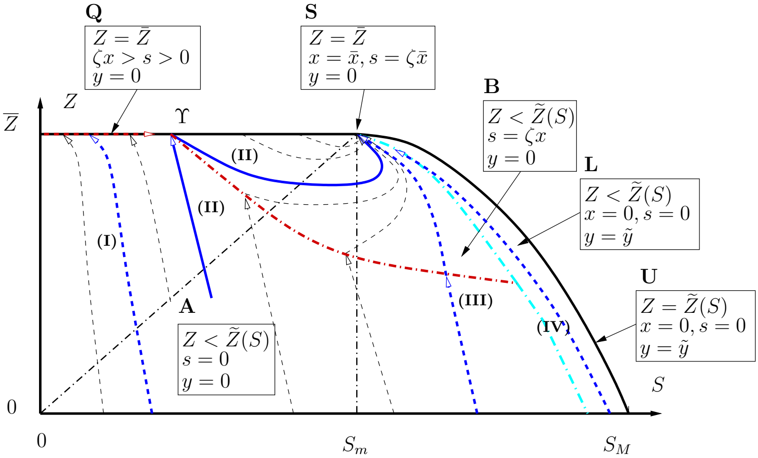
If the reference trajectory is optimal, then $D_1 + D_2$ must be positive. Asymptotically when Δt and Δx tend to 0, this is:

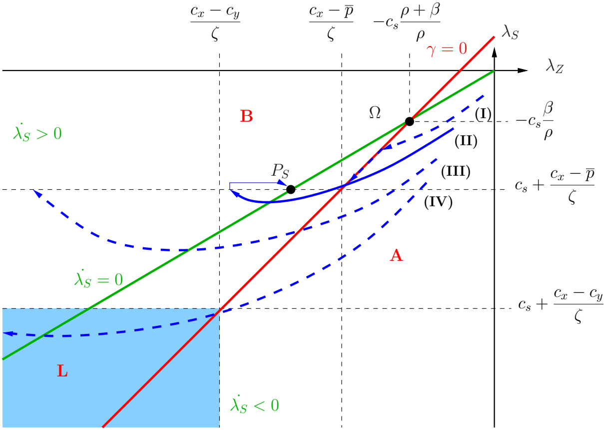
$$c_s \;\leq\; \frac{\rho}{\rho + \beta}\,\frac{\overline{p} - c_x}{\zeta} \;=\; \hat{c}_s\ .$$

Cheap CSS (small c_s)

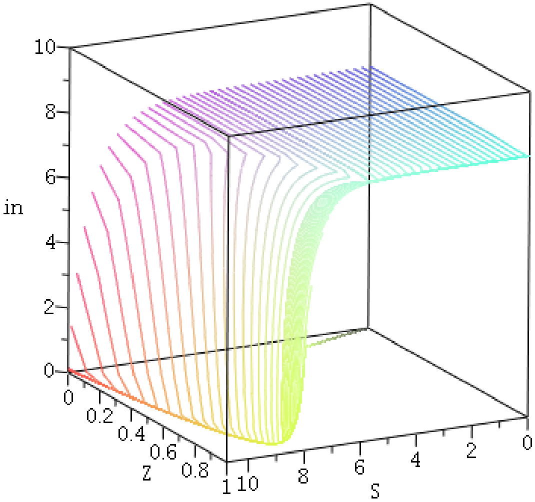
Phase S terminal. Jump of λ_Z at $(S_m, \overline{Z})$. $x = 0$ in the interior.



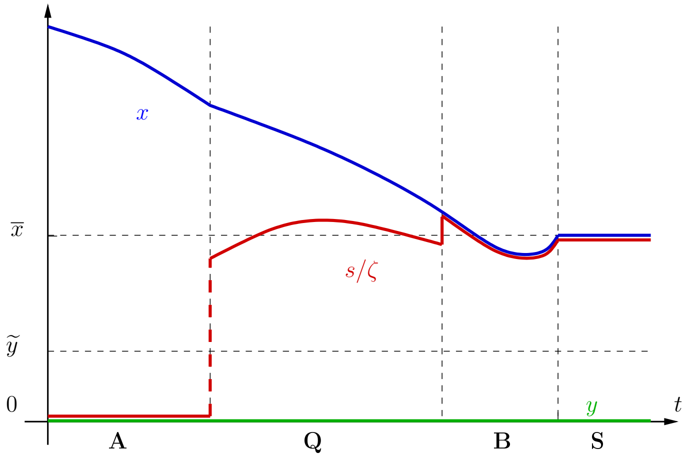
Small c_s, evolution of adjoint variables



Small c_s: value function



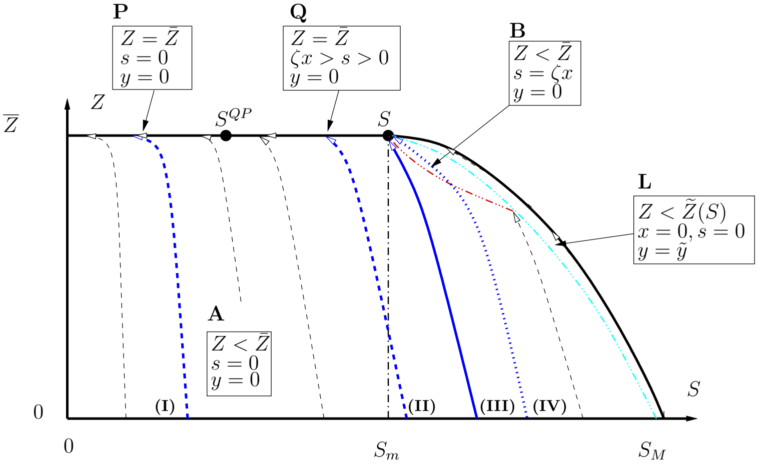
Consumption, sequestration and energy price evolution when c_s is small



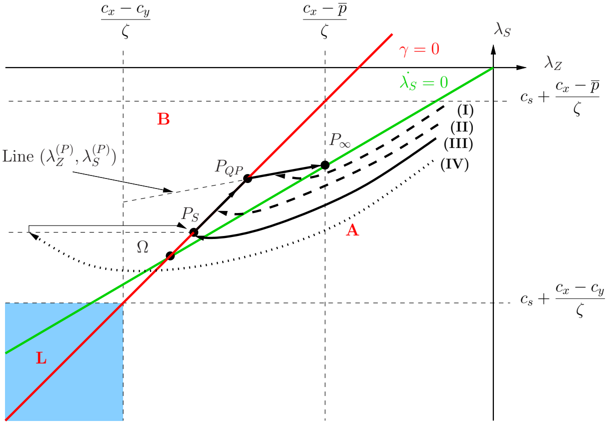
Non monotonicity

Medium-Inf c_s

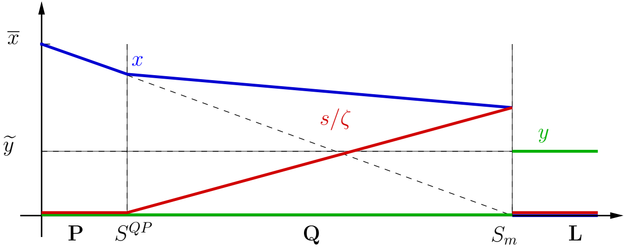
Change of direction on Phase Q. Phase P (terminal) appears. Jump of λ_Z at $(S_m, \overline{Z})$.



Medium-Inf c_s, evolution of adjoint variables



Consumption, sequestration and energy price evolution, Medium-Inf c_s

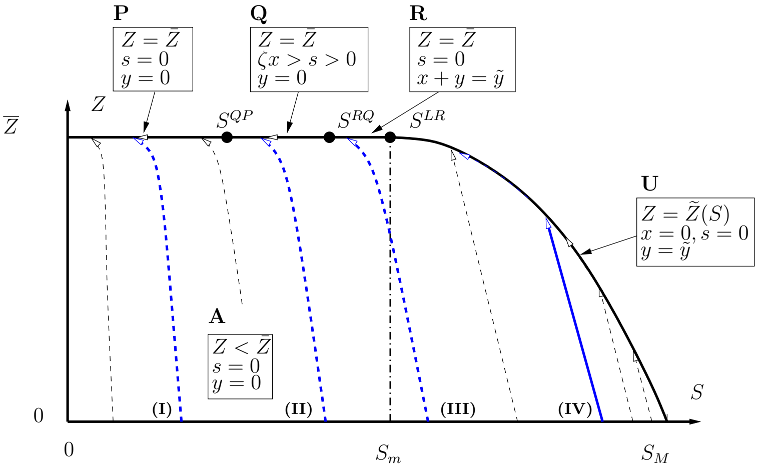


$c_{sQ} \geq c_s > \hat{c}_s$

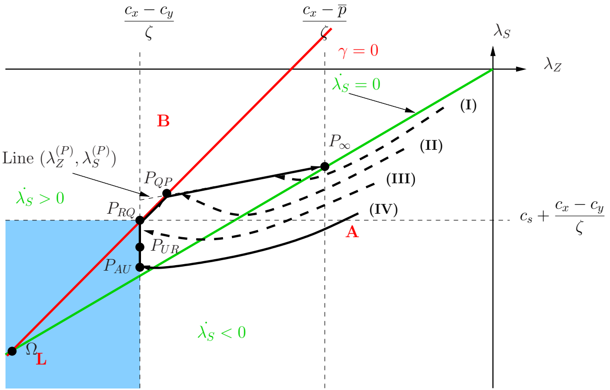
Discontinuity

Medium-Sup c_s

Need to have $y > 0$ *and* $x > 0$ (Phase R). No more jumps of λ_Z. Phase B disappears. Trajectories follow curve $\widetilde{Z}$.

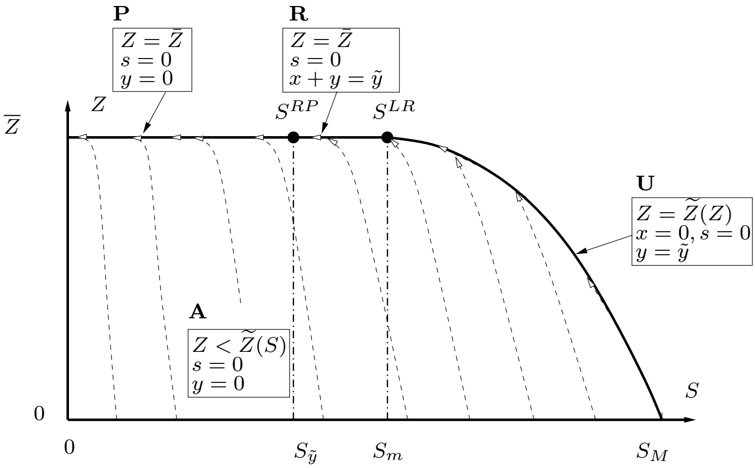


Medium-Sup c_s, evolution of adjoint variables

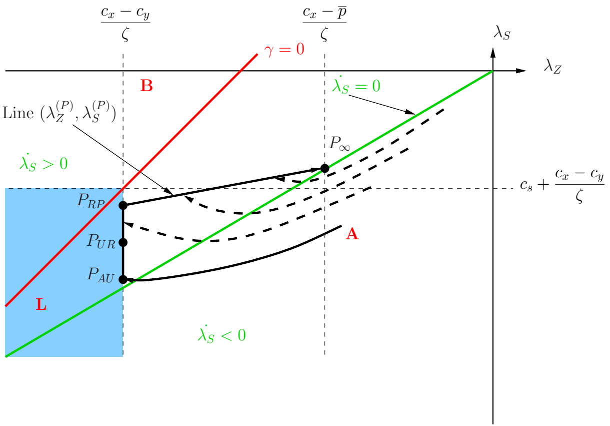


Expensive CSS (large values of c_s)

Phase Q disappears. Capture is so expensive in this case that $s(t) = 0$ at all times. The model is equivalent to one where capture is not possible at all.



Large c_s, evolution of adjoint variables



The limiting value for c_s:

$$c_{sm} \quad = \quad \frac{c_y - c_x}{\zeta} + \frac{\beta}{\zeta}\int_0^\infty e^{-(\rho+\beta)v}\left(c_x - u'(\overline{x} - \frac{\beta}{\zeta}S_{\widetilde{y}}e^{-\beta v})\right)\mathrm{d}v$$

Conclusions and work to do

- We can solve the optimal control problem and classify the different optimal solutions for all initial situation.
- Endogenous admissibility domain: not every possible configuration of atmospheric and sequestered stock is acceptable.
- Results confirm that the presence of leakage does reduce the economic incentive of sequestration.
- Explicit (or almost explicit) formulas explaining the different optimal solution depending on cost of sequestration, rate of leakage and discount factor.
- Optimal consumption path are very different with respect to the benchmark situation (without leakage), in particular energy prices can be non monotonous and discontinuous.

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Now that we have all the solutions we can try to exploit more the economic interpretations

The influence of the leakage rate β

When $\beta = 0$, $X = +\infty$, S is “free”: $\lambda_S = 0$.
Three cases for c_s. Note: $\hat{c}_s = (\overline{p} - c_x)/\zeta$.

$c_s \geq \hat{c}_s$: no capture, $x = \overline{x}$, S constant, $Z = \overline{Z}$;

$0 \leq c_s < \hat{c}_s$: $x = q^d(c_x + \zeta c_s)$, capture $s = x - \overline{x}$, $Z = \overline{Z}$;

$c_s < 0$: full capture $s = \zeta x$, $x = q^d(c_x + \zeta c_s)$, $Z < \overline{Z}$.

When $\beta > 0$, the situation is not so clear-cut:

$c_s \geq \hat{c}_s$: capture may be still optimal

$0 \leq c_s < \hat{c}_s$: no capture may be optimal at the ceiling, whereas capture may be optimal under the ceiling

$c_s < 0$: no capture may be optimal.

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