The hidden hyperbolic structure of the Internet

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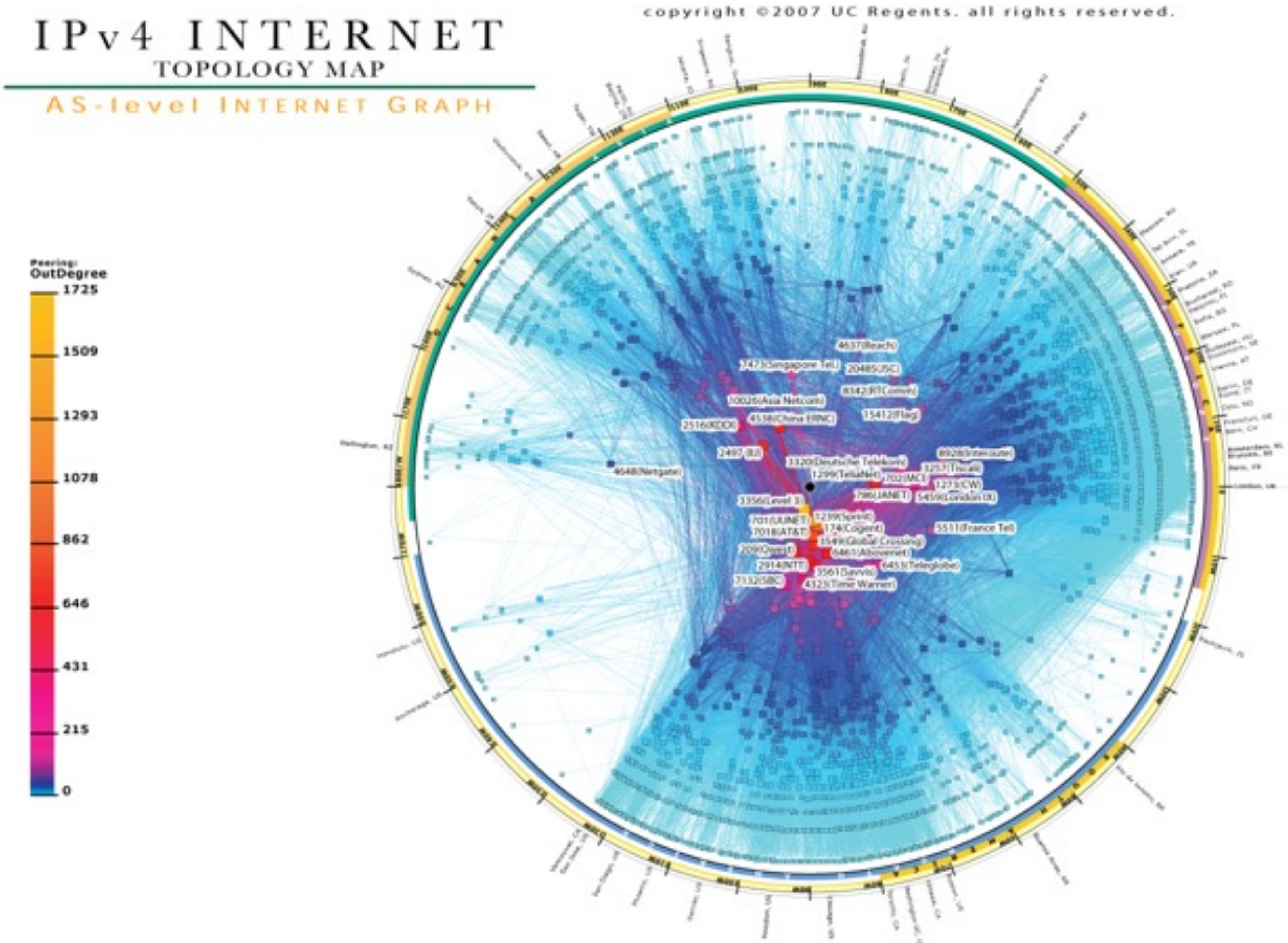
How is information routed in the Internet?



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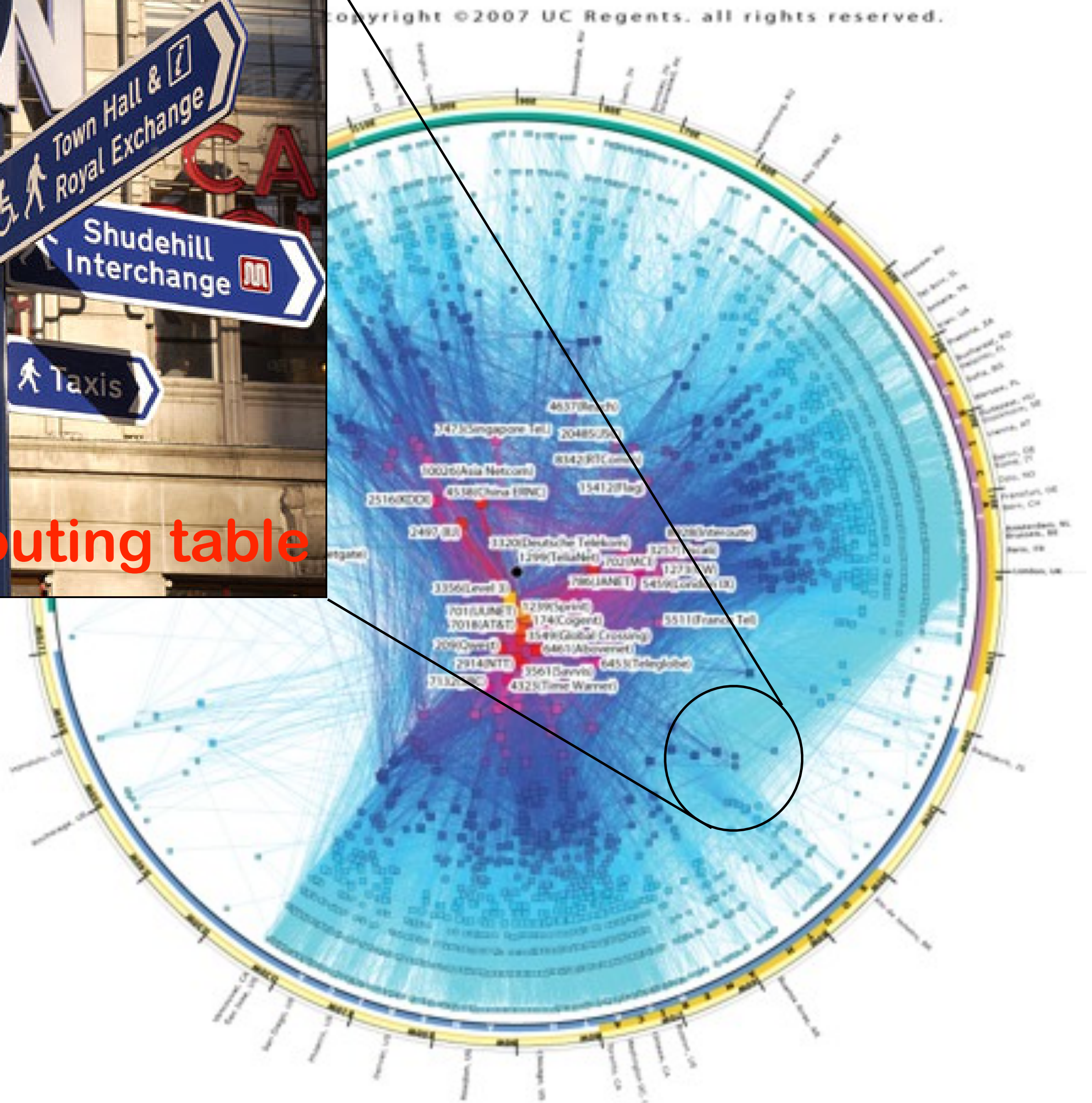

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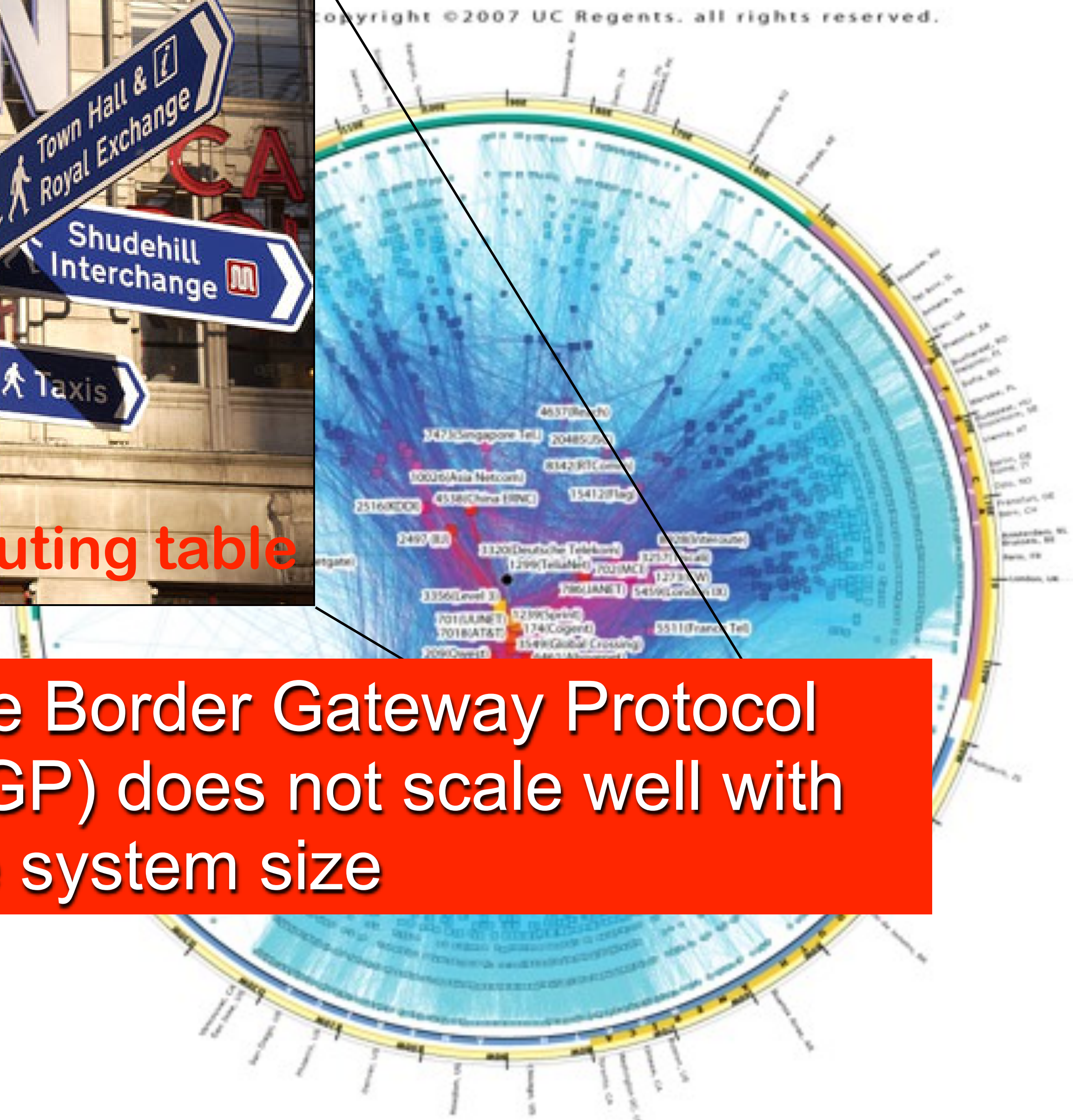
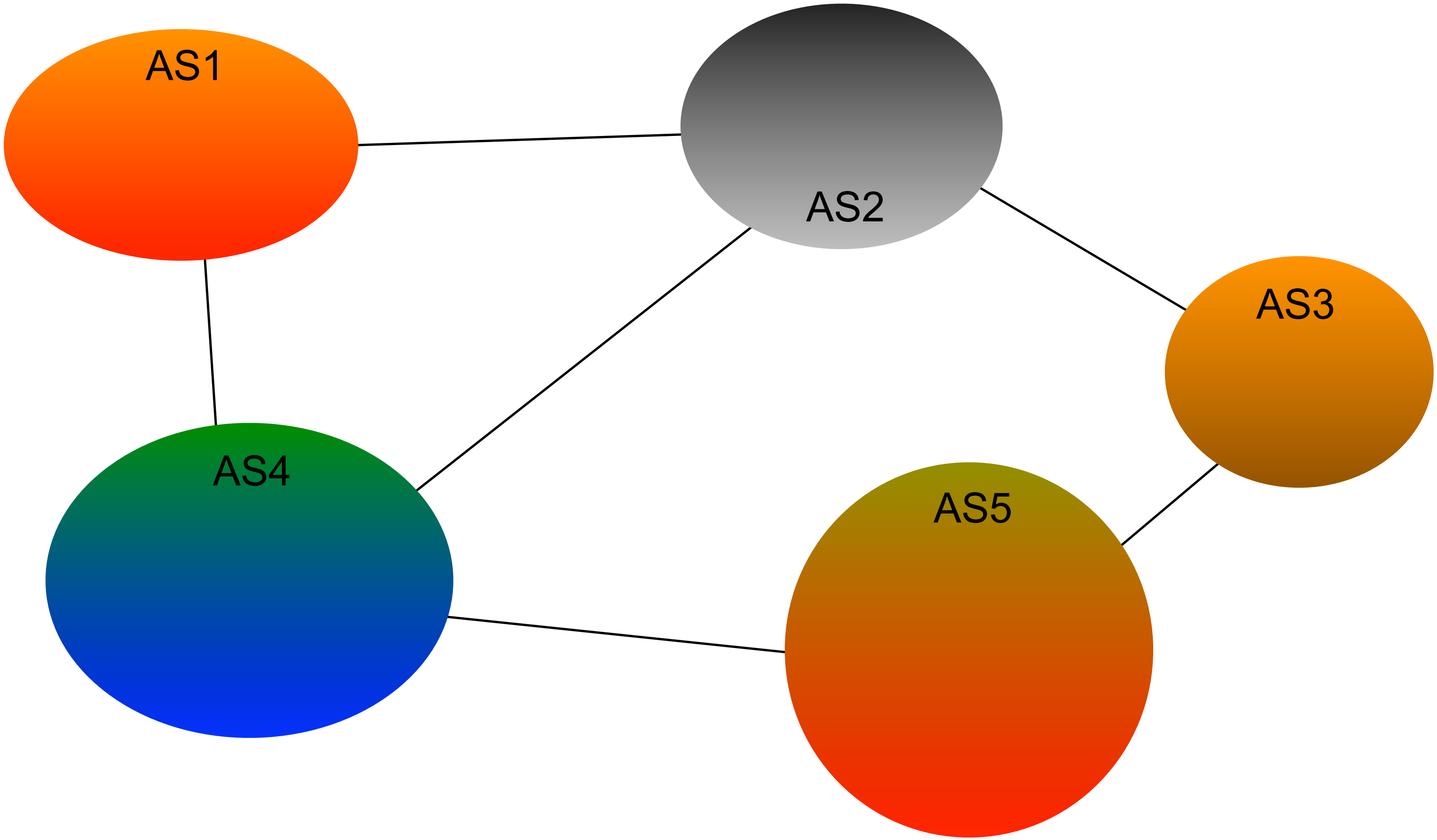


The Border Gateway Protocol (BGP) does not scale well with the system size

- ☑ Knowledge of the full topology of the graph
- ☑ Dynamical updates after single events



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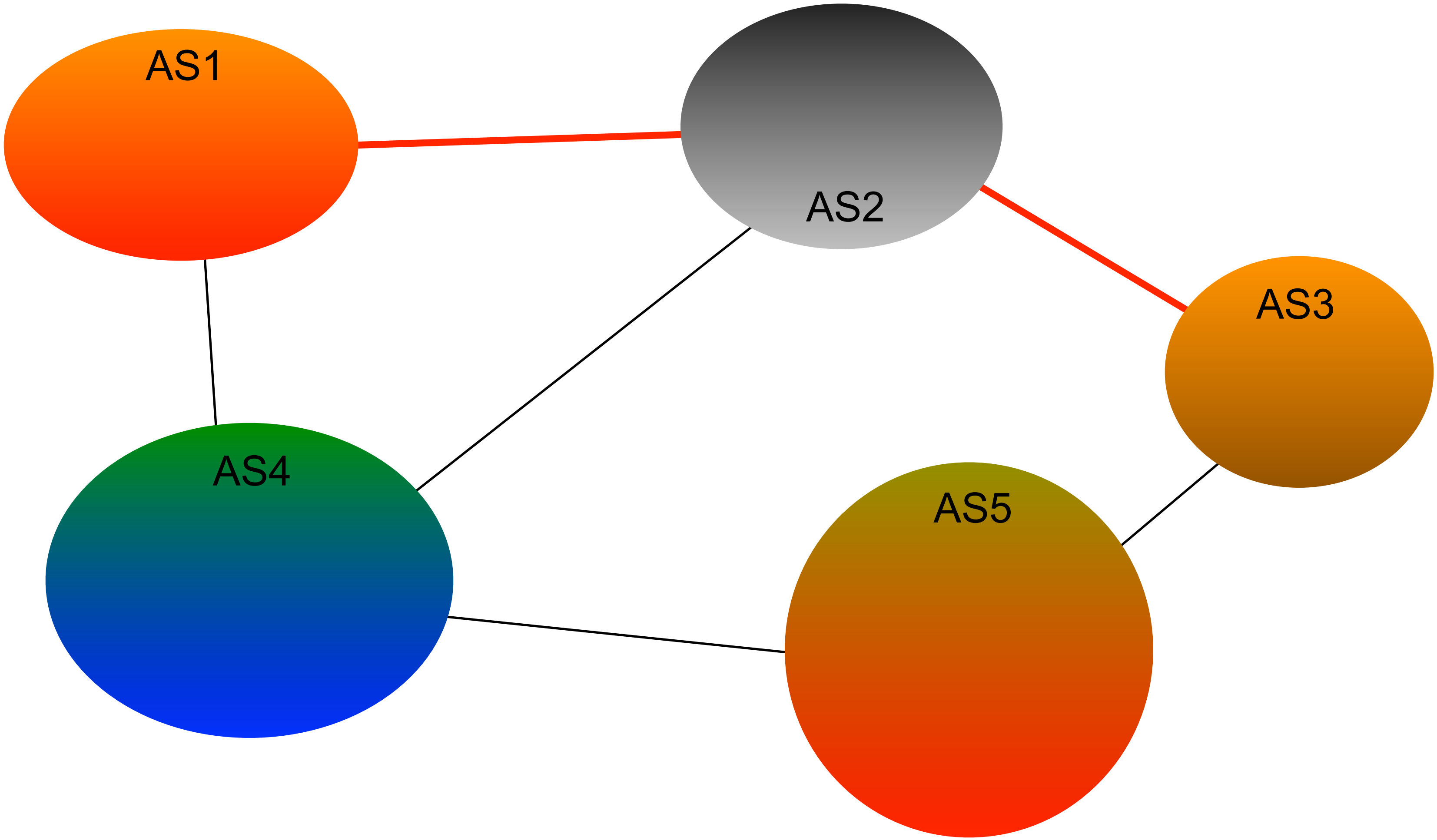

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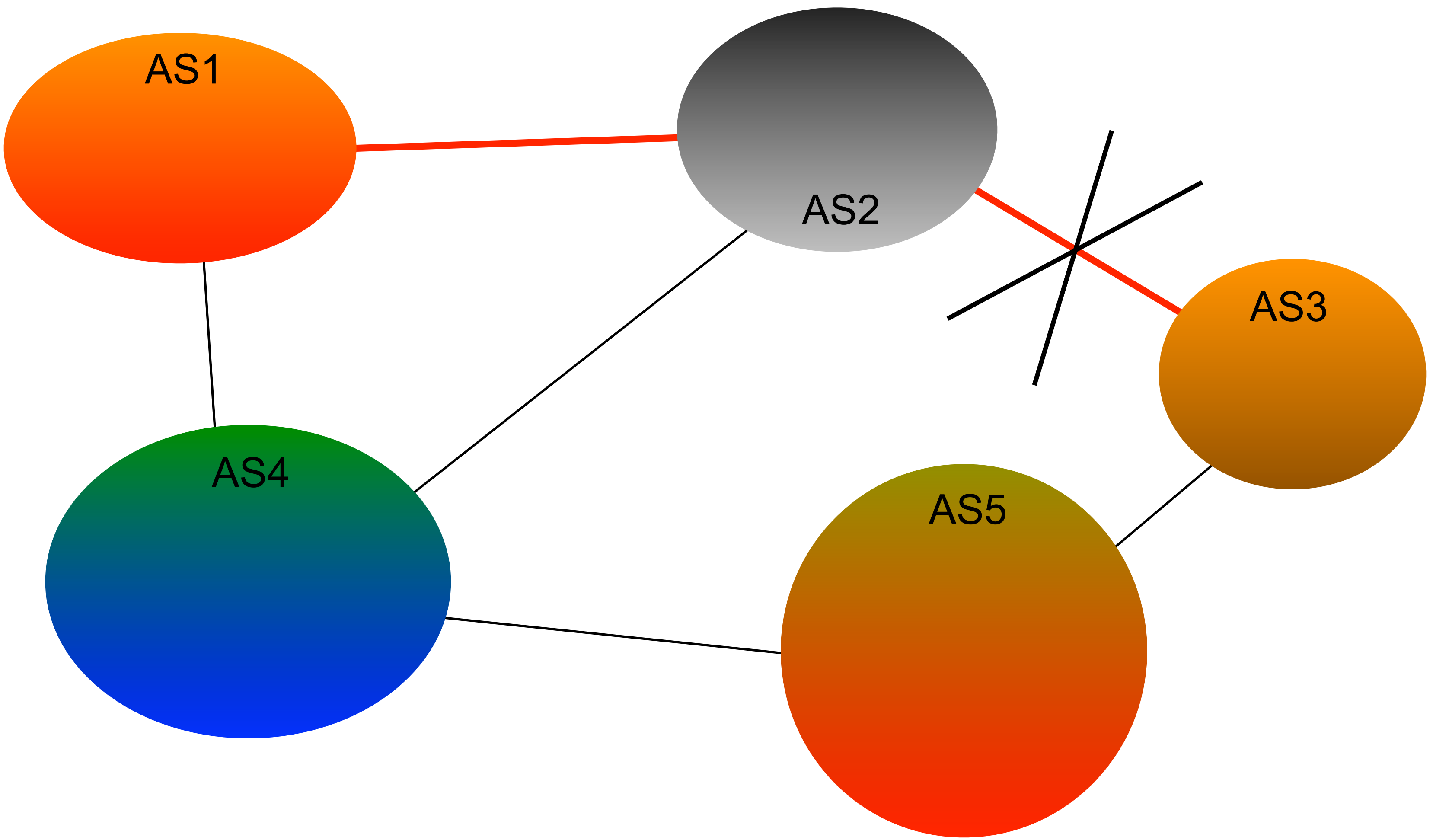

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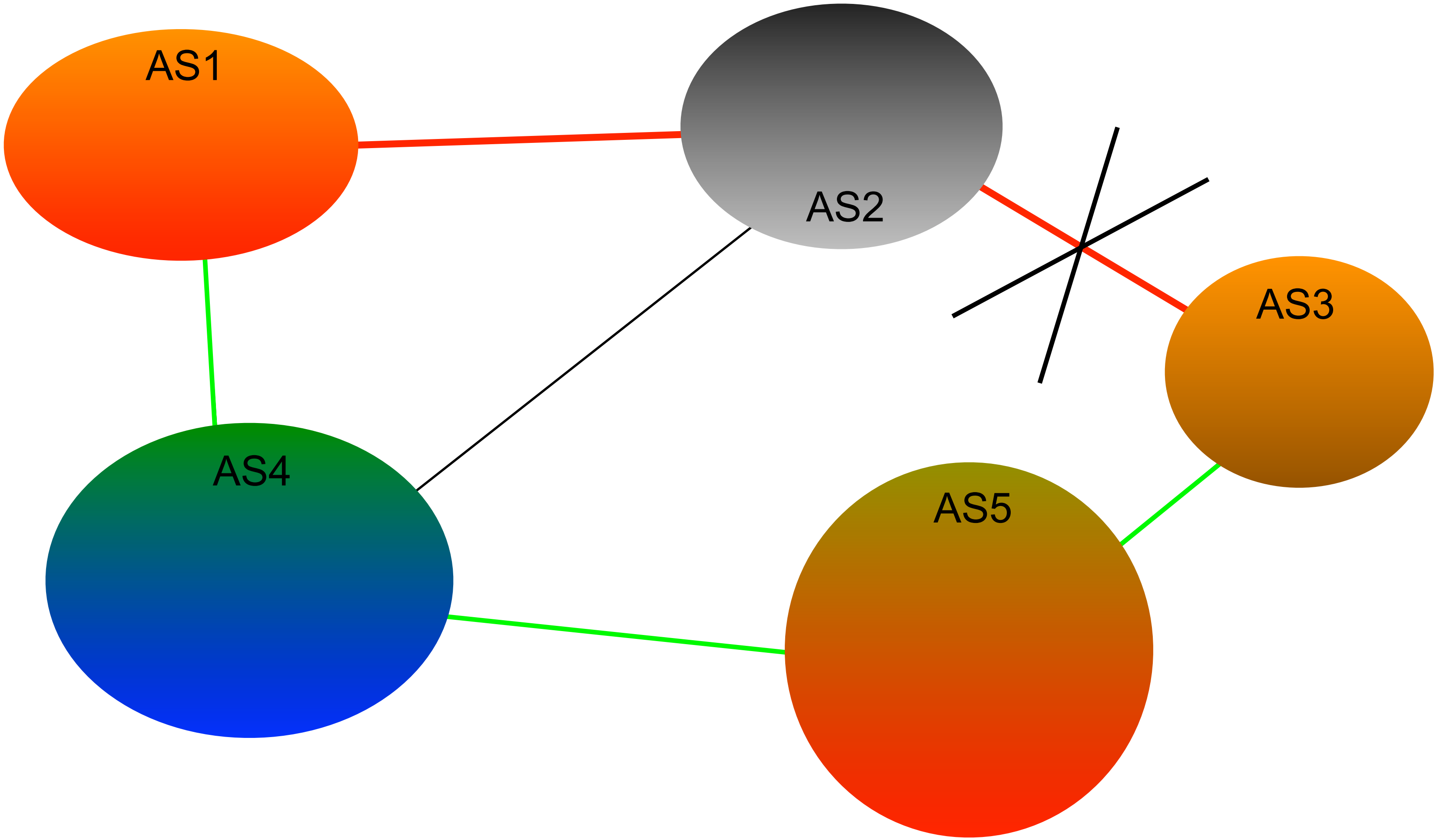

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according to the *Internet Architecture Board*, this is one of the most (if not the most) fundamental scalability limitation of our primary communication technologies, including the Internet.

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Trillion-dollar question!!!

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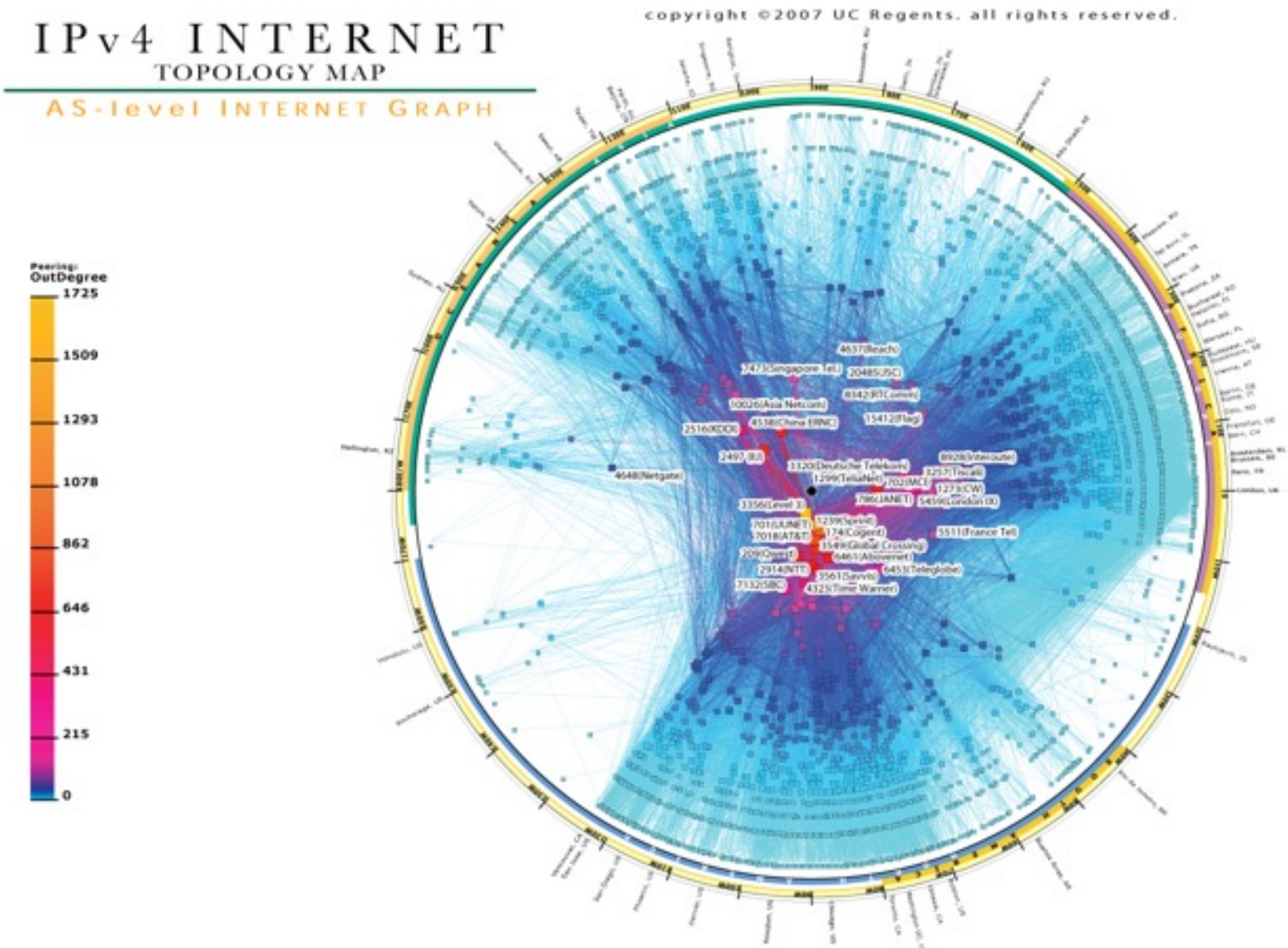
the AS level map of the Internet



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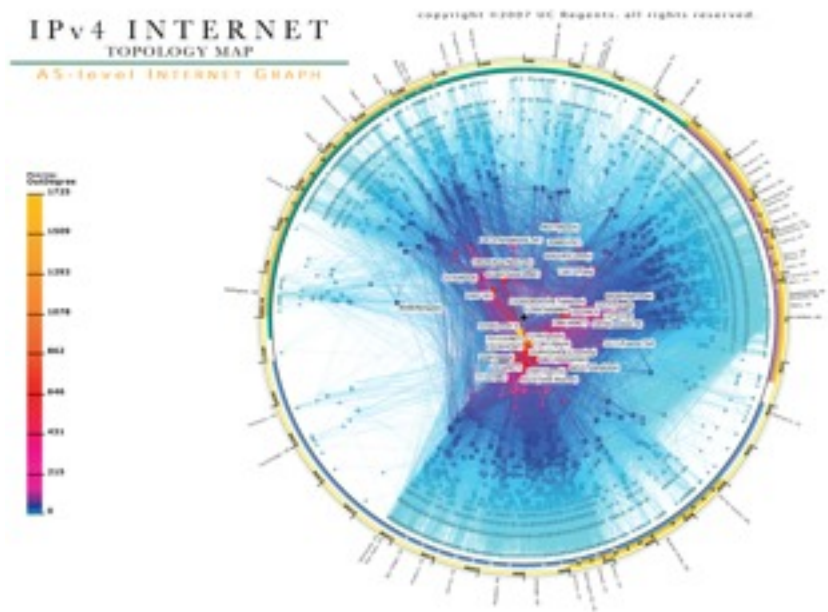

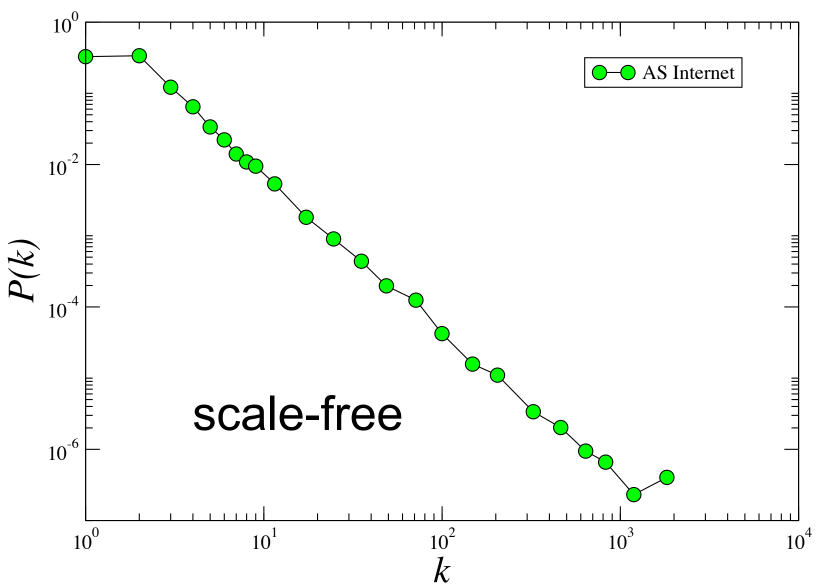

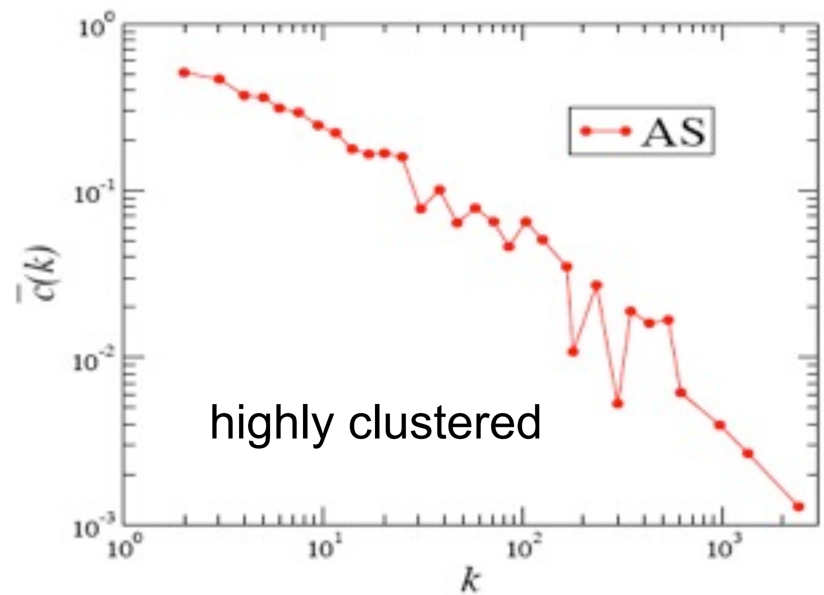

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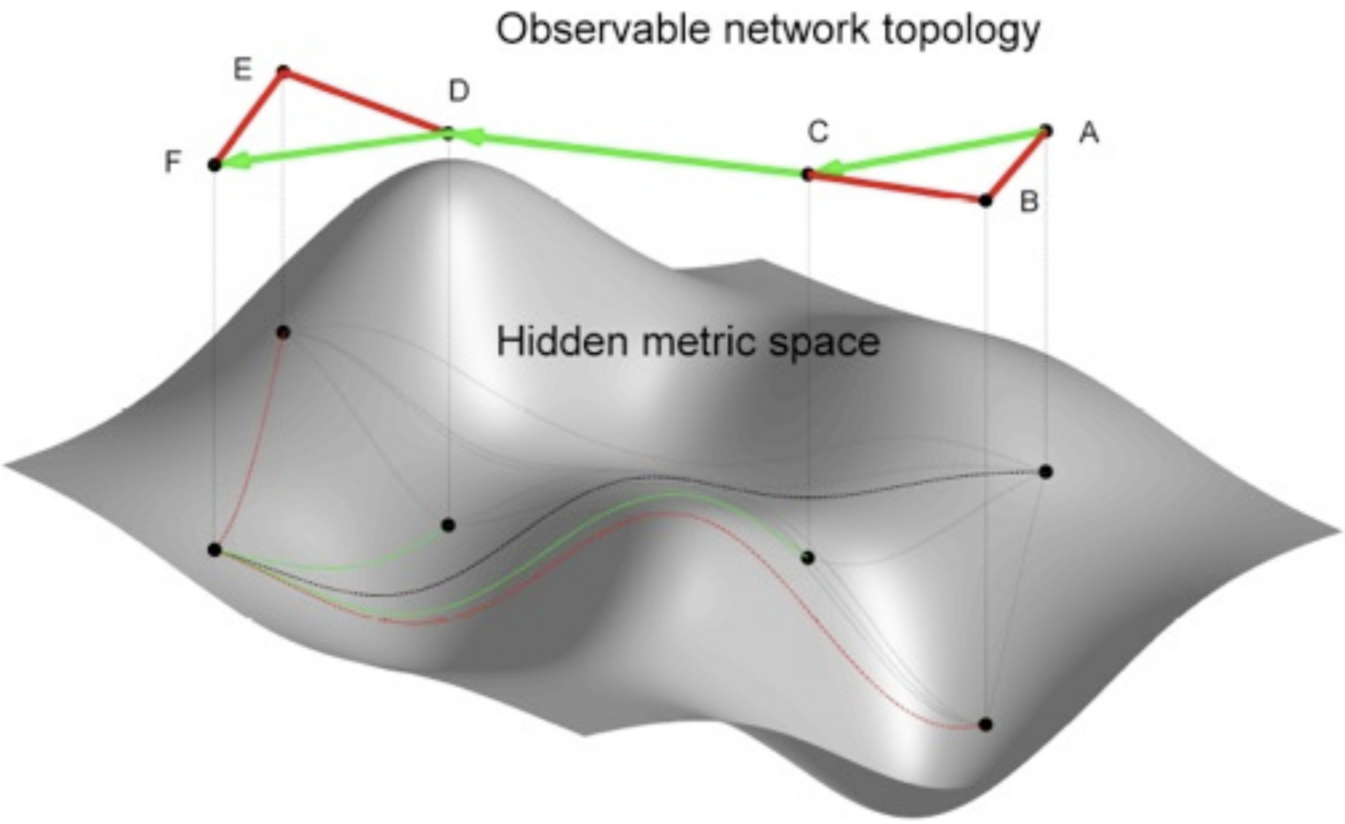

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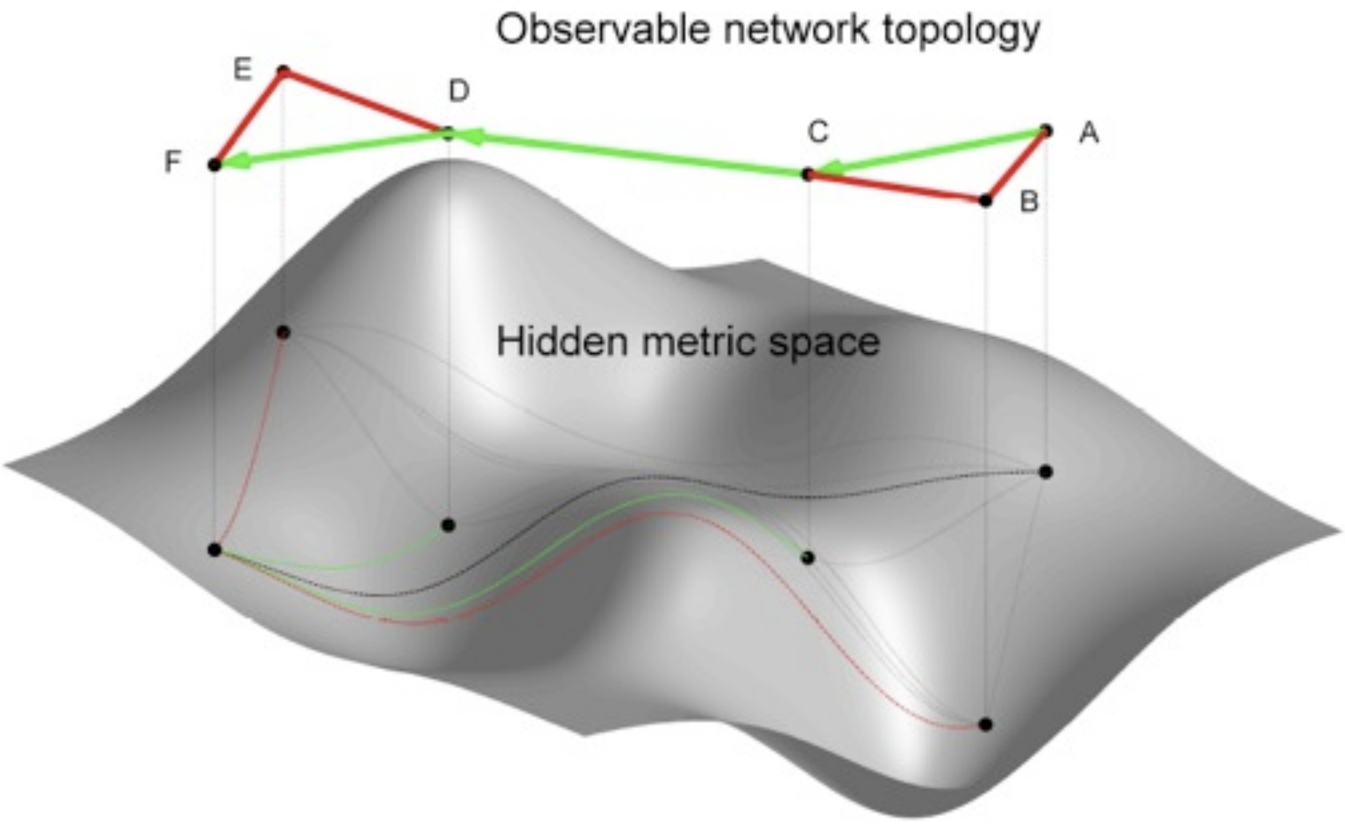

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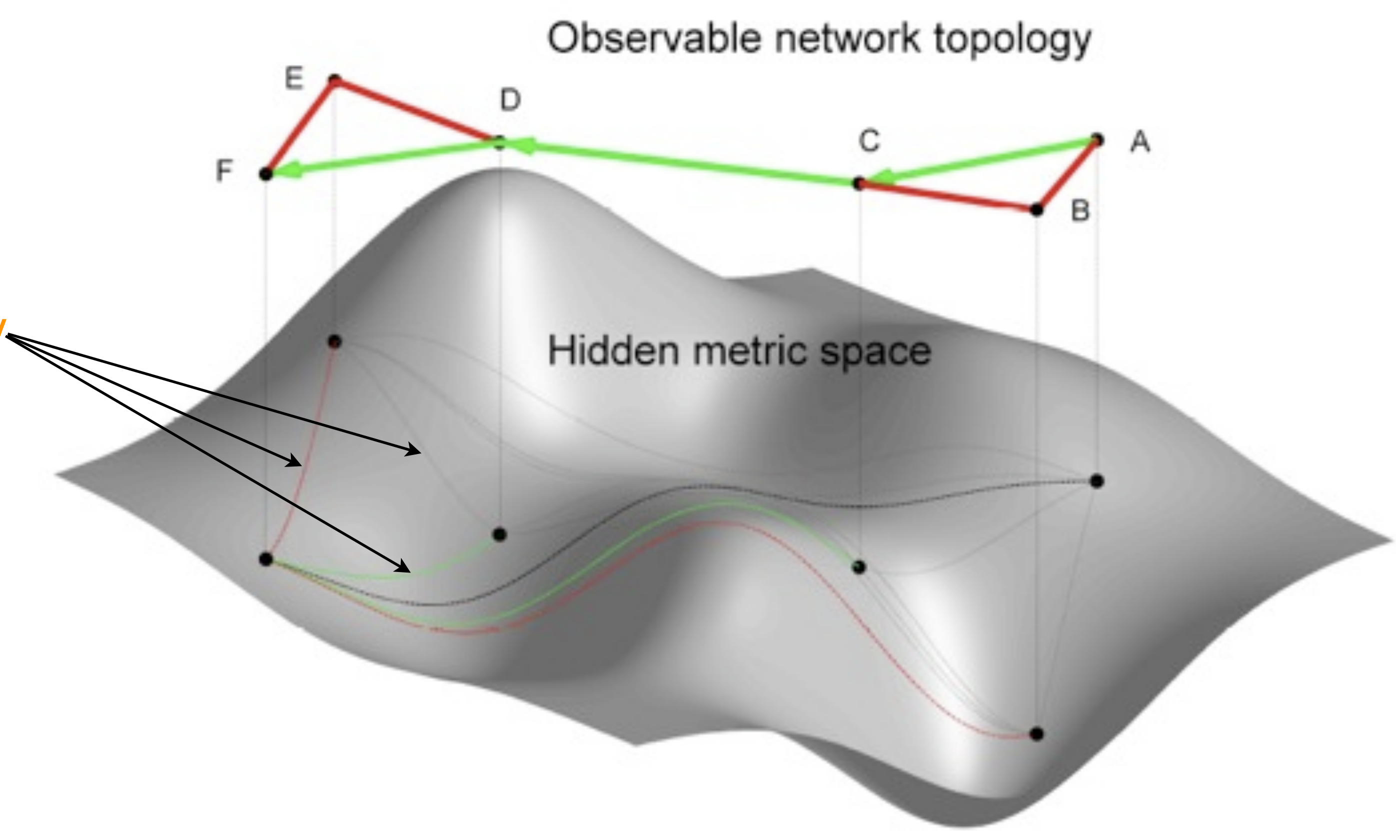

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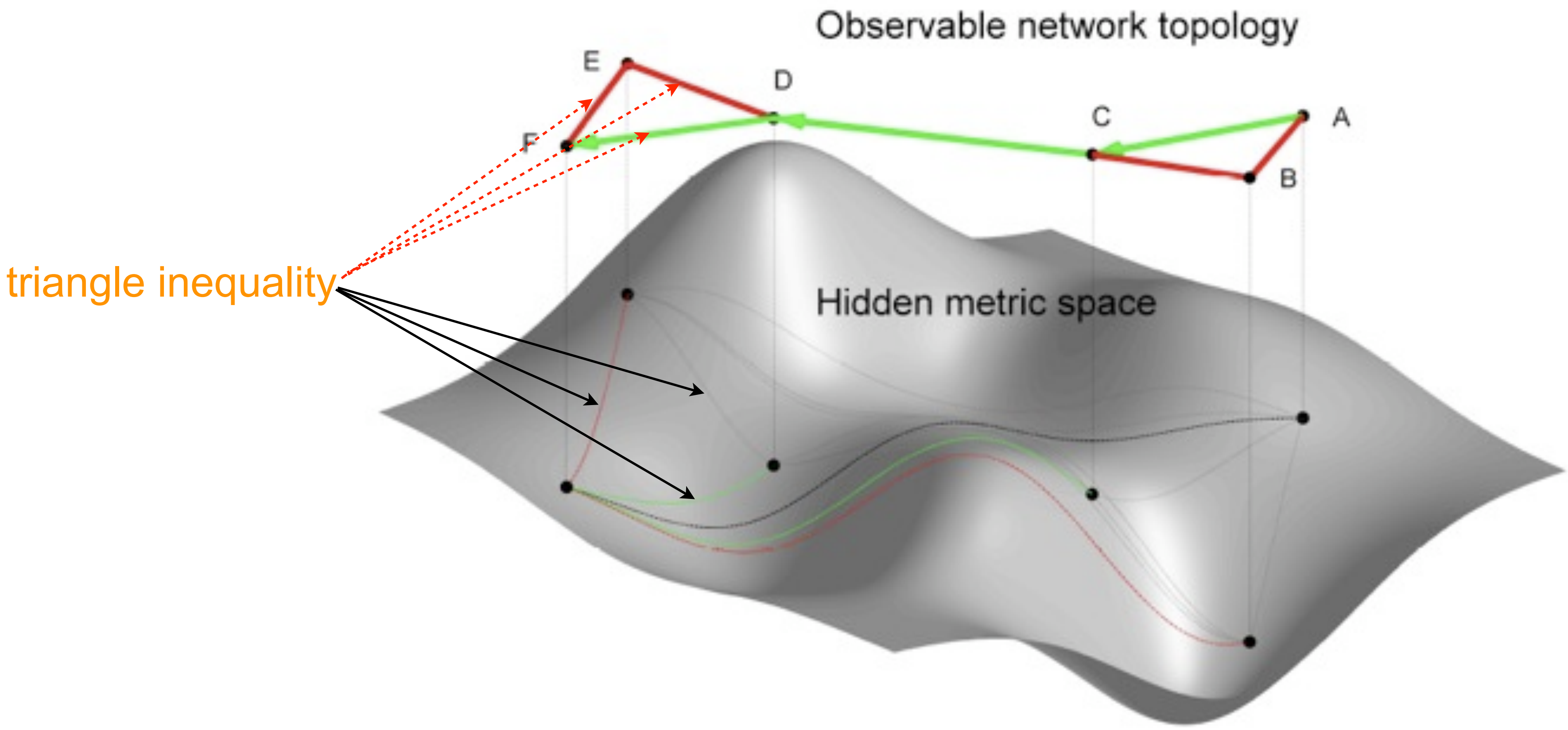

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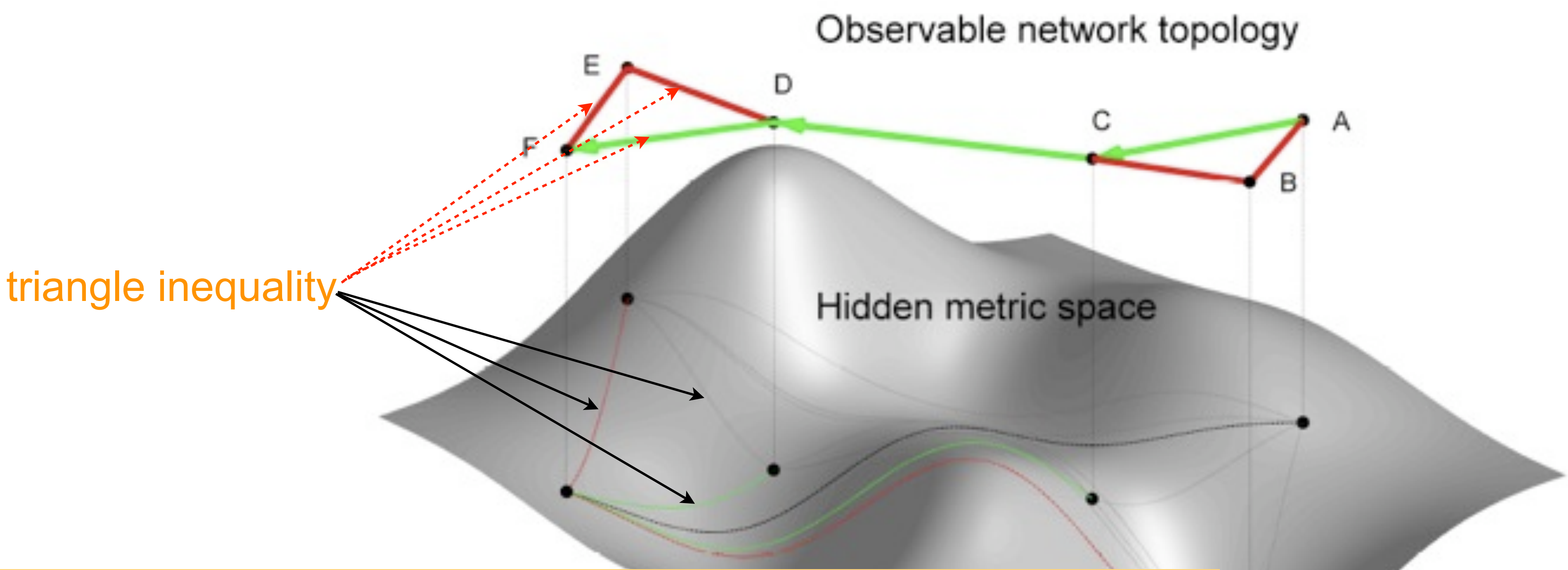

We can use distances to route information packets

Greedy Routing

Big questions

Is greedy routing a feasible mechanism in large networks?

What are the topological requirements for that to happen?

Can we really map real networks into metric spaces?

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WE NEED MODELS

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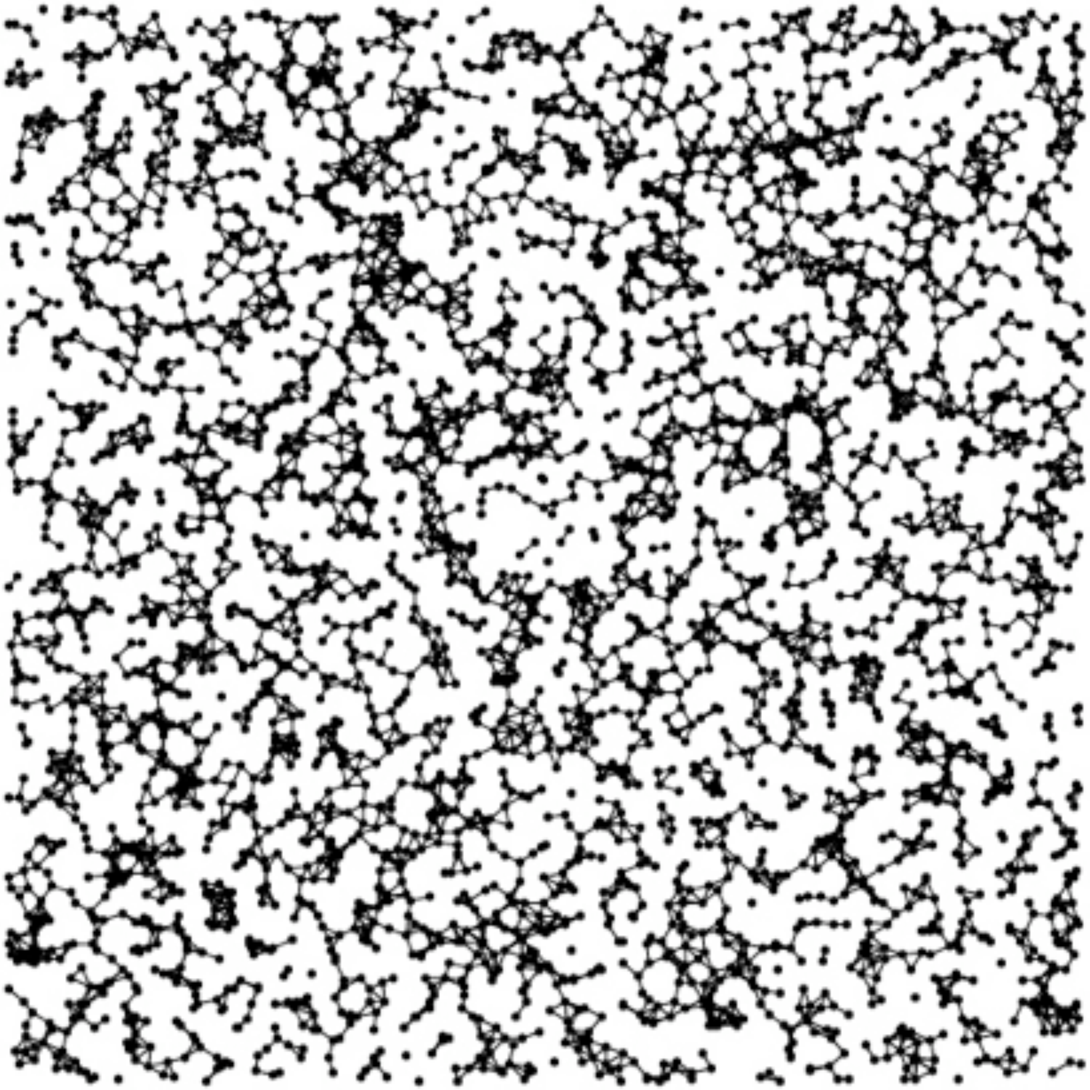

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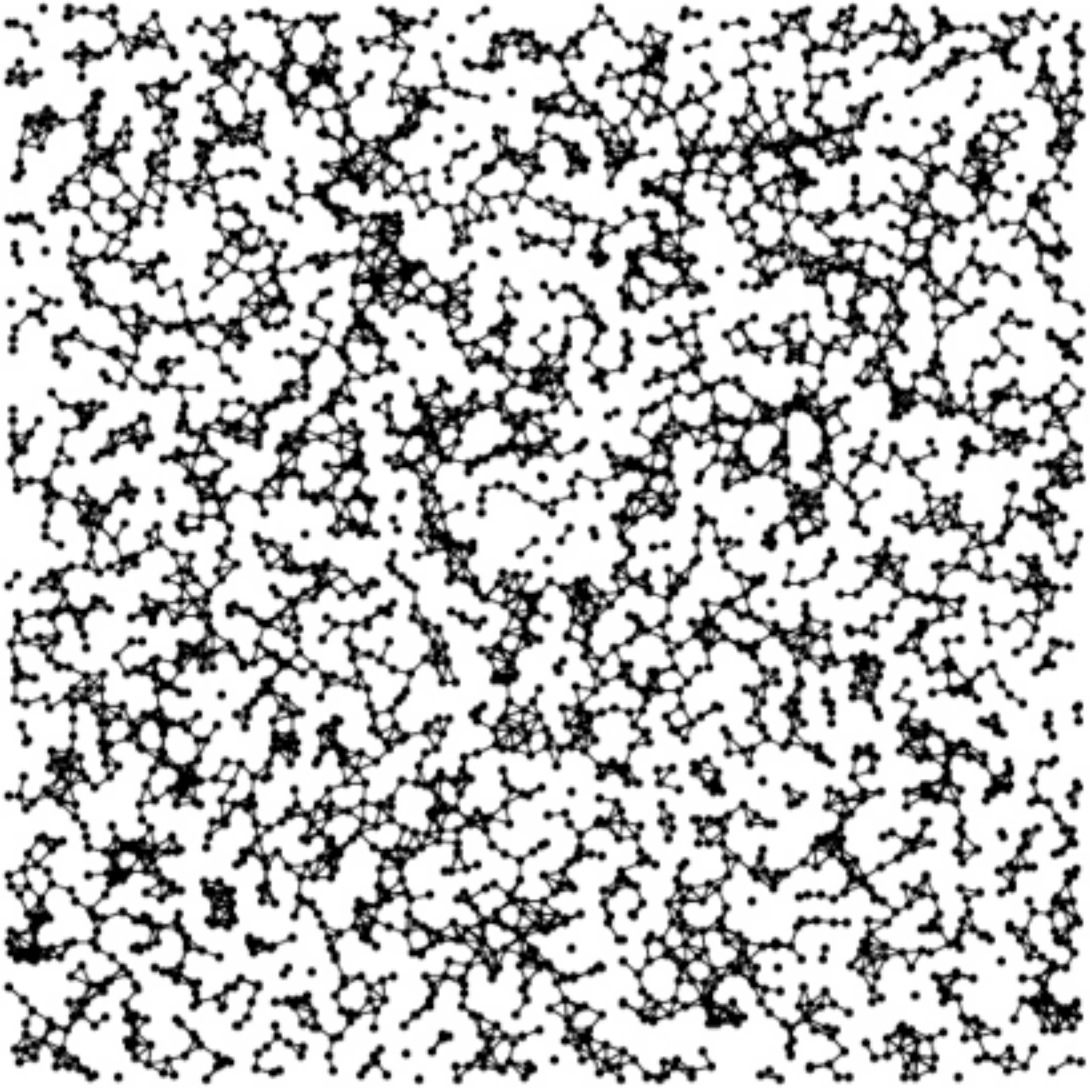

Distribute points in a plane using a Poisson process or whatever you like

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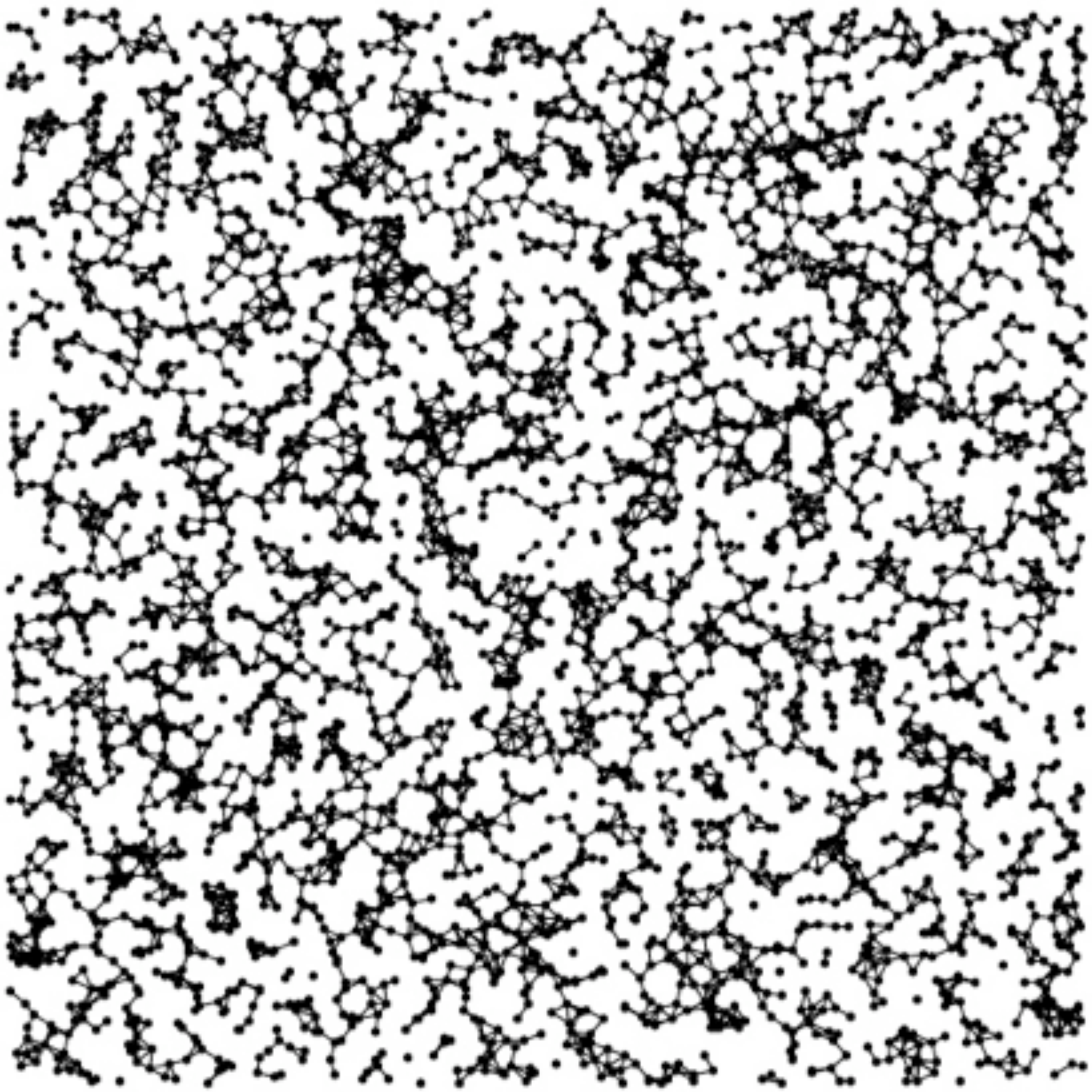

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Connect points that are below a critical distance

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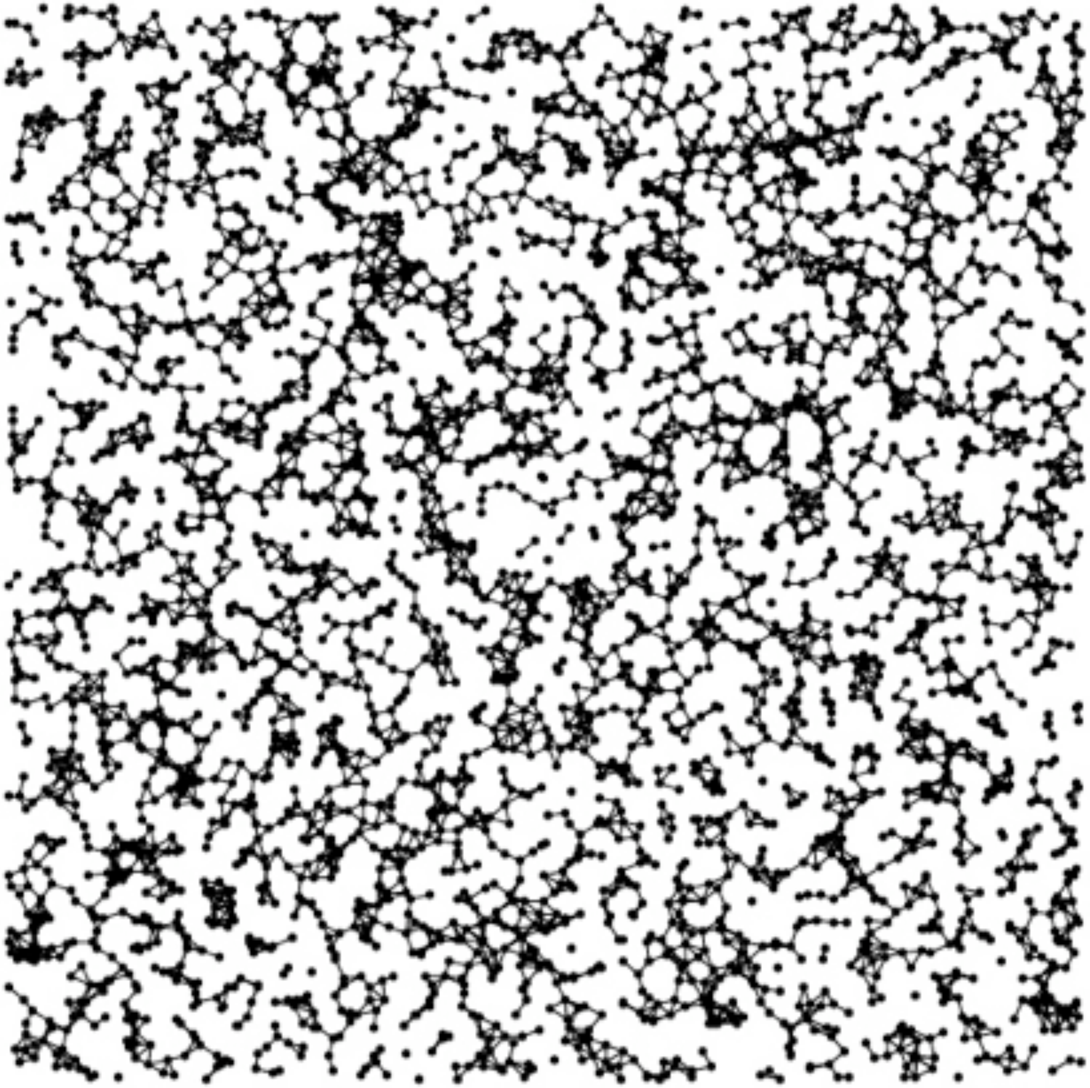

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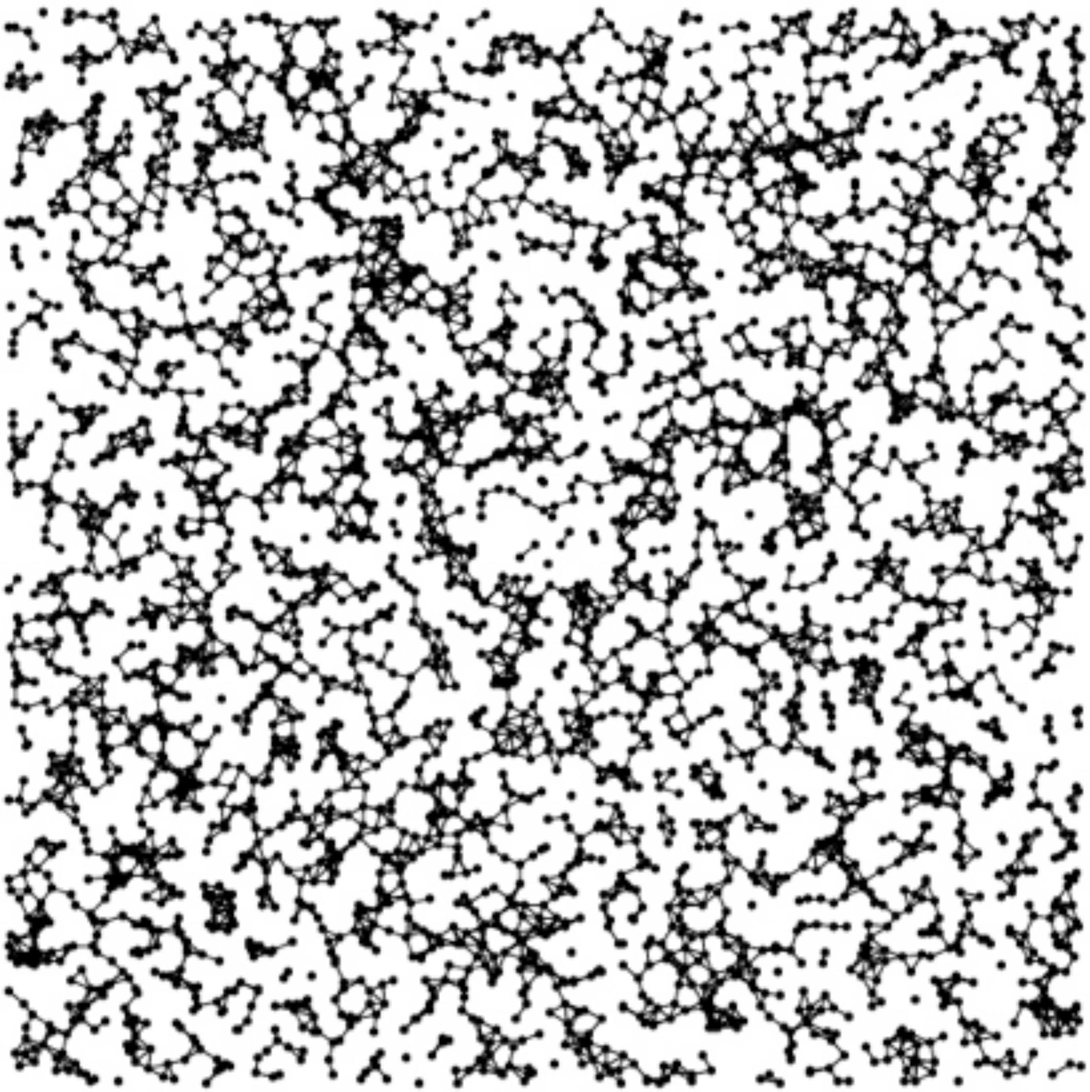

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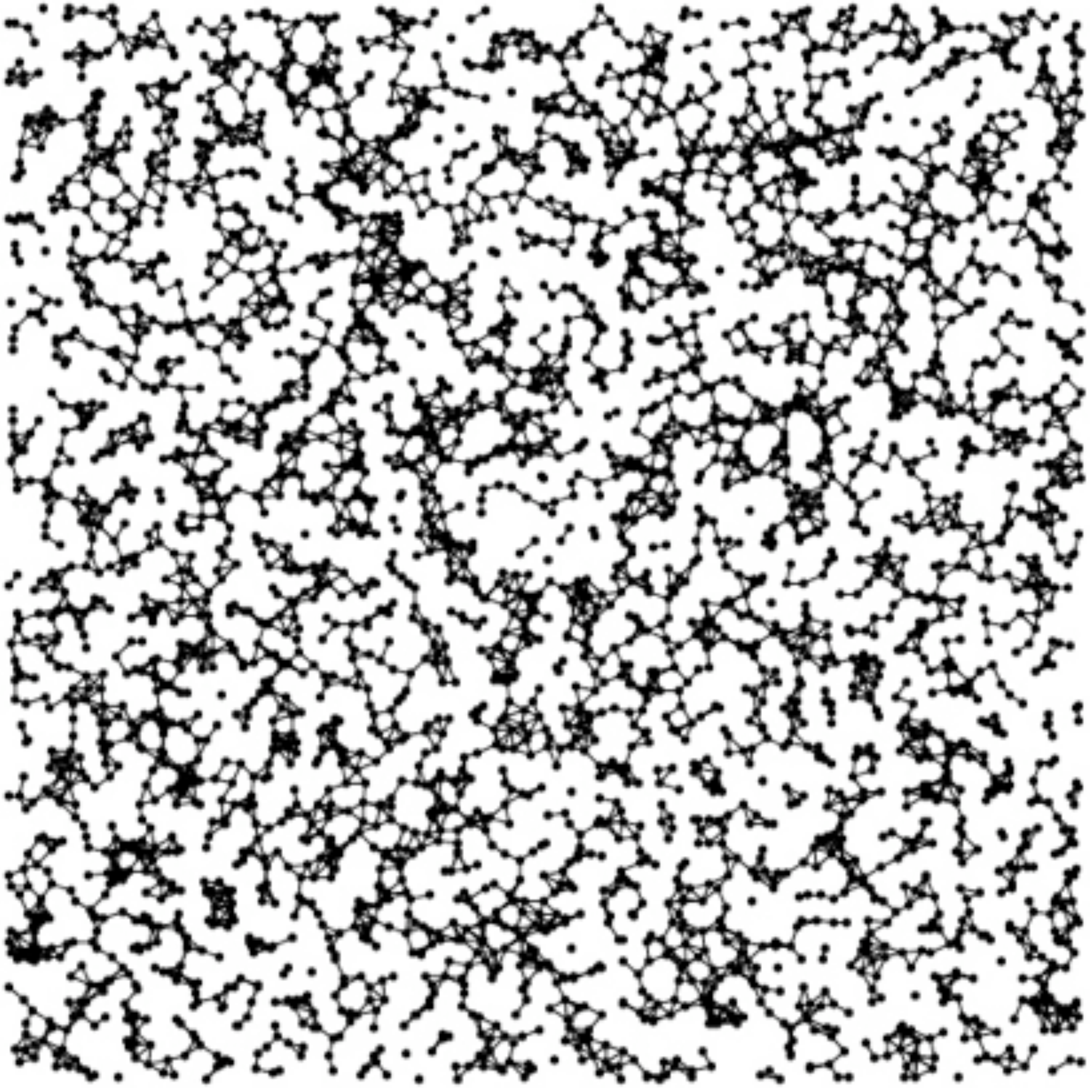

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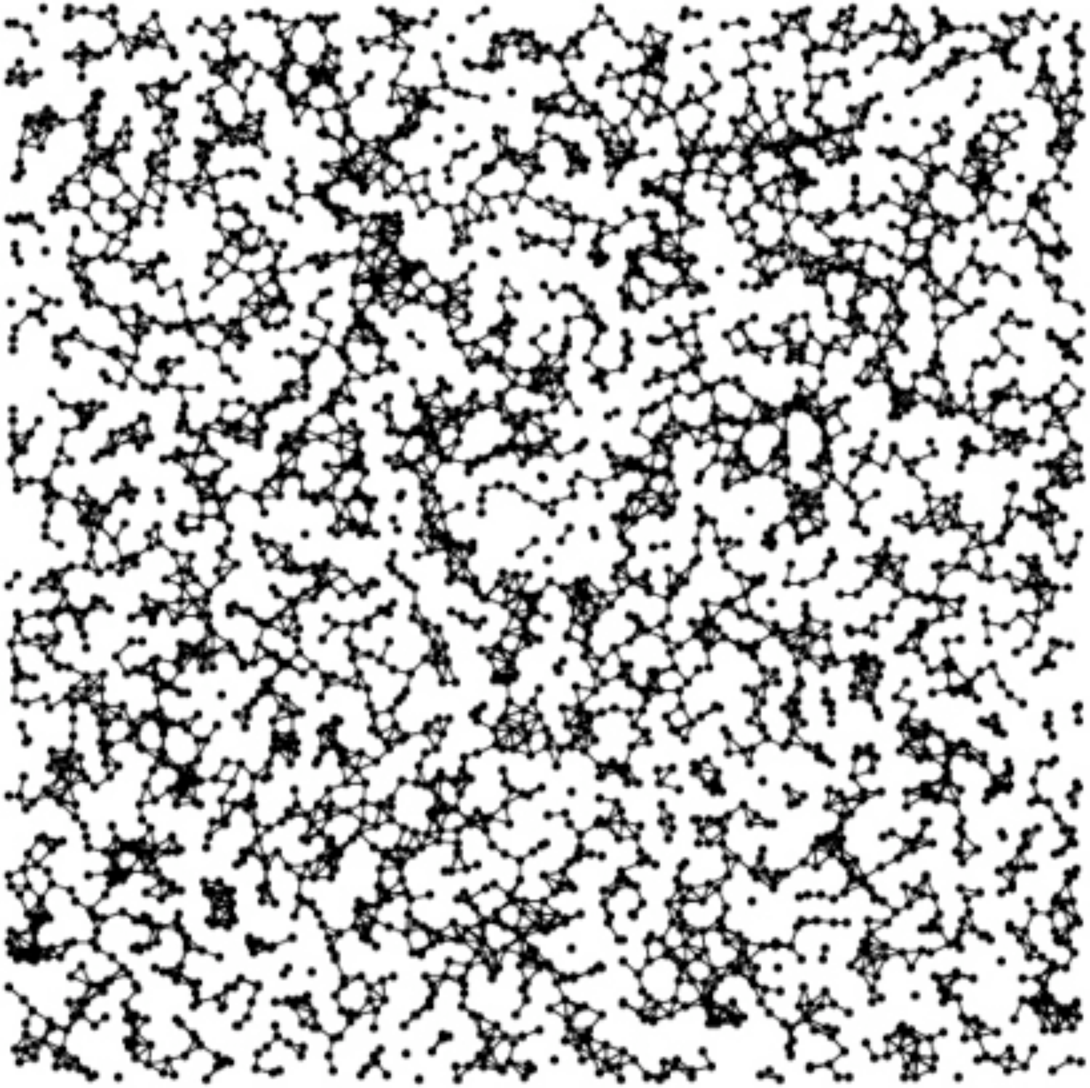

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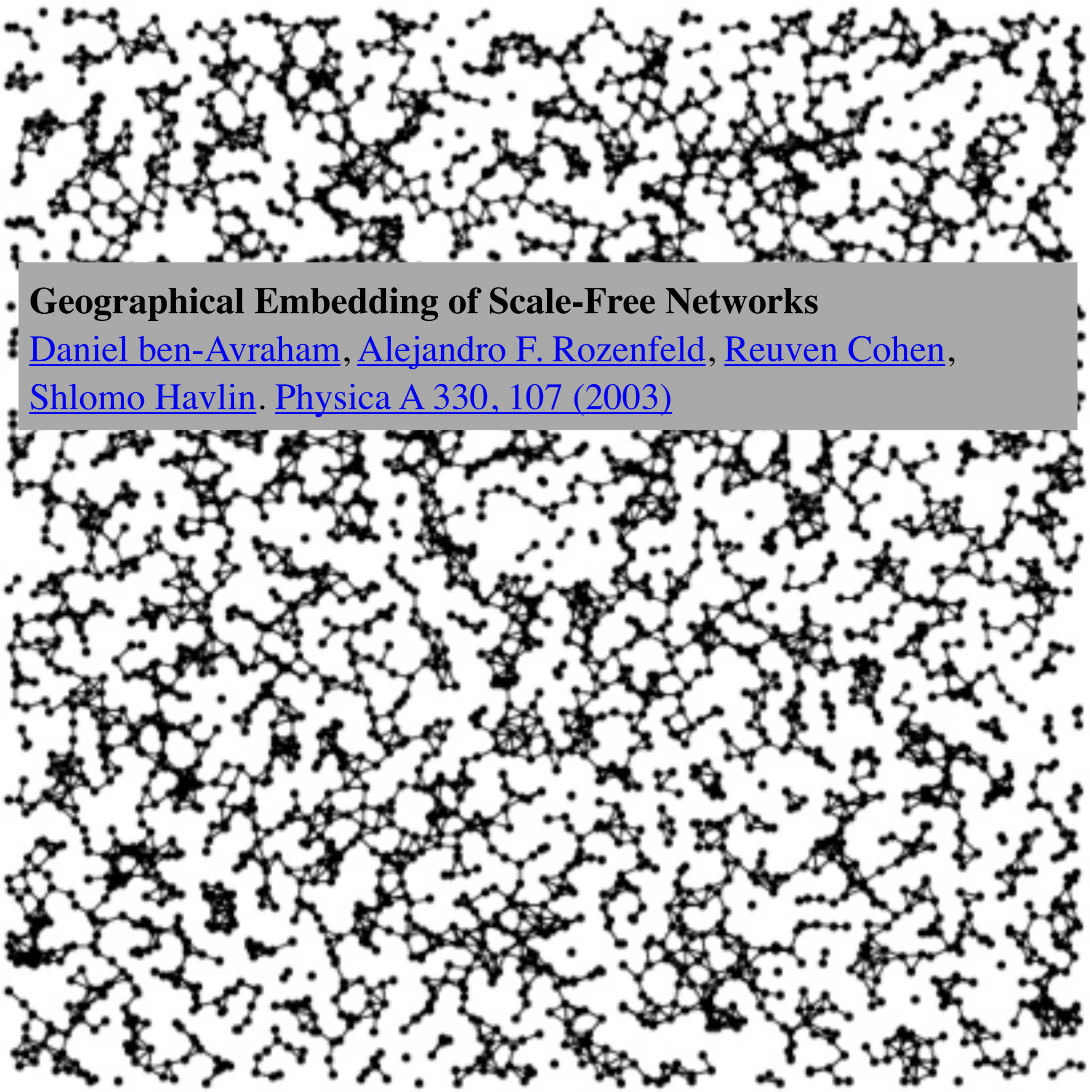

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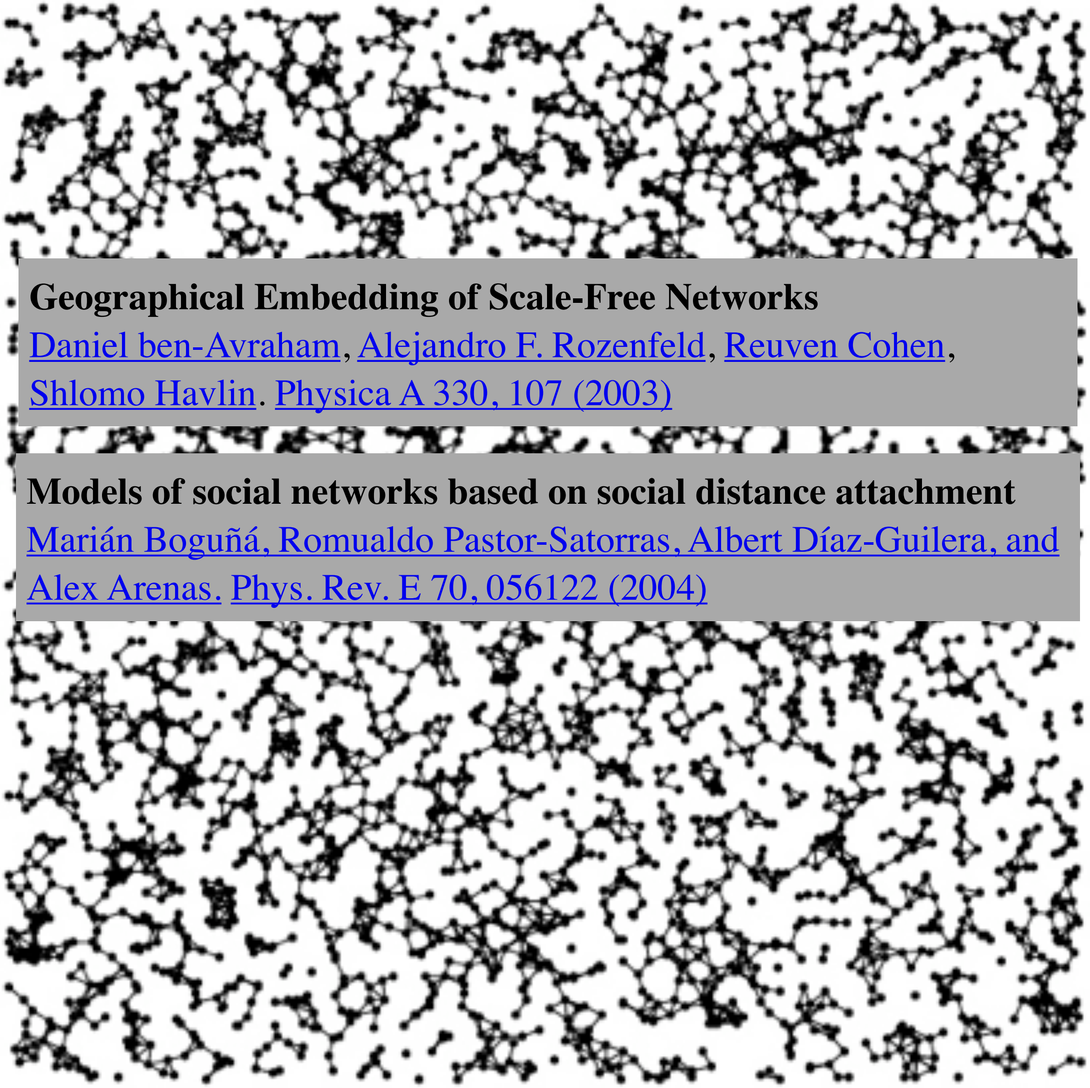

Geographical Embedding of Scale-Free Networks
Daniel ben-Avraham, Alejandro F. Rozenfeld, Reuven Cohen, Shlomo Havlin. Physica A 330, 107 (2003)

Models of social networks based on social distance attachment
Marián Boguñá, Romualdo Pastor-Satorras, Albert Díaz-Guilera, and Alex Arenas. Phys. Rev. E 70, 056122 (2004)

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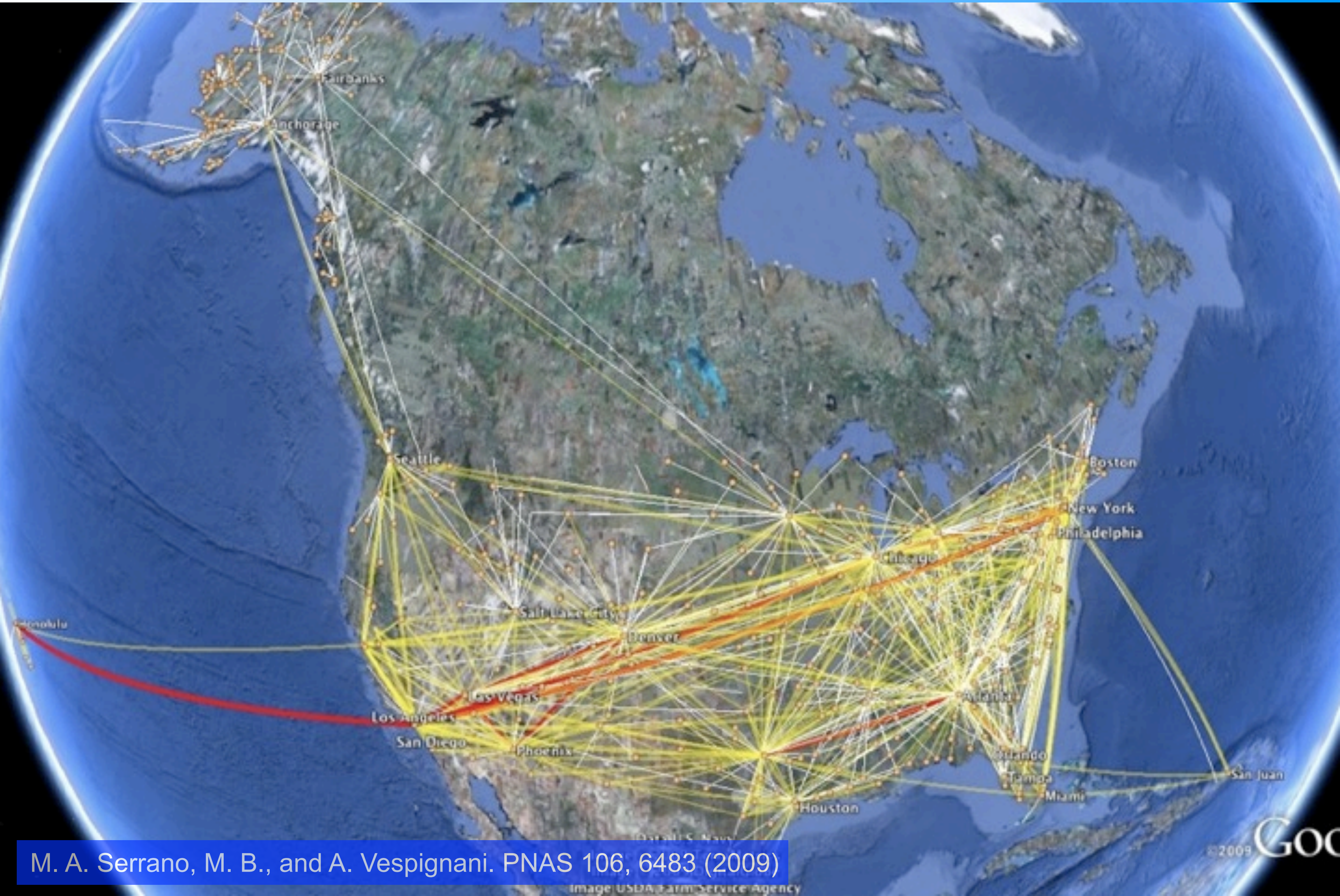
US Airport Network



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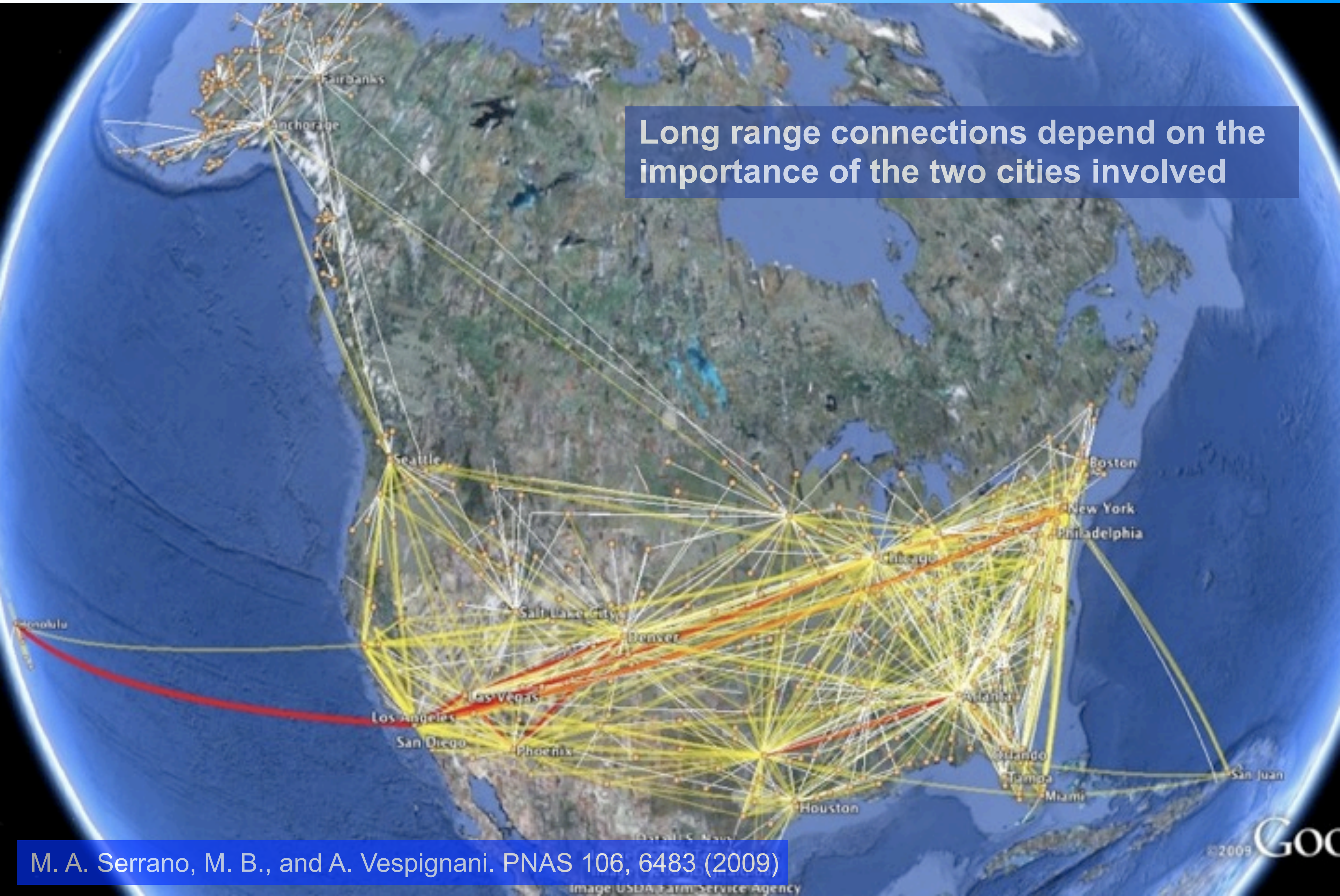

Long range connections depend on the importance of the two cities involved

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Cities' importance is an intrinsic property

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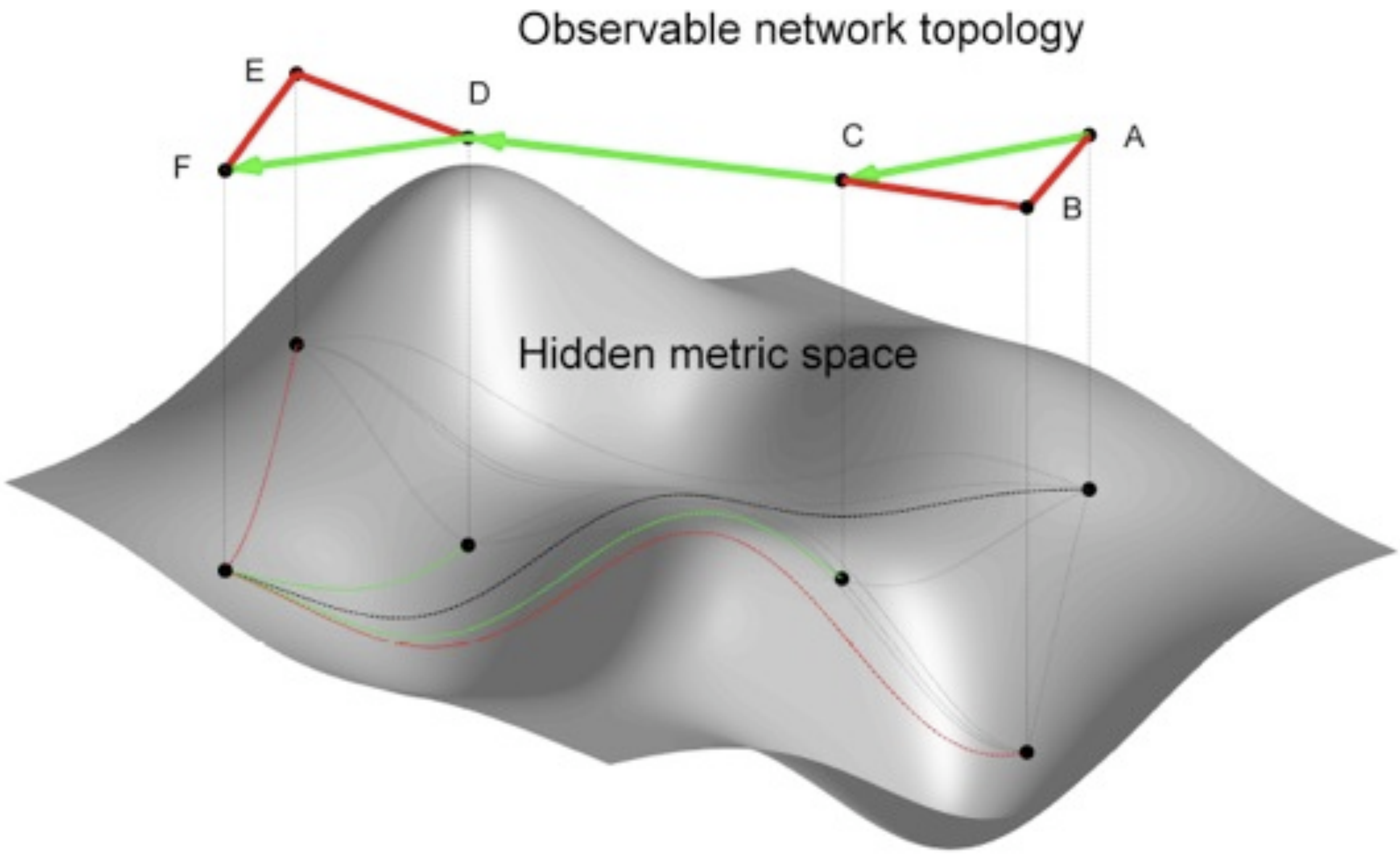

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Connection probability

The Newtonian model

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$$d_c(\kappa,\kappa') = \propto (\kappa\kappa')^{1/D}$$

Friendly people make connections more easily

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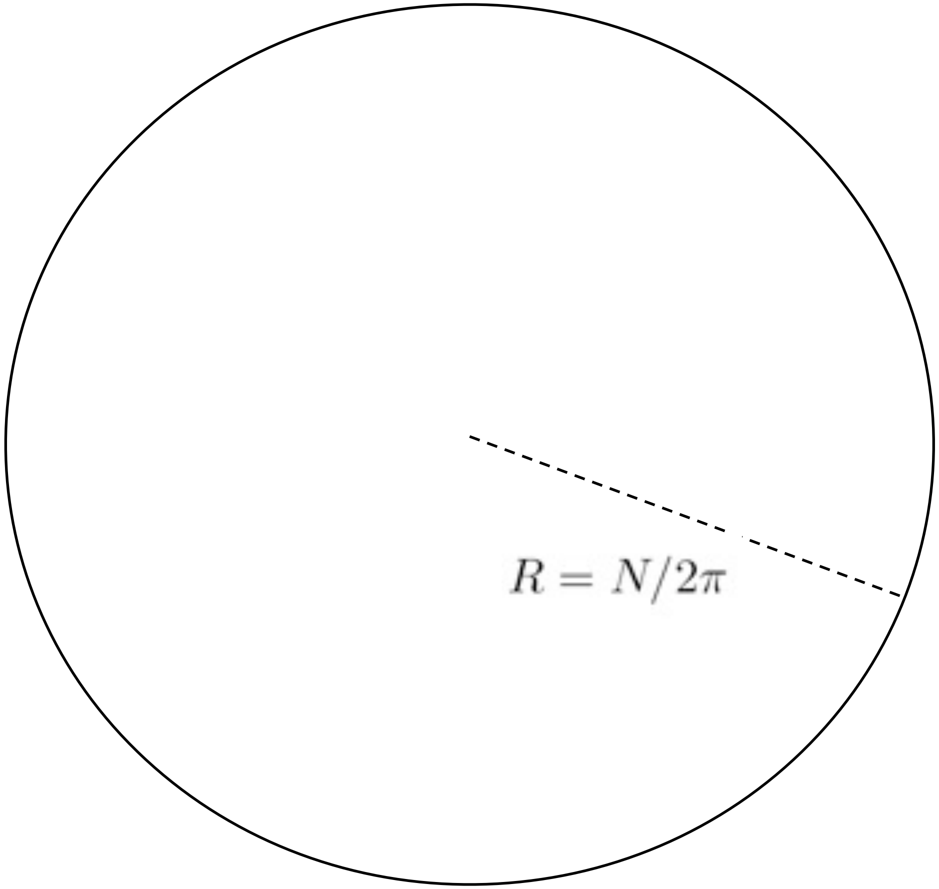
$$\rho(\kappa) \propto \kappa^{-\gamma} \rightarrow P(k) \propto k^{-\gamma}$$

The Newtonian-S^1 model



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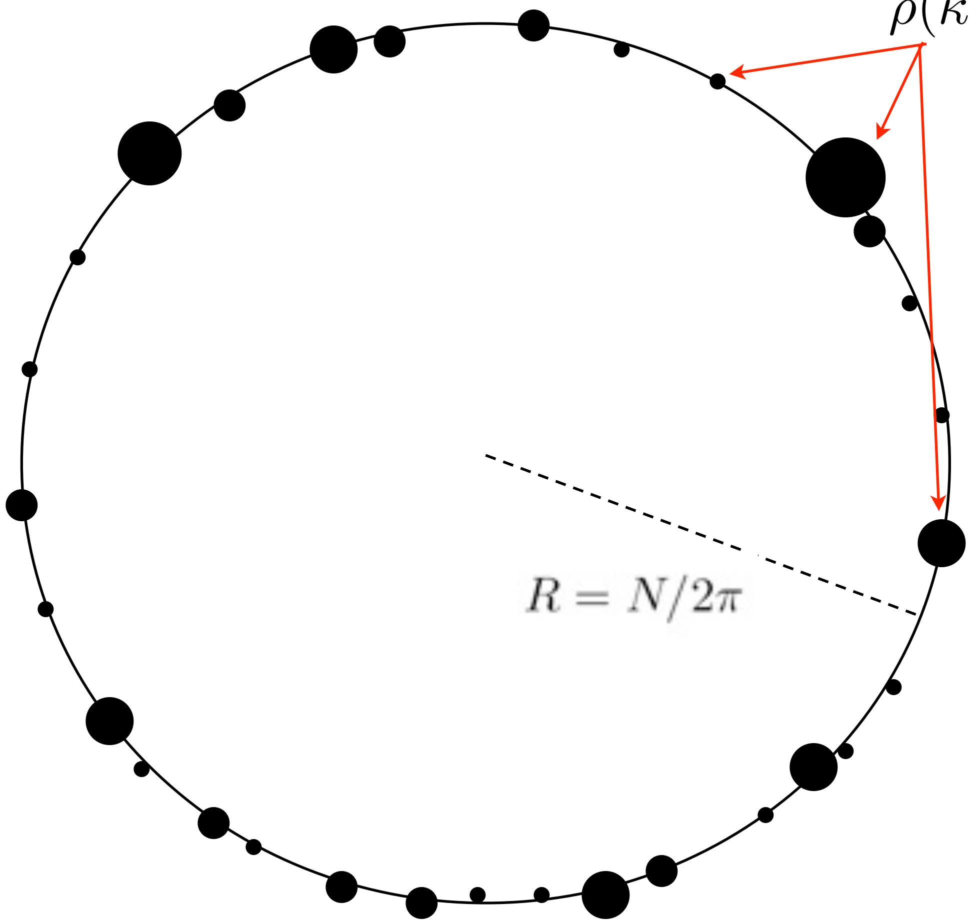

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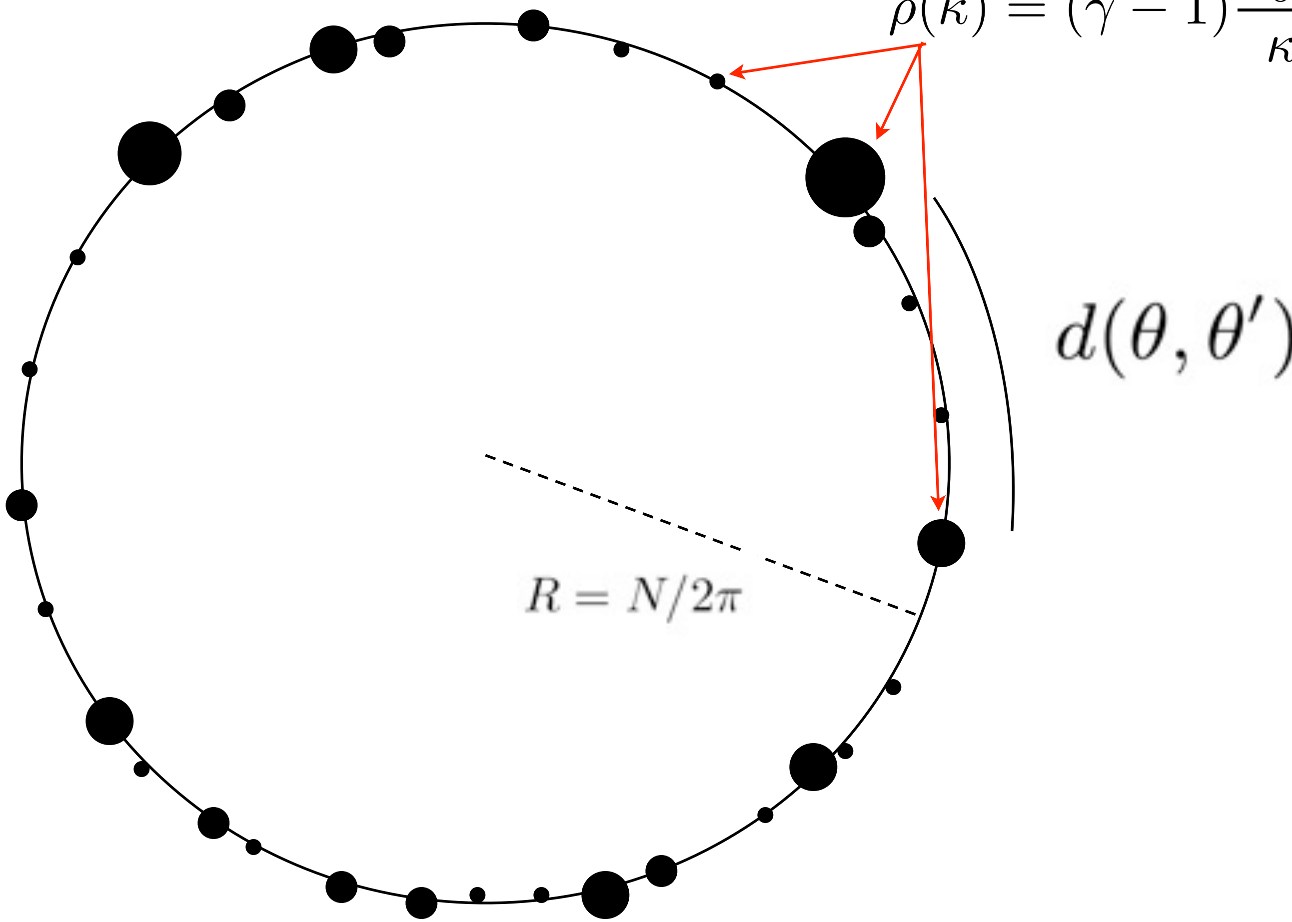

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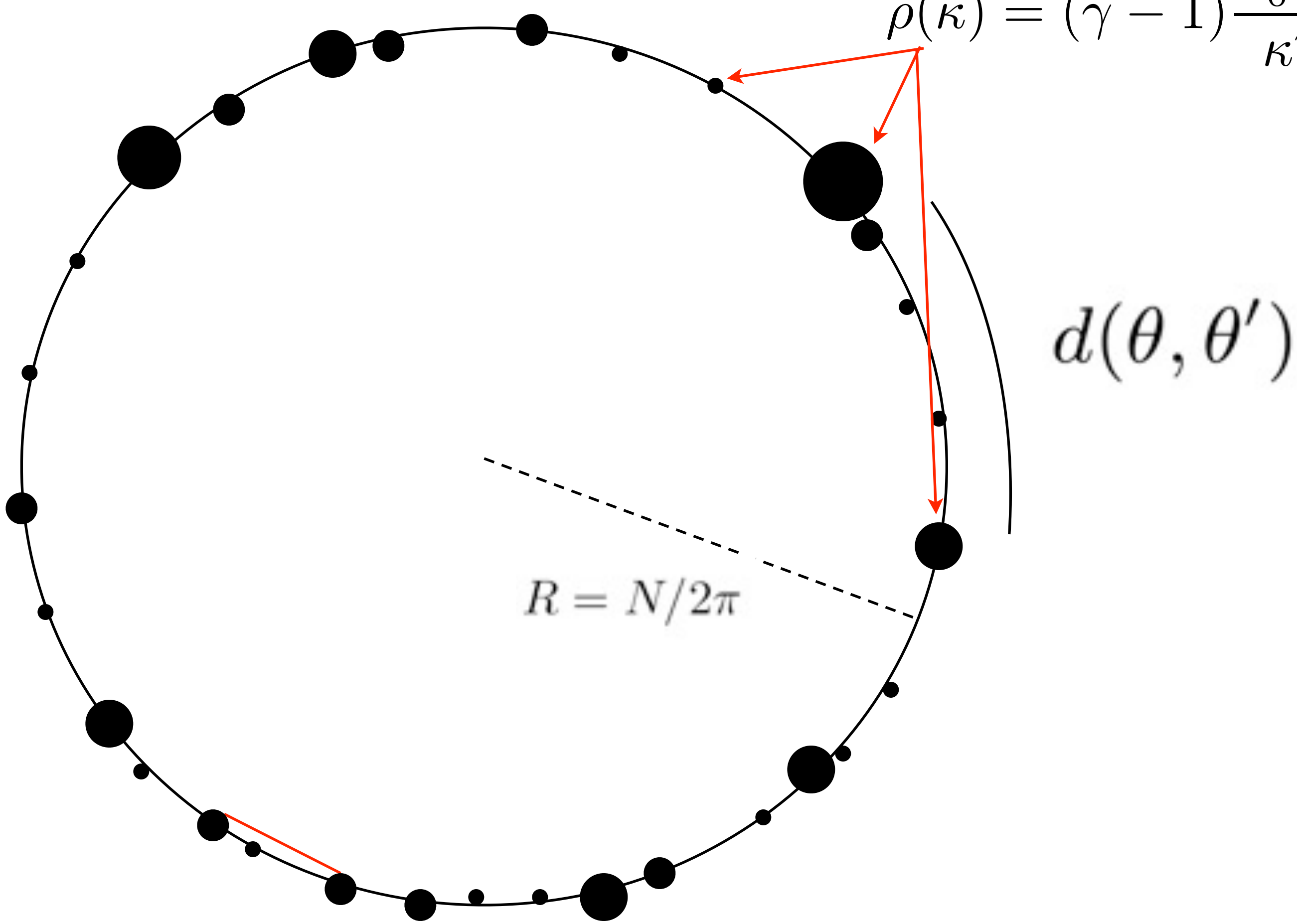

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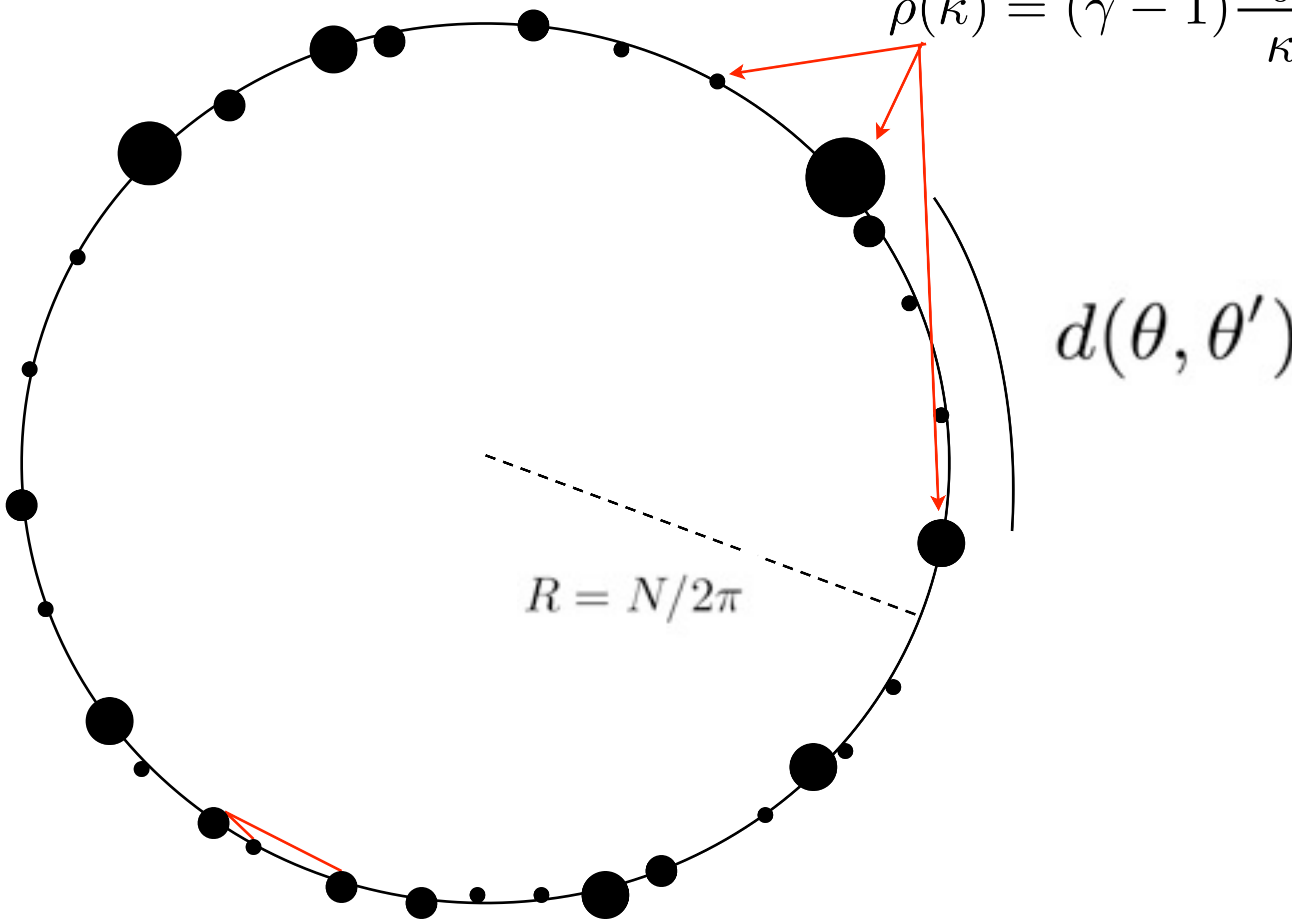

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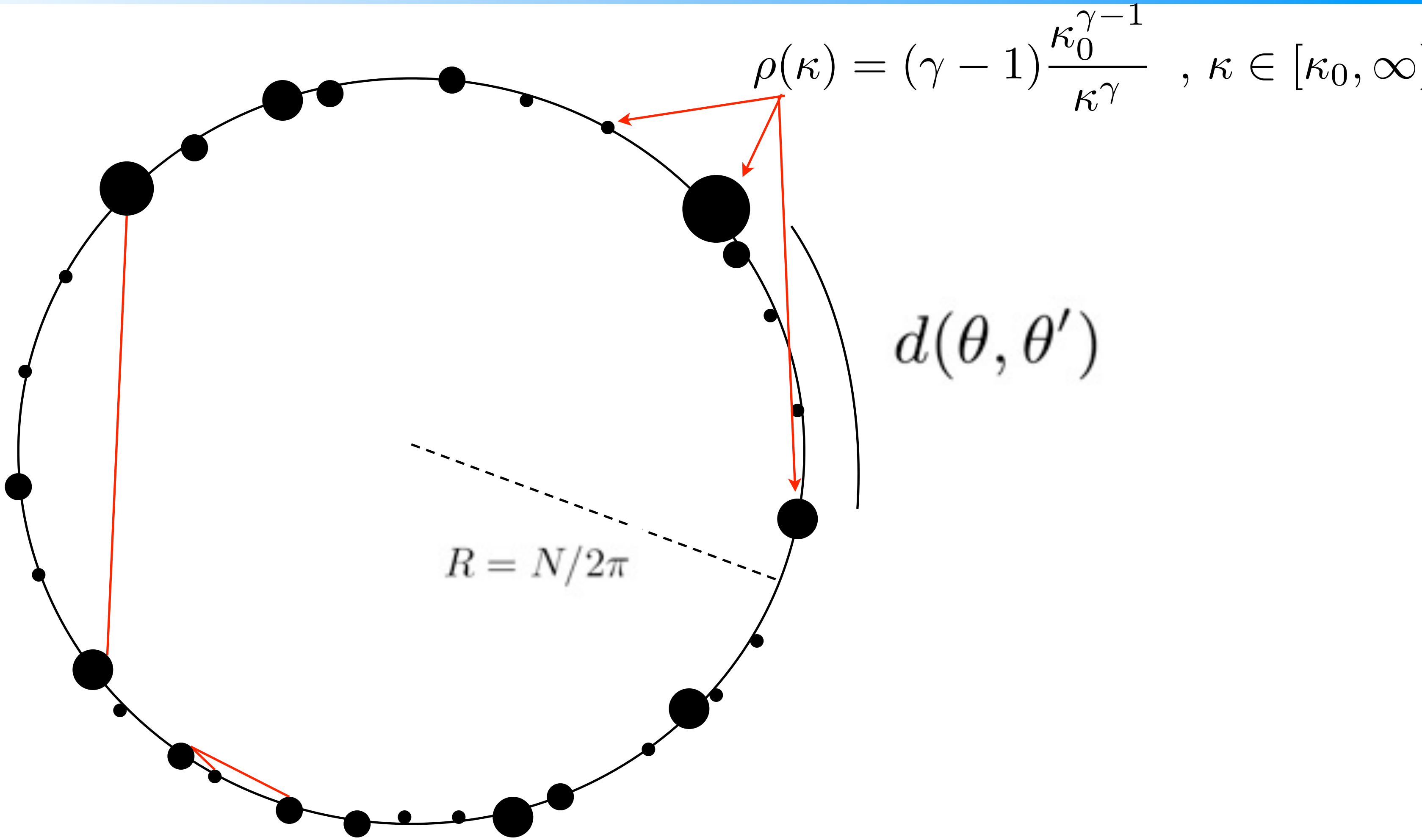

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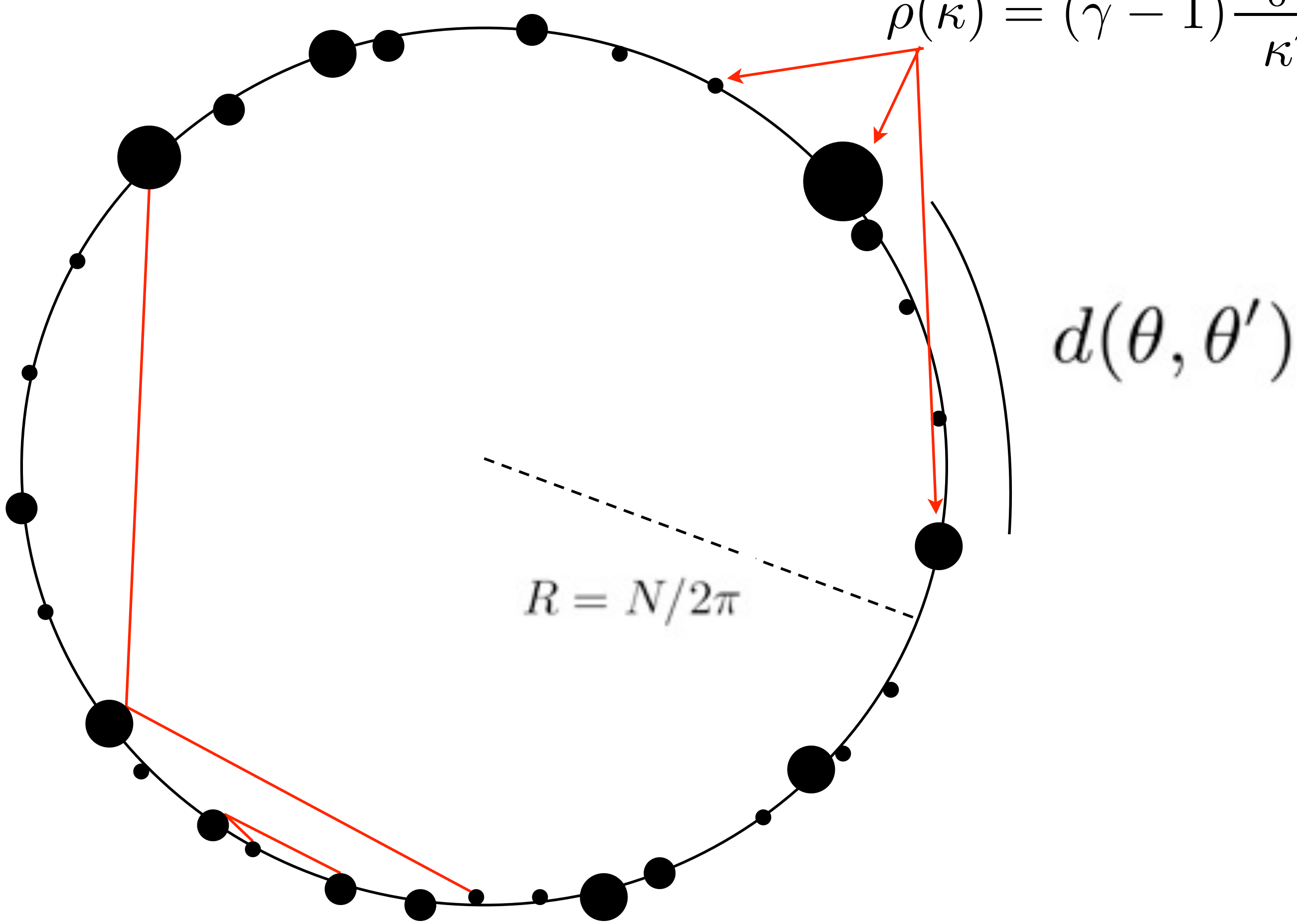

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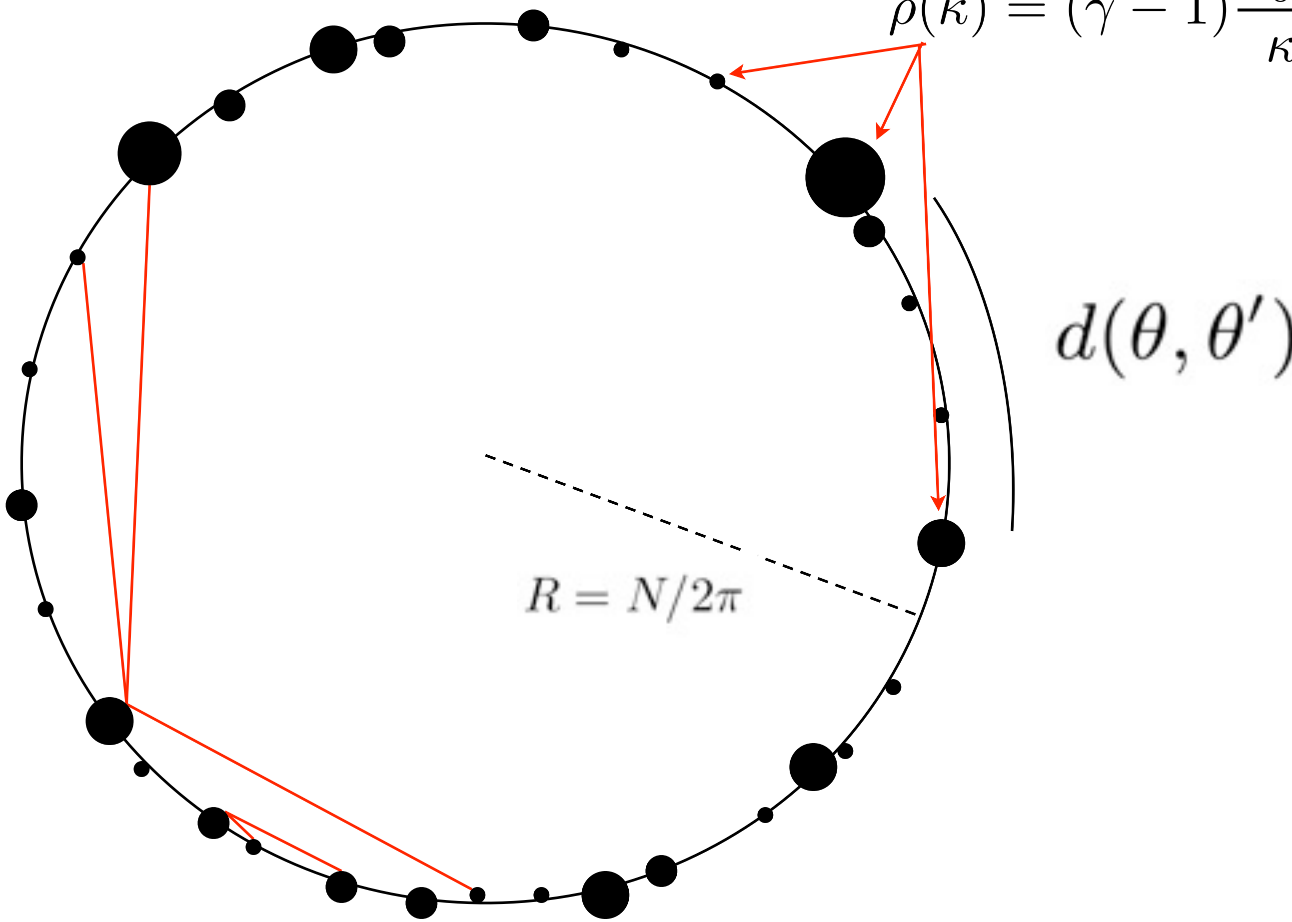

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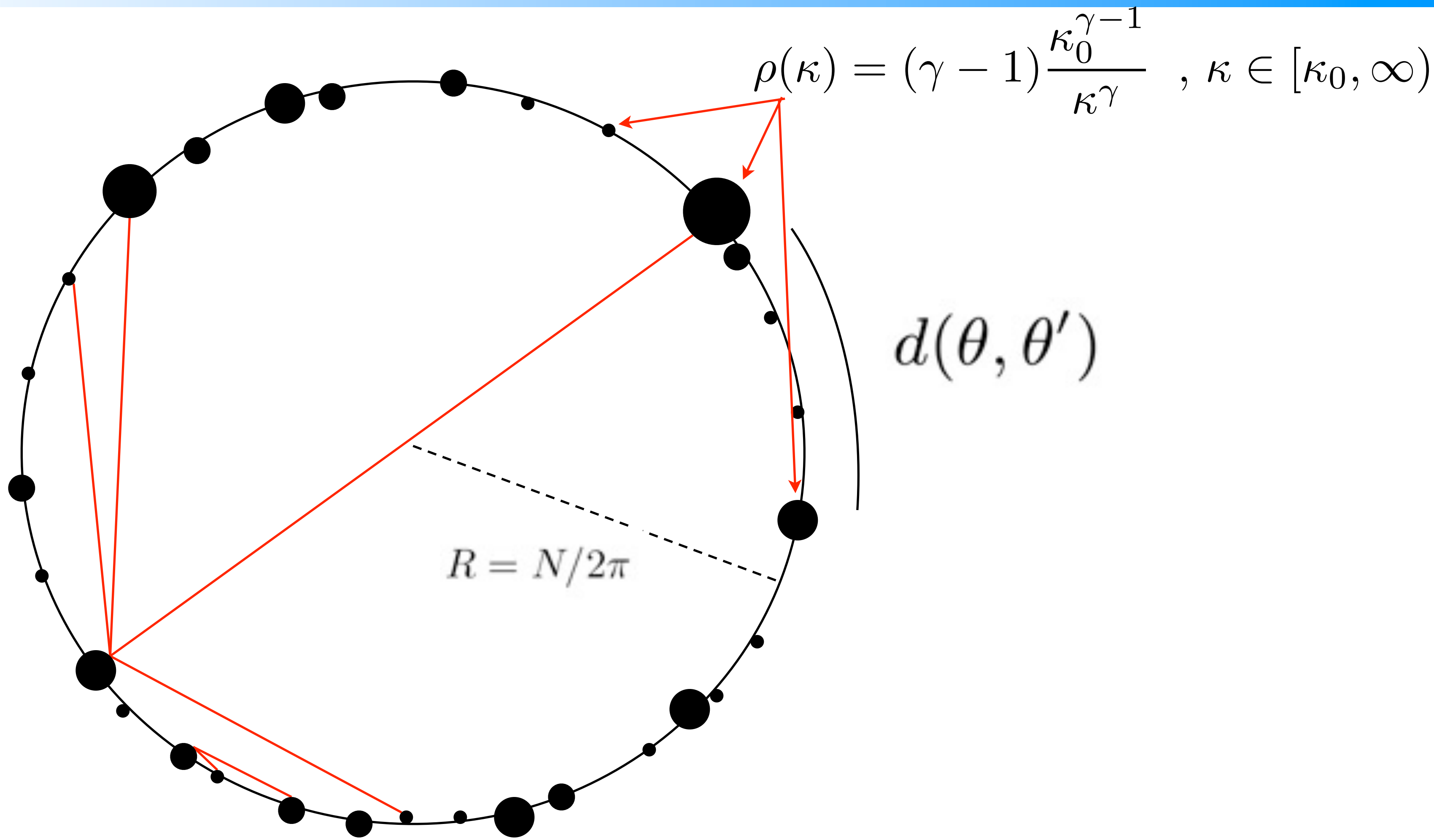

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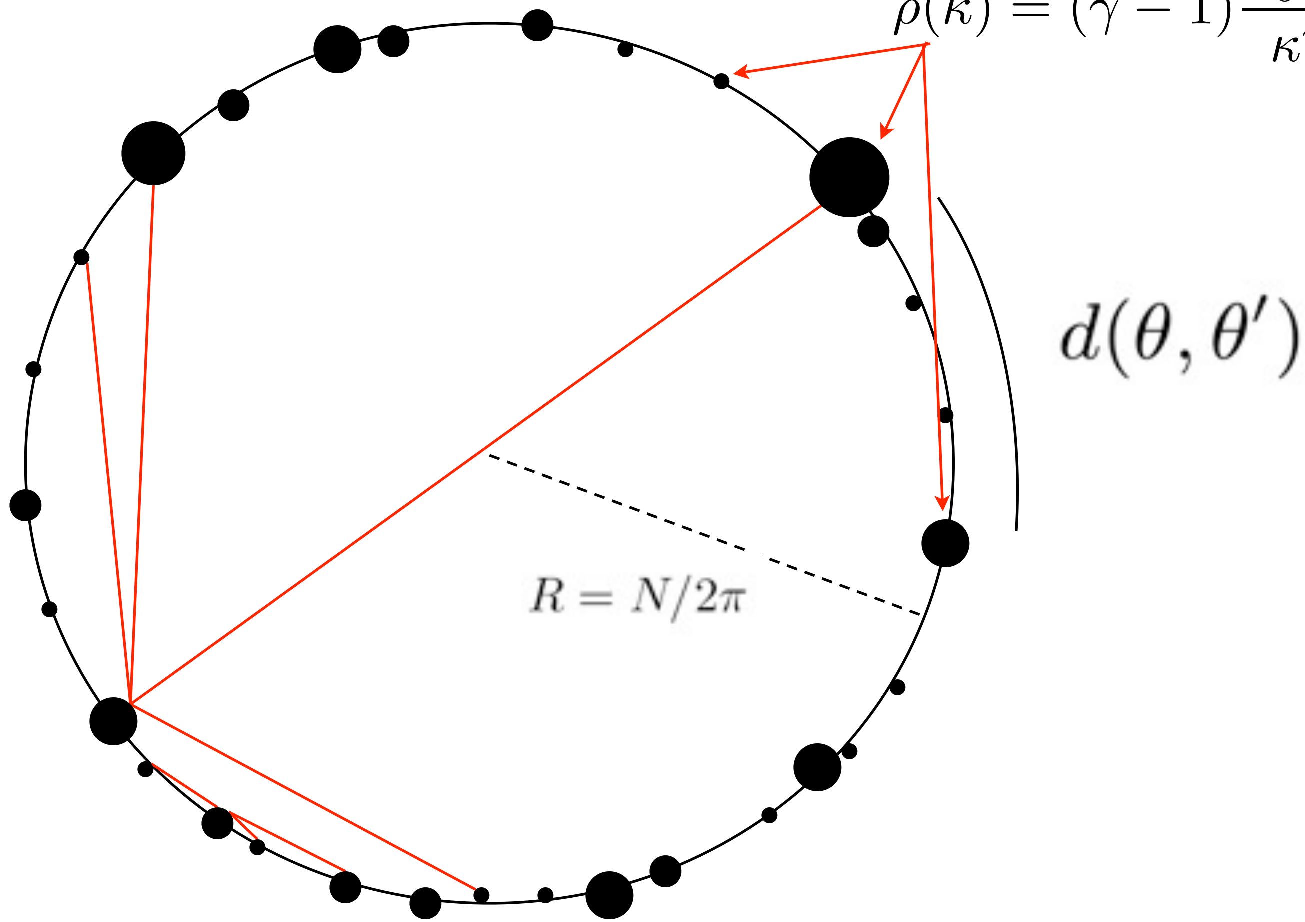

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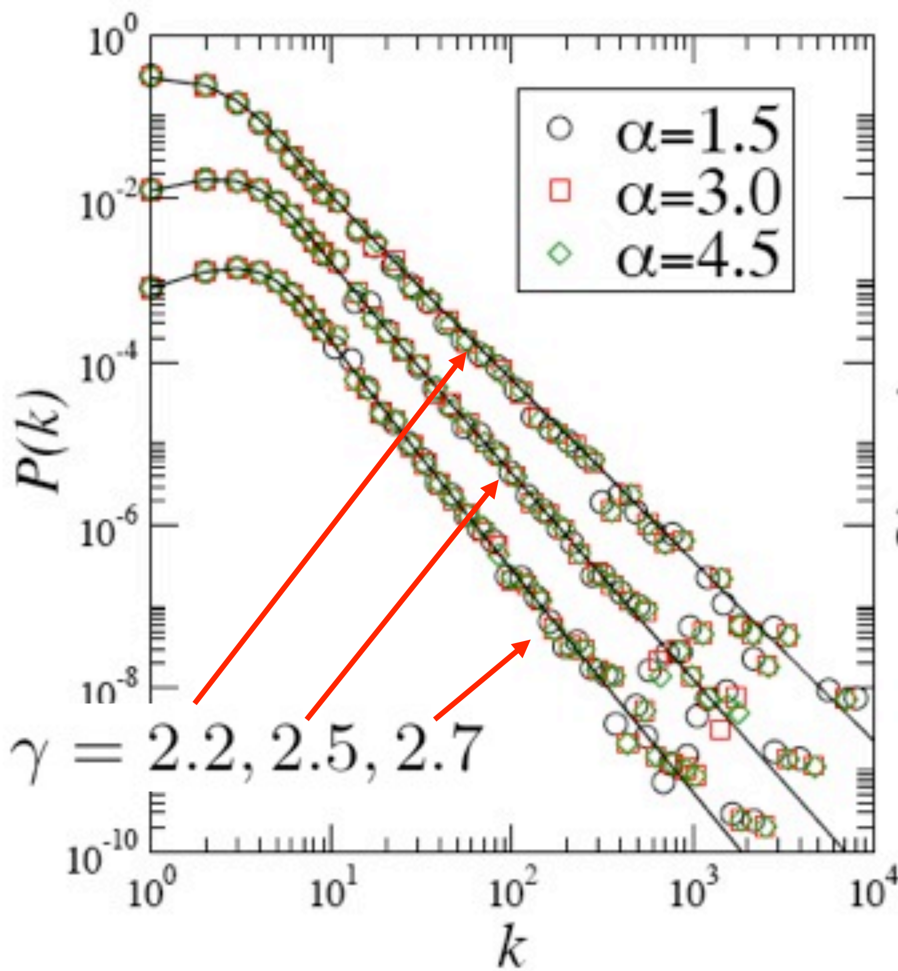

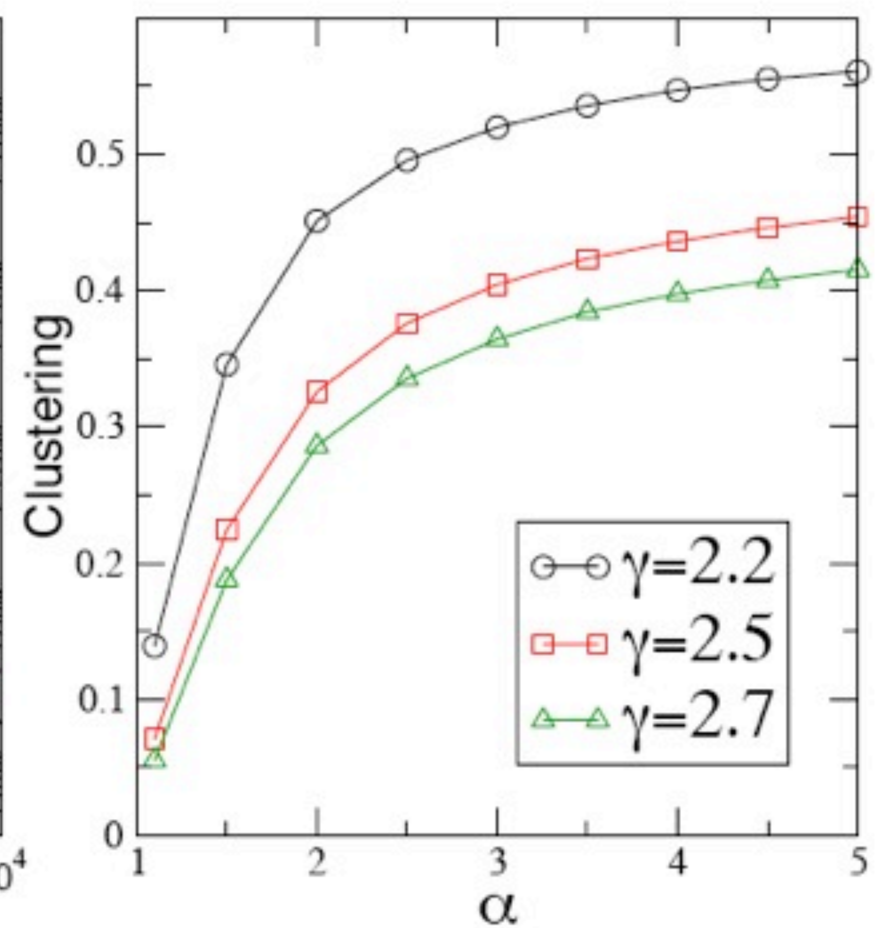

Naoki Masuda, Hiroyoshi Miwa, and Norio Konno. Phys. Rev. E 71, 036108 (2005)
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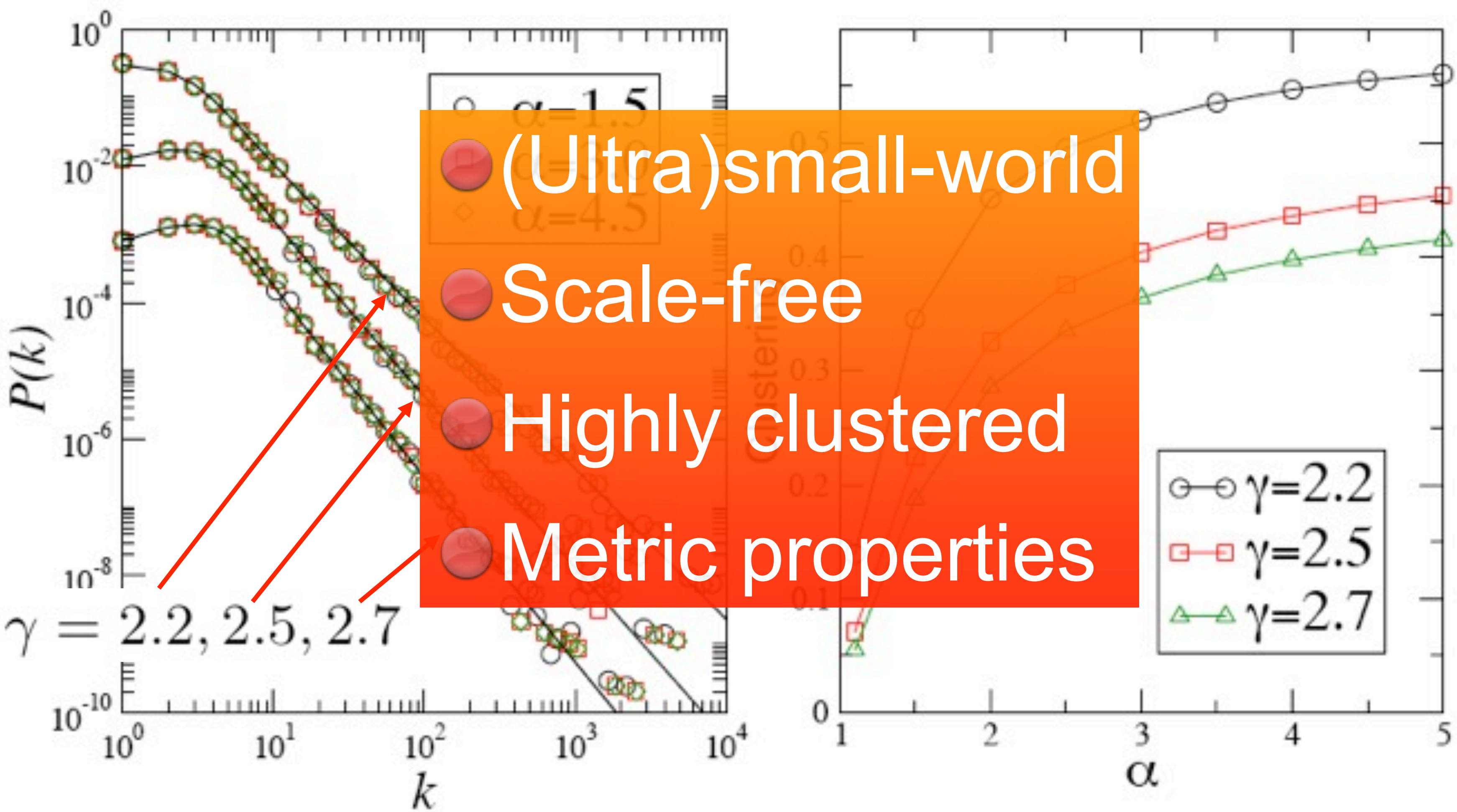

- (Ultra)small-world
- Scale-free
- Highly clustered
- Metric properties

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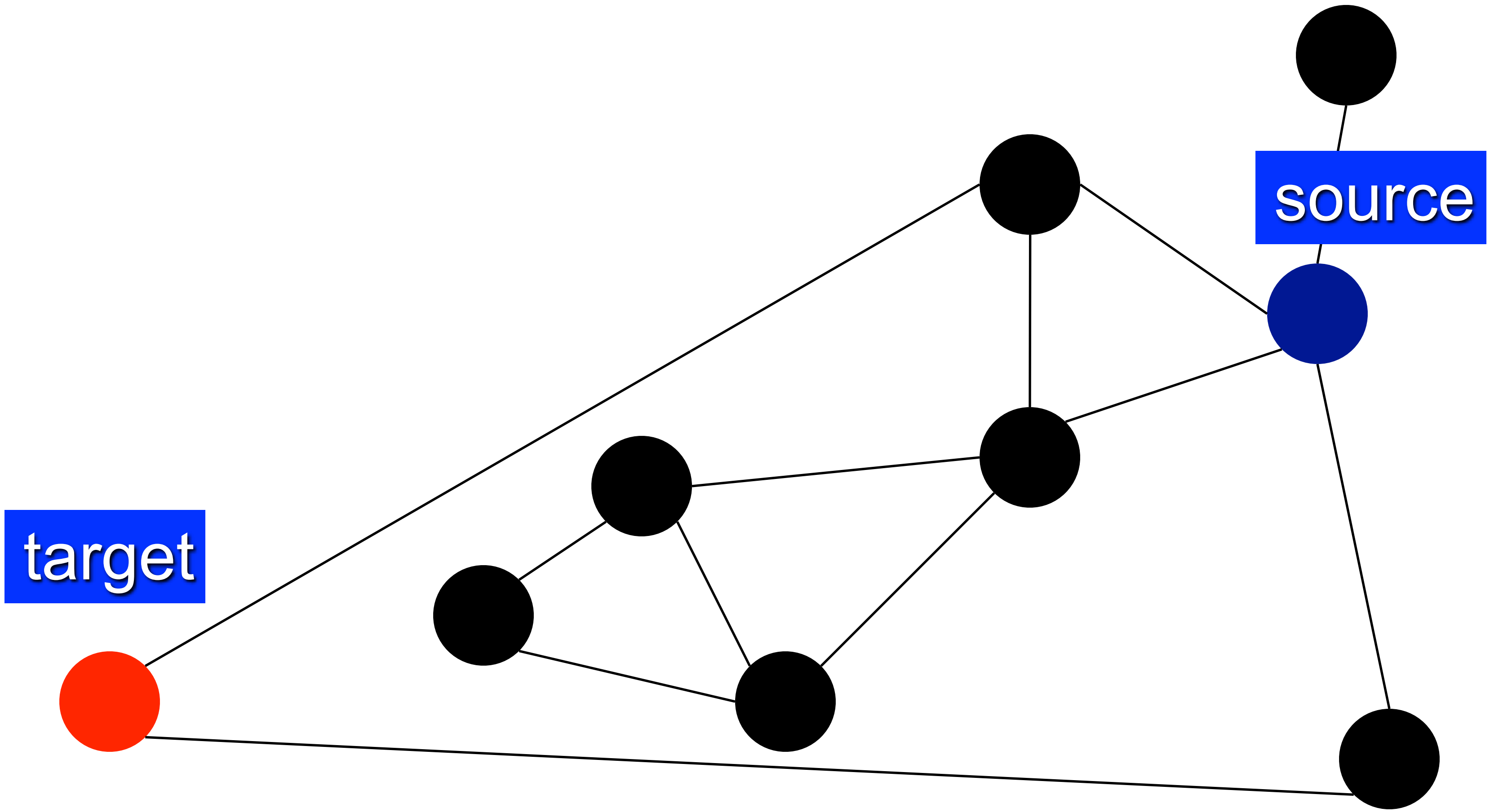
Greedy routing: Success probability

Messages can get trapped in terminal vertices



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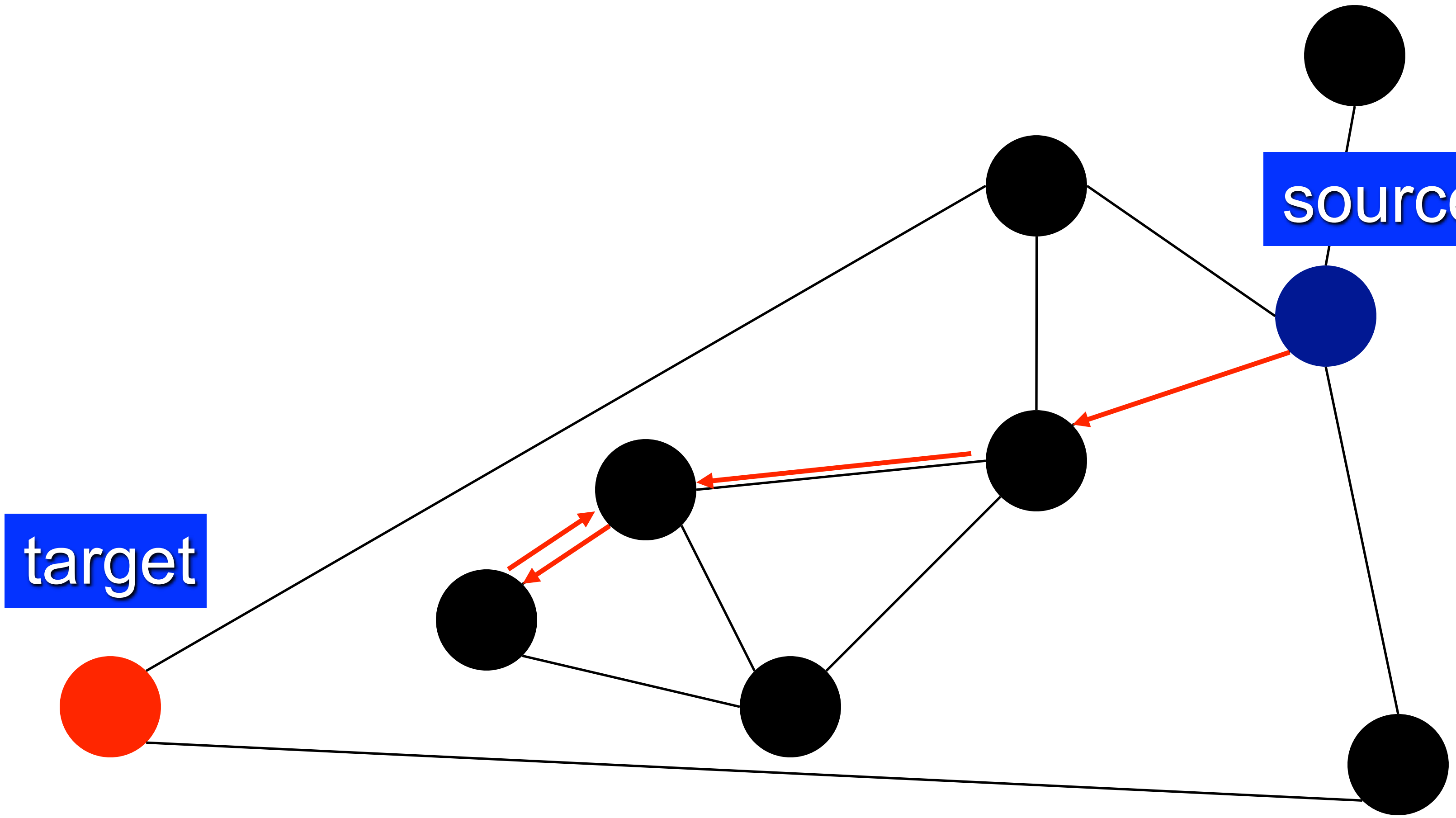

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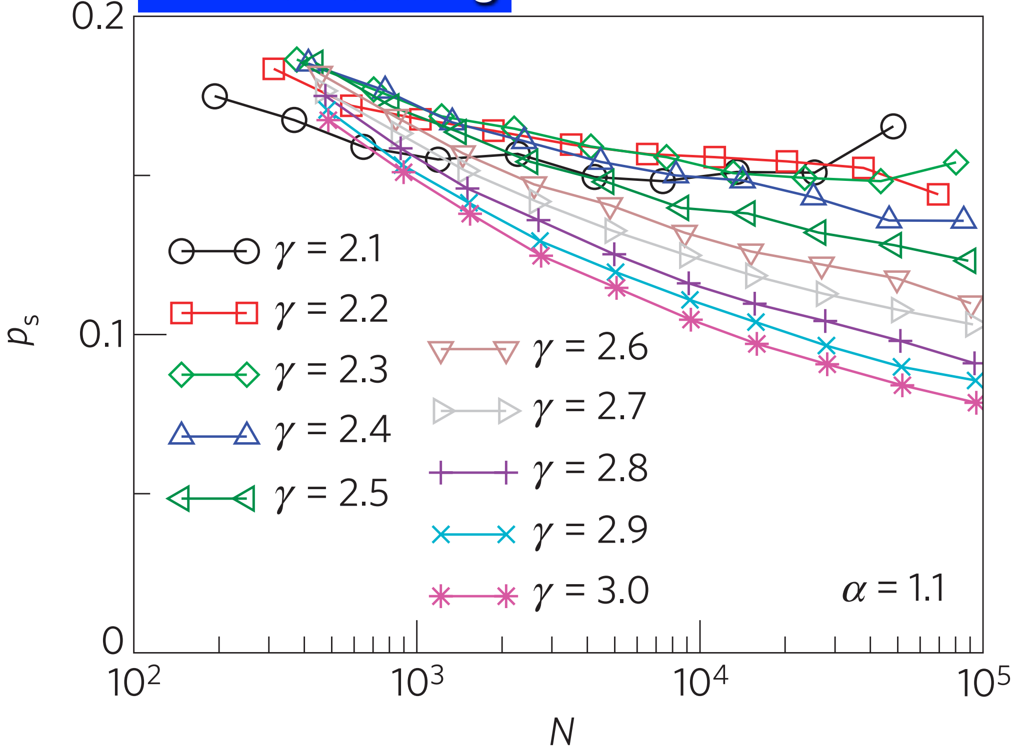

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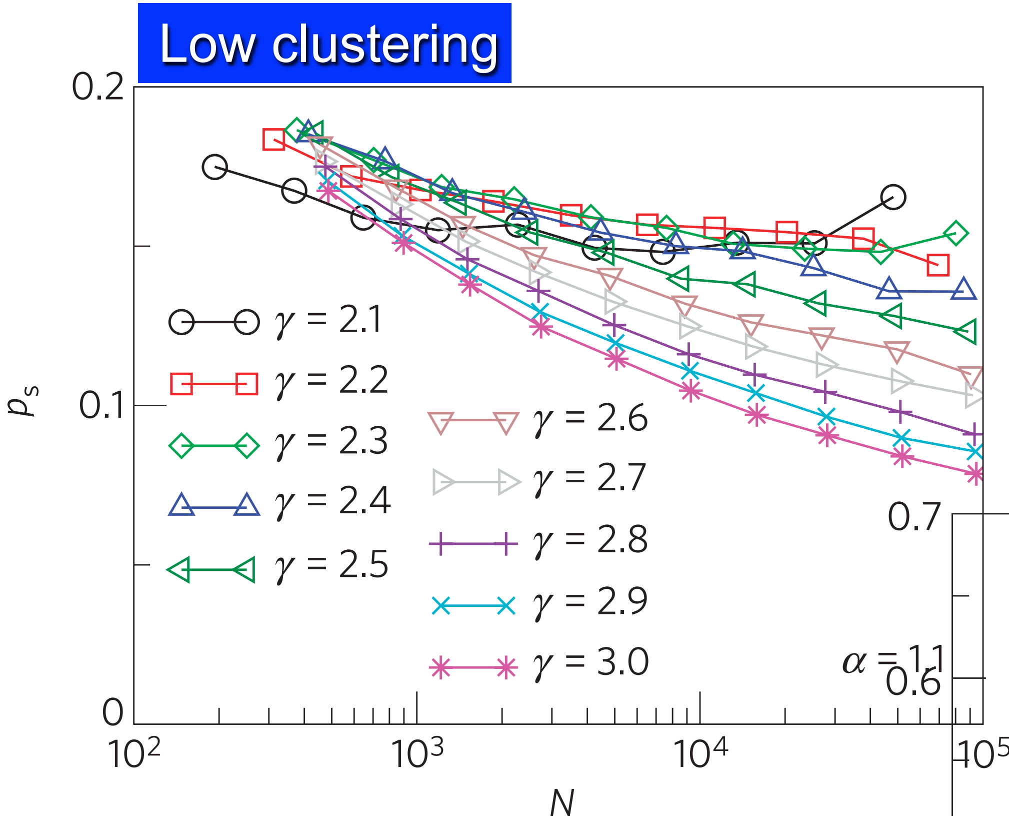
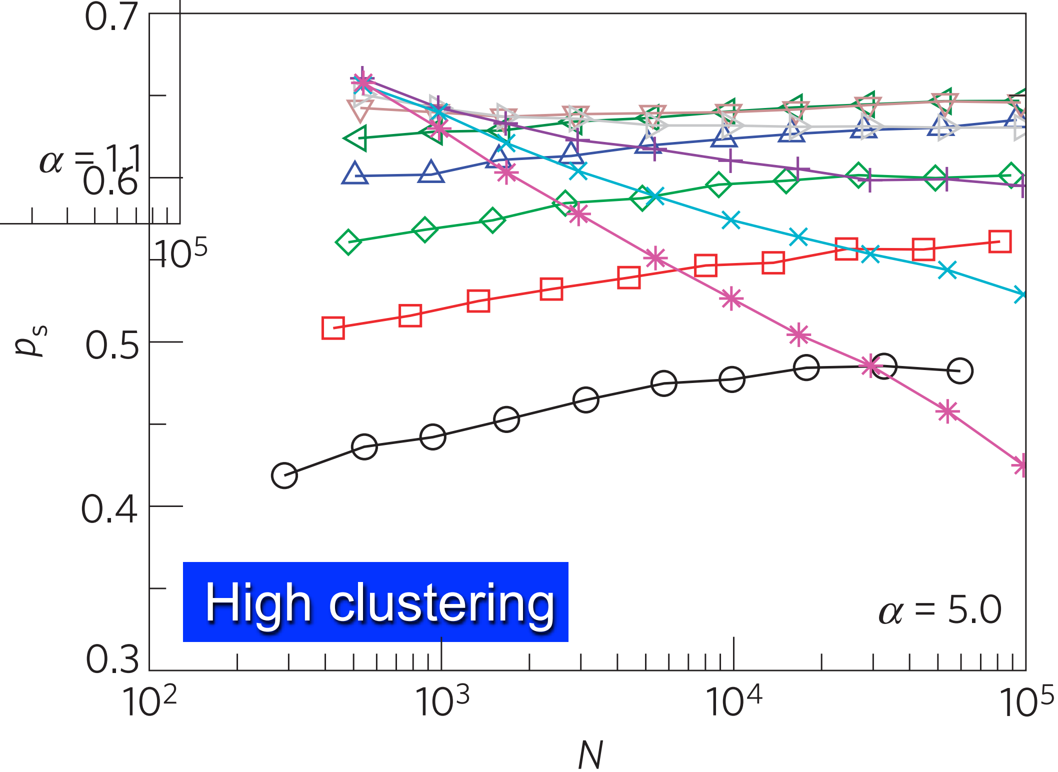
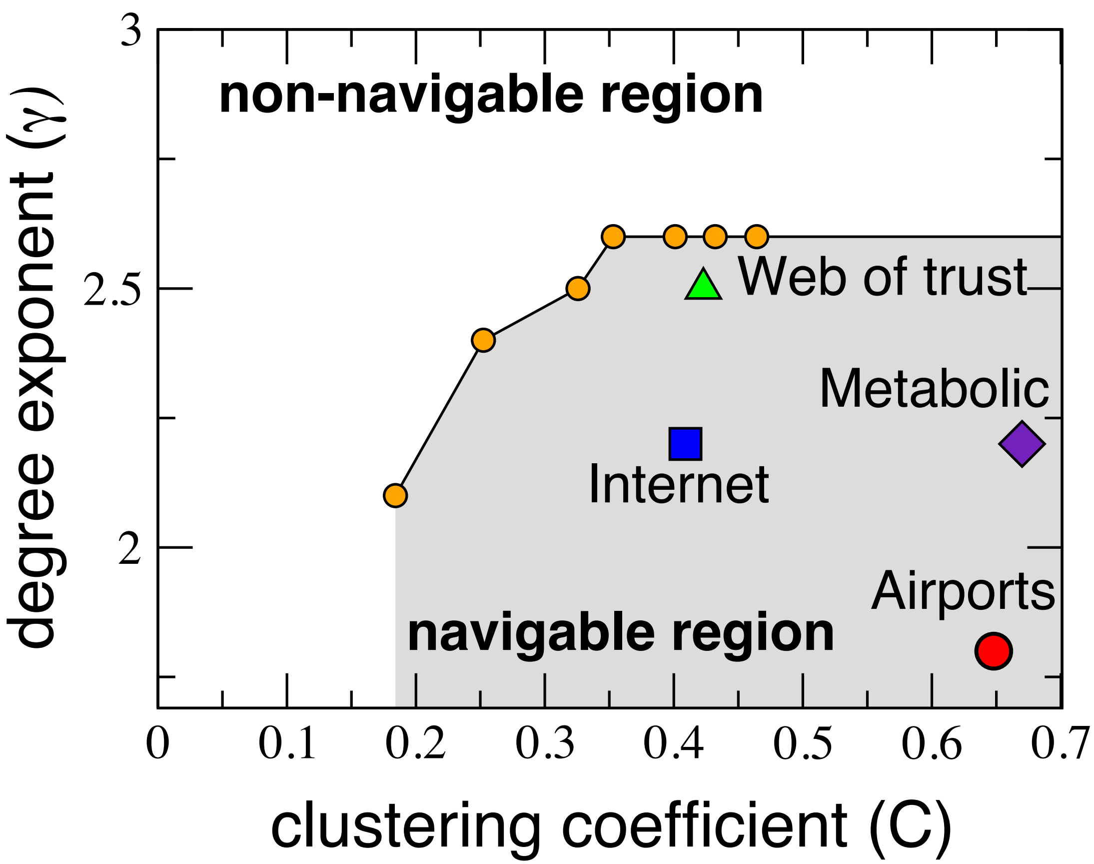




Greedy routing:navigable region

- **The more heterogeneous and clustered, the more efficient the navigability**



M. B., D. Krioukov, and kc claffy. Nature Physics 5, 74 (2009)
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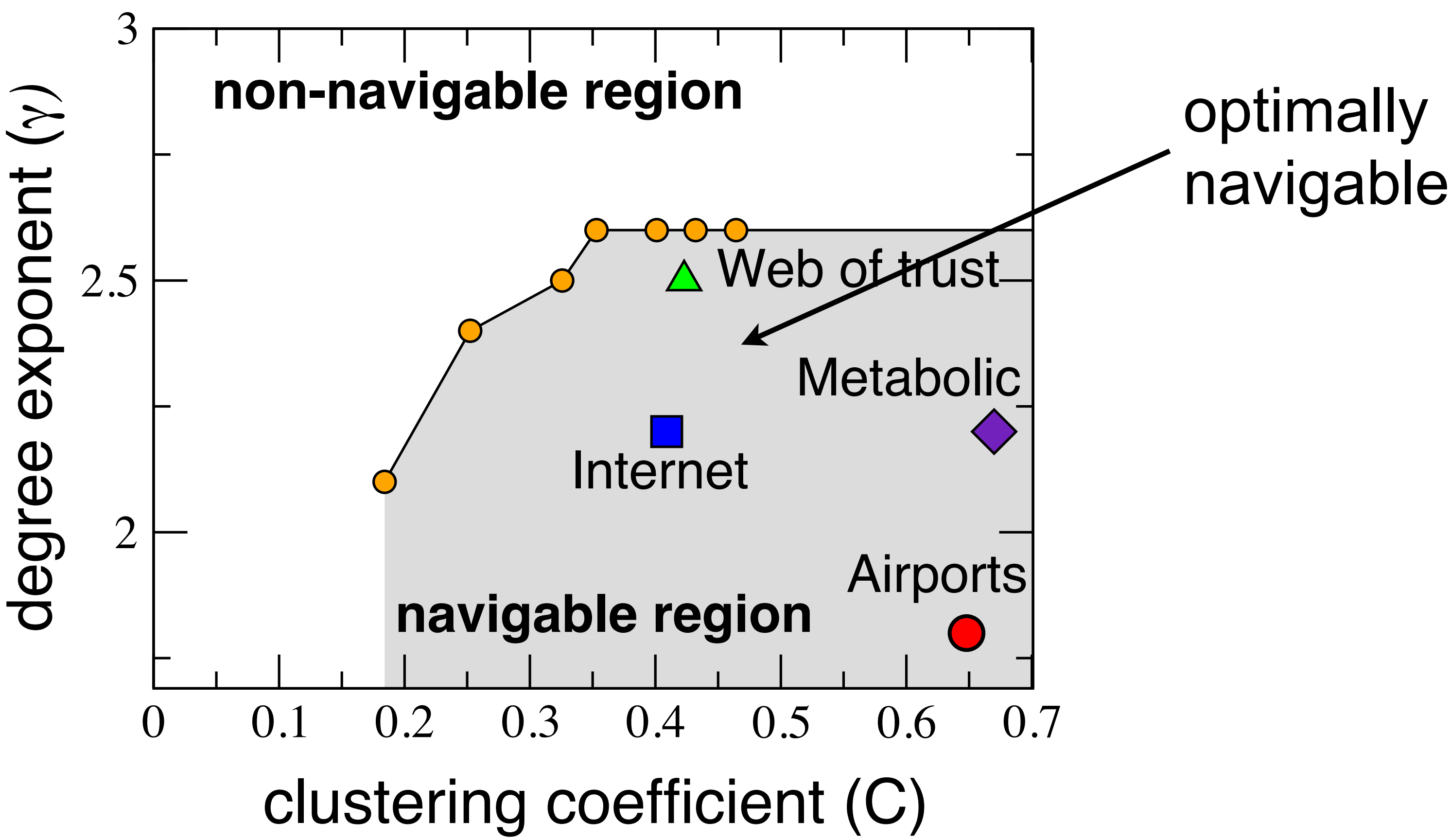

M. B., D. Krioukov, and kc claffy. Nature Physics 5, 74 (2009)
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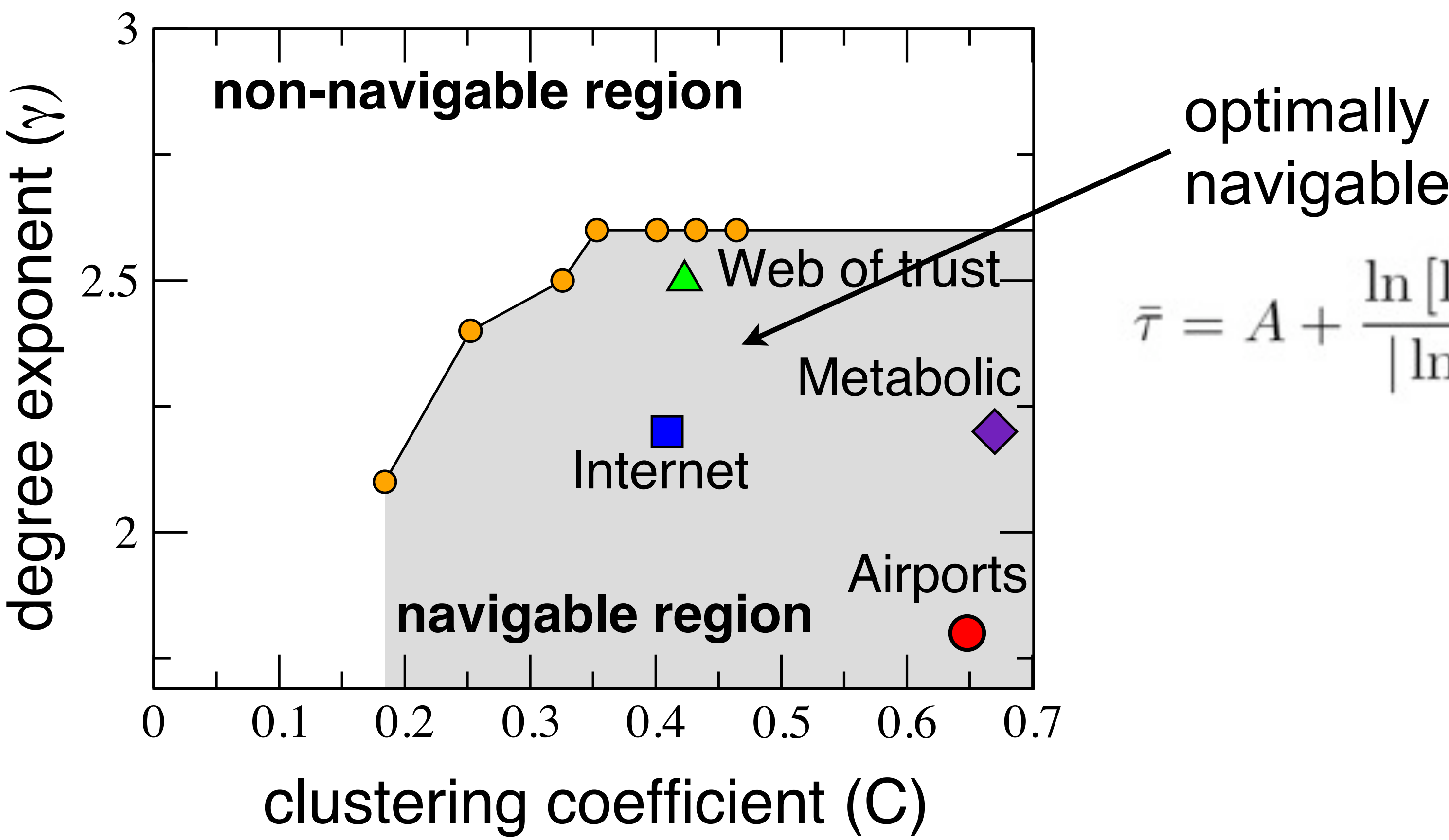

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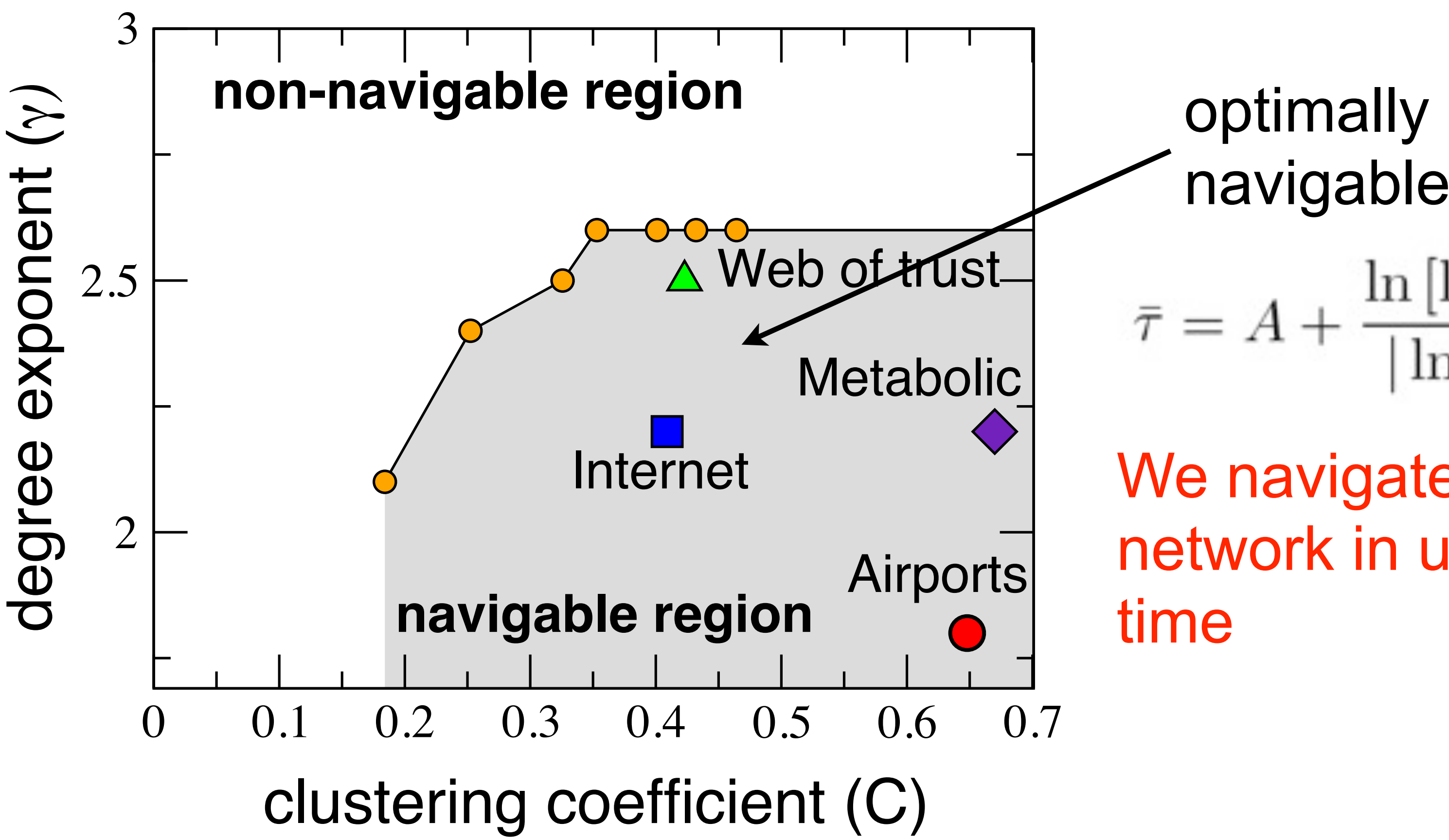

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We navigate the network in ultrashort time

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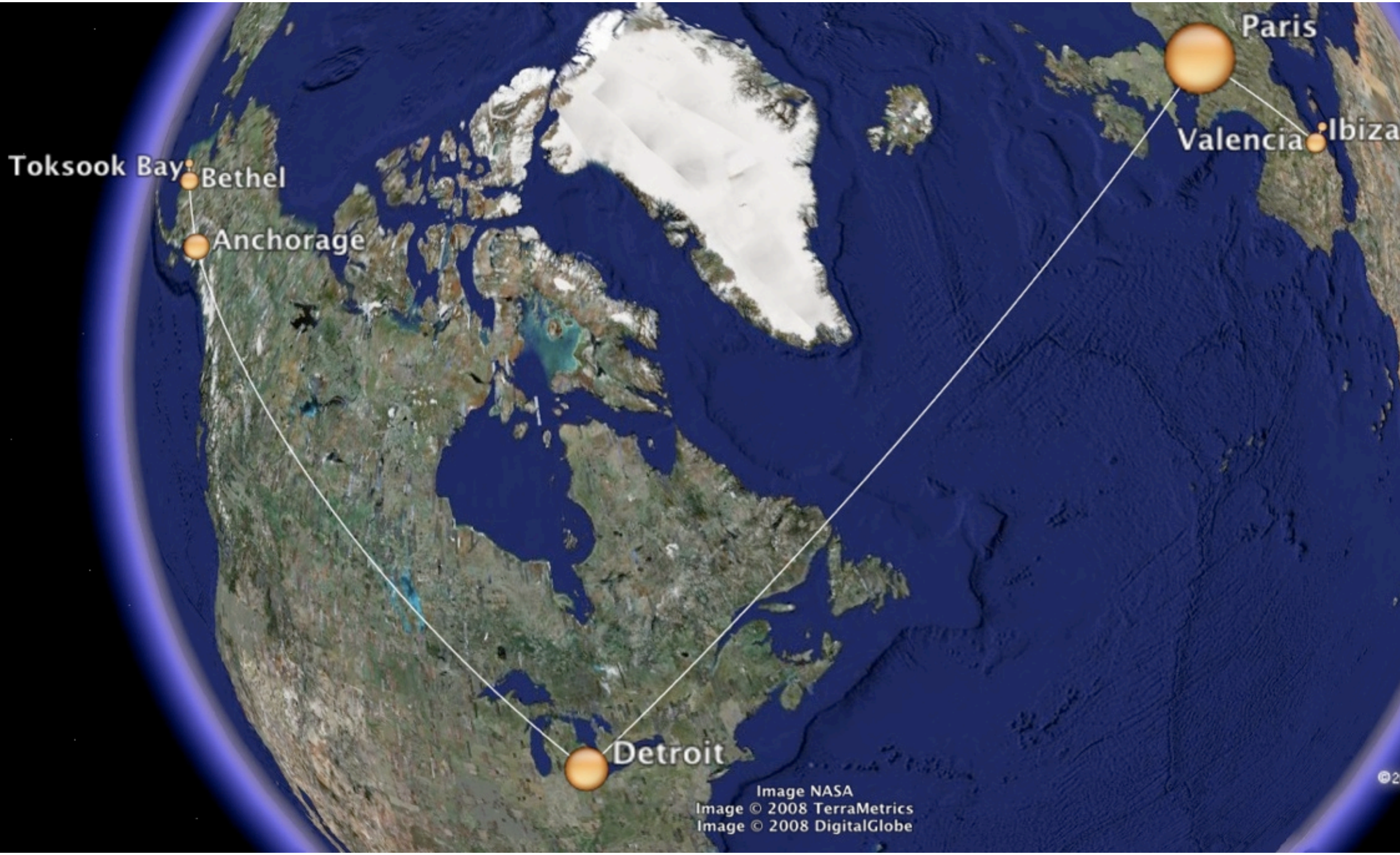
Greedy routing in the airport network



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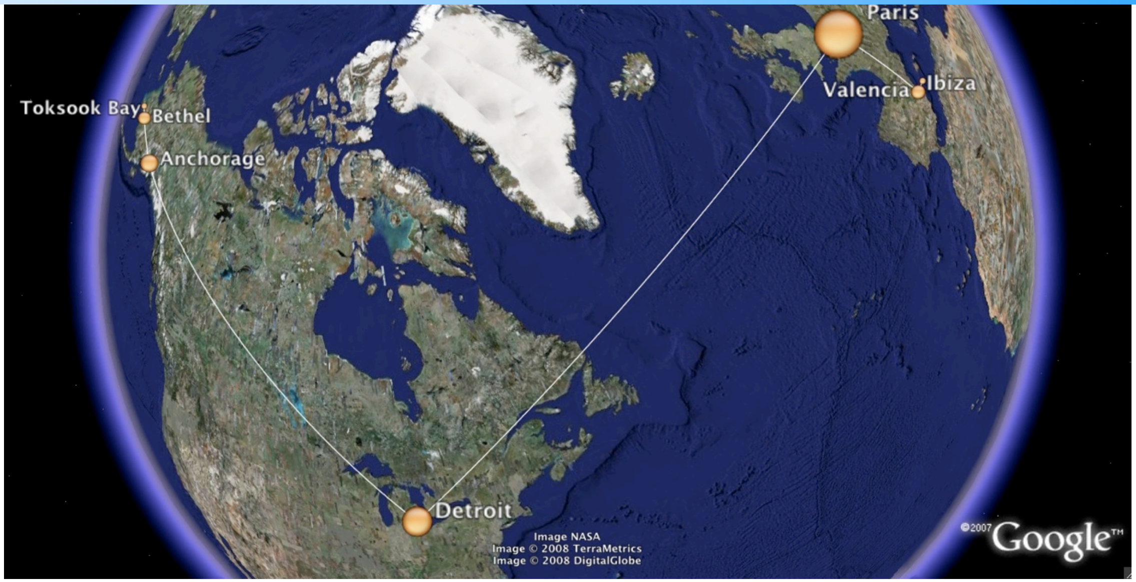

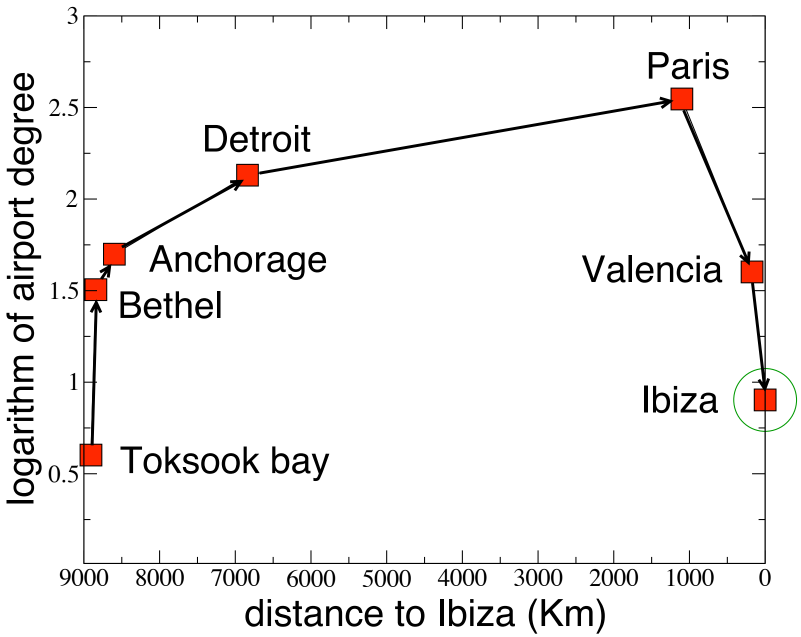

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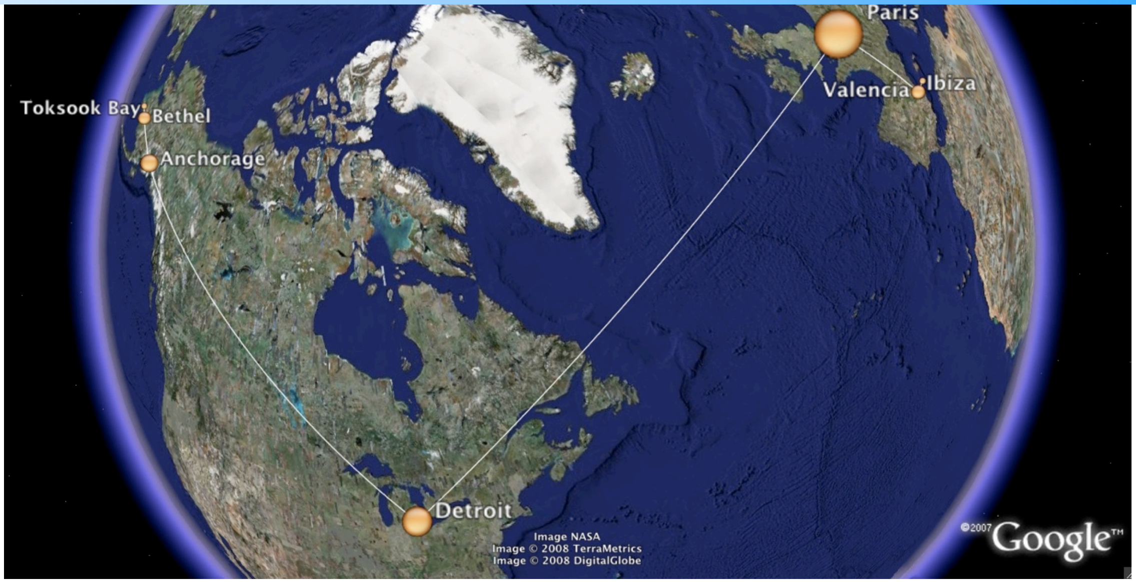
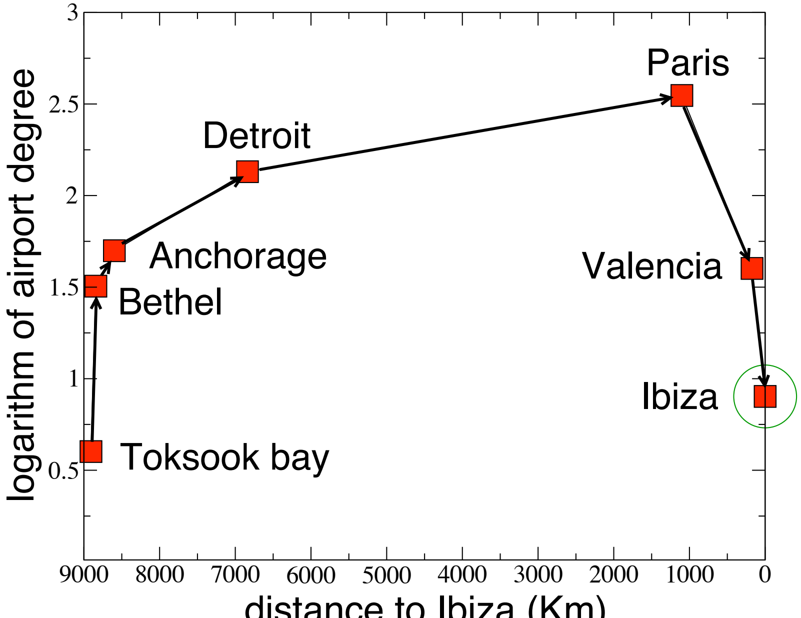
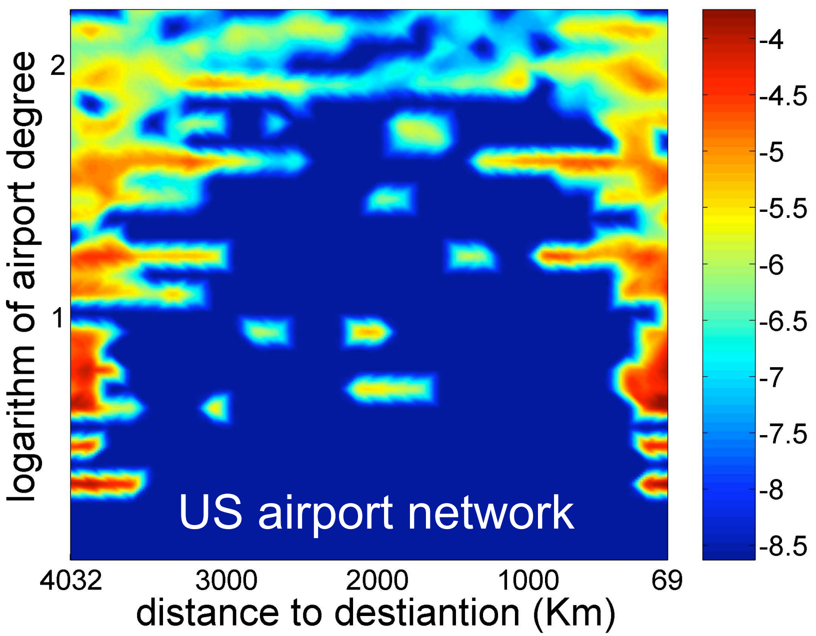
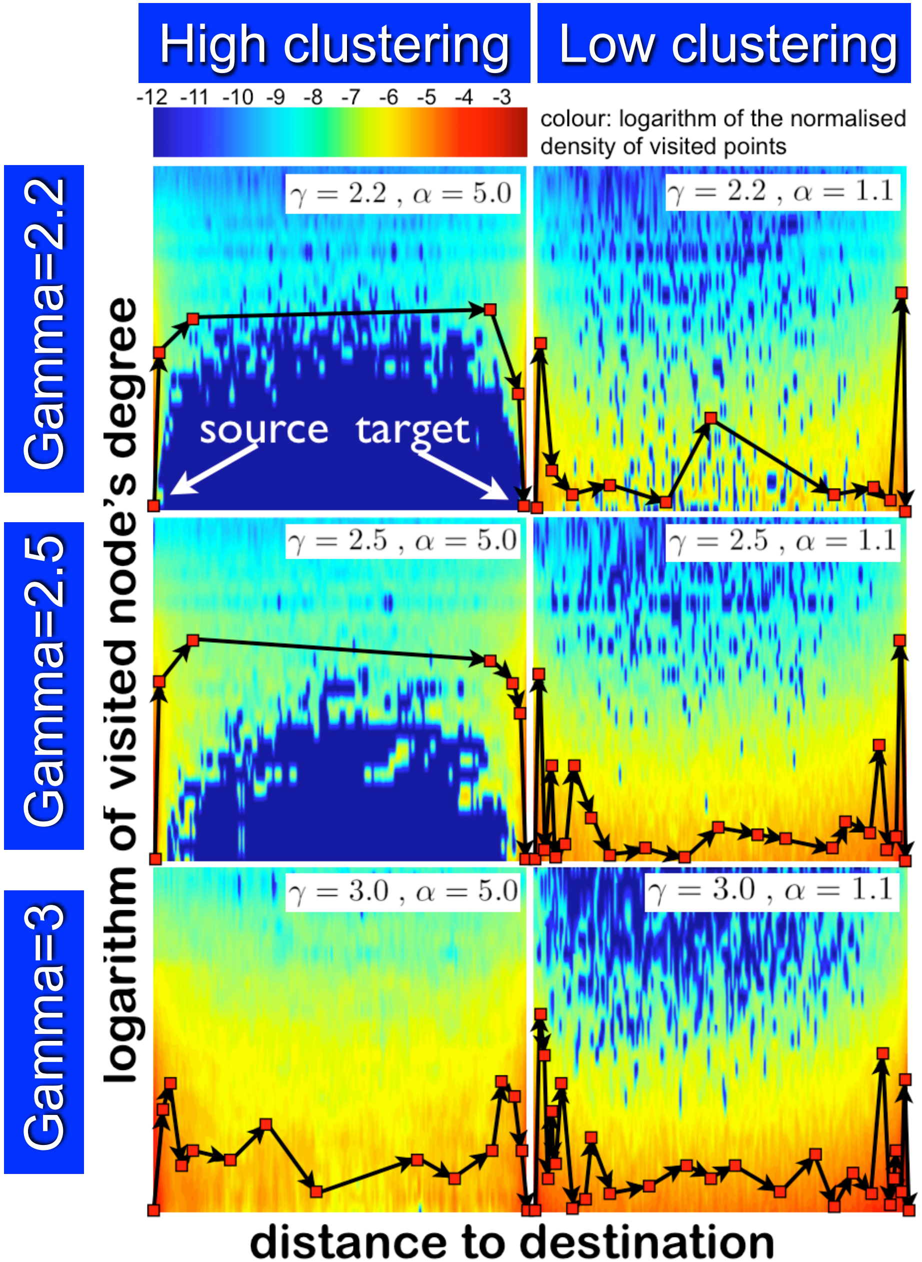






Phase space


High clustering
Low clustering
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3
colour: logarithm of the normalised density of visited points
Gamma=2.2
Gamma=2.5
Gamma=3
$\gamma = 2.2$, $\alpha = 5.0$
$\gamma = 2.2$, $\alpha = 1.1$
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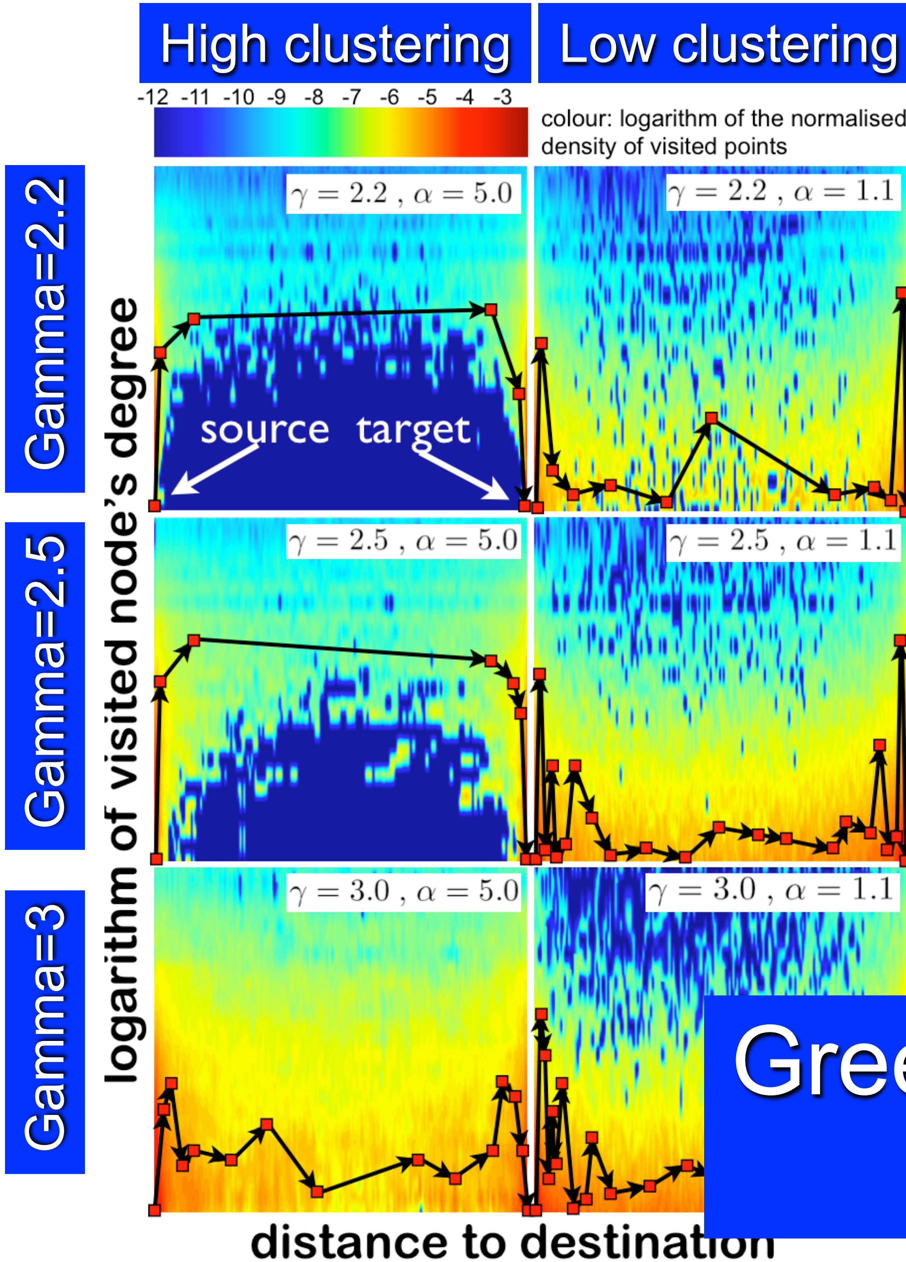
source
target
logarithm of visited node's degree
distance to destination

Phase space



Greedy routing is scalable

Still, the model does not yield perfect navigability. Not all pairs of nodes are connected by greedy routing paths

The reason is that the model introduces a non-geometric ingredient

For purely geometric graphs, the main problem comes from the small-world effect

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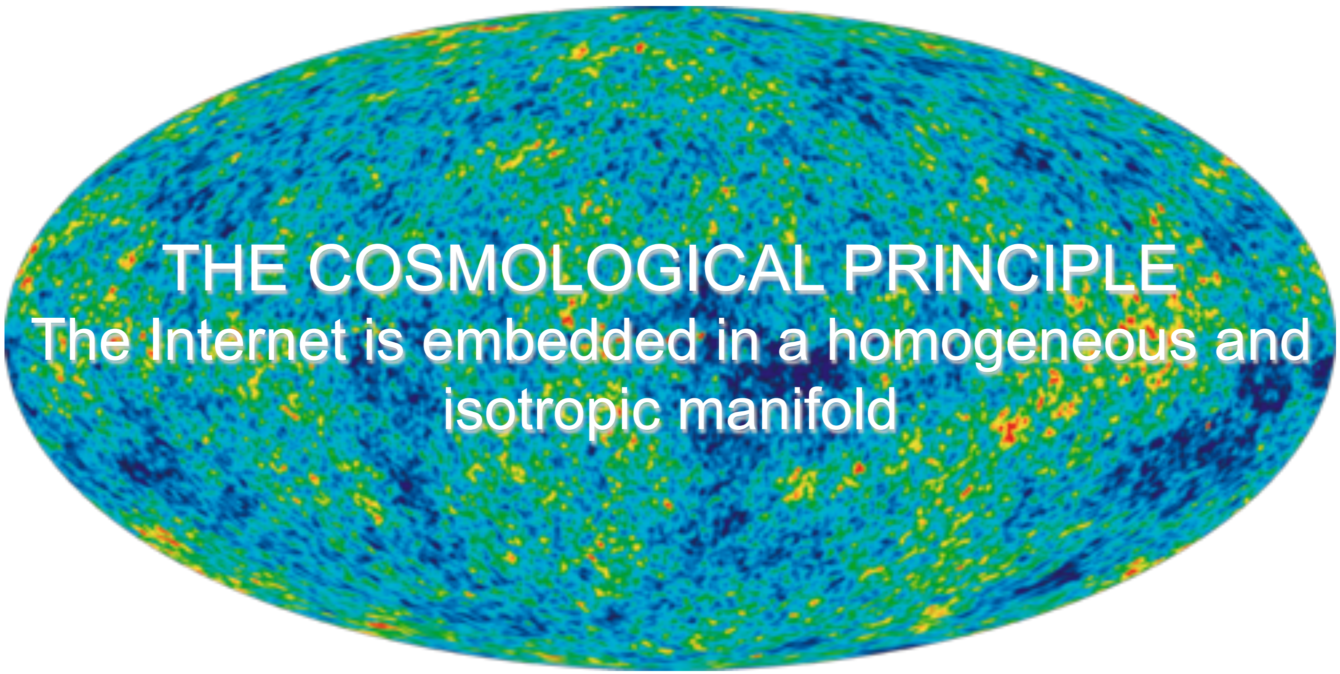
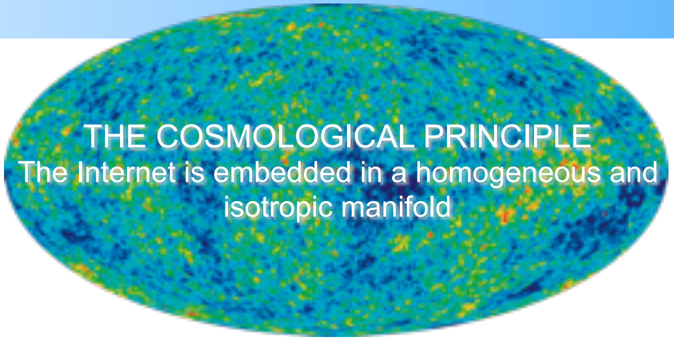
in Euclidean spaces it goes as

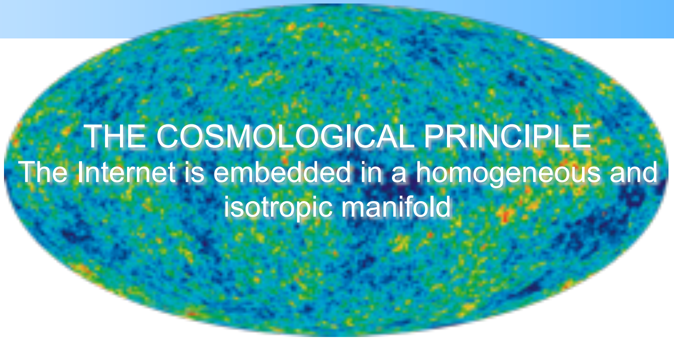
$$N(r) \sim r^D$$


THE COSMOLOGICAL PRINCIPLE
The Internet is embedded in a homogeneous and isotropic manifold

The cosmological principle

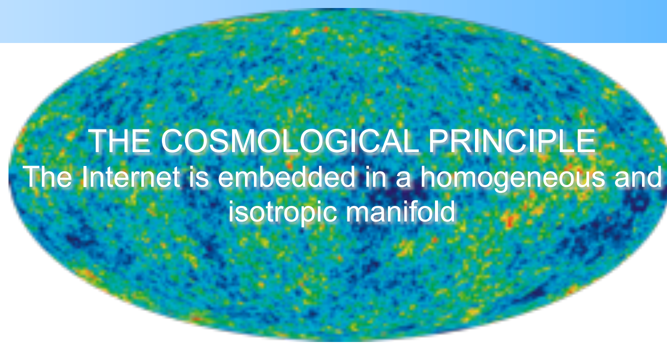


The cosmological principle



There are three types of homogeneous and isotropic spaces with constant curvature. For instance, in 2D

The cosmological principle



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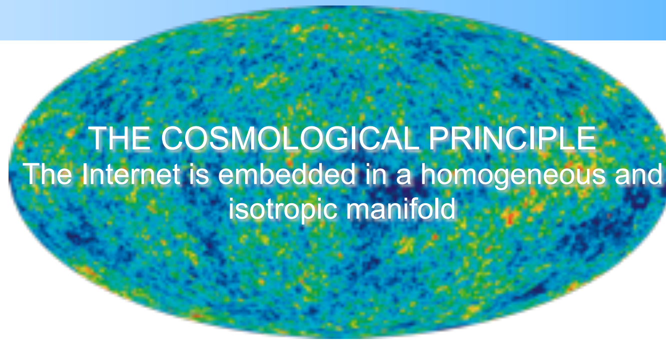

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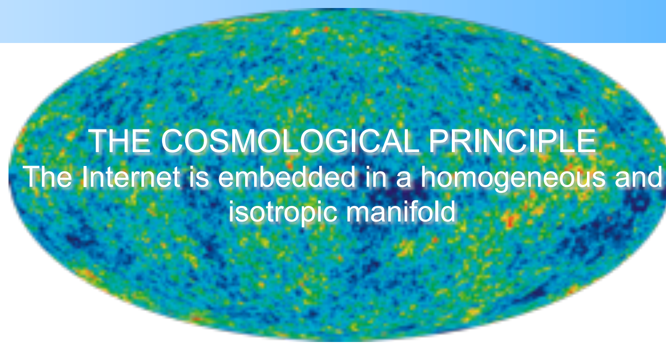

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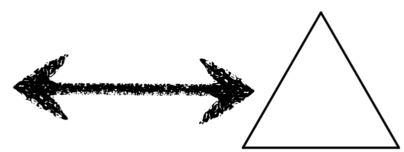
- Euclidean spaces $K=0$ $V(r) \sim r^2$ 

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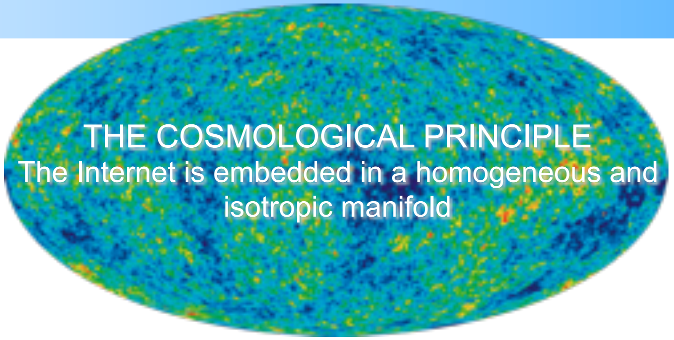

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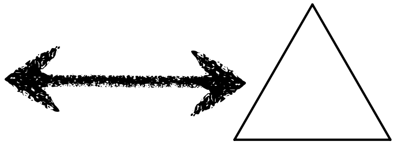
- Euclidean spaces $K=0$ $V(r) \sim r^2$ 
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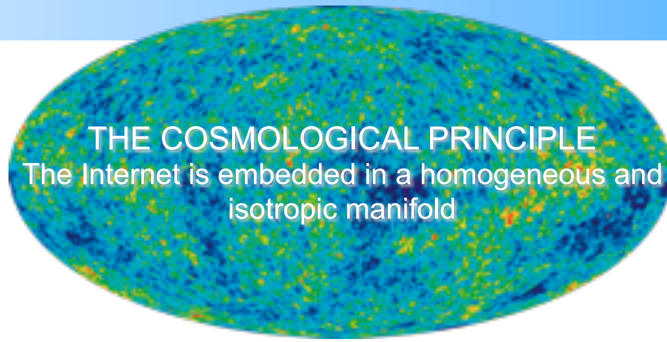

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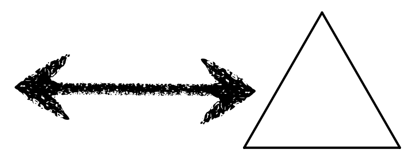
- Euclidean spaces $K=0$ $V(r) \sim r^2$ 
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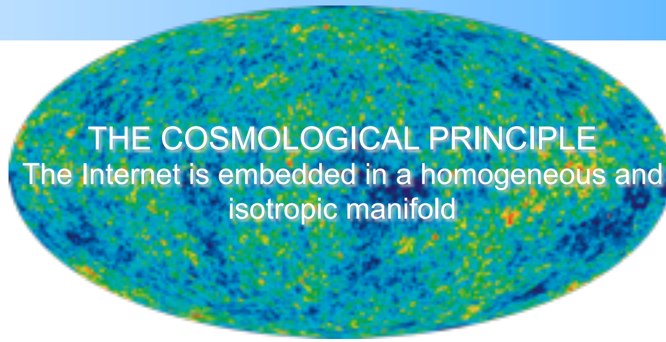

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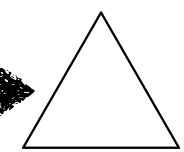
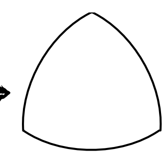
- Euclidean spaces   *K=0* $V(r) \sim r^2$ ⟷ 
- spherical spaces   *K>0* $V(r) \sim 1 - \cos r$ ⟷ 

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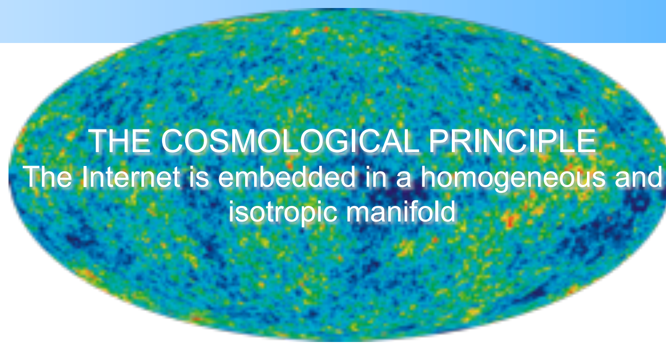

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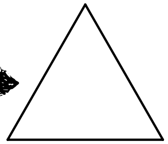
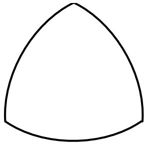
- Euclidean spaces $K=0$ $V(r) \sim r^2$ ⟷ 
- spherical spaces $K>0$ $V(r) \sim 1 - \cos r$ ⟷ 
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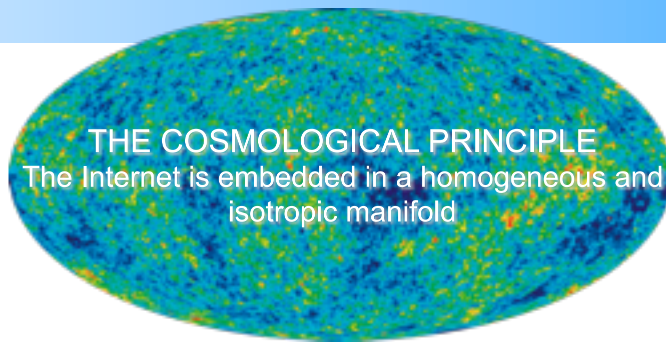

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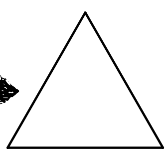
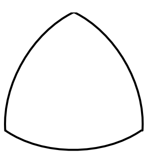
- Euclidean spaces $K=0$ $V(r) \sim r^2$ ⟷ 
- spherical spaces $K>0$ $V(r) \sim 1 - \cos r$ ⟷ 
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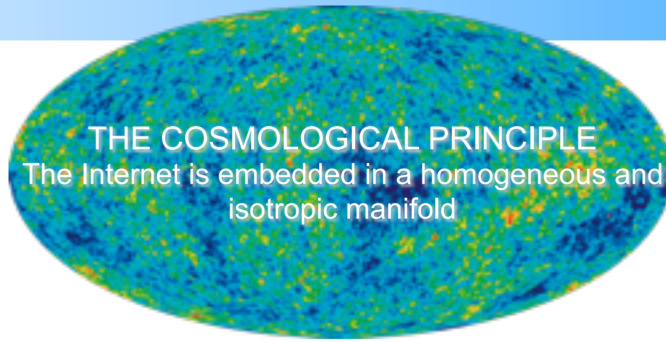

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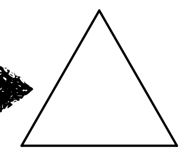
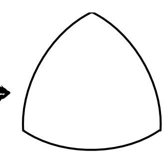
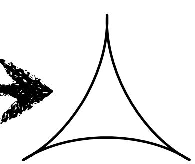
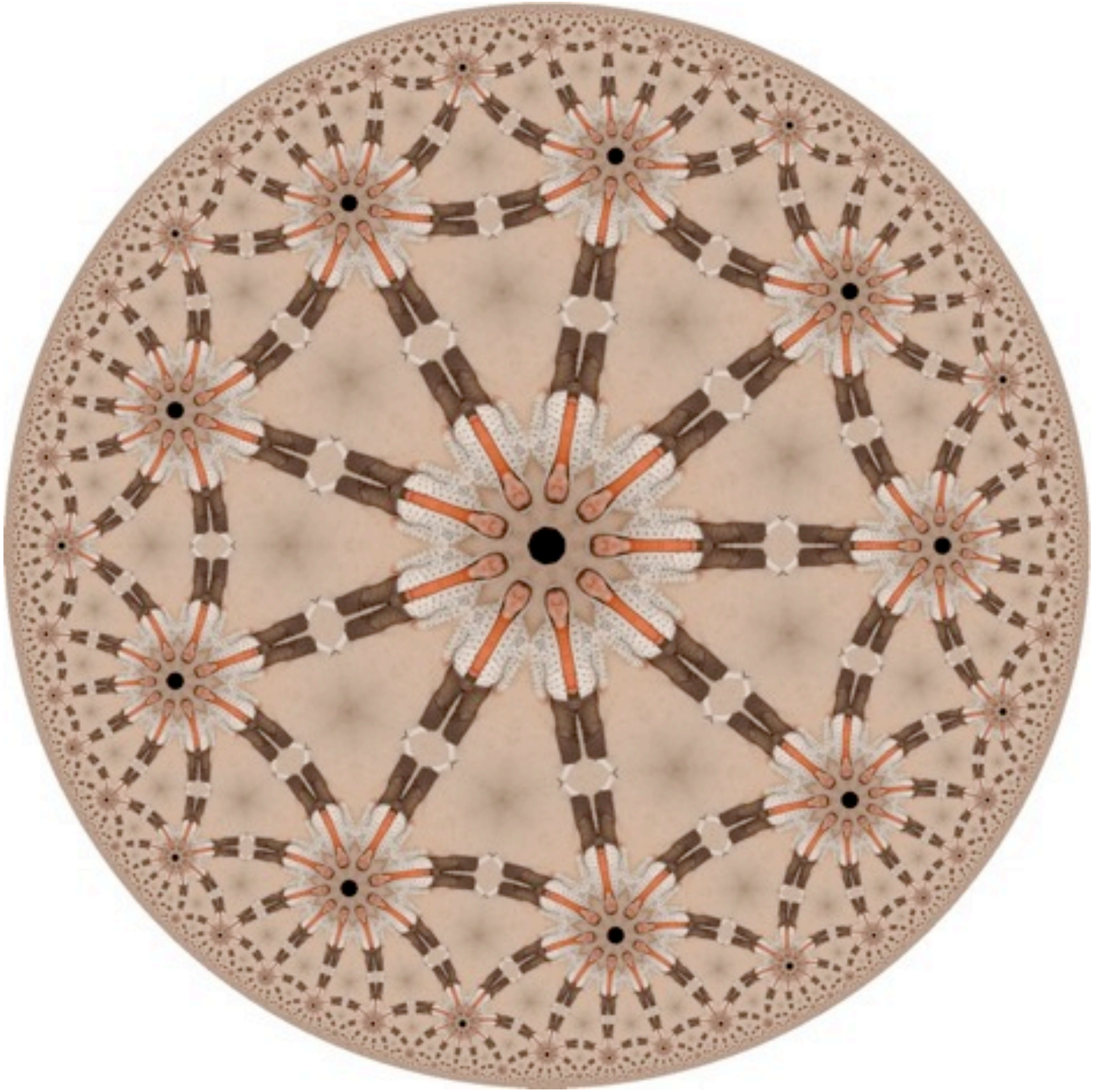
- Euclidean spaces $K=0$ $V(r) \sim r^2$ ⟷ 
- spherical spaces $K>0$ $V(r) \sim 1 - \cos r$ ⟷ 
- hyperbolic spaces $K<0$ $V(r) \sim \cosh r - 1$ ⟷ 

Poincare model of the hyperbolic plane



The simplest model in H^2. The Einsteinian-H^2 model

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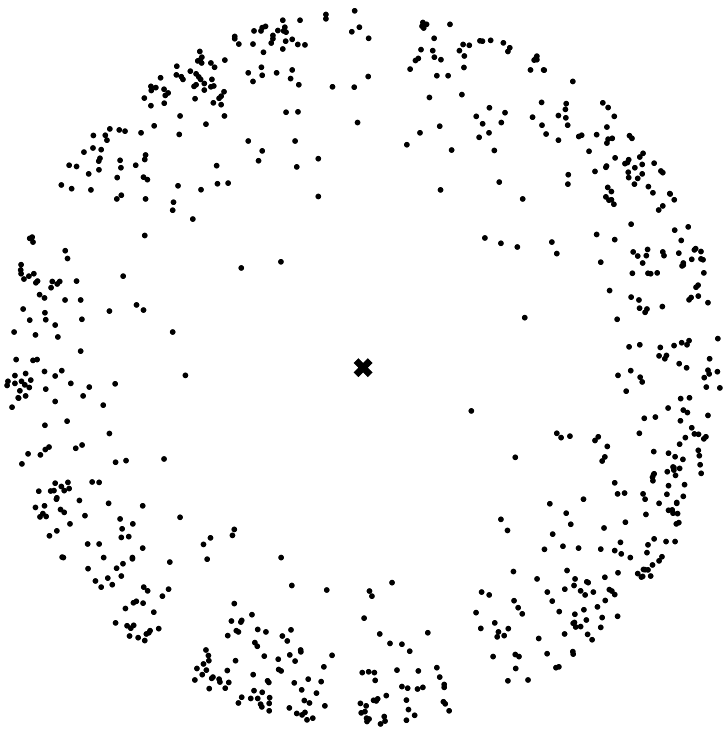
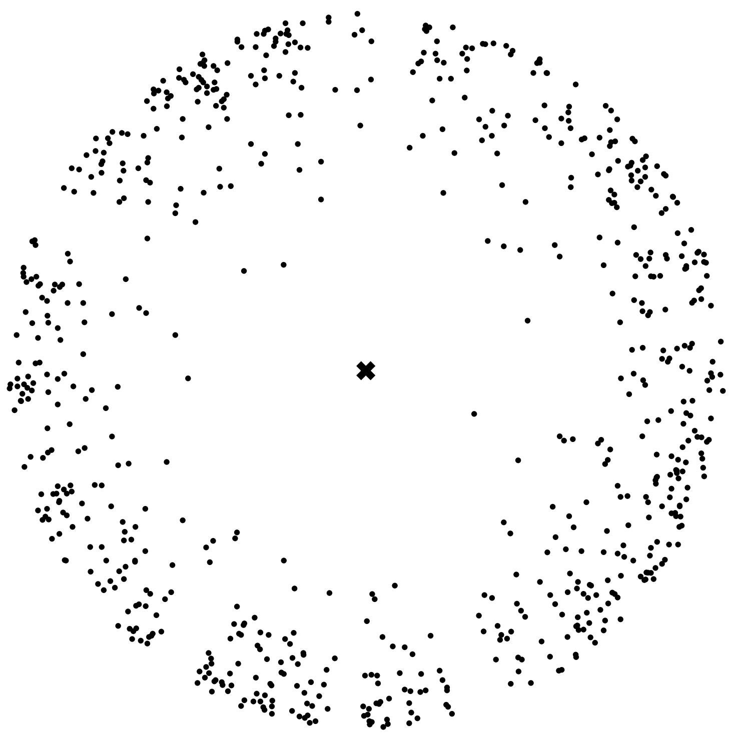


The simplest model in H²: The Einsteinian-H² model



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homogeneous distribution of points in the hyperbolic plane

$$p(x) = \Theta(R - x)$$

nodes at hyperbolic distances smaller than R become connected

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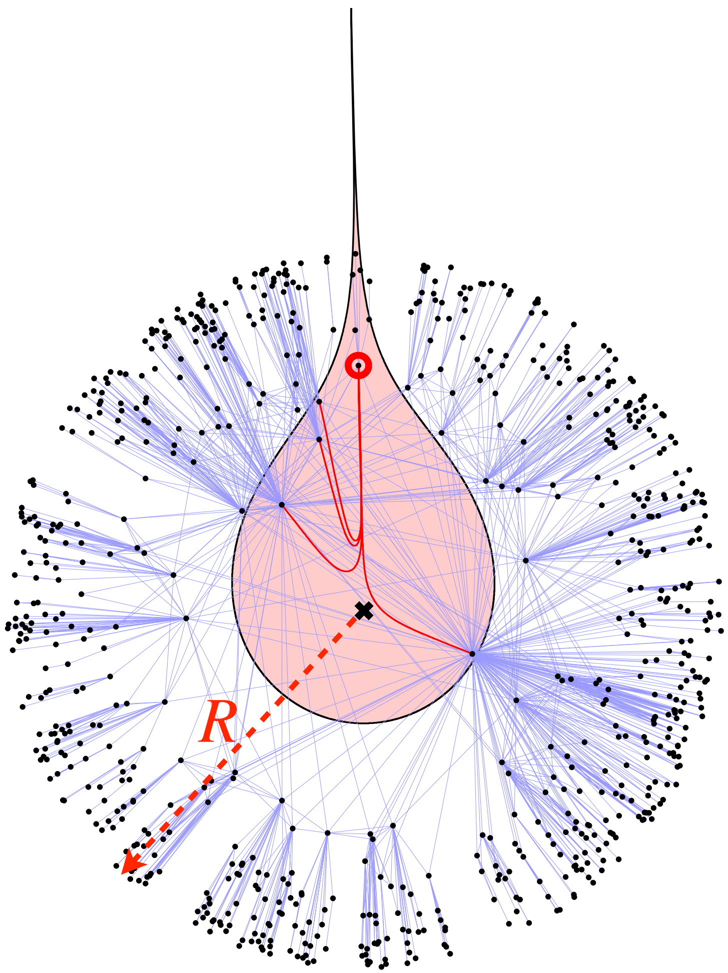

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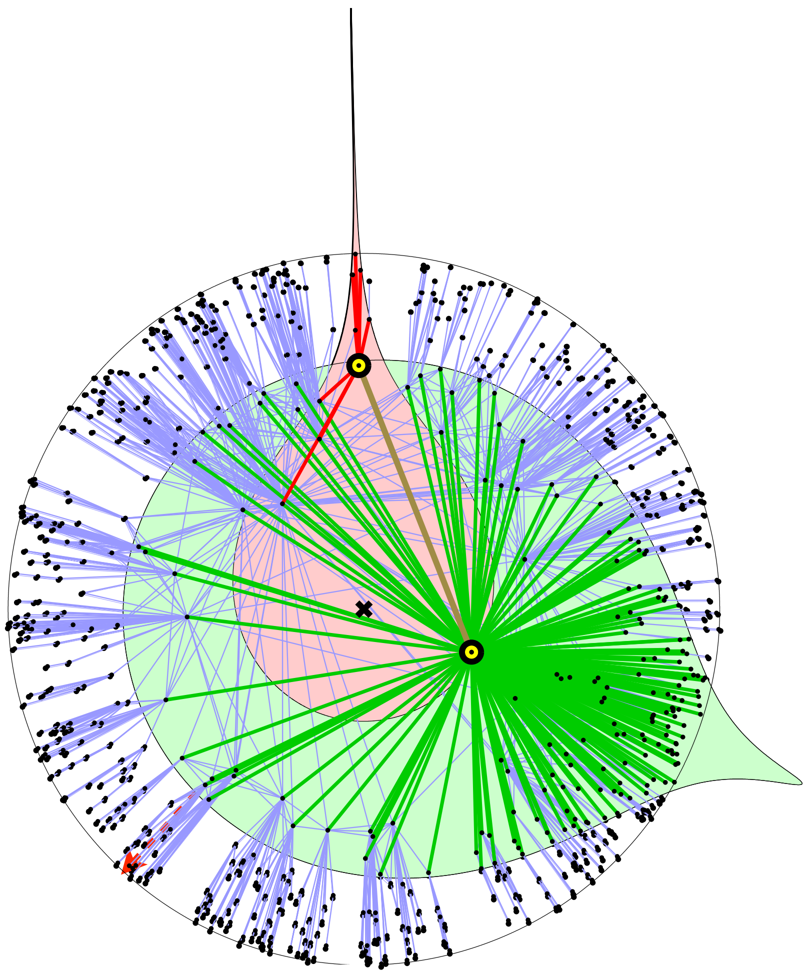

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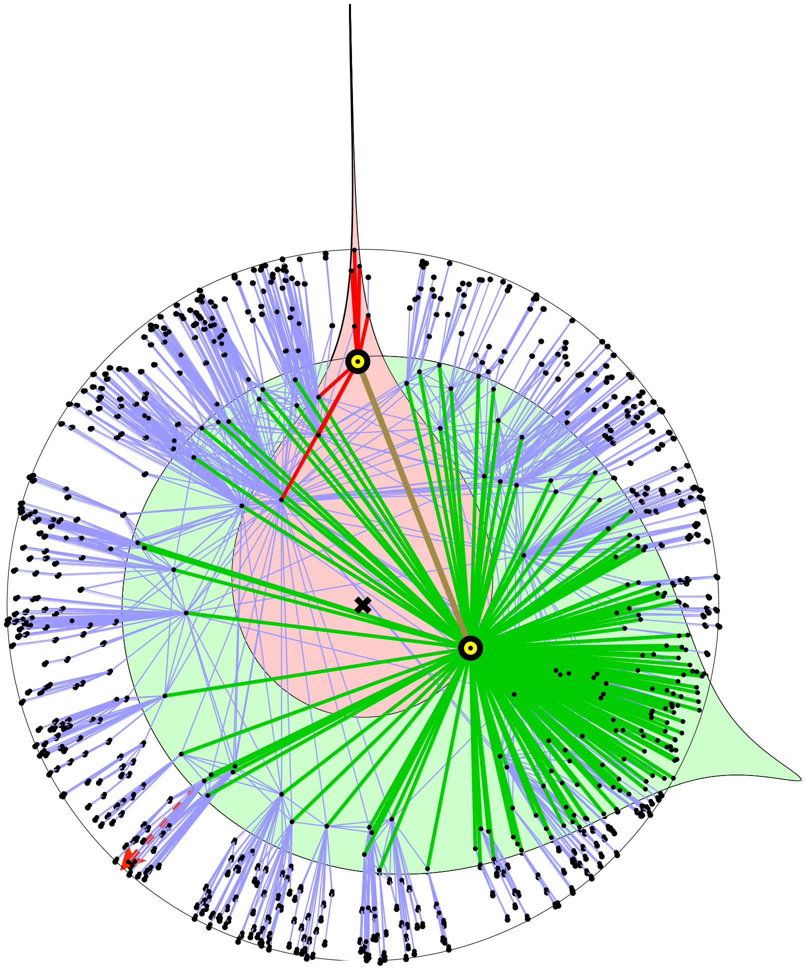

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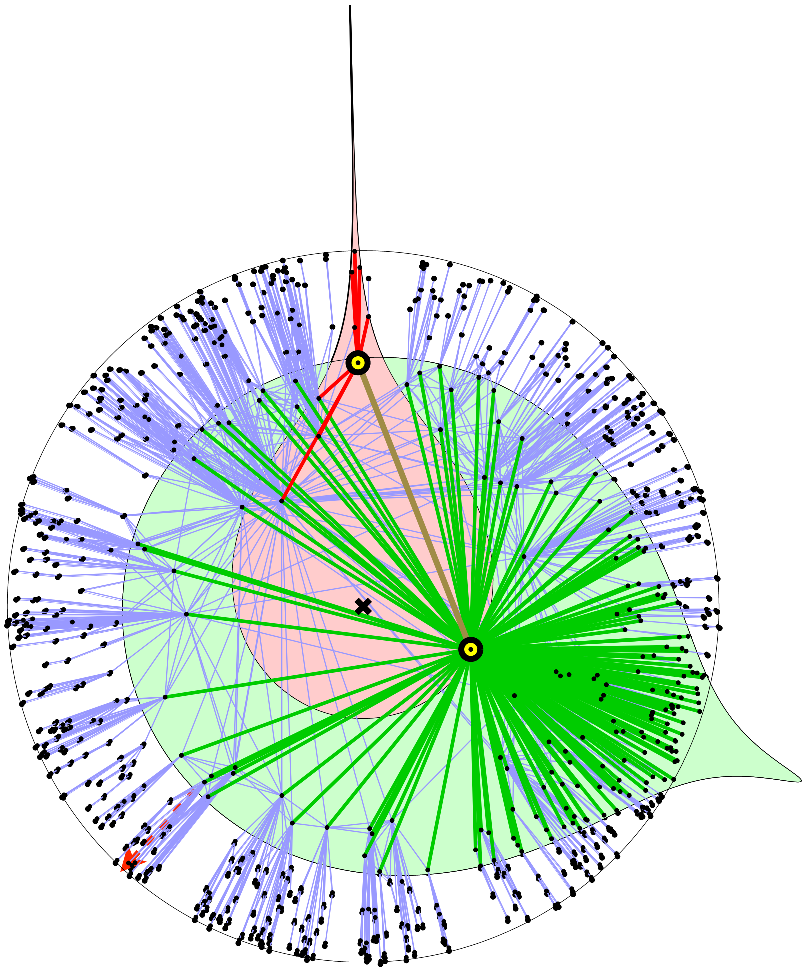

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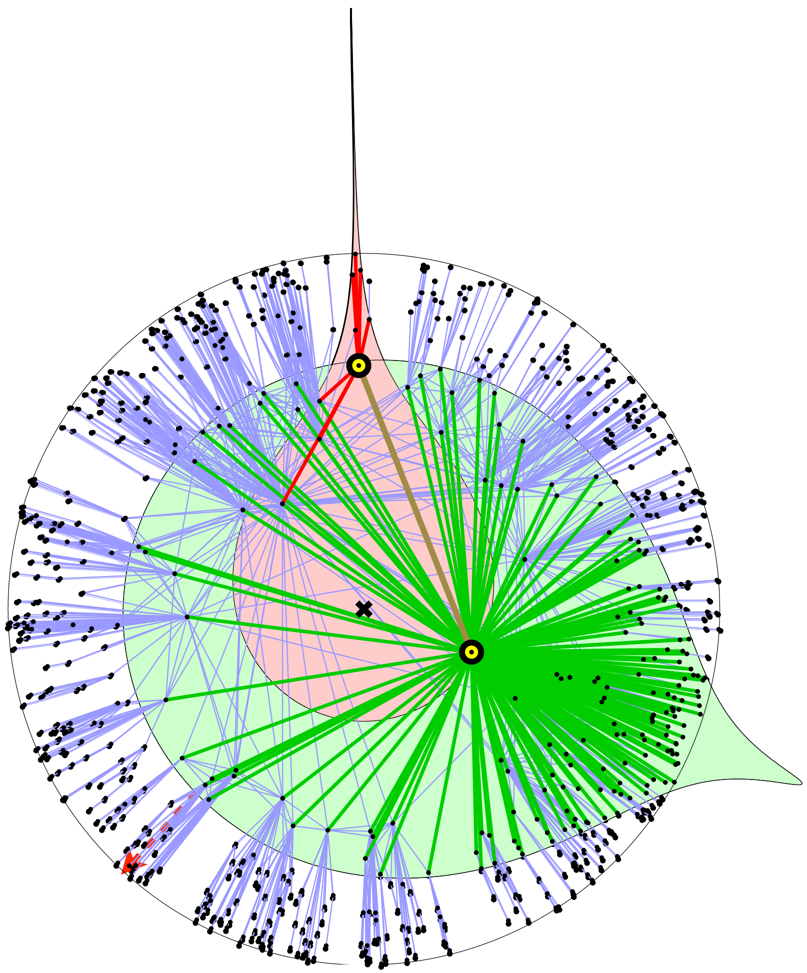

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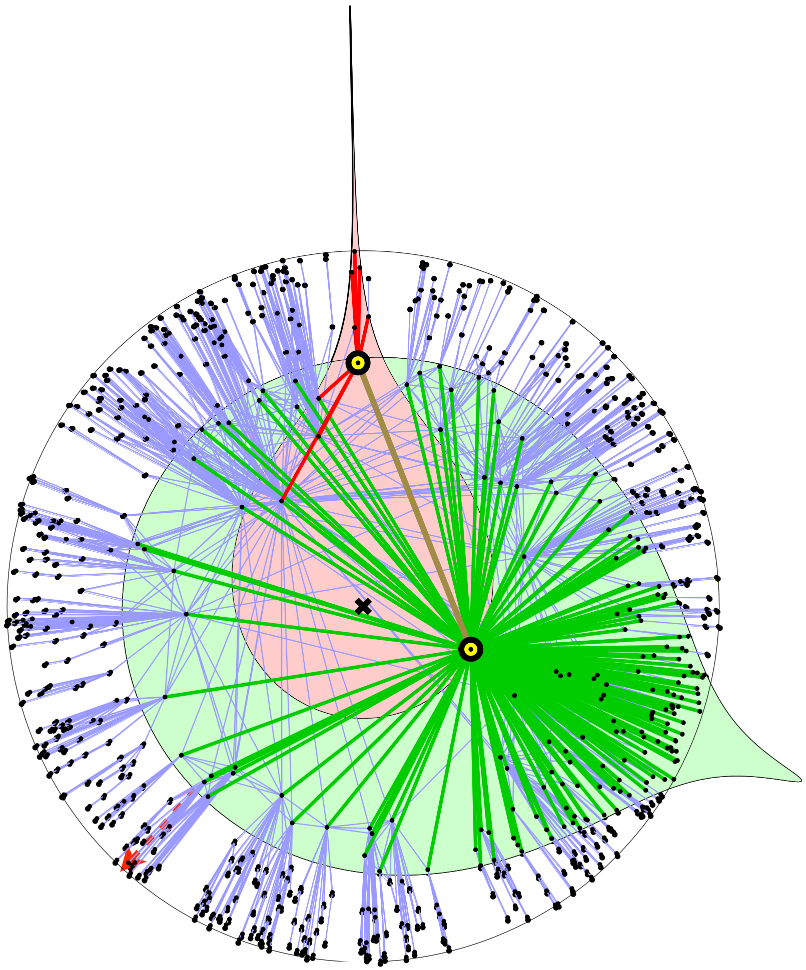

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$$P(k) \sim k^{-\gamma}, \quad \text{with } \gamma = \begin{cases} 2\alpha + 1 & \text{if } \alpha \geqslant \frac{1}{2} \\ 2 & \text{if } \alpha \leqslant \frac{1}{2} \end{cases}$$

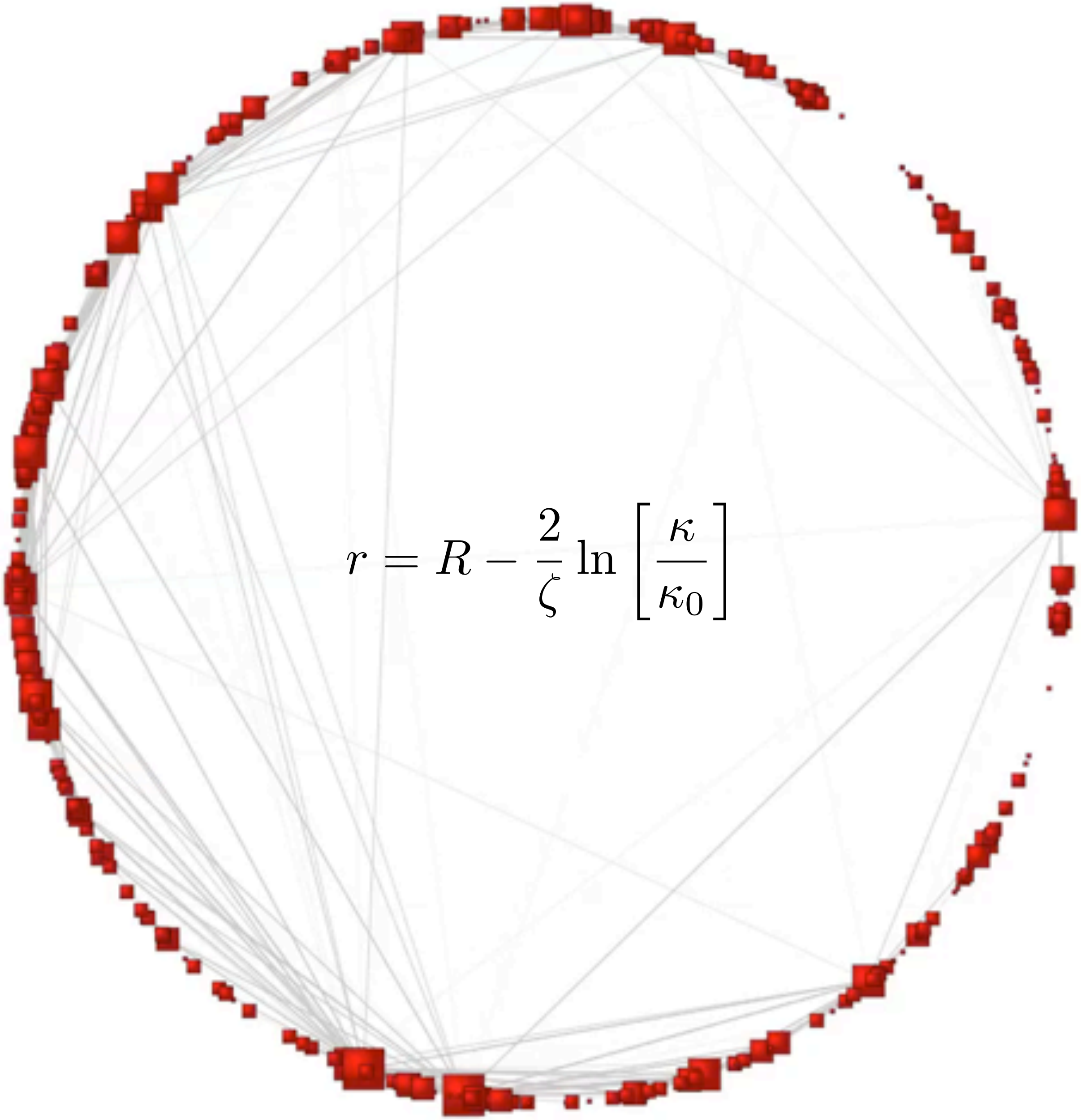
Which transformation goes from

$$\rho(\kappa) = (\gamma - 1)\frac{\kappa_0^{\gamma-1}}{\kappa^\gamma}$$

to this?

$$\rho(r) = \frac{\alpha \sinh \alpha r}{\cosh \alpha R - 1}$$

Newtonian-S^1 vs Einsteinian-H^2



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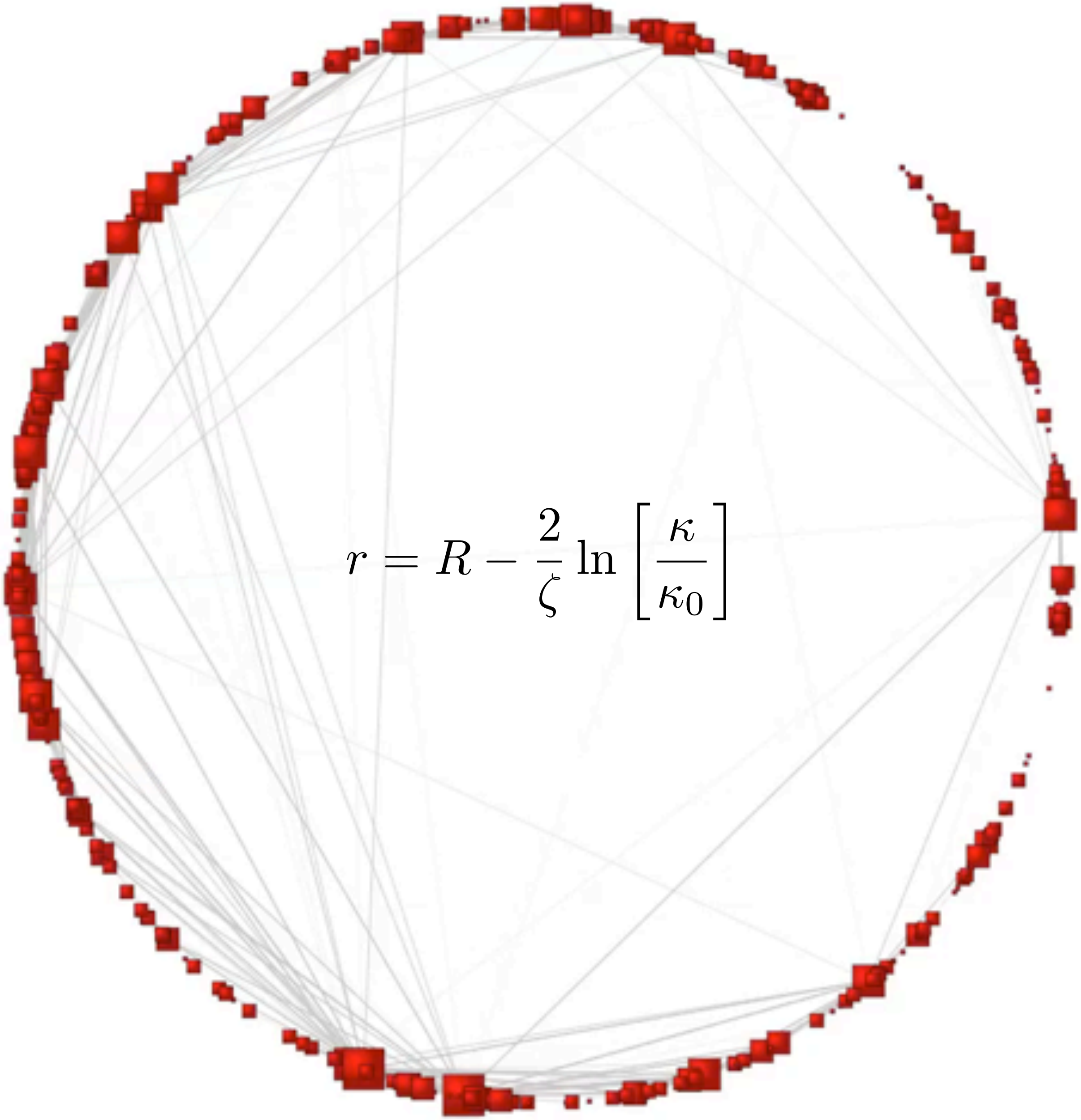

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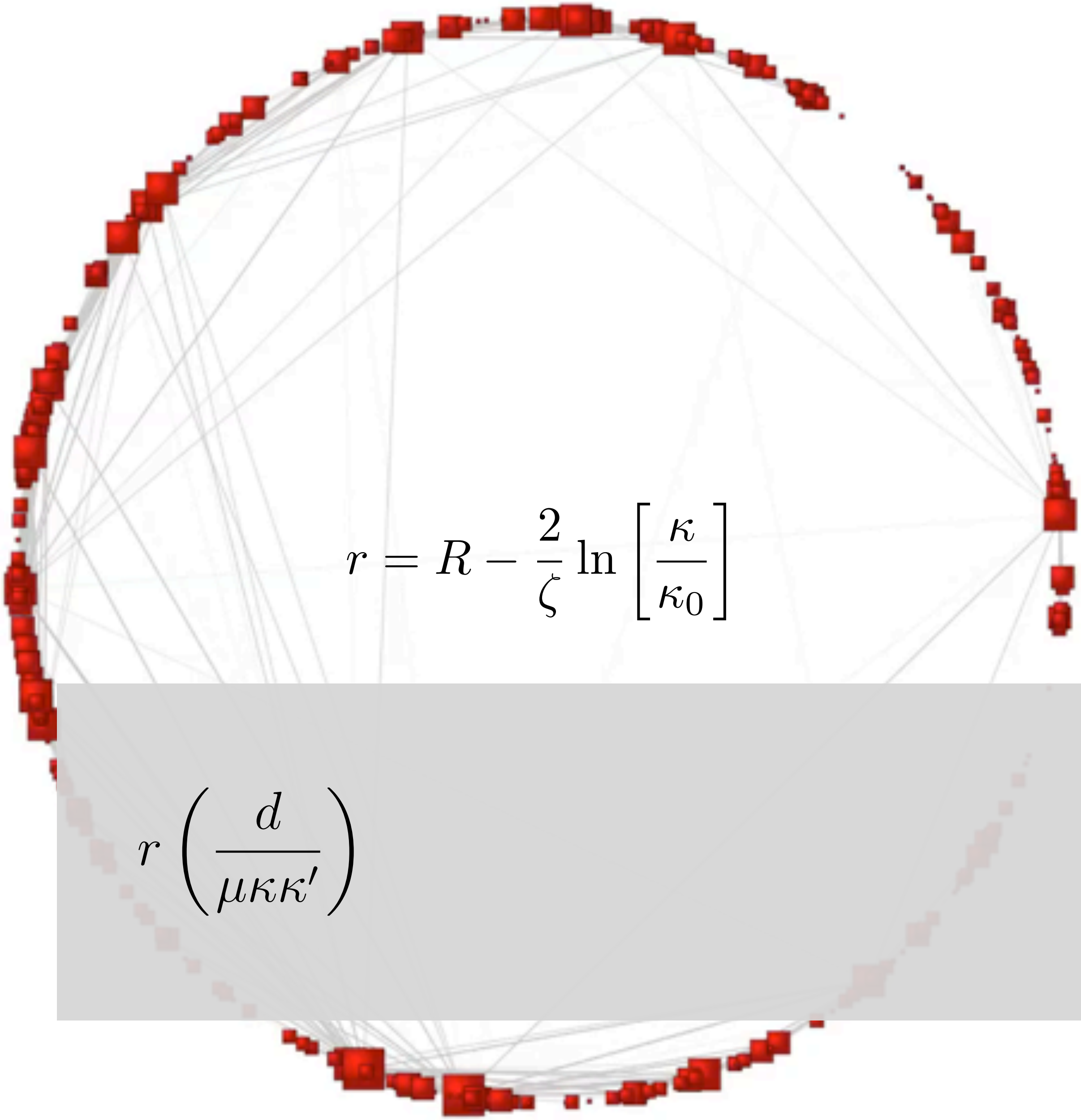

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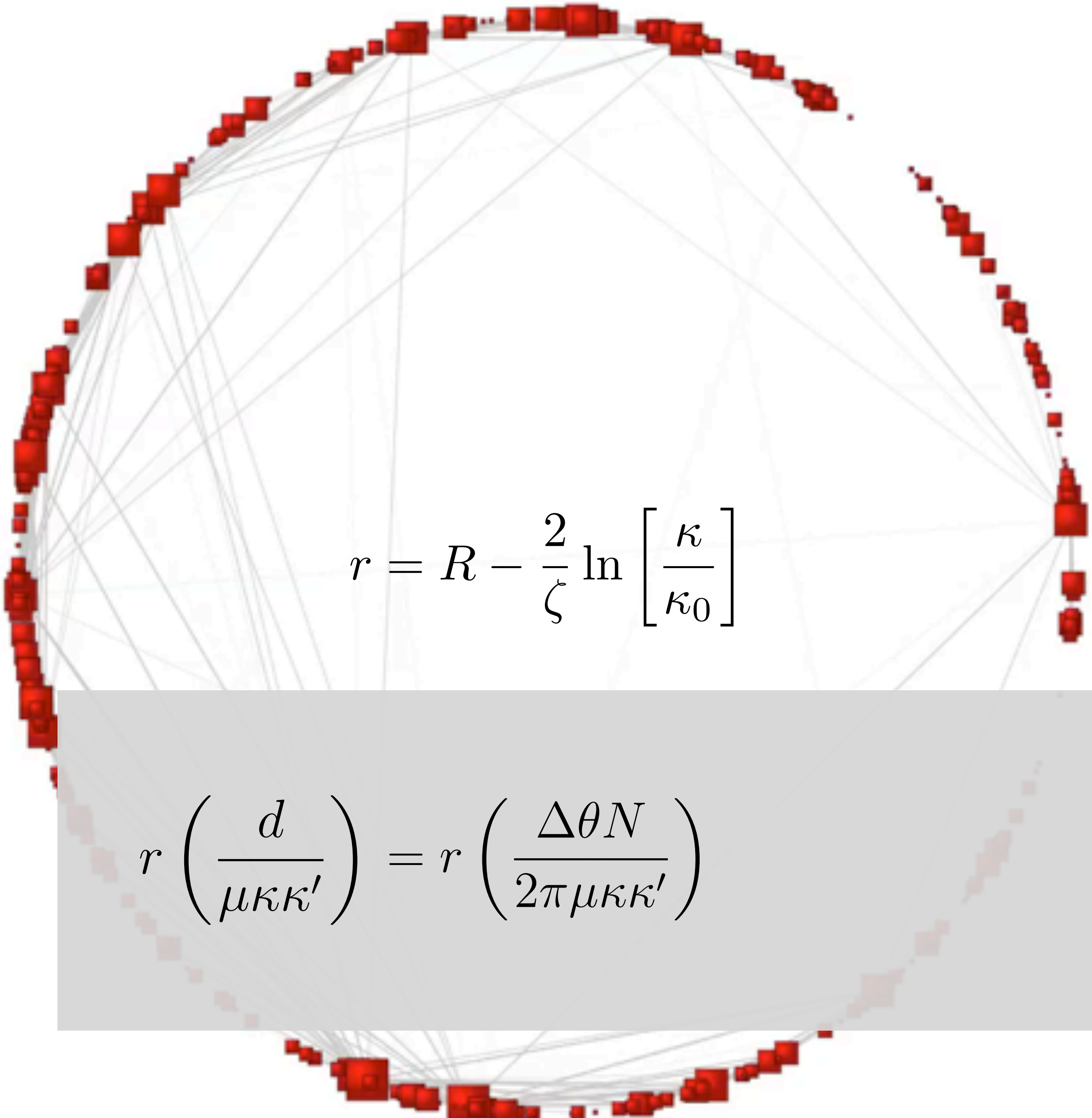

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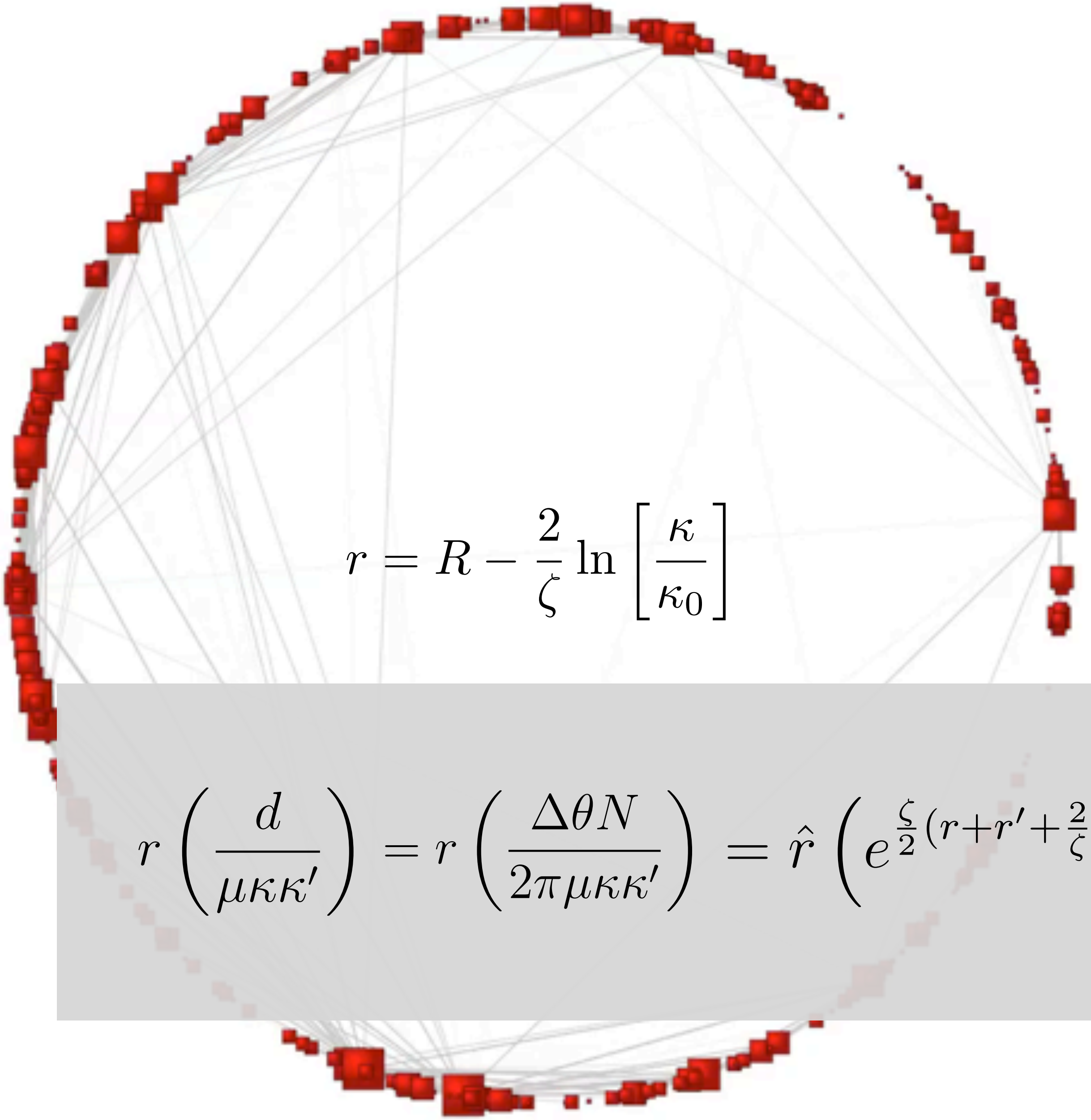
$$r\left(\frac{d}{\mu\kappa\kappa'}\right) = r\left(\frac{\Delta\theta N}{2\pi\mu\kappa\kappa'}\right)$$

Newtonian-S^1 vs Einsteinian-H^2



Which transformation goes from

$$\rho(\kappa) = (\gamma - 1)\frac{\kappa_0^{\gamma-1}}{\kappa^{\gamma}}$$

to this?

$$\rho(r) = \frac{\alpha \sinh \alpha r}{\cosh \alpha R - 1}$$

$$r = R - \frac{2}{\zeta} \ln \left[\frac{\kappa}{\kappa_0}\right]$$

$$r\left(\frac{d}{\mu\kappa\kappa'}\right) = r\left(\frac{\Delta\theta N}{2\pi\mu\kappa\kappa'}\right) = \hat{r}\left(e^{\frac{\zeta}{2}(r+r'+\frac{2}{\zeta}\ln\frac{\Delta\theta}{2}-R)}\right)$$

Newtonian-S^1

Einsteinian-H^2

Newtonian-S¹

$$\hat{r}\left(e^{\frac{\zeta}{2}(r+r'+\frac{2}{\zeta}\ln\frac{\Delta\theta}{2}-R)}\right)$$

Einsteinian-H²

Newtonian-S^1 vs Einsteinian-H^2

Newtonian-S^1

$$\hat{r}\left(e^{\frac{\zeta}{2}\left(\underbrace{r+r'+\frac{2}{\zeta}\ln\frac{\Delta\theta}{2}}-R\right)}\right)$$

$$x = r + r' + \frac{2}{\zeta}\ln\frac{\Delta\theta}{2}$$

Einsteinian-H^2

Newtonian-S1

$$\hat{r}\left(e^{\frac{\zeta}{2}\underbrace{(r+r'+\frac{2}{\zeta}\ln\frac{\Delta\theta}{2}}-R)}\right)$$

$$x = r + r' + \frac{2}{\zeta}\ln\frac{\Delta\theta}{2}$$

Einsteinian-H2

the hyperbolic distance is very well approximated by

$$x = r + r' + \frac{2}{\zeta}\ln\sin\frac{\Delta\theta}{2}$$

Newtonian-S^1

$$\hat{r}\left(e^{\frac{\zeta}{2}\underbrace{(r+r'+\frac{2}{\zeta}\ln\frac{\Delta\theta}{2}}-R)}\right)$$

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$$x = r + r' + \frac{2}{\zeta}\ln\frac{\Delta\theta}{2}$$

$$\hat{r}\left(e^{\frac{\zeta}{2}(x-R)}\right)$$

$$\hat{r}(z)\begin{cases}\text{cte} & z \ll 1 \\ 0 & z \gg 1\end{cases}$$

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$$\hat{r}\left(e^{\frac{\zeta}{2}(x-R)}\right)$$

$$\hat{r}(z)\begin{cases}\text{cte} & z \ll 1\\ 0 & z \gg 1\end{cases}$$

Einsteinian-H²

the hyperbolic distance is very well approximated by

$$x = r + r' + \frac{2}{\zeta}\ln\sin\frac{\Delta\theta}{2}$$

$$p(x) = \Theta(R - x)$$

Newtonian-S^1

$$\hat{r}\left(e^{\frac{\zeta}{2}\left(r+r'+\frac{2}{\zeta}\ln\frac{\Delta\theta}{2}-R\right)}\right)$$

$$x = r + r' + \frac{2}{\zeta}\ln\frac{\Delta\theta}{2}$$

$$\hat{r}\left(e^{\frac{\zeta}{2}(x-R)}\right)$$

$$\hat{r}(z)\begin{cases}\text{cte} & z \ll 1\\ 0 & z \gg 1\end{cases}$$

Einsteinian-H^2

the hyperbolic distance is very well approximated by

$$x = r + r' + \frac{2}{\zeta}\ln\sin\frac{\Delta\theta}{2}$$

$$p(x) = \Theta(R - x)$$

Newtonian-S^1

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$$x = r + r' + \frac{2}{\zeta}\ln\frac{\Delta\theta}{2}$$

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Einsteinian-H^2

the hyperbolic distance is very well approximated by

$$x = r + r' + \frac{2}{\zeta}\ln\sin\frac{\Delta\theta}{2}$$

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Newtonian-S^1 and Einteinian-H^2 are isomorphic

The important conclusion is

$$\hat{r}\left(e^{\frac{\zeta}{2}(x-R)}\right) \qquad \hat{r}(z)\begin{cases} \text{cte} & z \ll 1 \\ 0 & z \gg 1 \end{cases}$$

Fermi distribution

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a very natural candidate is

$$\hat{r}(z) = \frac{1}{1 + z^{1/T}}$$

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Fermi distribution

$$p(x) = \frac{1}{1 + e^{\frac{\zeta}{2T}(x-R)}}$$

Fermi distribution

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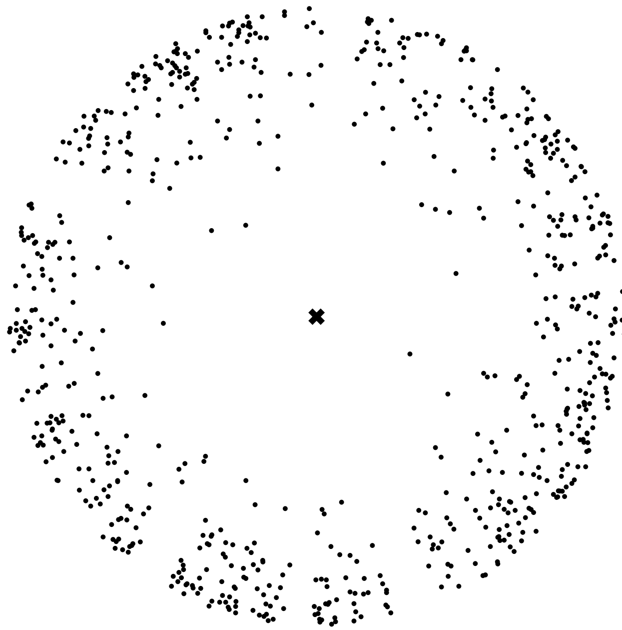

edges are fermions that can occupy any of the N(N-1)/2 possible states

Fermi distribution

The important conclusion is

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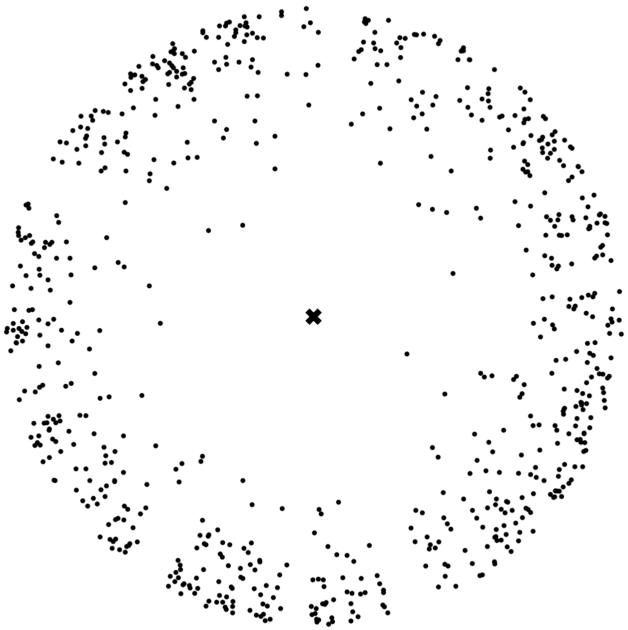

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Fermi distribution

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edges are fermions that can occupy any of the N(N-1)/2 possible states

hyperbolic distance is like the energy of the state

Fermi distribution

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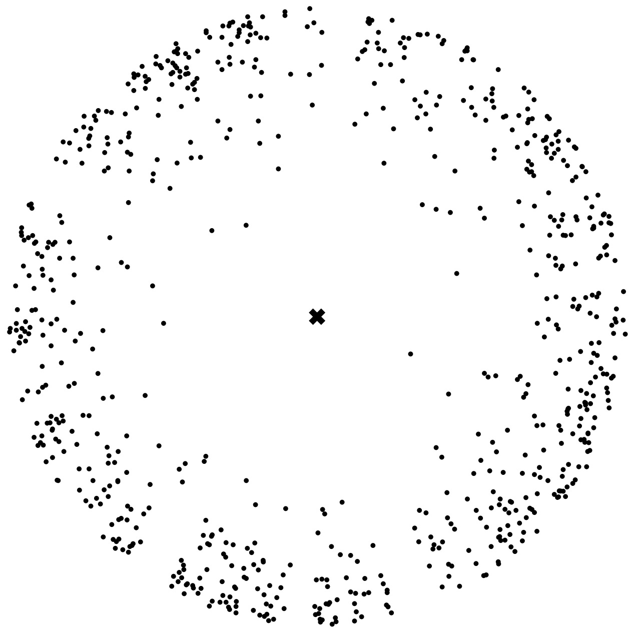

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Fermi distribution

$$p(x) = \frac{1}{1+e^{\frac{\zeta}{2T}(x-R)}}$$

edges are fermions that can occupy any of the N(N-1)/2 possible states

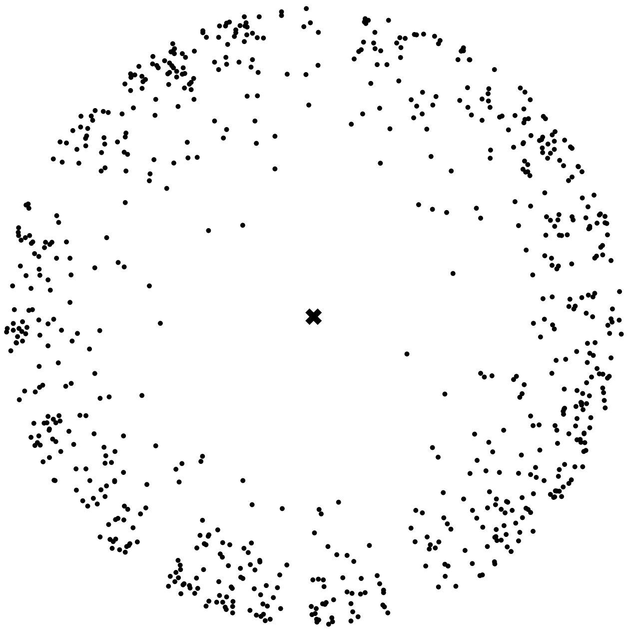
chemical potential

hyperbolic distance is like the energy of the state

Fermi distribution

The important conclusion is

$$\hat{r}\left(e^{\frac{\zeta}{2}(x-R)}\right) \qquad \hat{r}(z)\begin{cases} \text{cte} & z \ll 1 \\ 0 & z \gg 1 \end{cases}$$



a very natural candidate is

$$\hat{r}(z) = \frac{1}{1 + z^{1/T}}$$

Fermi distribution

$$p(x) = \frac{1}{1 + e^{\frac{\zeta}{2T}(x-R)}}$$

edges are fermions that can occupy any of the N(N-1)/2 possible states

temperature

hyperbolic distance is like the energy of the state

chemical potential

Degeneracy of the energy level x
(distance pairs probability density)

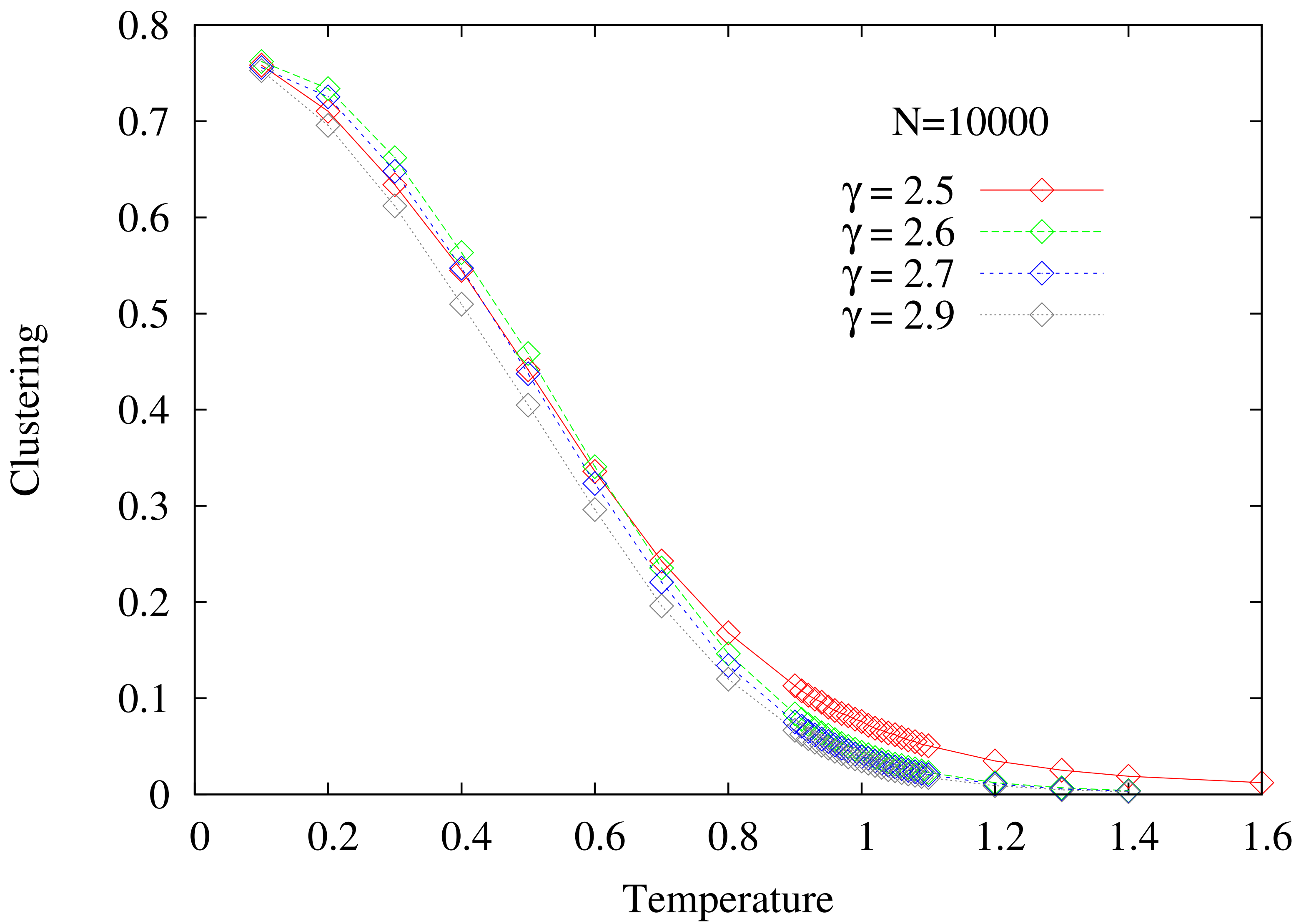
$$M = \binom{N}{2} \int_0^{2R} g(x) p(x) dx$$

Number of fermions (edges)

By fixing *M*, we obtain the chemical potential *R* at any temperature *T*, even above the critical one

Metropolis-Hasting simulations

Clustering undergoes a phase transition at Tc=1



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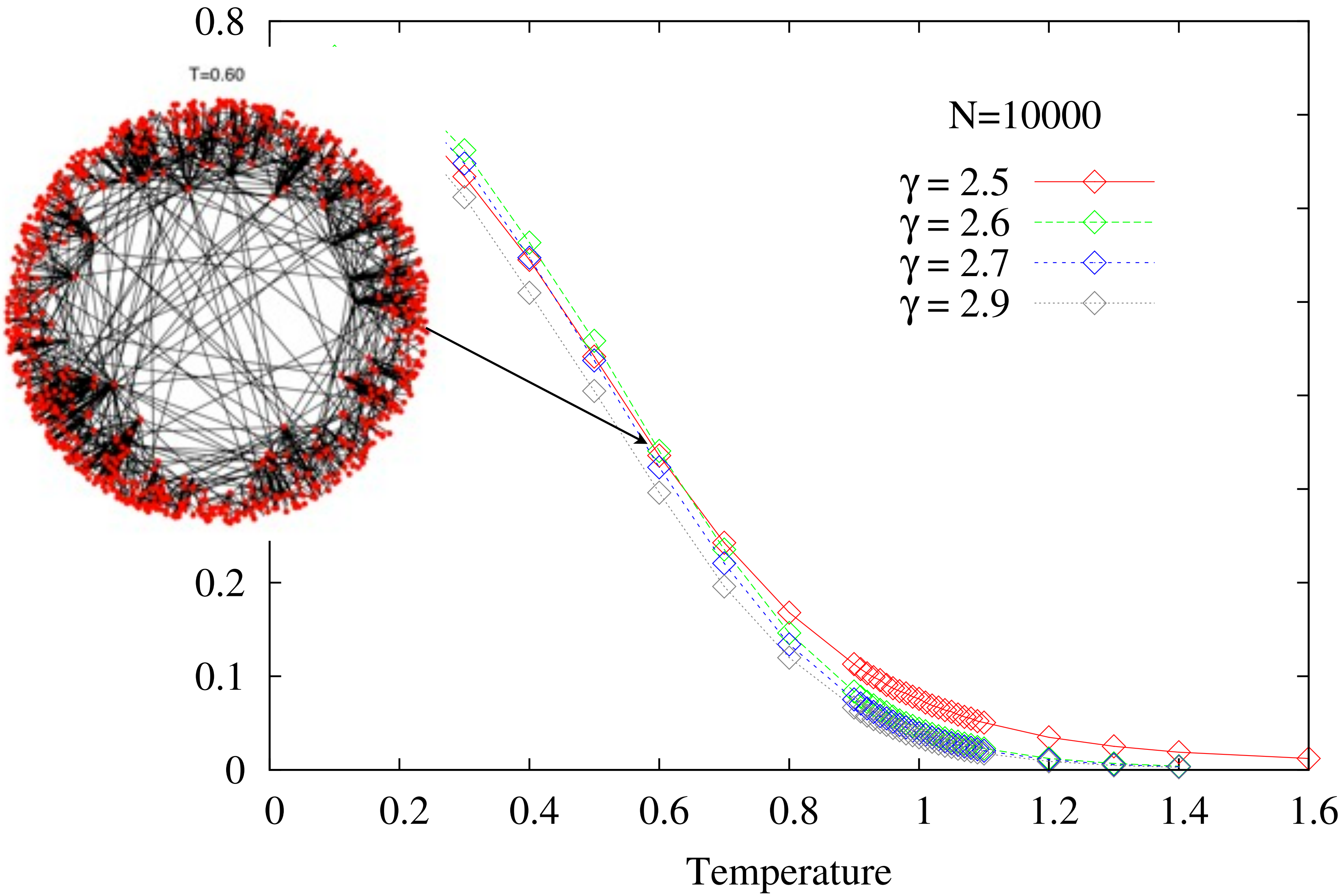

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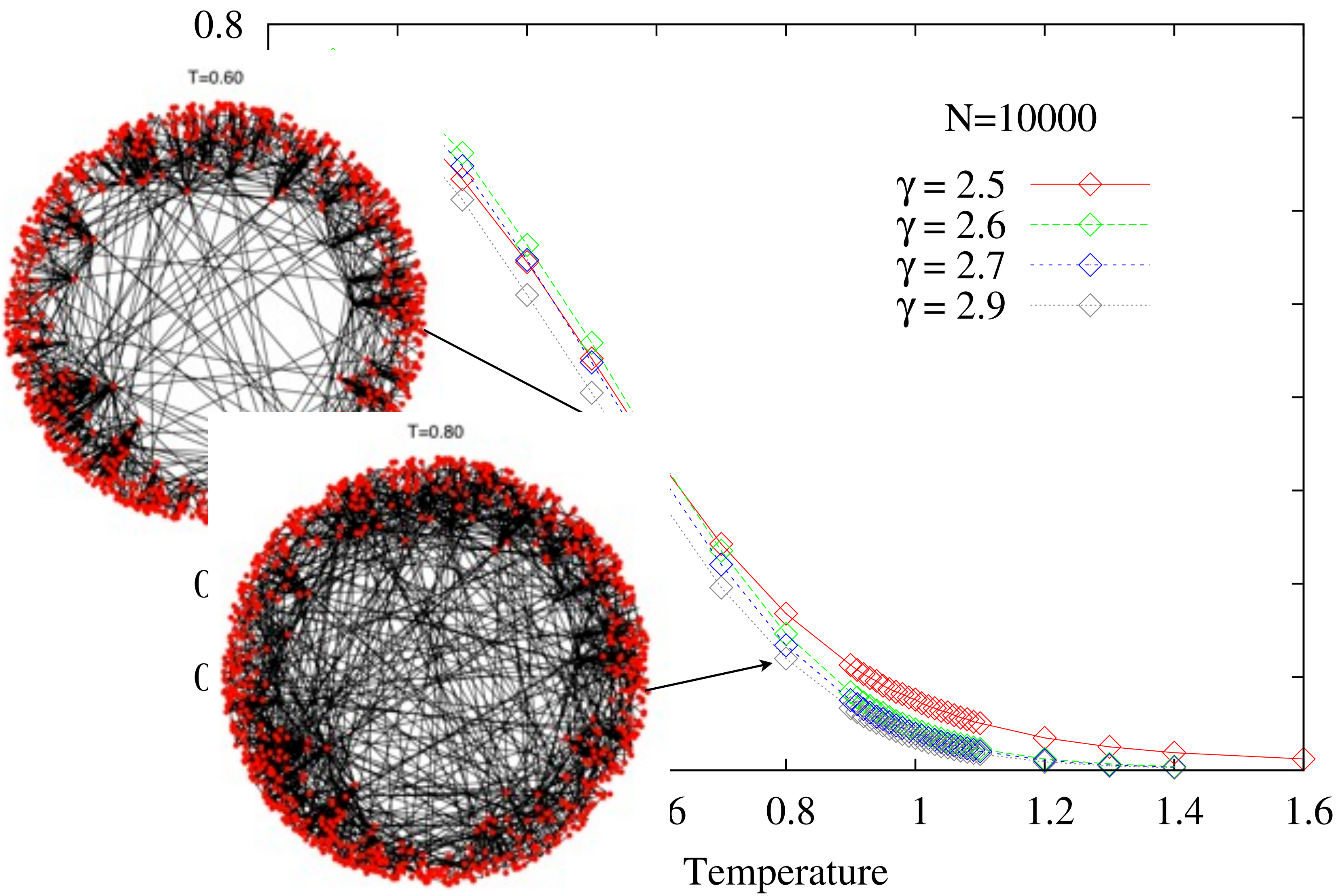
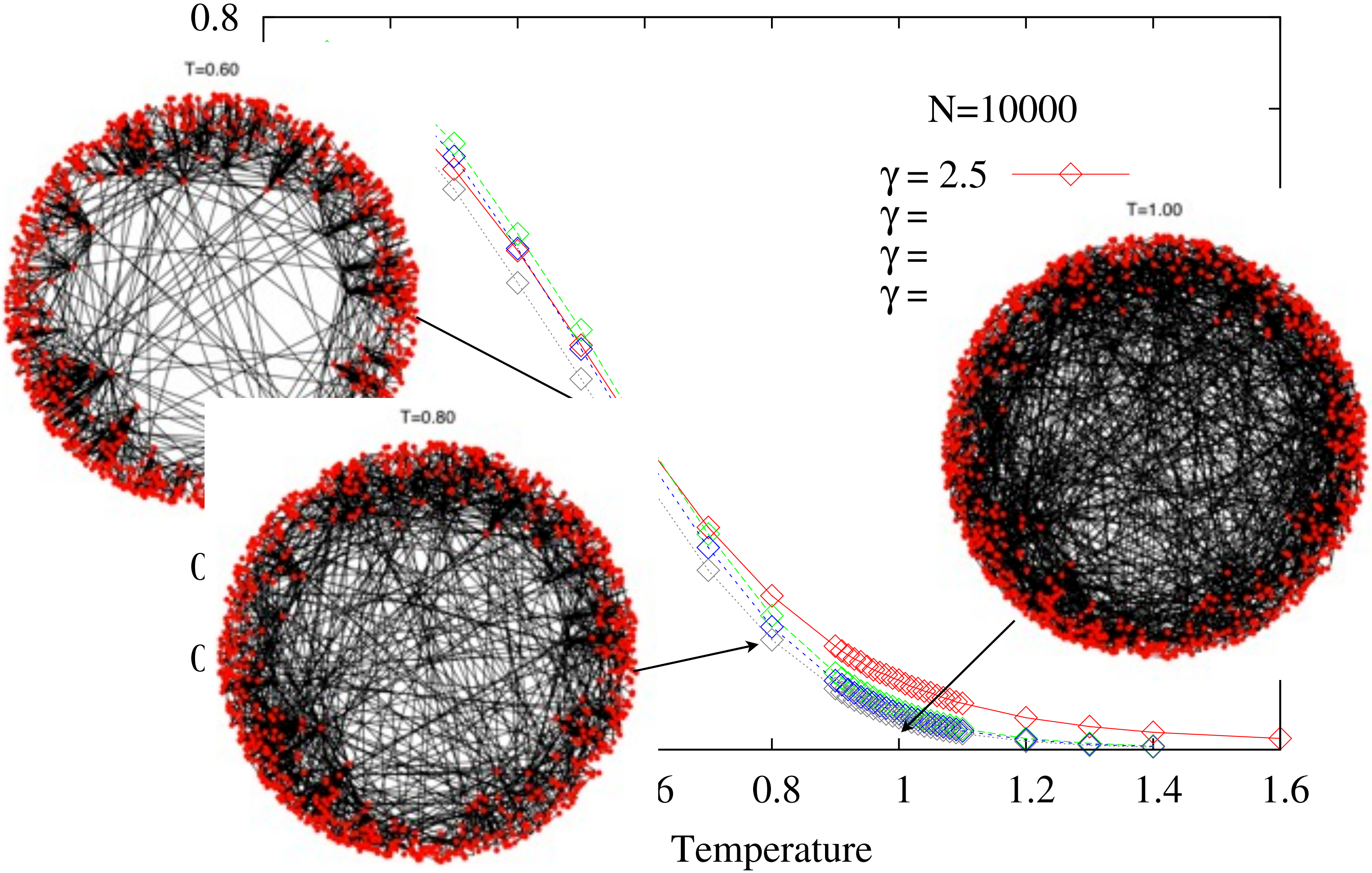
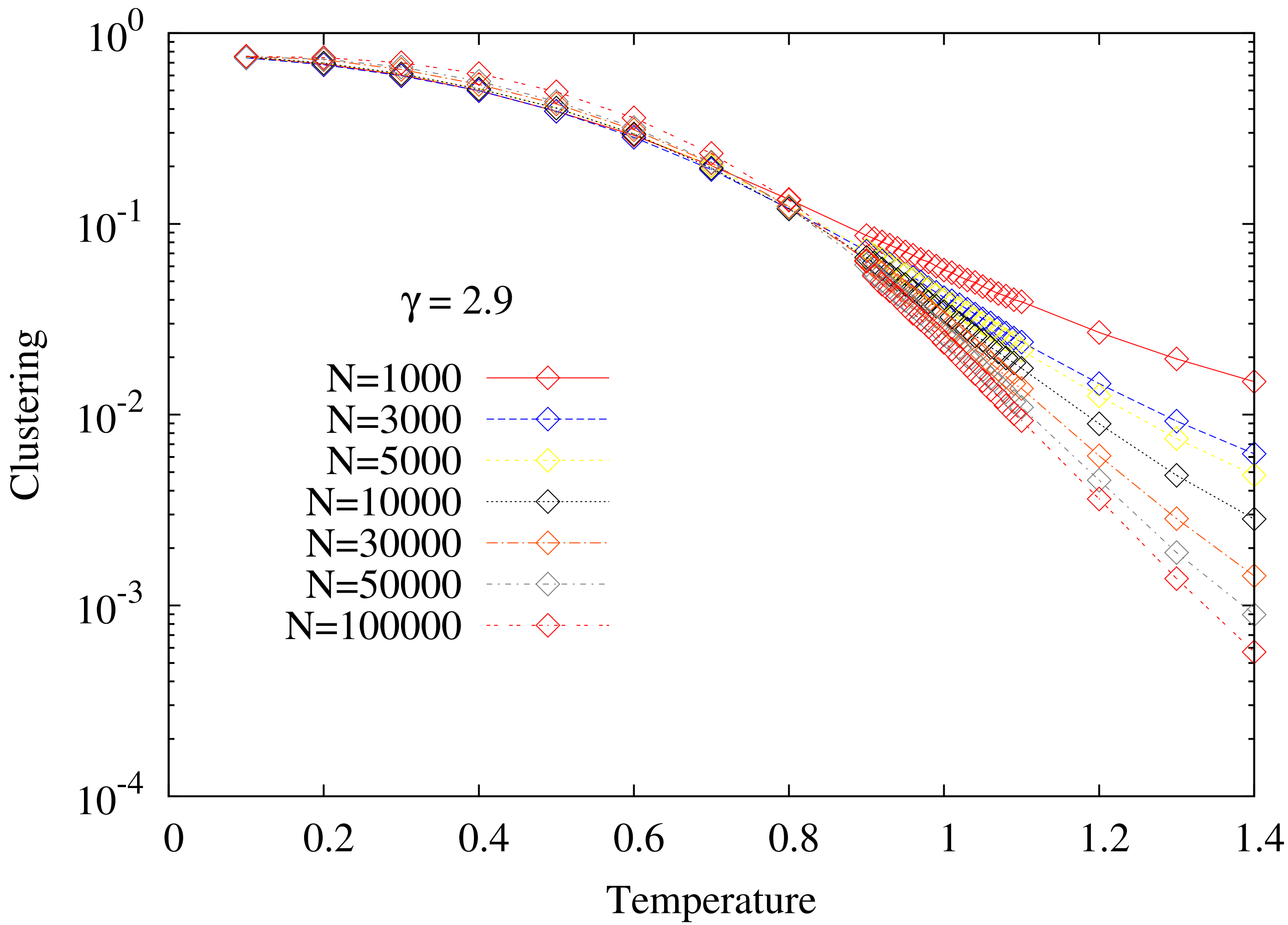


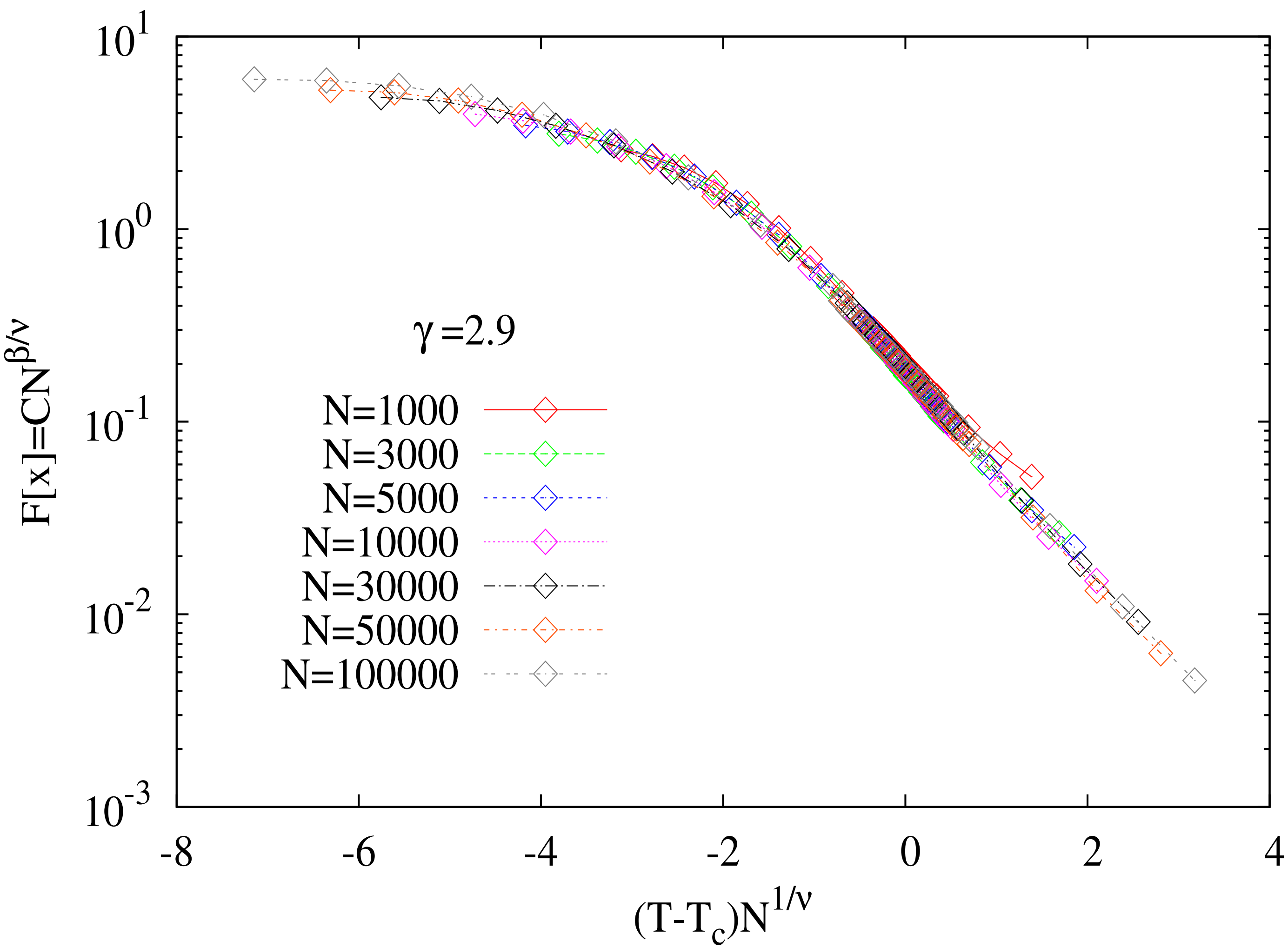
Clustering undergoes a phase transition at Tc=1



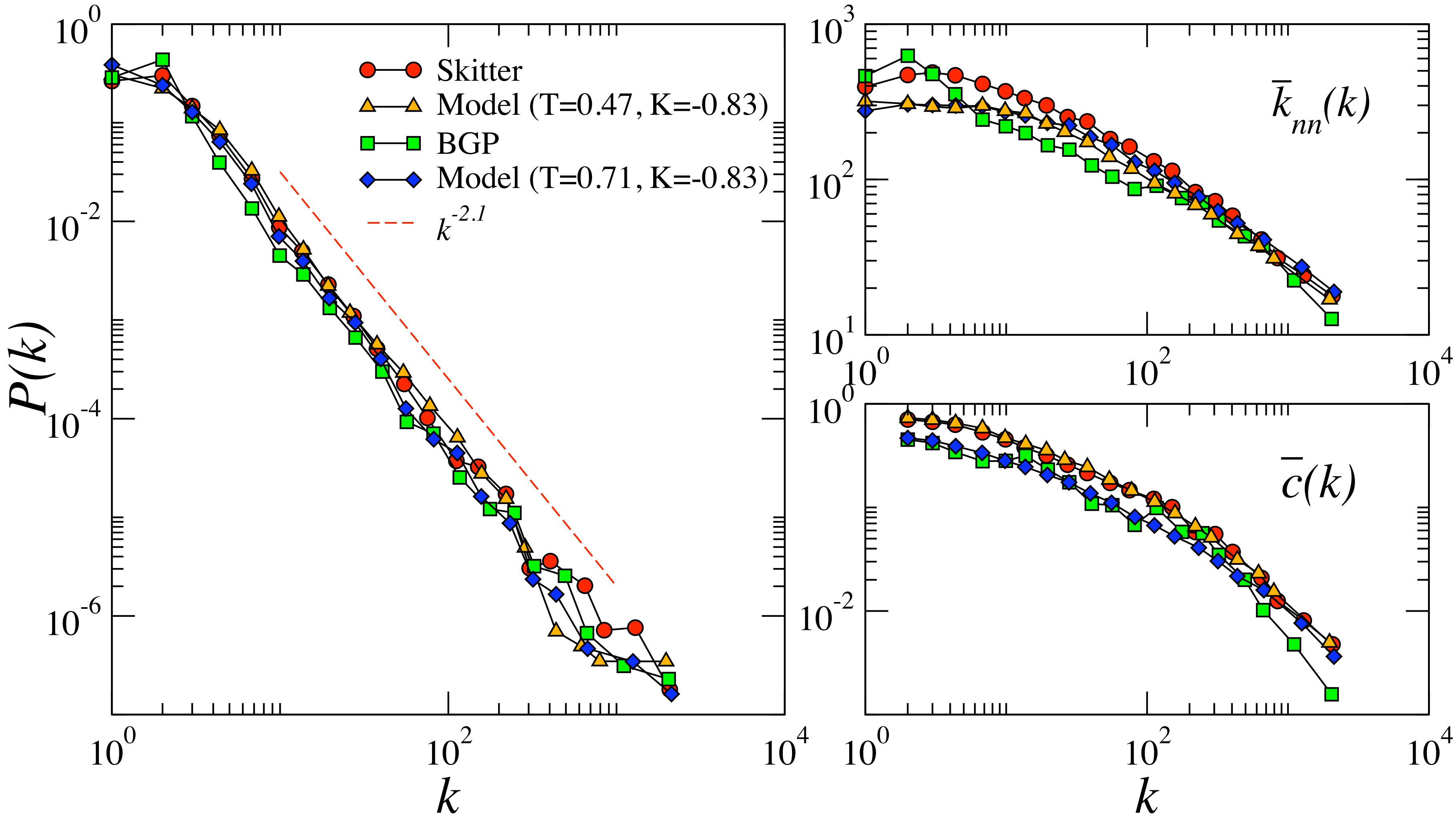
Finite Size Scaling holds



Finite Size Scaling holds

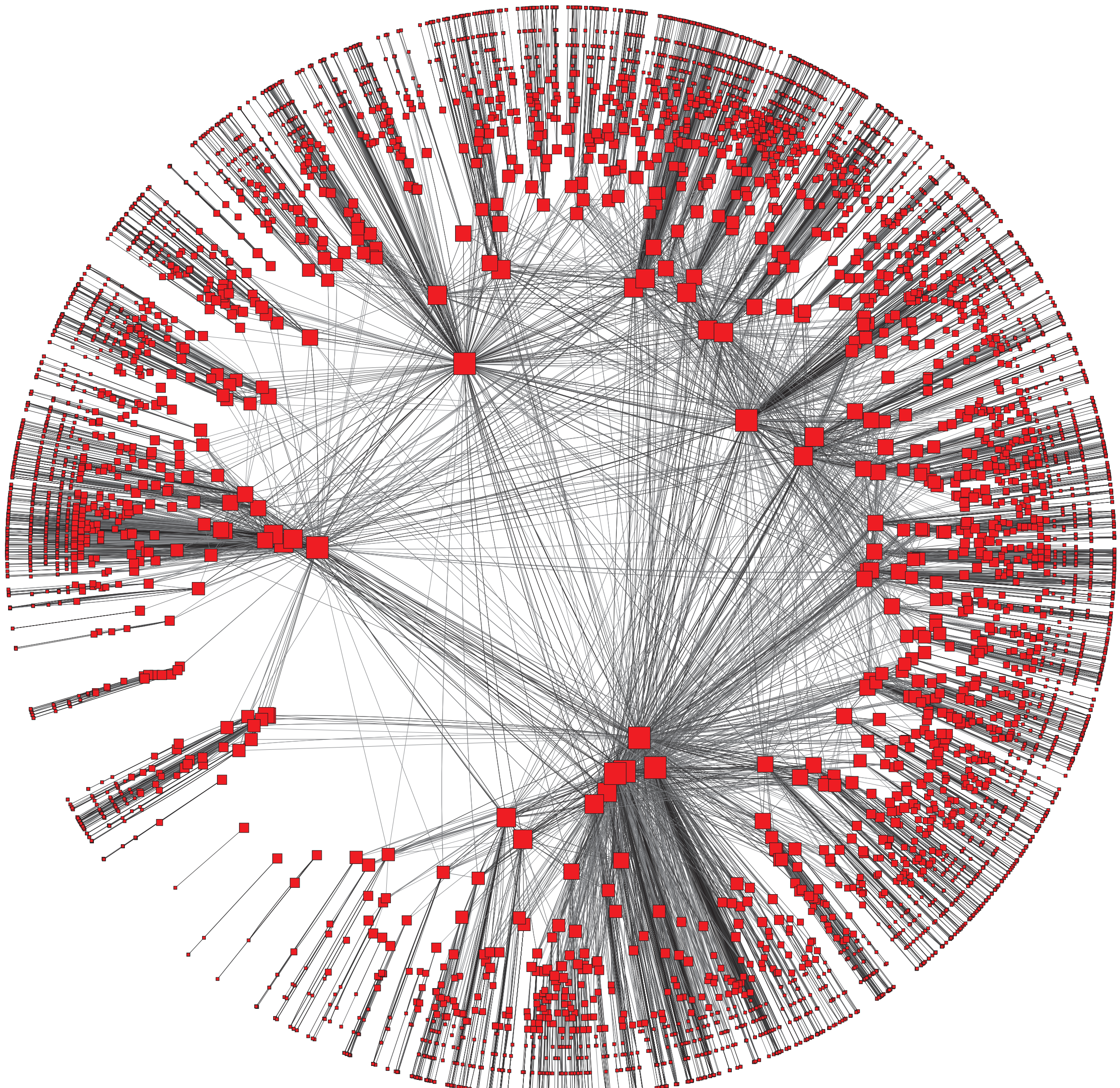


Comparing with the real Internet AS map

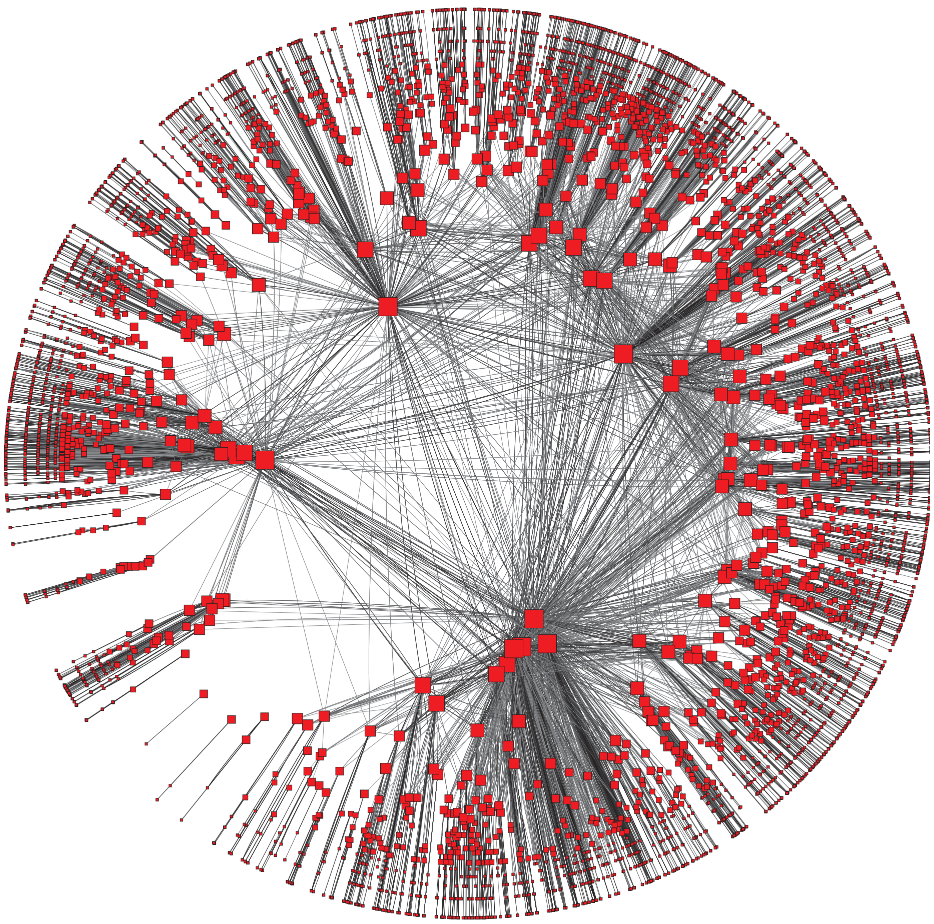


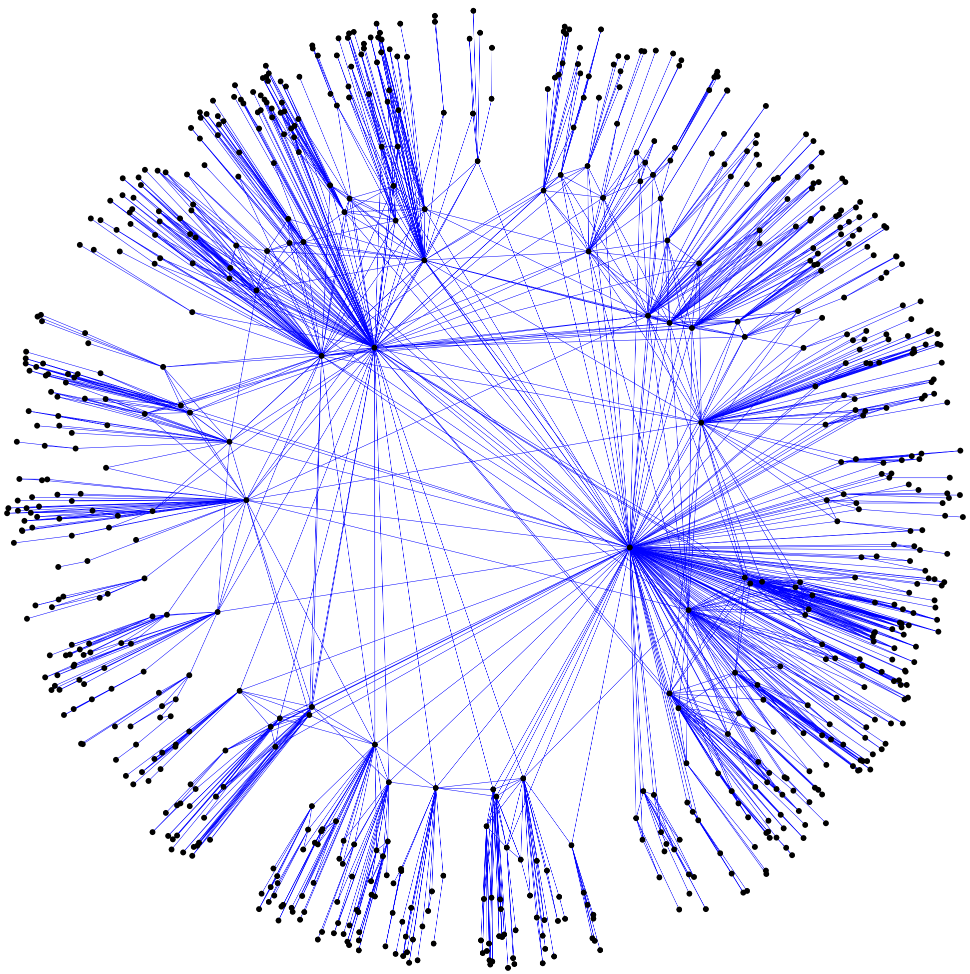
Curvature and temperature of complex networks
D. Krioukov, F. Papadopoulos, A. Vahdat, and M. B. Phys. Rev. E 80, 035101(R) (2009)

Embedding of the real AS Internet graph



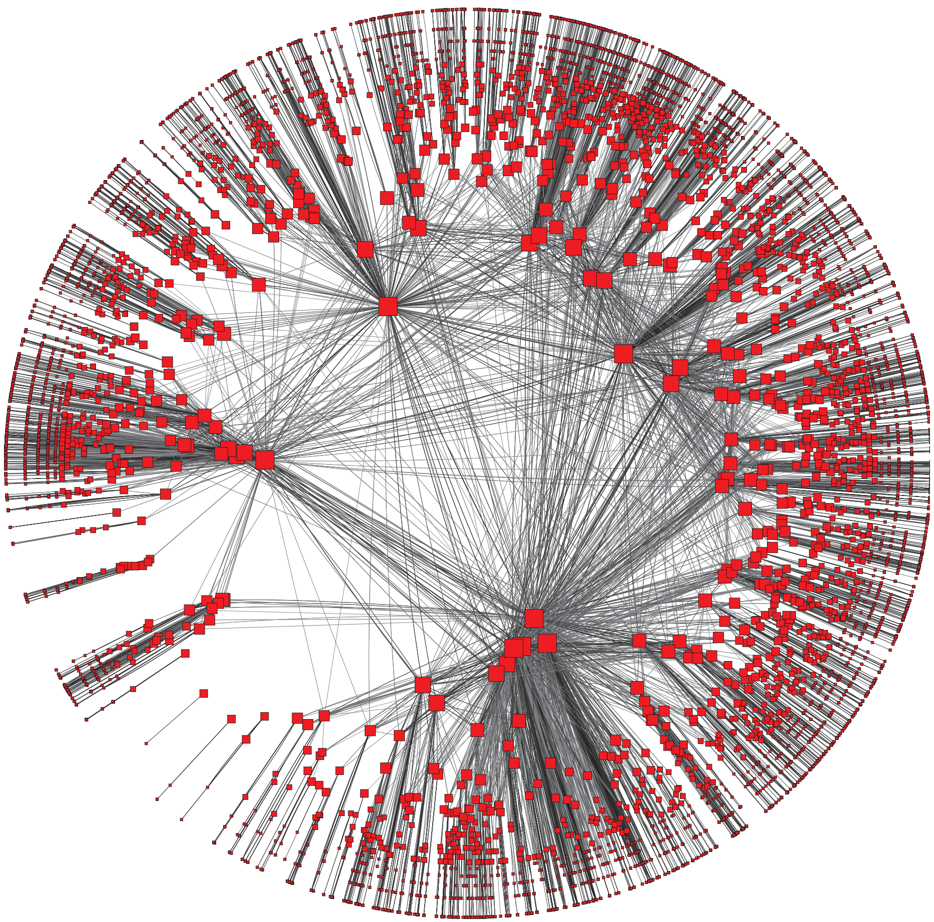
Embedding of the real AS Internet graph



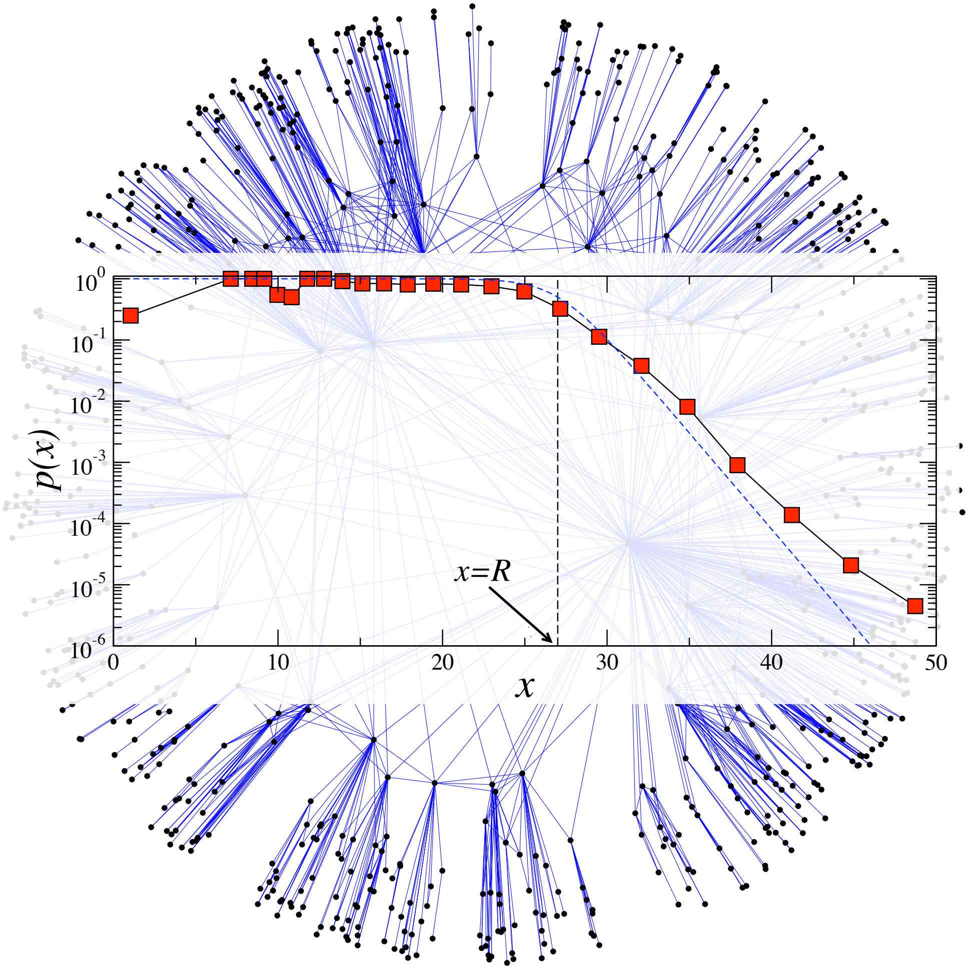


Embedding of the real AS Internet graph

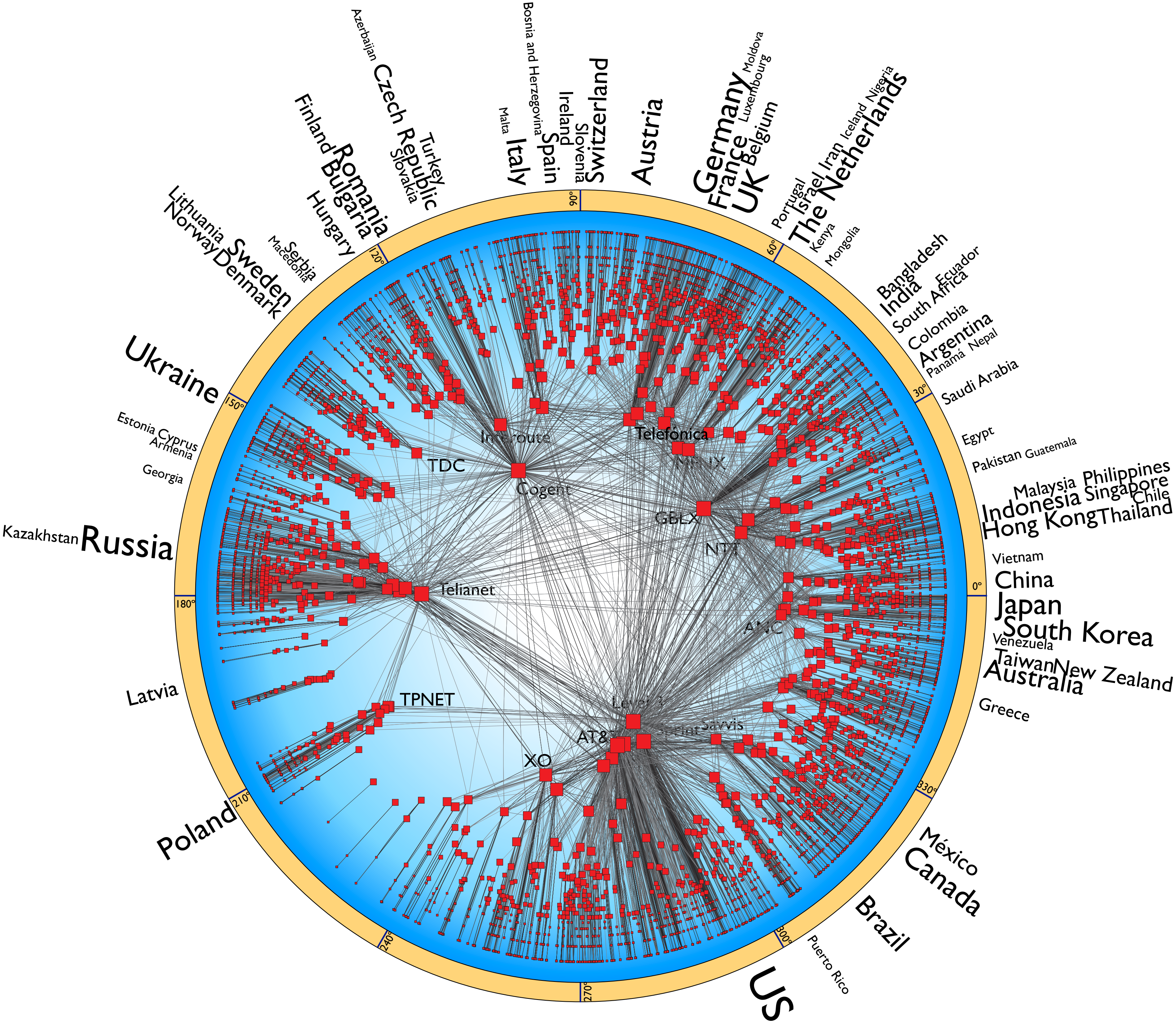
Real AS Internet graph



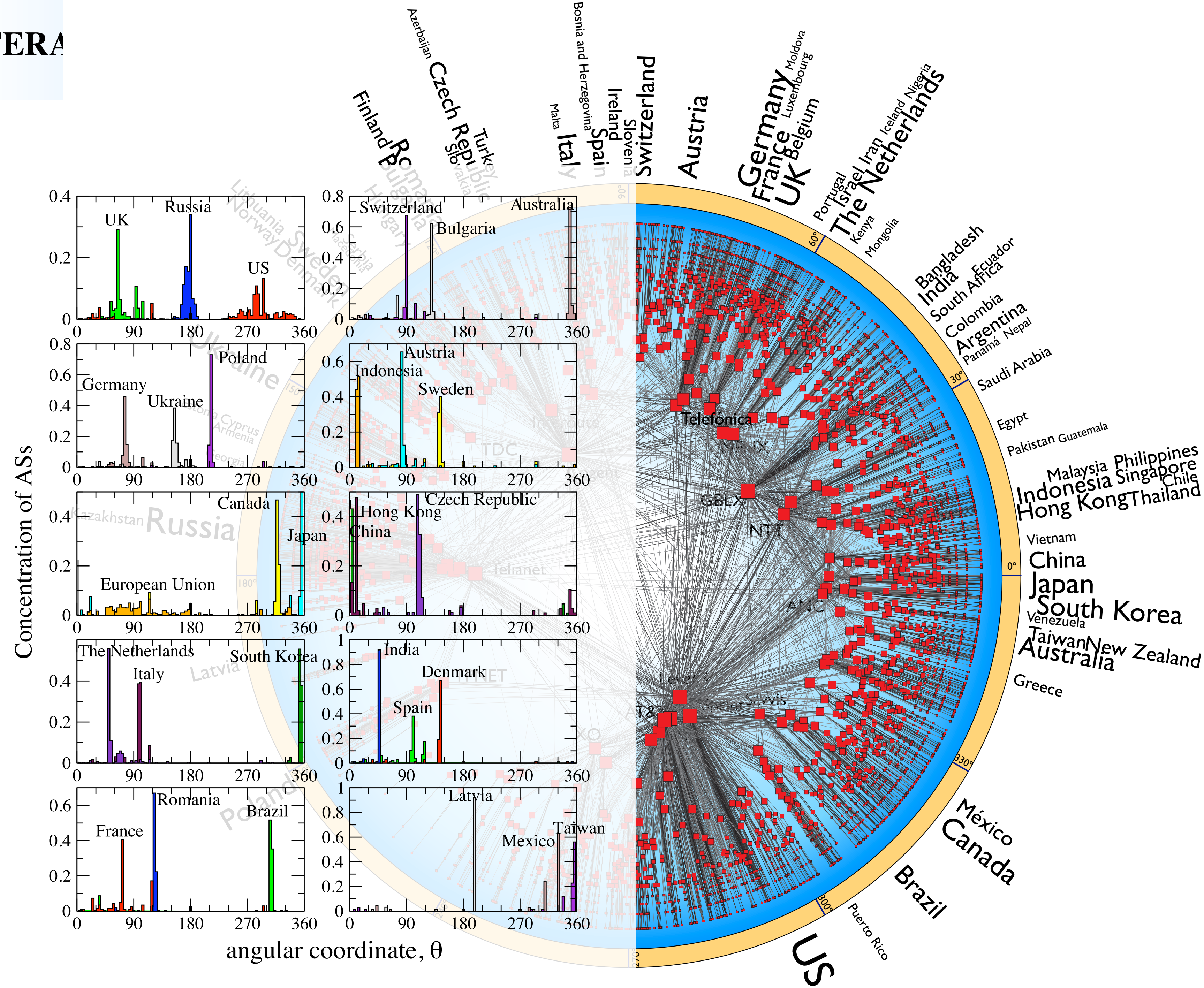
Einsteinian-H2 Model



TERA

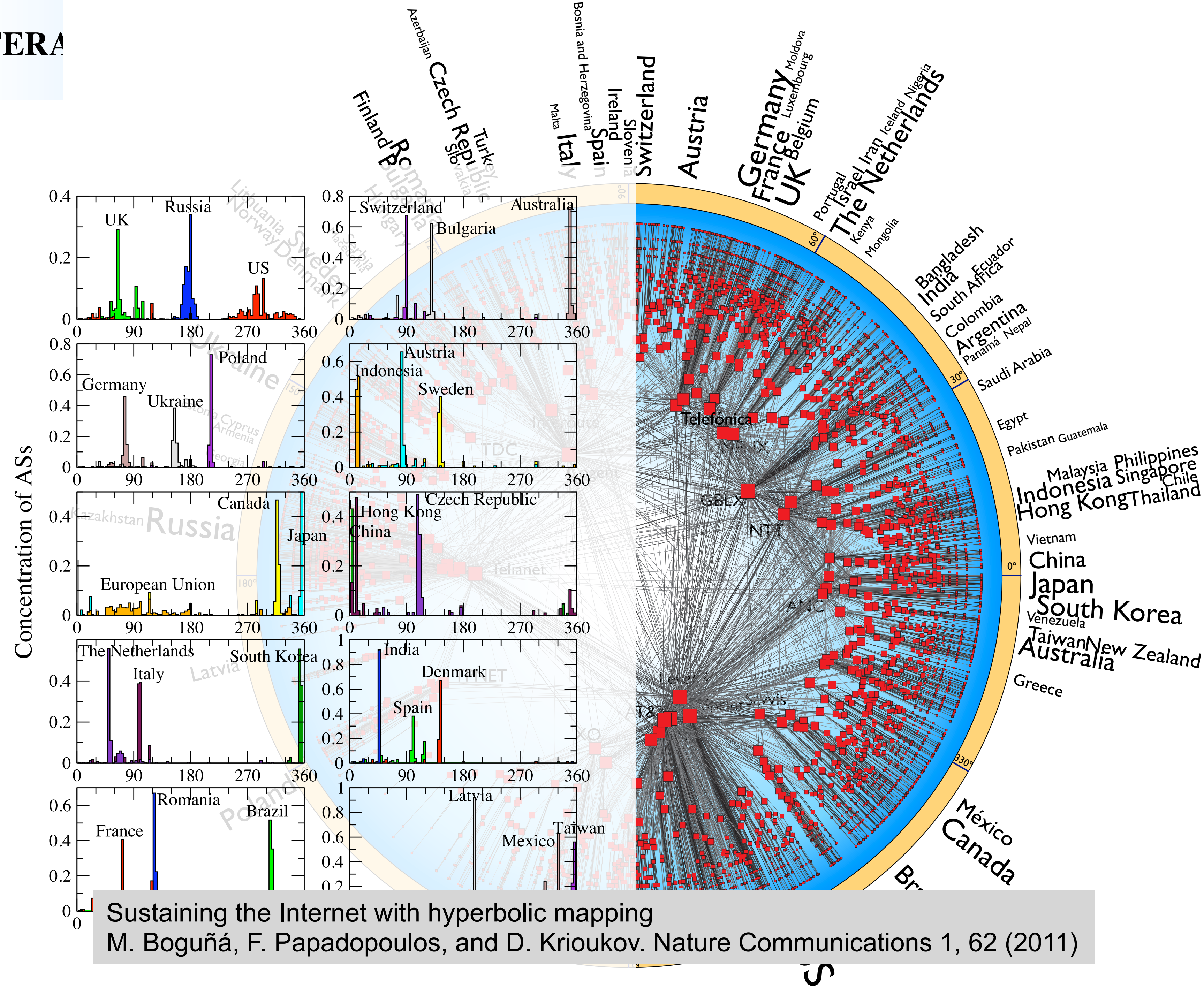

Telianet
TDC
Interoute
Cogent
Telefonica
MFNX
GBLX
NTT
ANC
Level 3
AT&T
Sprint
Savvis
XO
TPNET
US
Puerto Rico
Brazil
Canada
México
Greece
Australia
New Zealand
Taiwan
Venezuela
South Korea
Japan
China
Vietnam
Hong Kong
Thailand
Indonesia
Singapore
Chile
Malaysia
Philippines
Pakistan
Guatemala
Egypt
Saudi Arabia
Panamá
Nepal
Argentina
Colombia
South Africa
Ecuador
India
Bangladesh
Mongolia
Kenya
The Netherlands
Iran
Iceland
Nigeria
Israel
Portugal
UK
France
Germany
Belgium
Luxembourg
Moldova
Austria
Switzerland
Slovenia
Ireland
Spain
Bosnia and Herzegovina
Italy
Malta
Czech Republic
Azerbaijan
Turkey
Slovakia
Romania
Bulgaria
Finland
Hungary
Serbia
Macedonia
Sweden
Denmark
Norway
Lithuania
Ukraine
Estonia
Cyprus
Armenia
Georgia
Russia
Kazakhstan
Latvia
Poland
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°

TERA

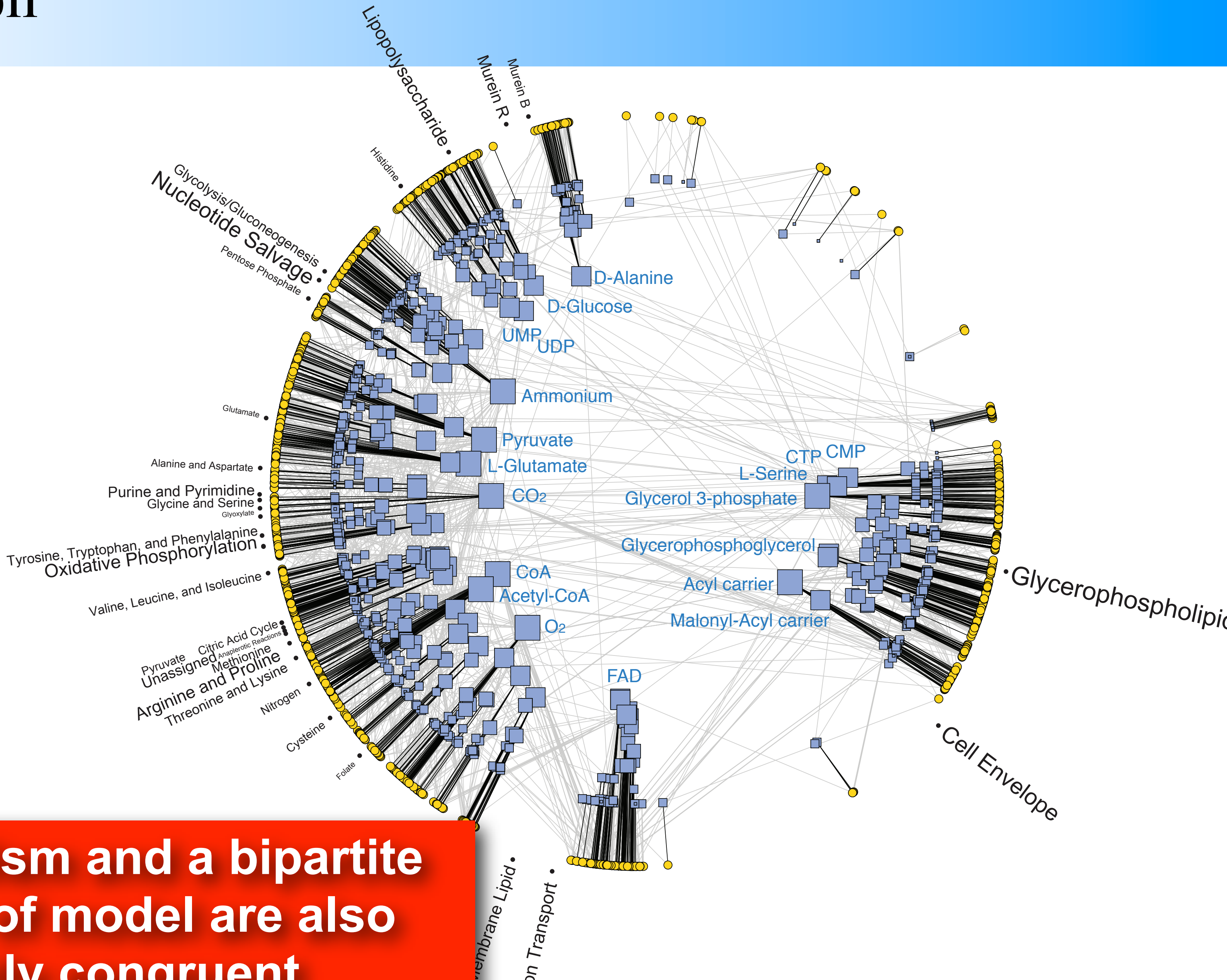


Concentration of ASs
angular coordinate, θ
UK
Russia
US
Switzerland
Bulgaria
Australia
Germany
Ukraine
Poland
Austria
Indonesia
Sweden
Canada
Japan
European Union
Czech Republic
Hong Kong
China
The Netherlands
Italy
South Korea
India
Denmark
Spain
France
Romania
Brazil
Latvia
Mexico
Taiwan
Telefonica
GBLX
NTT
Level 3
Savvis
60°
30°
0°
330°
300°
Finland
Czech Republic
Azerbaijan
Turkey
Slovakia
Bosnia and Herzegovina
Malta
Italy
Spain
Ireland
Slovenia
Switzerland
Austria
Germany
France
UK
Belgium
Luxembourg
Moldova
Portugal
Israel
Iran
Iceland
Nigeria
The Netherlands
Kenya
Mongolia
Bangladesh
India
South Africa
Ecuador
Colombia
Argentina
Panama
Nepal
Saudi Arabia
Egypt
Pakistan
Guatemala
Malaysia
Philippines
Indonesia
Singapore
Chile
Hong Kong
Thailand
Vietnam
China
Japan
South Korea
Venezuela
Taiwan
New Zealand
Australia
Greece
México
Canada
Brazil
Puerto Rico
US

TERA



Sustaining the Internet with hyperbolic mapping
M. Boguñá, F. Papadopoulos, and D. Krioukov. Nature Communications 1, 62 (2011)



Metabolism and a bipartite version of model are also extremelly congruent

M. A. Serrano, M. Boguñá, and F. Sagués, arXiv:1109.1934

summary

- A new class of models with metric properties that can be used to model real systems like the Internet
- Navigation is optimally efficient in the Einsteinian-H^2 model
- Metric properties are also a connection with the community structure of the network
- Embedding of the real Internet graph offers a readily available solution for inter-domain routing.

A million thanks to my collaborators in these works

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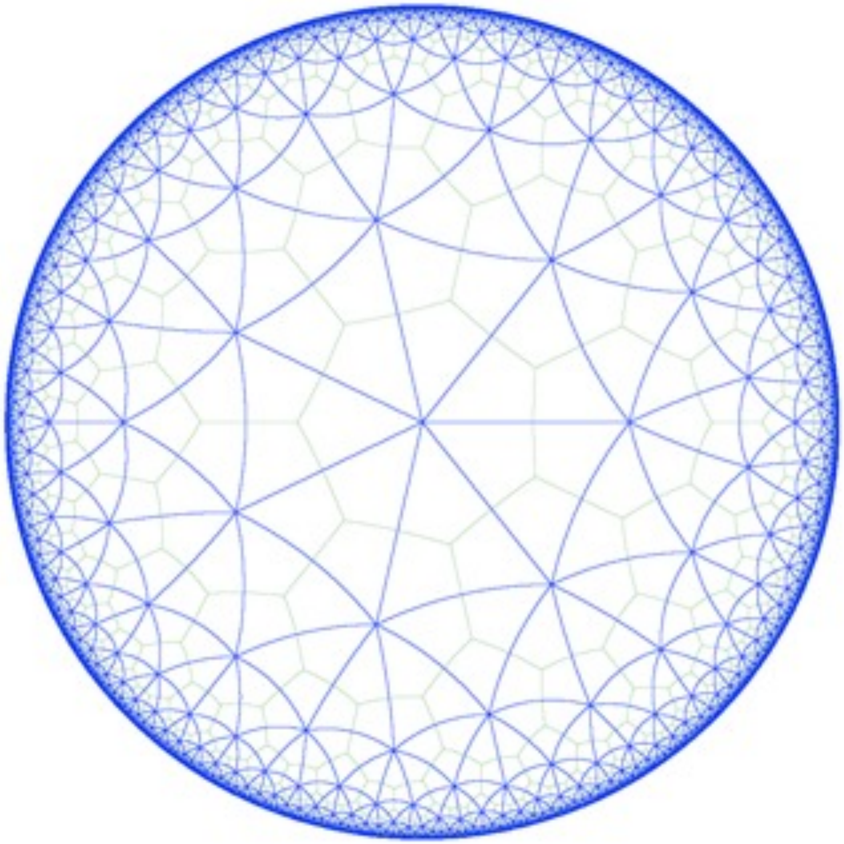
UCSD, USA

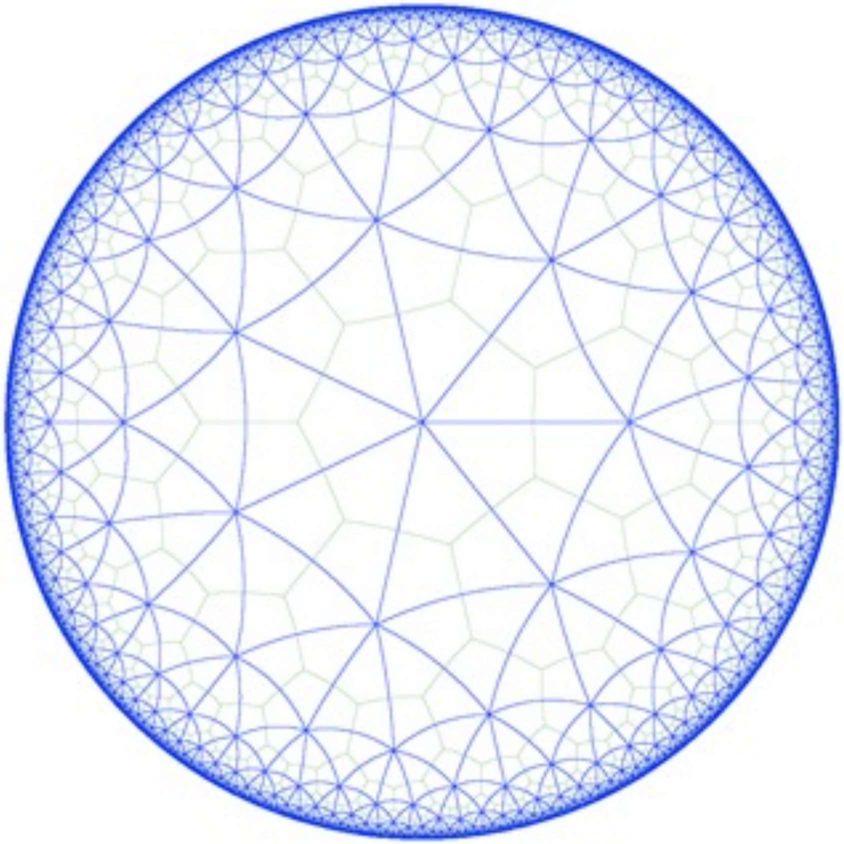
and Fragkiskos Papadopoulos
Department of Electrical and Computer Engineering
University of Cyprus, Cyprus

Poincaré representation of H^2



Tesselation with uniform hexagons

Poincaré representation of H^2



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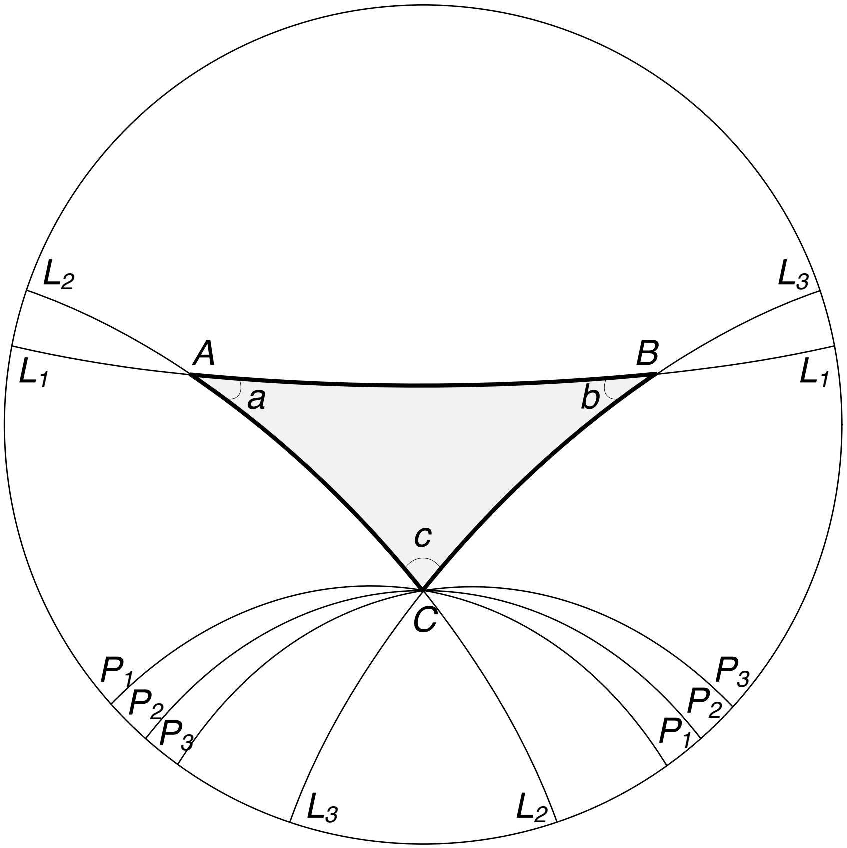

Infinite number of geodesic lines going through C and parallel to L_1