

Performance Evaluation of Networks
Part II Course
Master UBINET
Exam example

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The points marked for each exercise give an indication of the relative importance.

Ex. 1 — Voting (10 points)

In the faraway country Fickleland, elections are going to be held in order to choose the president between the two following candidates: Sharky and Holly. The $N = 1000$ voters are confused and uncertain as usual. At the moment S_0 of them think to vote for Sharky and $H_0 = N - S_0$ for Holly. Their opinion formation can be modeled as follows. Every hour one randomly selected voter A talks with two others, B and C. If B and C share the same voting opinion, then A assumes their opinion with probability p , otherwise A maintains his/her own.

1. Show that the system can be modeled as a Discrete Markov Chain.
2. Show that a Mean-Field limit can be correctly derived when N diverges. Write the corresponding system of Ordinary Differential Equations (ODE).
3. Assuming to know the solution of the ODE, how can you approximate the average number of voters for each candidate after 10000 hours.
4. Describe qualitatively the behavior of the solutions for different initial conditions. Determine in particular the equilibrium points of the ODE system. Do these equilibrium points correspond to stationary distributions for the Markov Chain?
5. If you can only simulate the system up to a size $N = 100$, describe which experiment you could carry on to evaluate if the Mean-Field approximation is satisfactory for $N = 1000$.

Ex. 2 — (2 points)

Solve the following games:

	A	B	C	D
A	-2	-3	5	-7
B	2	1	10	5
C	4	-5	-2	3

	A	B
A	(4,-2)	(2,4)
B	(1,6)	(3,0)

Ex. 3 — GSP and VCG auctions (4 points)

In an ads auction for a given keyword, there are three possible positions with expected click rates per-day 12, 5 and 1. Four companies bid for these positions. They value one click respectively 8\$, 7\$, 5\$ and 4\$. (Hint: one can consider that there is a fourth possible position with expected click rate equal to 0.)

1. In the case of a Generalized Second Price (GSP) auction, do the following bids produce a Nash equilibrium $b_1 = 6.5$, $b_2 = 7$, $b_3 = 5$, $b_4 = 4$?
2. Do you think that a payment from company x to company y to change company y 's bid may be advantageous for both of them? if so, may you provide an example?
3. In the case of a VCG auction, how are the ads priced? How does the seller's revenue changes in comparison to the GSP auction studied above?

Ex. 4 — 3rd-price auction (2 points)

Consider the following auction: the person with the highest bid gets the good and pays for it the third highest bid. Is truthful bidding a dominant strategy?

Ex. 5 — Power-Law (3 points)

1. What is the definition of a power-law graph?
2. Which of the following graphs is or can be power-law: a complete graph, a grid, an Erdos-Renyi graph, a Watts-Strogatz graph?
3. Does a power-law graph exhibit particular properties for epidemics and/or navigation by random walks?