Performance Evaluation – Master UBINET
Assignment 3

This assignment will not be graded.

Ex. 1 — In the faraway country Fickleland, elections are going to be held in order to choose the president between the two following candidates: Sharky and Holly. The $N = 1000$ voters are confused and uncertain as usual. At the moment S_0 of them think to vote for Sharky and $H_0 = N - S_0$ for Holly. Their opinion formation can be modeled as follows. Every hour one randomly selected voter A talks with two others, B and C. If B and C share the same voting opinion, then A assumes their opinion with probability p, otherwise A maintains his/her own.

1. Show that the system can be modeled as a Discrete Markov Chain.
2. Show that a Mean-Field limit can be correctly derived when N diverges. Write the corresponding system of Ordinary Differential Equations (ODE).
3. Assuming to know the solution of the ODE, how can you approximate the average number of voters for each candidate after 10000 hours.
4. Describe qualitatively the behavior of the solutions for different initial conditions. Determine in particular the equilibrium points of the ODE system. Do these equilibrium points correspond to stationary distributions for the Markov Chain?
5. If you can only simulate the system up to a size $N = 100$, describe which experiment you could carry on to evaluate if the Mean-Field approximation is satisfactory for $N = 1000$.

Ex. 2 — N mobile users are in a region covered by two Base Stations (BSs). They can decide to associate to one or to the other in order to maximize the Signal-to-Noise-plus-Interference Ratio (SNIR) for the signal coming from the base station. Let us denote N the channel noise power, S_i^{BS} the power of the signal transmitted by BS i ($i = 1, 2$), S_j^M the power of the signal transmitted by mobile $j = 1, 2, \ldots N$, N_i the set of mobiles associated to BS i, and $b(j) \in \{1, 2\}$ the index of the BS to which mobile j

is associated. If all the channel gains are assumed to be unitary, the SNIR for the mobile j is:

$$SNIR_j = \frac{S^{BS}_{b(j)}}{N + \sum_{n \in N_{b(j)} - \{j\}} S^M_j}.$$

1. Propose a game theoretical model to study the association problem.

2. Determine if there are pure-strategy Nash Equilibria for $N = 4$, $S^{BS}_1 = 7$, $S^{BS}_2 = 2$, $N = 1$.
3. Do you find any simple algorithm for the users to reach dynamically one Nash equilibrium? (Hint: can you define any potential function for the game?)

Ex. 3 — In an ads auction for a given keyword, there are three possible positions with expected click rates per-day 18, 11 and 2. Three companies bid for these positions. They value one click respectively 12$, 6$ and 4$.

1. In the case of a Generalized Second Price (GSP) auction, do truthful bids ($b_1 = 12$,$b_2 = 6$,$b_3 = 5$) produce a Nash Equilibrium in this case? and the following bids $b_1 = 12$, $b_2 = 4$,$b_3 = 5$?
2. In the case of a VCG auction, how are the ads priced? How is it the seller's revenue in comparison to the seller's revenue at the equilibria identified for the GSP?