Distributed Optimization and Games

Auctions

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INRIA – EPI Maestro
31 January 2018

Our starting problem

- We want to give an object to the person who values it the most, i.e.

$$\text{maximize} \quad \sum_{i=1}^{N} x_i v_i$$

$$\text{subject to} \quad \sum_{i=1}^{N} x_i = 1$$

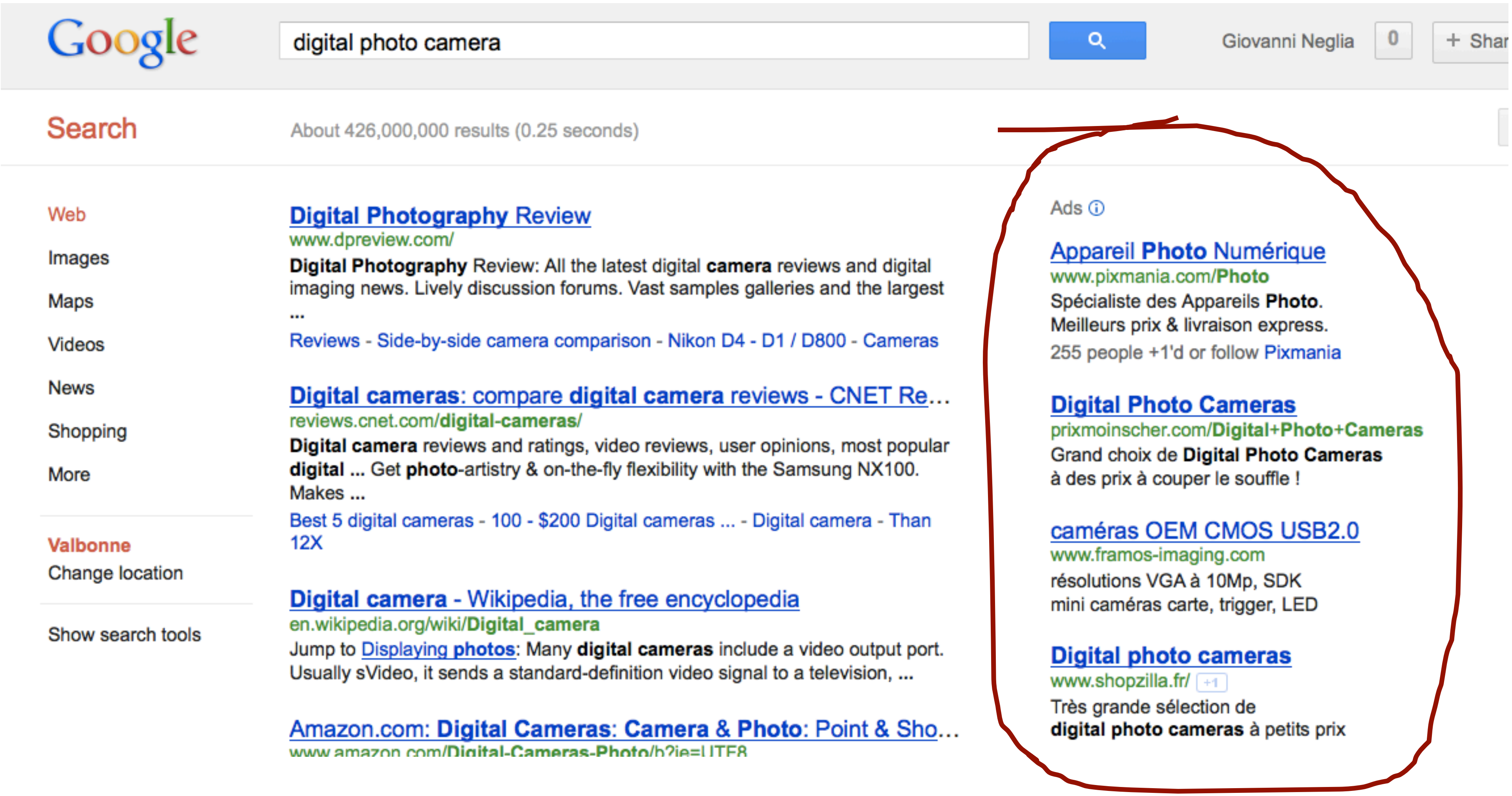
$$\text{over} \quad x_i \in \{0,1\}$$

- Difficulty: we do not know values v_i …
- and we cannot ask to people (they would lie)
- Solution: auctions, but we need to introduce money

Types of auctions

- 1st price & descending bids (Dutch auctions)
- 2nd price & ascending bids (English auctions)

Google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Très grande sélection de
digital photo cameras à petits prix

How it works

- Companies bid for keywords
- On the basis of the bids Google puts their link on a given position (first ads get more clicks)
- Companies are charged a given cost for each click (the cost depends on all the bids)
- Why Google adopted this solution:
  - It has no idea about the value of a click…
  - It lets the companies reveal it

Some numbers

- ≈ 90% of Google revenues from ads
  - 2014 out of 66 billions$
  - 2016 out of 89 billions$ abc.xyz/investor/
- Costs
  - "calligraphy pens" $1.70
  - "Loan consolidation" $50
  - "mesothelioma" $50 per click
- Click fraud problem

Outline

- Preliminaries
  - Auctions
  - Matching markets
- Possible approaches to ads pricing
- Google mechanism

- References
  - Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,10,15

Game Theoretic Model

- N players (the bidders)
- Strategies/actions: b_i is player i's bid
- For player i the good has value v_i
- p_i is player i's payment if he gets the good
- Utility:
  - $v_i - p_i$ if player i gets the good
  - 0 otherwise
- Assumption here: values v_i are *independent* and *private*
  - i.e. very particular goods for which there is not a reference price

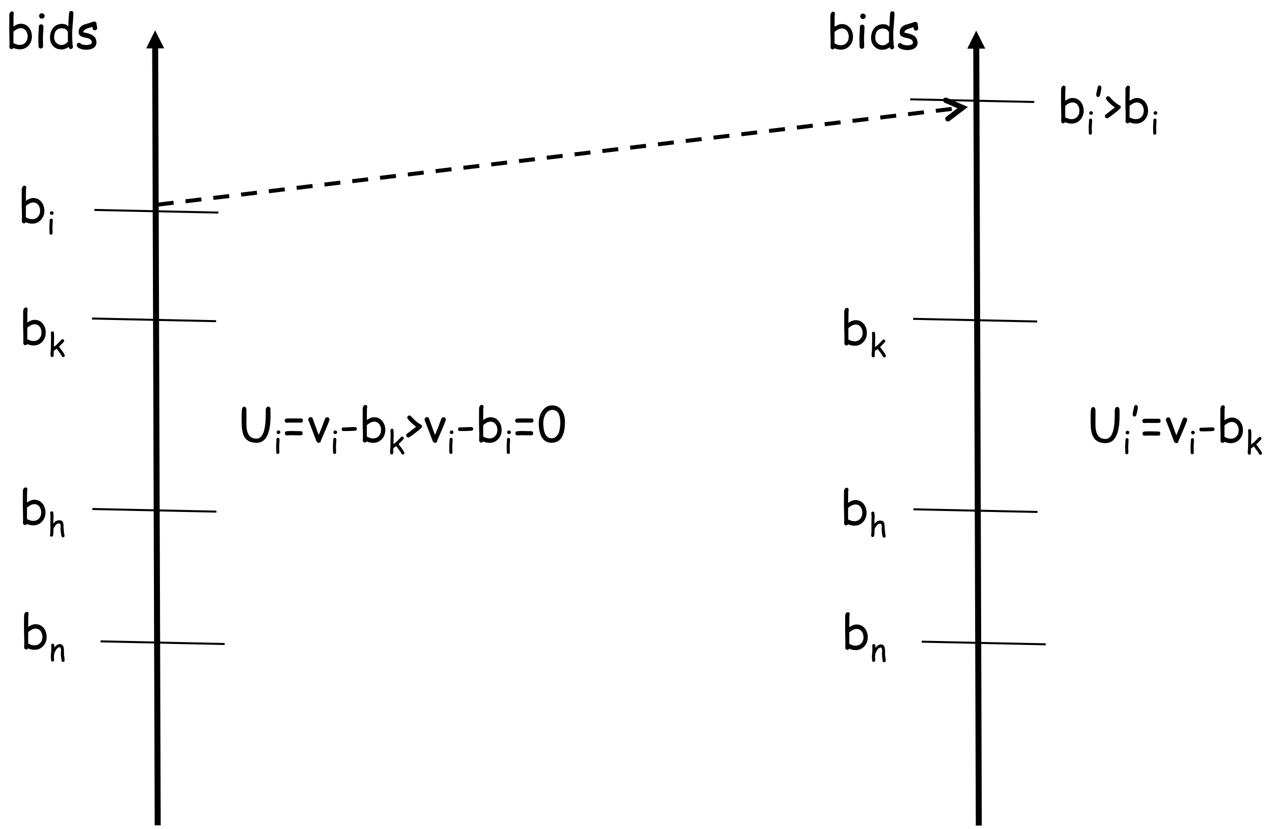
Game Theoretic Model

- N players (the bidders)
- Strategies: b_i is player i's bid
- Utility:
  - v_i-b_i if player i gets the good
  - 0 otherwise
- Difficulties:
  - Utilities of other players are unknown!
  - Better to model the strategy space as continuous (differently from the games we looked at)

2nd price auction

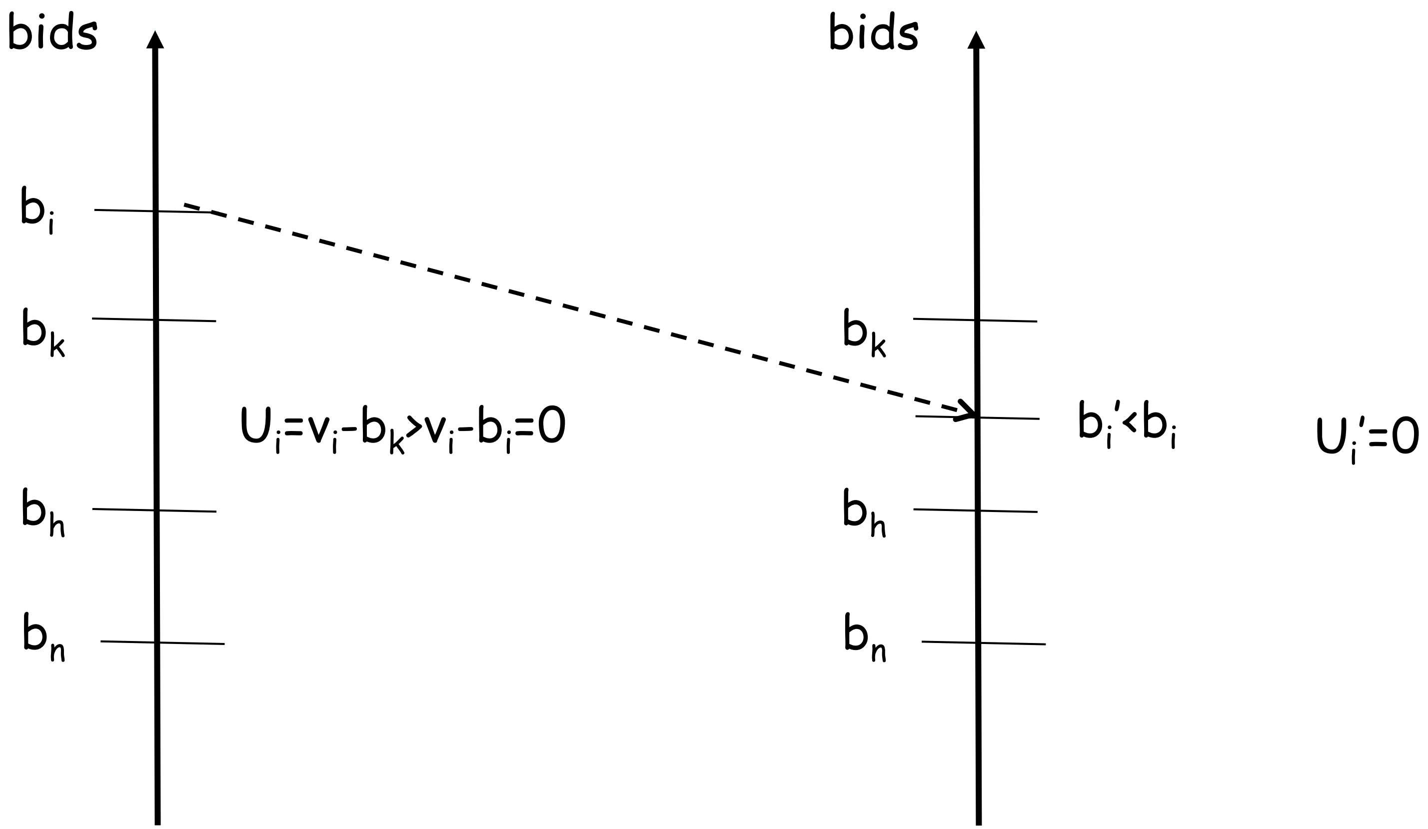
- Player with the highest bid gets the good and pays a price equal to the 2nd highest bid
- There is a dominant strategies
  - I.e. a strategy that is more convenient independently from what the other players do
  - **Be truthful**, i.e. bid how much you evaluate the good ($b_i=v_i$)
  - Social optimality: the bidder who value the good the most gets it!

$b_i = v_i$ is the highest bid



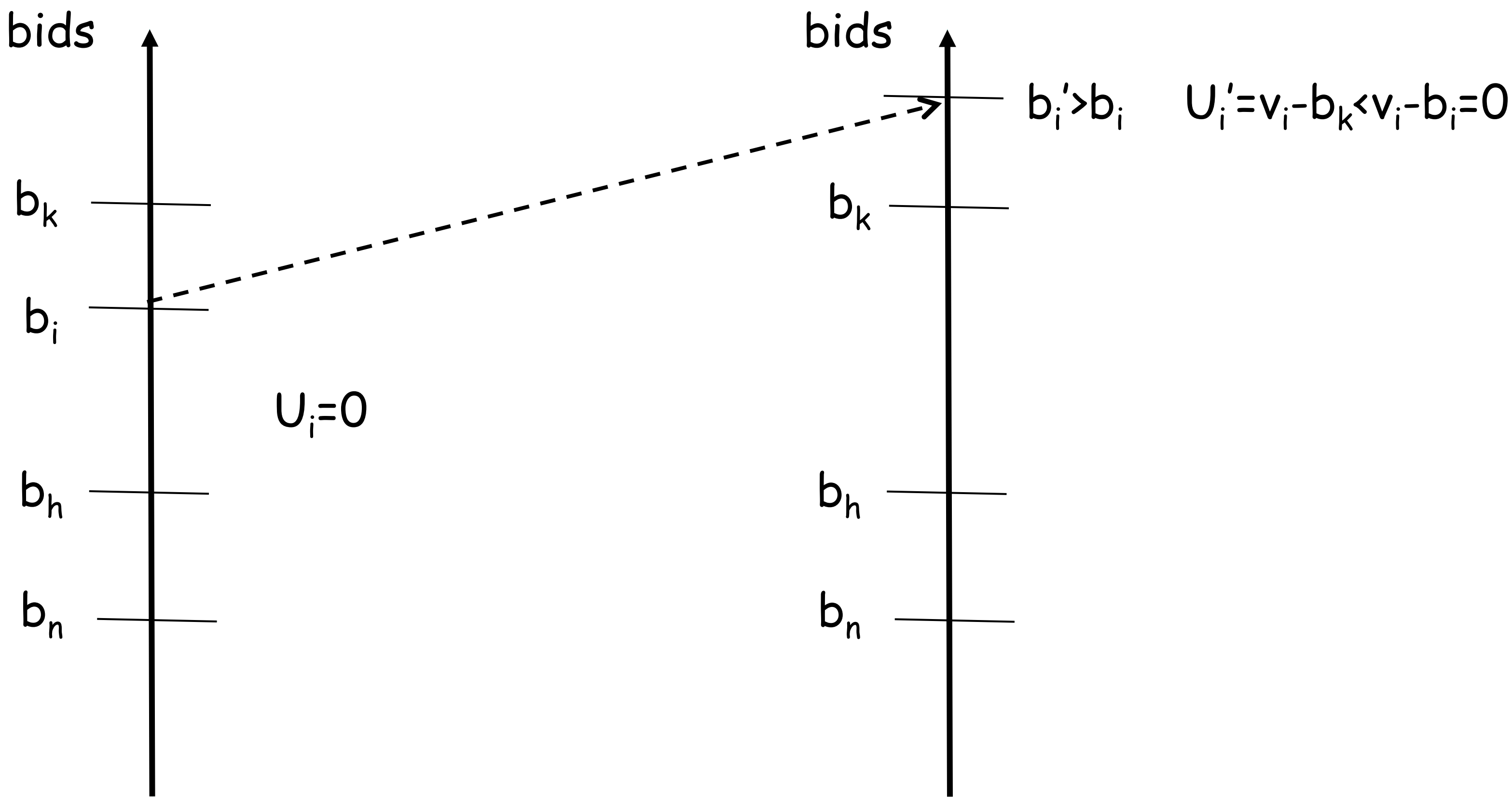
Bidding more than v_i is not convenient

$b_i = v_i$ is the highest bid



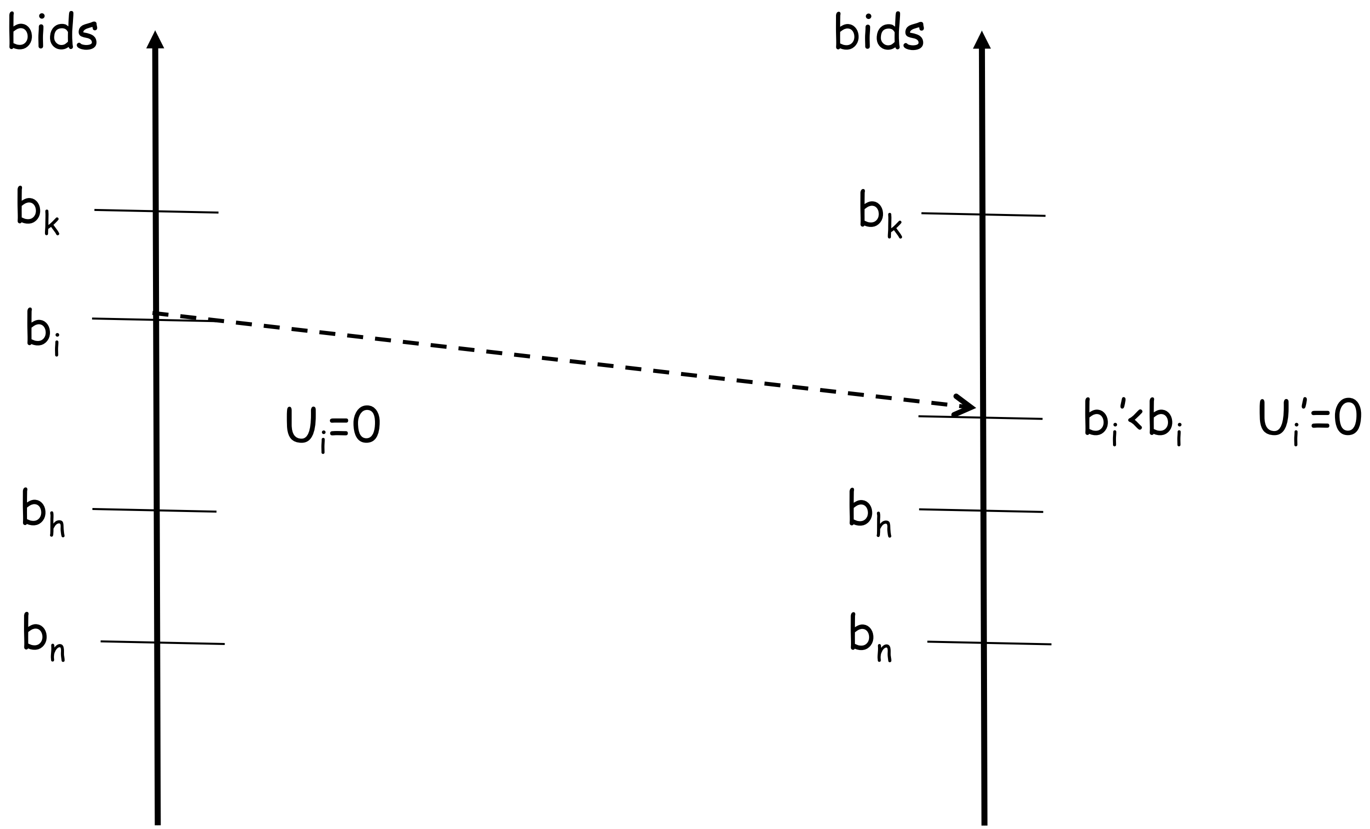
Bidding less than v_i is not convenient (may be unconvenient)

$b_i=v_i$ is not the highest bid



Bidding more than v_i is not convenient (may be unconvenient)

$b_i=v_i$ is not the highest bid



Bidding less than v_i is not convenient

Seller revenue

- N bidders
- Values are independent random values between 0 and 1
- Expected i^{th} largest utility is (N+1-i)/(N+1)
- Expected seller revenue is (N-1)/(N+1)

1st price auction

- Player with the highest bid gets the good and pays a price equal to her/his bid
- Being truthful is not a dominant strategy anymore!
  - Consider for example if I knew other players' utilities
- How to study it?

1^{st} price auction

- Assumption: for each player the other values are i.i.d. random variables between 0 and 1
  - to overcome the fact that utilities are unknown
- Player i's strategy is a function s() mapping value v_i to a bid b_i
  - s() strictly increasing, differentiable function
  - $0 \leq s(v) \leq v \rightarrow s(0)=0$
- We investigate if there is a strategy s() common to all the players that leads to a Nash equilibrium

1^{st} price auction

- Assumption: for each player the other values are i.i.d. random variables between 0 and 1
- Player i's strategy is a function s() mapping value v_i to a bid b_i
- Expected payoff of player i if all the players plays s():
  - $U_i(s(v_1),\ldots s(v_i),\ldots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$

prob. i wins

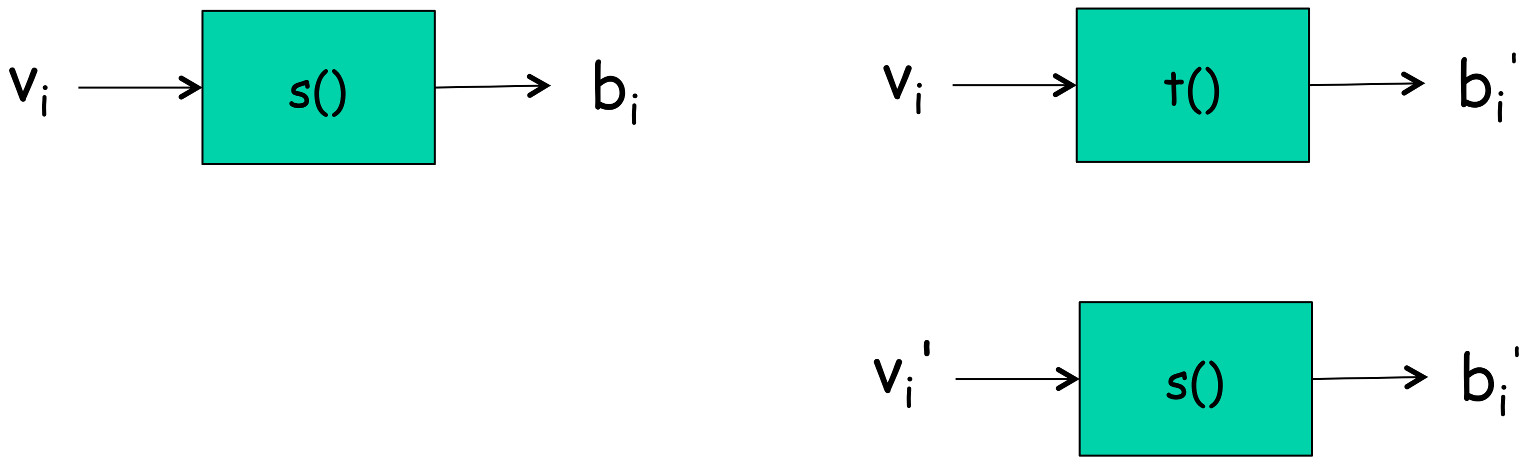
i's payoff if he/she wins

1^{st} price auction

- Expected payoff of player i if all the players play s():
  - $U_i(s(v_1),\ldots s(v_i),\ldots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$
- What if i plays a different strategy t()?
  - If all players playing s() is a NE, then :
  - $U_i(s(v_1),\ldots s(v_i),\ldots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$

    $\geq s^{-1}(t(v_i))^{N-1} (v_i - t(v_i)) = U_i(s(v_1),\ldots t(v_i),\ldots s(v_N))$
- Difficult to check for all the possible functions t() different from s()
- Help from the **revelation principle**

The Revelation Principle



- All the strategies are equivalent to bidder i supplying to s() a different value of v_i

1^{st} price auction

- Expected payoff of player i if all the players plays s():
  - $U_i(s(v_1),\dots s(v_i),\dots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$
- What if i plays a different strategy t()?
- By the revelation principle:
  - $U_i(s(v_1),\dots t(v_i),\dots s(v_N)) =_{eq} U_i(s(v_1),\dots s(v),\dots s(v_N)) = v^{N-1} (v_i - s(v))$
- If $v_i^{N-1} (v_i - s(v_i)) \geq v^{N-1} (v_i - s(v))$ for each v (and for each v_i)
  - Then all players playing s() is a NE

1^{st} price auction

- If $v_i^{N-1}(v_i - s(v_i)) \geq v^{N-1}(v_i - s(v))$ for each v (and for each v_i)
  - Then all players playing s() is a NE
- $f(v) = v_i^{N-1}(v_i - s(v_i)) - v^{N-1}(v_i - s(v))$ is minimized for $v = v_i$
- $f'(v) = 0$ for $v = v_i$,
  - i.e. $(N-1)\, v_i^{N-2}(v_i - s(v_i)) - v_i^{N-1} s'(v_i) = 0$ for each v_i
  - $s'(v_i) = (N-1)(1 - s(v_i)/v_i)$, $s(0)=0$
  - Solution: $s(v_i) = (N-1)/N\, v_i$

1^{st} price auction

- All players bidding according to
  $s(v) = (N-1)/N\ v$ is a NE
- Remarks
  - They are not truthful
  - The more they are, the higher they should bid
- Expected seller revenue
  - $((N-1)/N)\ E[v_{max}] = ((N-1)/N)\ (N/(N+1)) = (N-1)/(N+1)$
  - Identical to 2^{nd} price auction!
  - A general revenue equivalence principle

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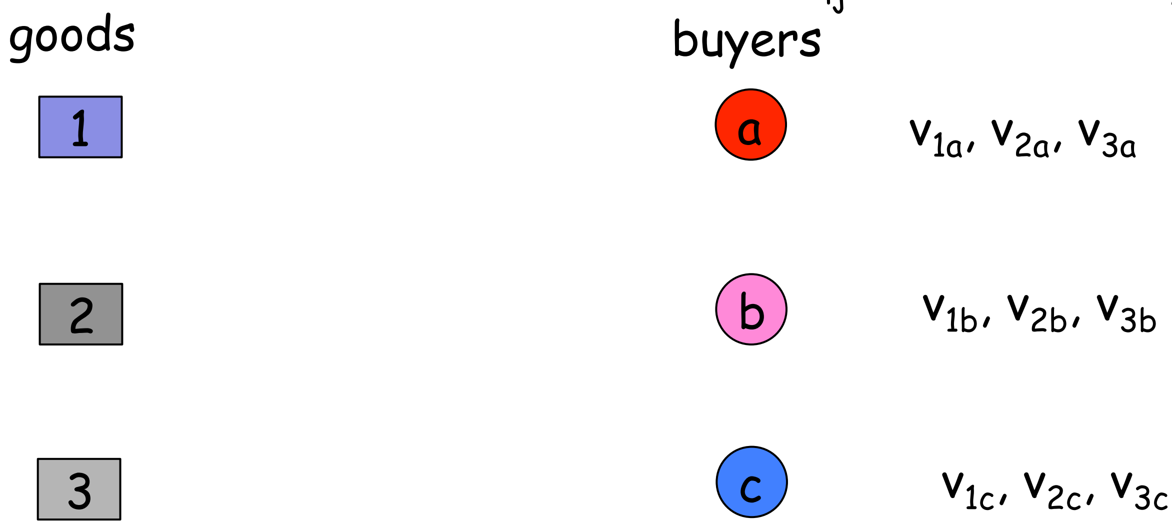
Matching Markets

v_{ij}: value that buyer j gives to good i



How to match a set of different goods to a set of buyers with different evaluations

Matching Markets

v_{ij}: value that buyer j gives to good i

v_{1a}, v_{2a}, v_{3a}

v_{1b}, v_{2b}, v_{3b}

v_{1c}, v_{2c}, v_{3c}

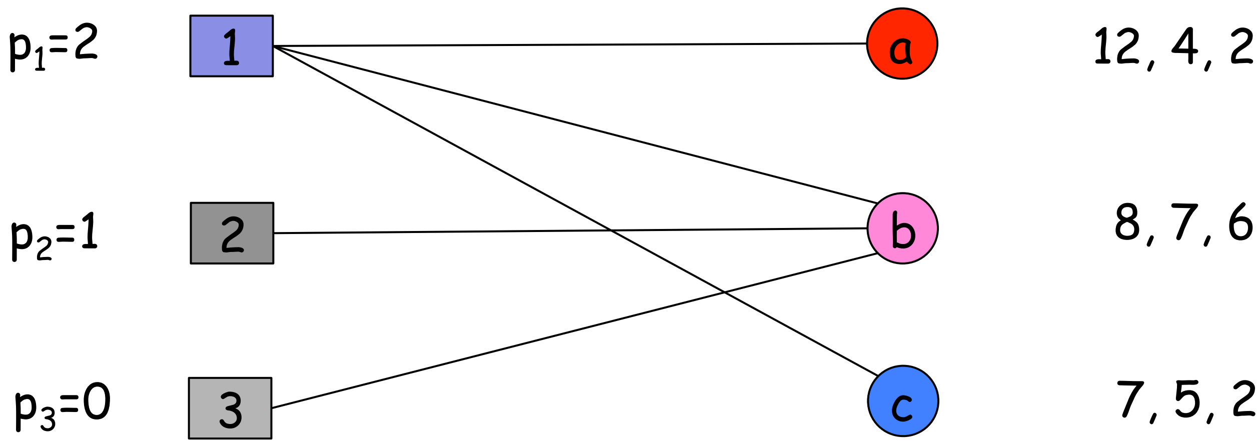
How to match a set of different goods to a set of buyers with different evaluations

maximize $$\sum_{i,j=1}^{N} x_{ij} v_{ij}$$

subject to $$\sum_{j=1}^{N} x_{ij} = 1, \quad \sum_{i=1}^{N} x_{ij} = 1,$$

over $$x_{ij} \in \{0,1\}$$

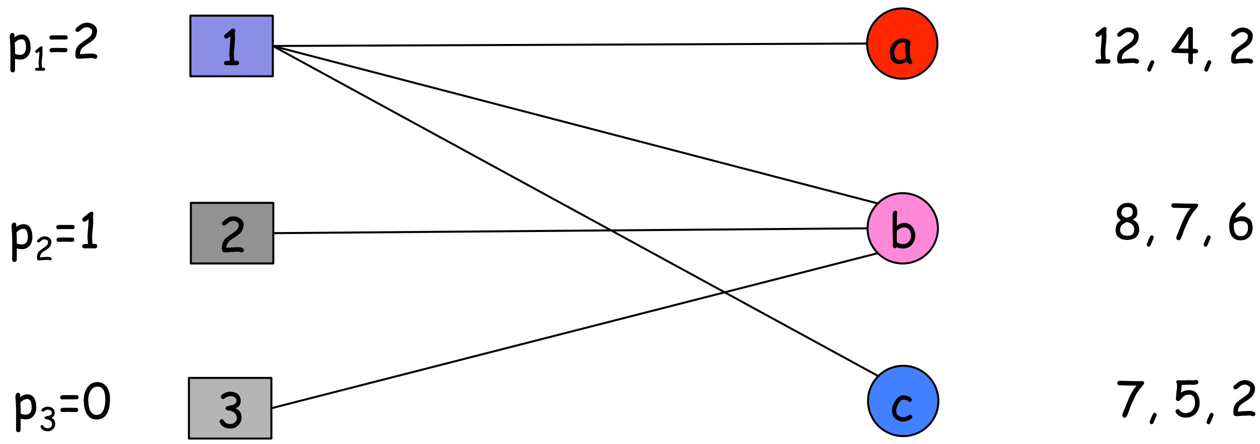
Matching Markets



Which goods buyers like most? Preferred seller graph

How to match a set of different goods to a set of buyers with different evaluations

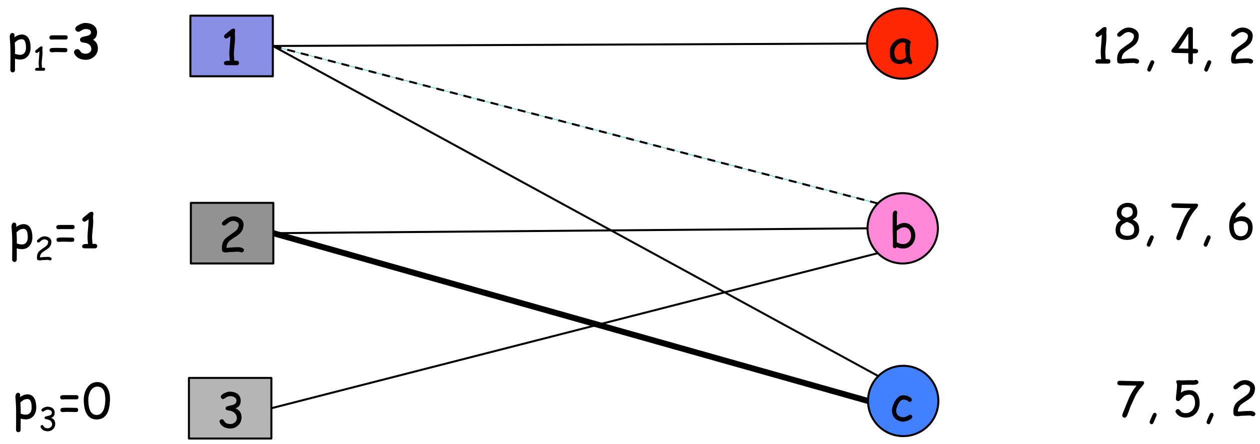
Matching Markets



Which goods buyers like most? Preferred seller graph

- Given the prices, look for a perfect matching on the preferred seller graph
- There is no such matching for this graph

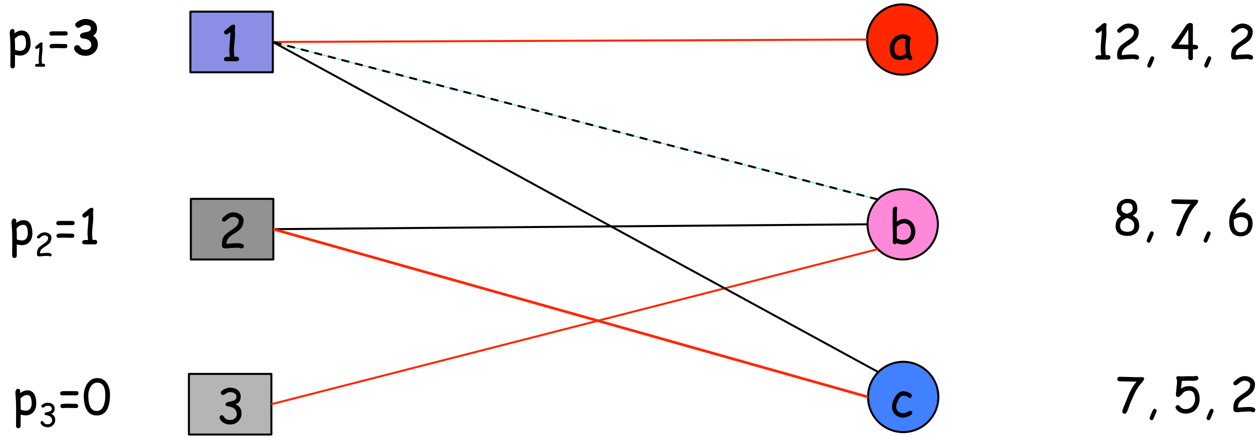
Matching Markets



Which goods buyers like most? Preferred seller graph

- But with different prices, there is

Matching Markets



Which goods buyers like most? Preferred seller graph

- But with different prices, there is
- Such prices are **market clearing prices**

Market Clearing Prices

- They always exist
  - And can be easily calculated if valuations are known
- They are socially optimal in the sense that
  - they achieve the maximum total valuation of any assignment of sellers to buyers
  - Or, equivalently, they maximize the sum of all the payoffs in the network (both sellers and buyers)