Distributed Optimization and Games

Introduction to Game Theory

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Outline

□ Preliminaries
  ○ Auctions
  ○ Matching markets

□ Possible approaches to ads pricing

□ **Google mechanism**

□ **References**
  ○ Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,10,**15**

Google's GSP auction

- Generalized Second Price
- Once all the bids are collected $b_1 > b_2 > ... b_N$
- Company i pays b_{i+1}
- In the case of a single good (position), GSP is equivalent to a 2nd price auction, and also to VCG
- But why Google wanted to implement something different???

GSP properties

☐ Truth-telling may not be an equilibrium

GSP example

Ads positions   companies

1 $r_1=10$ 1 $v_1=7$

2 $r_2=4$ 2 $v_2=6$

3 $r_3=0$ 3 $v_3=1$

r_i: click rate for an ad in position i v_i: value that company i
(assumed to be independent   gives to a click
from the ad and known a priori)

❑ If each player bids its true evaluation, 1 gets a payoff equal to 10

❑ If 1 bids 5, 1 gets a payoff equal to 24

GSP properties

☐ Truth-telling may not be an equilibrium
☐ There is always at least 1 socially optimal NE

GSP example

Ads positions   companies

$\boxed{1}$ $r_1 = 10$ ① $v_1 = 7$

$\boxed{2}$ $r_2 = 4$ ② $v_2 = 6$

$\boxed{3}$ $r_3 = 0$ ③ $v_3 = 1$

r_i: click rate for an ad in position i v_i: value that company i
(assumed to be independent   gives to a click
from the ad and known a priori)

❑ Multiple NE
- 1 bids 5, 2 bids 4 and 3 bids 2
- 1 bids 3, 2 bids 5 and 3 bids 1

GSP properties

- ❑ Truth-telling may not be an equilibrium
- ❑ There is always at least 1 socially optimal NE
- ❑ Revenues can be higher or lower than VCG
  - ○ Attention: the revenue equivalence principle does not hold for auctions with multiple goods!
  - ○ Google was targeting higher revenues…
  - ○ … not clear if they did the right choice.

GSP example

Ads positions

1    $r_1=10$

2    $r_2=4$

3    $r_3=0$

companies

1    $v_1=7$

2    $v_2=6$

3    $v_3=1$

☐ **Multiple NE**
  - ○ 1 bids 5, 2 bids 4, 3 bids 2 ➔ google's revenue=48
  - ○ 1 bids 3, 2 bids 5, 3 bids 1 ➔ google's revenue=34

☐ **With VCG, google's revenue=44**

Other issues

☐ Click rates are unknown and depend on the ad!

   ○ Concrete risk: low-quality advertiser bidding high may reduce the search engine's revenue

   ○ Google's solution: introduce and ad-quality factor taking into account actual click rate, relevance of the page and its ranking

      • Google is very secretive about how to calculate it => the market is more opaque

☐ Complex queries, nobody paid for

   ○ Usually engines extrapolate from simpler bids