Distributed Optimization and Games

Introduction to Game Theory

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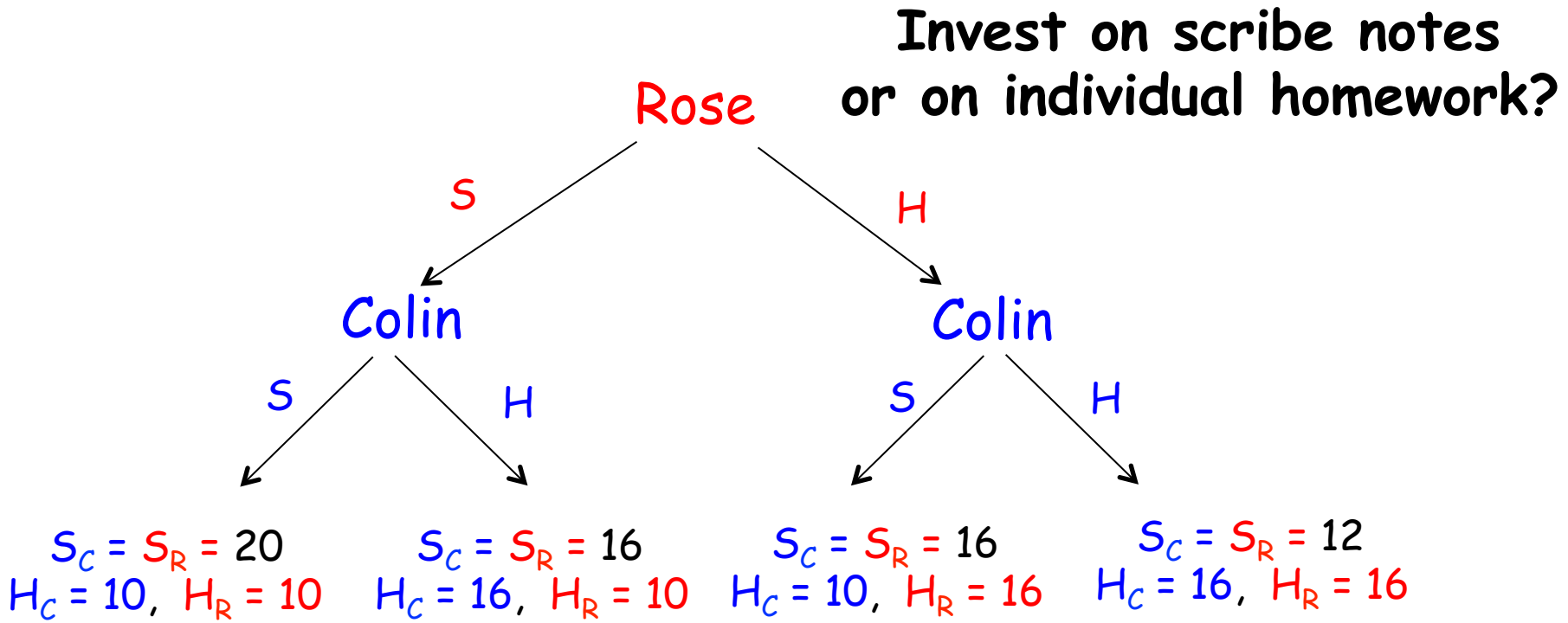
What is Game Theory About?

- Mathematical/Logical analysis of situations of conflict and cooperation



- Goal: to prescribe how rational players should act

What is a Game?

- A Game consists of
  - at least two players
  - a set of strategies for each player
  - a preference relation over possible outcomes
- Player is general entity
  - individual, company, nation, protocol, animal, etc
- Strategies
  - actions which a player chooses to follow
- Outcome
  - determined by mutual choice of strategies
- Preference relation
  - modeled as utility (payoff) over set of outcomes

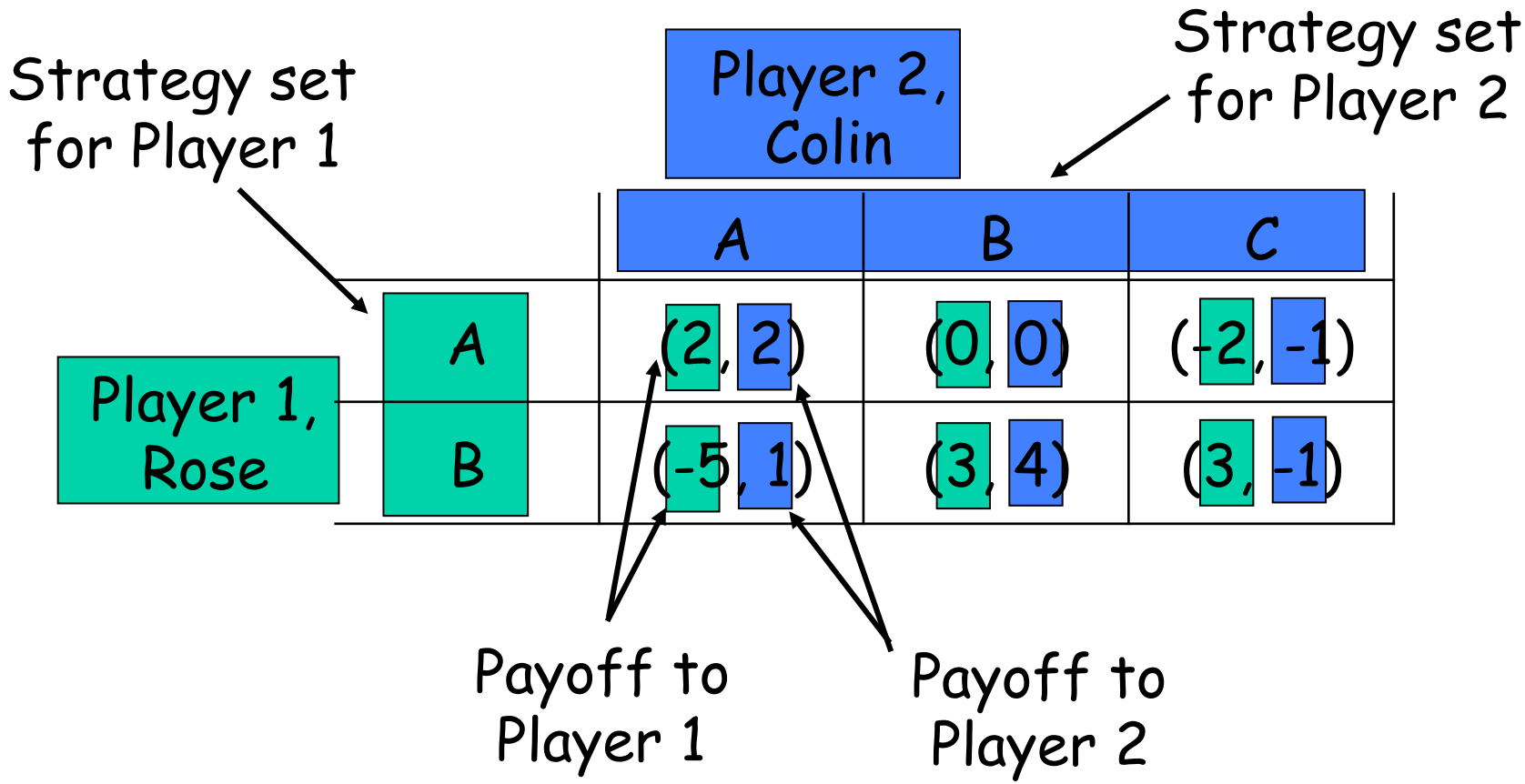
Short history of GT

- Forerunners:
  - Waldegrave's first minimax mixed strategy solution to a 2-person game (1713), Cournot's duopoly (1838), Zermelo's theorem on chess (1913), Borel's minimax solution for 2-person games with 3 or 5 strategies (20s)
- 1928: von Neumann's theorem on two-person zero-sum games
- 1944: von Neumann and Morgenstern, Theory of Games and Economic Behaviour
- 1950-53: Nash's contributions (Nash equilibrium, bargaining theory)
- 1952-53: Shapley and Gillies' core (basic concept in cooperative GT)
- 60s: Aumann's extends cooperative GT to non-transferable utility games
- 1967-68: Harsanyi's theory of games of incomplete information
- 1972: Maynard Smith's concept of an Evolutionarily Stable Strategy
- Nobel prizes in economics
  - 1994 to Nash, Harsanyi and Selten for "their pioneering analysis of equilibria in the theory of non-cooperative games"
  - 2005 to Aumann and Schelling "for having enhanced our understanding of conflict and cooperation through game-theory analysis"
  - 2012 to Roth and Shapley "for the theory of stable allocations and the practice of market design"
- Movies:
  - 2001 "A beautiful mind" on John Nash's life
- See also:
  - www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm

Applications of Game Theory

- Economy
- Politics (vote, coalitions)
- Biology (Darwin's principle, evolutionary GT)
- Anthropology
- War
- Management-labor arbitration
- Philosophy (morality and free will)
- National Football league draft
- "Recently" applied to computer networks
  - Nagle, RFC 970, 1985: "datagram networks as a multi-player game"
  - wider interest starting around 2000

Matrix Game (Normal form)



- Simultaneous play
  - players analyze the game and then write their strategy on a piece of paper

Students' game

		Colin	
		S	H
Rose	S	15, 15	13, 16
	H	16, 13	14, 14

More Formal Game Definition

- Normal form (strategic) game
  - a finite set N of players
  - a set strategies S_i for each player $i \in N$
  - payoff function $u_i(s)$ for each player $i \in N$
    - where $s \in S = \times_{j \in N} S_j$ is an outcome
    - sometimes also $u_i(A, B, \ldots) \quad A \in S_1, B \in S_2, \ldots$
    - $u_i : S \rightarrow \Re$