Regeneration: a New Algorithm in Numerical Algebraic Geometry

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Numerical algebraic geometry is concerned with the numerical solution of systems of polynomial equations. At its core is polynomial continuation, a technique useful for finding isolated roots. Numerical algebraic geometry extends this capability to find solution sets of any dimension using finite sets of points, called witness points, as surrogates for positive dimensional components. Algorithms are available to factor these witness sets into irreducible components. Furthermore, operations on algebraic sets, most notably intersection, can be carried out by homotopies that operate on their witness sets. In earlier work with Jan Verschelde, we showed how this capability could be used to efficiently compute solution sets equation-by-equation, i.e., by successive intersection of the varieties for the individual equations via "diagonal homotopy." Interestingly, this can be easily modified to find only the isolated solutions of a system, sometimes more efficiently than a direct application of the original isolated root finding methods upon which numerical algebraic geometry is built. Regeneration is a new algorithm that also finds isolated solutions equation-by-equation but by a substantially different, and simpler, method than the diagonal homotopy. We will show that for large systems the approach is more efficient than the previously best method for finding isolated roots, the polyhedral homotopy.

This talk will briefly review the highlights of numerical algebraic geometry and then describe the new regeneration algorithm. This algorithm is publicly available as one option in Bertini, a software package for numerical algebraic geometry developed by the authors and Daniel Bates.