Perfect Point Configurations

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A point configuration S in $\mathbf{R}^n$ is perfect if its vanishing ideal has the property that whenever a linear polynomial f is non-negative over S, it can be written as a sum of squares of linear polynomials modulo the ideal. For instance, the vertices of a hypercube is a perfect configuration and a graph is perfect (in the graph theory sense) if and only if the characteristic vectors of its stable sets form a perfect point configuration. In this work we characterize zero-dimensional radical ideals in the polynomial ring whose real varieties are perfect, answering a question of Lovász. Several related results emerge which use tools from Groebner basis theory and semidefinite programming.