Moment Matrices, Trace Matrices and the Radical of Ideals

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Let I be an ideal in $\mathbb{C}[x_1, ..., x_m]$ generated by polynomials $f_1, \ldots, f_s$, and assume that the factor algebra $A := \mathbb{C}[x_1, ..., x_m]/I$ is finite dimensional over $\mathbb{C}$. In this talk we present a simple algorithm to compute matrices of traces of A with respect to some basis of A. Matrices of traces play an important role in real and complex algebraic geometry: for example their rank is equal to the number of distinct complex roots of I, and their signature is equal to the number of real roots of I. Previous methods for computing matrices of traces include the use of Newton Sums, residues, or the computation of high powers of multiplication matrices of I.

Our method to compute the matrices of traces is very elementary, it uses only linear algebra on the Sylvester matrix of $f_1, \ldots, f_s$ and a polynomial J, where J is a generalization of the Jacobian of a well-constrained system, i.e. when s=m. These matrices of traces in turn allow us to compute a system of multiplication matrices of the radical ideal of I. These multiplication matrices are simultaneously diagonalizable with eigenvalues which are the coordinates of the distinct complex roots.

Finally, we also present an adaptation of the latter method for systems with approximate coefficients. In this case we assume that I has clusters of roots, and we use matrices of traces to compute an "approximate radical", an ideal which has one root for each of the clusters, in the center of gravity of the cluster. We give estimates for the precision of our algorithm.