On Stable Methods for Solving Polynomial Equations

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Solving polynomial equations with approximate coefficients is ubiquitous in many applications. Typically, in domain such as geometric modeling, we are interested in computing characteristic points on curves or surfaces which are given with incertitudes on the input coefficients. An important and difficult question is then to isolate the (real) roots of these equations and to analyse there multiplicities. To tackle this issue from an algebraic point of view, border basis methods have been introduced recently. Compared to Grobner basis computation, they yield representations of the quotient algebra, which are more stable from a numerical point of view: If the number of solutions does not change in a neighborhod of the input system, the computed basis stays invariant.

In this talk, we will recall the main properties of border basis, how they can be characterised, how they can be computed, how the syzygies are generated and give some examples of numerical computation of Border basis computations. This will be used to describe the varieties of algebras with fixed basis. We will provide some hint on the complex geometry of these varieties and describe a numerical algorithm which improve the numerical quality of a quotient algebra representation.