On bivariate toric polynomial absolute factorization

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The talk relates a joint work with M. Elkadi and M. Weimann.

Whereas rational factorization is only concerned by factors in $\mathbb{Q}[x_1, \ldots, x_n]$, absolute factorization provides all irreducible factors with coefficients in $\overline{\mathbb{Q}}$, the algebraic closure of $\mathbb{Q}$ (rationals). We concentrate on the bivariate case but our techniques naturally extend to the n variables case, $n \geq 2$.

During the last decade two main strategies have been quite successful. An algebraic approach with Ruppert-Gao matrix improved by Cheze and Lecerf. A geometric approach, based on a zero-sum criterion (traces), provides very efficient semi-numerical probabilistic algorithms able to deal with polynomials having degree up to 200, see Cheze Galligo 2005, and was derived from the study of a monodromy group.

The model of computation in these approaches is the following. The input is a polynomial with integer coefficients and the output is a list of polynomials with coefficients in an algebraic extension of $\mathbb{Q}$ which should also be computed. In order to determine these coefficients the strategy consists in embedding $\overline{\mathbb{Q}}$ in the complex field and representing approximations of these coefficients by bigfloats. Then conjugacy relations are used to recognize an algebraic presentation of an extension of $\mathbb{Q}$ and an exact algebraic presentation of the coefficients.

Here, we first reinterpret the vanishing traces criterion in the geometric approach as a consequence of Wood's theorem (1984) on algebraic interpolation of a family of analytic germs of curves. Second, we provide a generalization of Wood's theorem to the factorization of polynomials with fixed Newton polytopes. Third, we outline an algorithm for toric absolute factorization that we tested on examples.