# $\mathbb{C}^*$-actions and Kinematics

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Let $X$ be a nonsingular algebraic variety with a $\mathbb{C}^*$-action having a finite fixed point set. It is a classical result of Bialynicki-Birula that the action induces two decompositions of $X$ in invariant (Zariski) locally closed subsets, called the “plus” and “minus” decomposition, in short the B-B-decompositions. The invariant factors form a basis for the Chow ring of $X$. Thus the intersection of factors of complementary dimension determines the intersection of two subvarieties of $X$, of complementary dimension.

An algorithm (the intersection algorithm) to compute intersections of certain subvarieties of $X$ will be presented. The method uses homotopy continuation to conveniently move the factors coming from the two B-B-decompositions, and track back the intersection points.

Kinematics problems are typically concerned with finding solutions consisting of “special Euclidean transforms in 3-space”. Such transforms are in one to one correspondence with points on a non singular quadric hypersurface in $\mathbb{P}^7$, called the Study Quadric. This variety has a large choice of $\mathbb{C}^*$-actions.

The Inverse Kinematics Problem (IKP) of a general 6R can be reduced to finding the intersection points of two 3-dimensional subvarieties of the Study Quadric. Thus the intersection algorithm provides a new algorithm to solve the IKP of a general 6R.